2/17/26 Announcements

- HW3 Due Thurs 2/19 11:59 PM on Icon
- Quiz 3 Thurs 2/19 in Discussion
- Exam Questions?
- Post notes Online?

Problem 8: What is x_n based on the dataset shown below:

$$\{20, 35, 47, 51, 60, 80\}$$

a. 6
b. 20
c. 80
d. 293
e. None of the above

X_n is the value of the n-th data point. That is, the data are

$$\{X_1, X_2, \ldots, X_n\}$$

In this case $n = 6$ and

$X_6 = 80$

Problem 14: The probability a customer buys a product when coming into a particular store is 0.65. Assuming that each customer purchase is independent and three customers enter the store, what is the probability that **at most two** customers buy a product when they enter the store.

a. 0.0429
b. 0.2746
c. 0.7254
d. 0.9571
e. None of the choices listed

P(at most n-1) = 1 - P(all)

In this case n=3, P(b) = 0.65

So

$$P(\text{at most } 2) = 1 - P(b_1 \cap b_2 \cap b_3)$$
$$= 1 - (0.65)^3$$
$$= 0.7254$$

Problem 19: Suppose events A and B are *independent* where $P(A|B) = 0.2$ and $P(B|A) = 0.5$. Which of the following is/are true?

a. $P(A \cap B) = 0.10$
b. $P(A^C) = 0.9$
c. $P(A \cup B) = 0.6$
d. $P(A^C \cap B^C) = 0.36$
e. All of the above

Because independent

$P(A|B) = P(A)$, $P(B|A) = P(B)$

Therefore $P(A) = 0.2$, $P(B) = 0.5$

and because A & B independent,

$P(A \cap B) = P(A)P(B)$

$= (0.2)(0.5)$

$= 0.1$ ✓ (A)

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.2 + 0.5 - 0.1$

$= 0.6$ ✓ (C)

Note $P(A^C) = 0.8$

$P(A^C \cap B^C) = 1 - P(A \cup B) = 0.4$

- *) Brief Review
- In a given population we have a parameter of interest
- Sample is drawn and a statistic is calculated.
- Ex) Population mean & sample mean, Population variance & sample variance
- Sample space $S = \{s_1, s_2, \dots\}$ collection of all outcomes where an event A is a subset
- Probability is a function mapping

events $\rightarrow [0, 1]$

- A random variable maps outcomes → real numbers that respects the probability of events

Def) Expected value

$E(X) = \sum x_i P(X = x_i)$

"weighted" mean of the random variable

Ex)

Given the following table

X	1	2	3
$P(X=x)$	0.5	0.3	0.2

$E(X) = 1(0.5) + 2(0.3) + 3(0.2)$

$= 0.5 + 0.6 + 0.6 = 1.7$

Def) Variance

$$Var(X) = \sum (x_i - E(X))^2 P(X=x_i)$$
$$= E(X^2) - (E(X))^2$$

Where

$$E(X^2) = \sum x_i^2 P(X=x_i)$$

"spread" of random variable

Ex)

Given data above the variance is

$$Var(X) = (1-1.7)^2(0.5) + (2-1.7)^2(0.3) + (3-1.7)^2(0.2)$$
$$= 0.532$$

Problem 1: A marketing firm is evaluating a promotional campaign in which customers can receive different discount values based on a two-stage random process. First, a customer is randomly selected to qualify for a promotion. If the customer does not qualify, they receive no discount. If the customer qualifies, the customer is then randomly assigned one of several discount tiers.

- If the customer does not qualify, the discount value is $0. 3/6
- If the customer qualifies and is assigned a standard tier, the discount value is $5. 1/6
- If the customer qualifies and is assigned a premium tier, the discount value is $10. 1/6
- If the customer qualifies and is assigned a VIP tier, the discount value is $20. 1/6

Assume the qualification step is equally likely (qualify or not qualify), and the tier assignment among qualifying customers is equally likely.

a. Create a probability distribution table showing the possible discount values and their associated probabilities.

b. What is the expected value of the discount offered per customer?

c. What is the variance of the discount value?

d. If offering this promotion costs the company an average of $6 per customer, should the company implement the promotion from a financial standpoint?

(a)

X	0	5	10	20
$P(X=x)$	$3/6$	$1/6$	$1/6$	$1/6$

(b)

$E(X) = \sum x_i P(X=x)$

$= 0(3/6) + 5(1/6) + 10(1/6) +$
$20(1/6)$

$= 5/6 + 10/6 + 20/6$

$= 35/6 = 5\,5/6$

(c)

$Var(X) = \Sigma(x_i - E(x))^2 P(X = x_i)$

$= (0 - 35/6)^2 (3/6) +$
$(5 - 35/6)^2 (1/6) +$
$(10 - 35/6)^2 (1/6) +$
$(20 - 35/6)^2 (1/6)$

Or find $E(x^2)$

$E(x^2) = 0^2(3/6) + 5^2(1/6) +$
$10^2(1/6) + 20^2(1/6)$

$= 25(1/6) + 100(1/6) + 400(1/6)$

$= 525(1/6)$

$$\mathrm{Var}(X) = E(X^2) - (E(X))^2$$
$$= 525/6 - (35/6)^2$$
$$= 3150/36 - 1125/36$$
$$= 1925/36$$
$$\approx 53.47$$

d. If offering this promotion costs the company an average of $6 per customer, should the company implement the promotion from a financial standpoint?

Given max cost per customer = \$6. From (b) $E(X) = 5.83$

So its acceptable

- Def) Binomial Coefficient

$$\binom{n}{k} = \frac{n!}{k!(n-k)!}$$

ways to choose k from n

Ex) On a deserted island with 8 people. The ways to choose a 3-person leadership council is

$$\binom{8}{3} = \frac{8!}{3!(8-3)!}$$

$$= \frac{(8)(7)(6)5!}{3!(5!)}$$

$$= \frac{(8)(7)(6)}{(3)(2)(1)}$$

$$= 56$$

Problem 2: Answer the following questions using combinations:

a. If you have 5 trials, how many different ways are there to have 3 successes?

b. If you have 3 trials, how many different ways are there to have 2 successes?

c. If you have 3 trials, list the ways there are to have 2 successes. (This is listing all the outcomes from part b)

(a)

$$\binom{5}{3} = \frac{5!}{(3!)(5-3)!}$$

$$= \frac{5!}{(3!)(2!)}$$

$$= \frac{(5)(4)3!}{(3!)(2)}$$

$$= 20$$

(b)

$$\binom{3}{2} = \frac{3!}{(2!)(1!)}$$

$$= \frac{(3)(2!)}{(2!)} = 3$$

c. If you have 3 trials, list the ways there are to have 2 successes. (This is listing all the outcomes from part b)

1. $\{S, S, F\}$
2. $\{S, F, S\}$
3. $\{F, S, S\}$

- Def) Probability Distribution
Set of possible values $x_1, x_2, \ldots$ a random variable can take with $P(X = x_i)$

defined and

(i) $0 \le P(X=x_i) \le 1$

(ii) $\sum P(X=x_i) = 1$

Ex) Geometric distribution

$P(X=x|p) = (1-p)^{x-1} p$

where $x \in \{1, 2, 3, \dots\}$

Problem 3: A sales manager is evaluating the performance of a sales associate, Alex, during a short sales campaign. Each customer interaction results in either a successful sale or no sale. Historical data show that the probability Alex successfully closes a sale with any given customer is 0.6. During a particular campaign, Alex interacts with 5 customers and records the number of successful sales. This scenario is designed to model outcomes using the binomial distribution.

a. When Alex interacts with 5 customers, how many different ways are there to have 5 successful sales?

b. When Alex interacts with 5 customers, calculate the probability that Alex has 5 successful sales.

c. When Alex interacts with 5 customers, how many different ways are there to have 4 successful sales?

d. When Alex interacts with 5 customers, calculate the probability that Alex has 4 successful sales? (You will need your answer from part d.)

e. When Alex interacts with n customers, how many different ways are there to have x successful sales?

f. When Alex interacts with n customers, calculate the probability that Alex has x successful sales.

g. If the probability that Alex successfully closes a sale is 0.6, what is the probability that a customer interaction results in no sale?

h. If the probability of a successful sale is p, what is the probability of no sale?

i. **Ultra Important Question**: If Alex interacts with n customers, with probability p of success on each interaction, what is the probability that Alex has x successful sales?

a. When Alex interacts with 5 customers, how many different ways are there to have 5 successful sales?

b. When Alex interacts with 5 customers, calculate the probability that Alex has 5 successful sales.

(a) $\binom{5}{5} = \frac{5!}{5!(0!)} = 1$

(b) Prob 5 sales since indep,

$P(X=5) = (0.6)^5$

c. When Alex interacts with 5 customers, how many different ways are there to have 4 successful sales?

d. When Alex interacts with 5 customers, calculate the probability that Alex has 4 successful sales? (You will need your answer from part d.)

(c)

$\binom{5}{4} = \frac{5!}{4!\,1!} = \frac{(5)(4!)}{4!} = 5$

(d)

$P(X=4) = \binom{5}{4}(0.6)^4(0.4)^1$

e. When Alex interacts with n customers, how many different ways are there to have x successful sales?

f. When Alex interacts with n customers, calculate the probability that Alex has x successful sales.

(e) $\binom{n}{x} = \frac{n!}{x!\,(n-x)!}$

(f)

$P(X=x) = \binom{n}{x} (0.6)^x (0.4)^{n-x}$

h. If the probability of a successful sale is p, what is the probability of no sale?

i. **Ultra Important Question**: If Alex interacts with n customers, with probability p of success on each interaction, what is the probability that Alex has x successful sales?

(h)

$P(\text{Sale}) = p$

$\Rightarrow P(\text{no Sale}) = 1 - P(\text{sale})$

$= 1 - p$

(i)

$P(X=x_i) = \binom{n}{x} p^x (1-p)^{n-x}$

where $x \geq \{1, 2, 3, \ldots,$