Announcements

- HW2 Due Thurs Feb 5th 11:59 PM (Do before quiz)
- Quiz 2 During Discussion Thurs Feb 5th
- HW1 Missed Problems Due Feb 5th 11:59 PM (Email submission Subject: STAT 1030 tyler-Schmidt-1@UIOWA
- Exam 1 Wed Feb 11th 6:30-8:30 Location TBD 1 Crib Sheet Allowed.
- Lost Watter Bottle 240 SH

Def) Event

Collection of results/outcomes of a procedure.

Def) Sample Space

Set of all possible "simple" events. Denoted S or Ω

Example) 6-Sided die

Roll a 6-sided die once and record the number

Sample Space:

$S = \{1, 2, 3, 4, 5, 6\}$

Events are any subset, including

1) Roll a 3, $A_1 = \{3\}$

2) Roll at least a 5, $\bar{A_2} = \{5, 6\}$

3) Roll anything but a 2,

$A_3 = \{1, 3, 4, 5, 6\}$

- Def) Probability

The probability $P(A)$ is a function that maps events to a number $[0, 1]$ with

1) $P(\text{nothing}) = P(\emptyset) = 0$
2) $P(\text{everything}) = P(S) = 1$
3) $0 \leq P(A) \leq 1$
4) Probability sums across disjoint events

- Def) Complement

The complement of event A, A^c, is all other events in the sample space.

$P(A^c) = 1 - P(A)$

- Example) Six-sided die

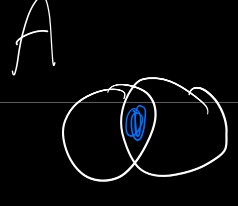
Recall $S = \{1,2,3,4,5,6\}$

Let A be rolling an even number, $A = \{2,4,6\}$

Then $A^c = \{1,3,5\}$

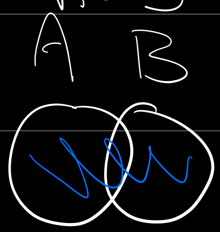
- Def) Intersection $A \cap B$

Outcomes in both A and B



∘ Def) Union $A \cup B$

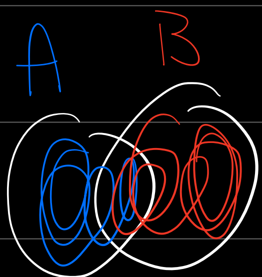
Outcomes in A or B

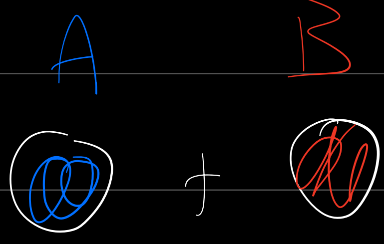


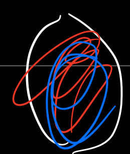
∘ ★) Important Formula

$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$

$$\Leftrightarrow P(A \cup B) = P(A) + P(B) - P(A \cap B)$$



$P(A) + P(B) =$ 

Problem is  intersection double counted.

1. Use the chart below to answer the following questions. In the following problems, imagine that you are rolling 2 six-sided die and adding the number of pips together.

Die 1

		1	2	3	4	5	6
Die 2	1	2	3	4	5	6	7
	2	3	4	5	6	7	8
	3	4	5	6	7	8	9
	4	5	6	7	8	9	10
	5	6	7	8	9	10	11
	6	7	8	9	10	11	12

a. What is the probability of rolling an 8?

b. What is the probability of rolling an 8 or a 10?

c. What is the probability of rolling an 8 and a 10 in the same roll? **Note:** Make sure your answer is a number.

d. What is the probability of rolling an 8 followed by rolling a 10?

e. What is the probability of rolling an 8 OR an even number? **Note:** There is some overlap you want to make sure you do not count twice.

(a) All outcomes (i.e. Die 1/2 = 1, 2, ..., 6) equally likely so we take the ratio

$$P(\text{roll } 8) = \frac{\text{ways to roll 8}}{\text{all ways}}$$

2. Let's say the probability of a household owning a dog is 0.4, the probability a household owns a cat is 0.3, and the probability that a household owns a dog AND a cat is 0.1. Here is a picture of Alex's dog, Ollie. He is a good boy!
    a. What is the probability that a household owns a dog OR a cat?

    b. What is the probability a household owns ONLY a dog?

$d = \text{dog}, \; c = \text{cat}$

$P(d) = 0.4, \; P(c) = 0.3$

$P(d \cap c) = 0.1,$

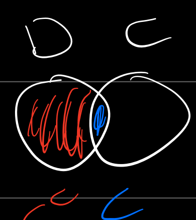
$P(d \cup c) = ?$

Using formula,

$P(d \cup c) = P(d) + P(c) - P(d \cap c)$

$= 0.4 + 0.3 - 0.1$

$= 0.6$



(b)

$P(d \cap c^c) = P(D) - P(D \cap C)$

$= 0.4 - 0.1 = 0.3$

• Def) Independence

Events A and B independent if (and only if)

$P(A \cap B) = P(A)P(B)$

That is, knowledge of B doesn't change the probability of A. ($P(A \cap B)/P(B) = P(A)$

5. A bowl has 3 red (R) chips and 7 (W) chips. Draw chips *with replacement*.

   a. Determine the probability that a red chip is obtained on the first draw and a white chip is obtained on the second draw.

   b. Determine the probability that a white chip is obtained on both draws.

(a) A_1 = Red Chip, A_2 = Blue Chip

these are independent, which means

$P(A_1 A_2) = P(A_1)P(A_2)$

$= 3/10 \cdot 7/10$

$= 21/100 = 0.21$

(b)

$P(W_1 W_2) = P(W_1) P(W_2)$

$= 7/10 \cdot 7/10$

$= 49/100 = 0.49$

6. Roll a die and then randomly draw a chip from a bowl containing 3 red and 7 white chips. Consider the events $A = Roll\ Even$ and $B = Draw\ Red$.

   a. Find $P(A \cap B)$

   b. Find $P(A^C \cap B^C)$

$P(A \cap B) = P(A) P(B)$

$= (1/2)(3/10)$

$= 3/20 = 0.15$

(b) $P(A^c \cap B^c) = P(A^c) P(B^c)$

$= (1/2)(7/10)$

$= 7/20 = 0.35$ ✓

7. Suppose a circuit board contains 3 modules. The probability that the first module works properly is 0.98, while the second and third modules work properly with probability 0.95 and 0.92 respectively. Modules are independent.

a. Find the probability all three modules work properly.

b. ***Difficult*** Find the probability that one or more modules work.

$P(M_1) = 0.98,$

$P(M_2) = 0.95,$

$P(M_3) = 0.92$

(a)

$P(M_1 \cap M_2 \cap M_3)$

$= P(M_1)\,P(M_2)\,P(M_3)$

$= (0.98)(0.95)(0.92)$

(b) $P(\text{At least one } M_i)^c$

$= P(M_1^c \cap M_2^c \cap M_3^c)$

$= (0.02)(0.05)(0.08)$

$= 0.0008$

Def) Conditional Probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

Informally: What is the probability of A given the restriction B.

Equiv Independent defined for $P(A|B) = P(A)$

Example) 6-Sided die

What is the probability of a 2 if the roll is even

A = roll a 2 = $\{2\}$

B = roll an even = $\{2, 4, 6\}$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1/6}{1/2} = \frac{1}{3}$$

What is the probability of a 1 given the roll is even?

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0}{1/2} = 0$$

3. A survey of 1056 homes in the Saratoga Springs, New York area found the following relationship between price and whether the house had a fireplace in 2006.

	Fireplace	
	No	Yes
Low (<$112k)	198	66
Med Low ($112 - $152k)	133	131
Med High ($152k - $207k)	65	199
High (>$207k)	31	233

a. What is the probability a house costs over $152k GIVEN it has a fireplace?

b. What is the probability a house has a fireplace GIVEN is costs less than $112k?

c. What is the probability a house costs less than $112k GIVEN that it has a fireplace?

(a)

	Fireplace	
	No	Yes
Low (<$112k)	198	66
Med Low ($112 - $152k)	133	131
Med High ($152k - $207k)	65	199
High (>$207k)	31	233

Def) Bayes Rule

$$P(A|B) = \frac{P(A\cap B)}{B}$$

$$= \frac{P(B|A)P(A)}{P(B)}$$

$$= \frac{P(B|A)\,P(A)}{P(B|A)P(A) + P(B|A^c)P(A^c)}$$

Answer the following question using Bayes Rule.

4. A company that manufactures and sells consumer video gameras sells two versions of their popular hard disck camera, a basic camera for $750 and a deluxe version for $1250. A total of 75% of the customers select the basic camera. Of those, 60% purchase the extended warranty for an additional $200. Of the people who buy the deluxe version, 90% purchase the extended warranty. Given that a customer purchases an extended warranty, what is the probability that they bought the deluxe version?

bc = basic cam, W = Warranty

We are given

$P(bc) = 0.75,$

$P(W|bc) = 0.6$

$P(W|dc) = 0.9$

$P(dc|W)?$

$$P(dc|W) = \frac{P(W|dc)\,P(dc)}{P(W|bc)\,P(bc) + P(W|dc)P(dc)}$$

Need to first find $P(dc)$

$P(dc) = 1 - P(bc)$

$= 1 - 0.75 = 0.25$