Estimation and Inference of Impulse Responses with Random Forests*
PRELIMINARY AND INCOMPLETE

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Abstract

This paper introduces methods to estimate and conduct inference on impulse responses without specification and estimation of the underlying system, which are particularly suited in the presence of a nonlinear structure. We use flexible machine learning techniques to estimate response values at each horizon of interest, as it is done with local projections. The advantages of our tree-based method over traditional available methods to estimate impulse responses are the following: (1) they automatically estimate nonlinear specifications without having to postulate a specific nonlinear model; (2) they allow for the inclusion of a large number of covariates that far exceed the limits of linear models; (3) they are more robust to model misspecification. We present Monte Carlo evidence and applications that make these advantages apparent.

Keywords: Impulse response functions, Statistical Inference, Machine Learning, Random Forest.

*The views presented here are solely those of the authors and do not necessarily represent those of Board of Governors of the Federal Reserve or of the Federal Reserve System.

1 Introduction

Linear vector autorregressive models (VARs) are used extensively to estimate impulse response functions, an object of great interest to macroeconomists (e.g. Sims (1980)). Furthermore these models have been very influential in estimating the dynamic dependencies of macroeconomic variables, inform the relevance of theoretical models and are regularly used to inform policy makers. VARs are flexible, easy to estimate and have had a fruitful career in macroeconomic forecasting.

Nevertheless these models are also well known to be misspecified when used to estimate macroeconomic impulse response functions. There has been extensive evidence that VARs are inadequate to describe the empirical behavior of macroeconomic time series (see for example Zellner et al. (1974), Tsay (1998), Cogley and Sargent (2001), Primiceri (2005), and in theory Aruoba et al. (2017)). As well as inadequate to describe the impulse responses emanating from theoretical models that are, in most cases, nonlinear (for example models that include an effective lower bound on nominal interest rates such as Gust et al. (2017) or Maliar and Maliar (2015), or that incorporate financial frictions such as Brunnermeier and Sannikov (2014) and He and Krishnamurthy (2013)).

An alternative to VARs for estimating impulse responses is the local projections method of Jordà (2005). This technique exhibit greater flexibility and fewer modelling assumptions tha VARs, and has gained in popularity as a result. However, as Kilian and Kim (2011) and Herbst and Johannsen (2020) note, local projection based impulse responses may be biased towards zero in small sample sizes. While Herbst and Johannsen (2020) present a bias correction, they do not take up the task of correcting the associated confidence intervals. As a result, researchers choosing to use local projections must weigh the trade-offs between modeling flexibility and biased results. An appealing feature of local projections is their ability to encompass nonlinear specifications. This process relies on the expertise of the user to select an adequate nonlinear structure which can be an insurmountable task when dealing with a large number of variables.

In this paper we propose a novel method to estimate impulse response functions for a vector of macroeconomic time series. Our approach does not need to specify the underlying system, and is particularly suited in the presence of a nonlinearites, while it replicates the results of of linear methods when the underlying dynamics are linear. We use the Wager and Athey (2018) Causal Forests extension of Breiman (2001) Random Forests, that allow us to estimate response values and conduct inference at each horizon of interest.

The advantages of our tree-based method over traditional available methods to estimate impulse responses are the following: tree-based models may estimate nonlinear specifications without having to postulate a specific nonlinear model; tree-based models allow for the inclusion of a large number of covariates to contribute to the estimation of the impulse response function at each horizon, far more covariates than the limits of linear models allow; tree-based models, due to their flexibility, are more robust to model misspecification, and their regularization has been shown to avoid over-fitting.

This paper proceeds as follows: section 2 defines impulse responses. Section 3 describes the Random Forest and Causal Forest algorithms. Section 4 describes our method to estimate and perform inference of impulse responses using Random Forests. Section 5 replicates the inflation-output trade-offs application of Jordà (2005) and compares tree-based impulse responses to those estimated via VARs and local projections. Section 6 demonstrates the flexible ability of tree-based models to estimate state-dependent impulse responses through an application to estimating the fiscal multiplier, replicating Auerbach and Gorodnichenko (2012). Section 7 shows tree-based models' ability to estimate impulse responses in high dimensional settings with an application to estimating the effects of a credit shock, replicating the results of Brunnermeier et al. (2017). Section 8 concludes.

2 Impulse responses

We define the time-t, s-step ahead impulse response as the difference between two forecasts that differ in their information set:

$$IR(t,s,d) = E[Y_{t+s}|X_{t-1}, v_t = d] - E[Y_t|X_{t-1}, v_t = 0] \tag{1}$$

where $E(.|.)$ is the conditional expectation operator and denotes the best mean square error (MSE) predictor, Y_t is a vector that contains the macroeconomic object of interest, X_{t-1} represents the history of macroeconomic quantities known before the arrival of the shock, v_t is the vector of disturbances to the economic system, d_i is a given macroeconomic shock. One can think of d as being the components of a matrix D emanating from an identification scheme that give a "structural" interpretation to the impulse responses of interest. In this paper, as in Jordà (2005), we are agnostic about identification and take d as a disturbance given to the econometrician.

In a linear setup, and assuming a well specified model, ordinary least squares is the best linear unbiased estimator and therefore the best candidate to approximate $E(.|.)$. Also the linear structure allows for recursive of sequential forecast to compute the s-step ahead impulse response, which simplifies the calculations. However, there are at least three concerns when using a linear models.

First, when the model is not well specified, imposing linearity may unintentionally bias the predicted values. Alternatively, the econometrician may carefully specify a non-linear model, but that process is hard to systematize in practice and more data is considered the curse of dimensionality forces a trade-off between careful specification and the size of the information set.

Second, when seeking to minimize the in-sample MSE of a direct projection forecast, one is likely to over-fit the model (see Elliott and Timmermann (2008)). An over-fit model too closely matches the observed data, in a sense ignoring the presence of noise in the time-series sample, and as a result may not reflect the underlying data generating process.

Finally, linearity imposes that responses are symmetric around zero, and that responses to shocks of different magnitudes deliver scaled version of responses. It is unlikely to be the case that responses are in symmetric and shape invariant in many macroeconomic object of interest.

In order to address these issues we propose a non-linear machine to robustly estimate impulse responses in high dimensional and non-linear settings. It should be noted that Local Projections (see Jordà (2005)) address part of the problems describe above and that are an improvement to estimating impulse responses with VARs. Our method, expands on local projections by allowing for an automatic estimation of the nonlinear responses as well as by being able to address large dimensional vectors, Section 4 addresses in more detail the relation of our method to local projections.

3 Random and Causal Forest

We use Random Causal Forests to compute our forecasts and therefore our impulse responses. Random Causal Forests, build on the Random Forest (see Breiman (2001)) which is an average of bagged simple decision trees (see Breiman et al. (1984)). These algorithms estimate a non-linear approximation of an unspecified underlying data generating process, and have been shown to be successful in out-of-sample forecasting (see Basak et al. (2019) for an application to asset returns, Jones et al. (2015) or Pike et al. (2019) for an application to firm-level bankruptcies, and Montero-Manso et al. (2018) or Pike and Vazquez-Grande (2020) for an application to forecast combinations).

However, the Random Forests estimates have suffered from a lack of well understood statistical properties, making them inaccessible to statistical inference. To overcome this limitation, Wager and Athey (2018) proposed a modified Random Forest, so-called Causal Forest, whose predictions are shown to be consistent and asymptotically Gaussian. This allow us to compute standard errors and confidence intervals, that are of first order importance in the economic evaluation of impulse responses.

This section will first describe the Random Forest and then the causal forest.

3.1 Random Forests

A single decision tree can be viewed as a function mapping each element of the domain to a neighborhood in the set of dependent variables, and then returning the average of that neighborhood. A bagged collection of trees such as a Random Forest (algorithm 1), by extension, may be thought of as a robust average of sampled averages from slightly varying neighborhoods. As a result, tree-based methods perform well "off-the-shelf" with little tuning, can easily handle nonlinear and sparse data, have intuitively evaluated variables of importance metrics, and can be extended to produce confidence intervals if desired (see Athey and Imbens (2019) for a discussion of tree-based methods for economics, Wager and Athey (2018) for an extensive literature review of tree-based methods for estimating treatment effects, Biau and Scornet (2015) for a summary of the statistical theory of tree-based methods, and Hastie et al. (2009) for a comprehensive statistically-oriented textbook treatment of of the subject).

A single decision tree, first introduced by Breiman et al. (1984), is characterized by a collection of partitions called "decision nodes." Given a sample of outcomes Y_t and independent variables (also called "covariates") X_{tk} for $k = 1, \ldots, K$ and $t = 1, \ldots, T$, the algorithm starts by choosing a variable and a threshold to split the sample into two subsamples, according to whether the values of the chosen variable fall above or below the threshold. The estimated outcome of the tree for the observations on each subsample is then set to the average outcome for each of the subsamples. The algorithm proceeds by creating more splits sequentially on each of the subsamples on a single independent variable at a time. At each step, the algorithm selects the covariate and threshold that maximize the log-likelihood for the resulting subsamples. The size of the tree (the number of splits) represents the complexity of the tree and is a regularization parameter to be chosen to improve accuracy and avoid overfitting.

Random Forests, first introduced by Breiman (2001), were proposed to improve the out-of-sample performance of the tree algorithm. By construction, trees provide a discontinuous predictive function: as values of the covariate cross thresholds the tree jumps to new predictive values. Random Forests induce smoothness by averaging over a large number of trees. These trees differ from each other in two ways. First, each tree is based not on the original sample, but on a random subsample of the data, that selects randomly a subset of independent variables. Second, the splits at each stage are not optimized over all possible covariates, but over the random subset of the covariates that changes with every split. These two modifications lead to sufficient variation in the trees so that the average is relatively smooth and more importantly, has better predictive power than a single tree. The maximum number of splits of trees and the size of subsamples are the regularization parameter to be chosen in this algorithm.

There are several advantages to using a Random Forest to estimate a conditional mean or probability distribution. 1) The true data generating process does not need to have a recognizable parametric form, and the Random Forest do not need to assume any recognizable parametric form. 2) The algorithm automatically detects and incorporates a broad range of non-linearities among covariates, include variable interactions, regime switching, structural breaks, and non-linear variable transformations. 3) Random Forest are an average of many simple decision trees estimated on bootstrapped sub-samples of the observations. As a result of the averaging, Random Forest are well regularized to prevent in-sample over-fitting. 4) Trees, and by extension, Random Forest, have been demonstrated to effectively ignore irrelevant information, making them robust in high dimensional setting. 5) As the algorithm is inherently non-linear, one can observe asymmetric effects due to differences in sign and magnitude of an input variable.

Algorithm 1: Random Forest

for $b = 1$ *to* B **do**
  Draw a bootstrap sample Z^* of size N from the training data;
  Grow a decision tree T_b to the bootstrapped data, by recursively repeating the following steps for each terminal node of the tree, until the minimum node size n_{min} is reached;
  while $n > n_{min}$ **do**
    Select m variables at random from the p variables.;
    Pick the best variable/split point among the m.;
    Split the node into two daughter nodes.;
  end
end
Output the ensemble of trees $T_{b_i}^{B}$;

Source: Hastie et al. (2009)

3.2 Causal Forests

While there are many advantages to Random Forest, there is one primary drawback: a lack of well understood distributional properties. However, Wager and Athey (2018) mitigate this disadvantage through establishing a class of Random Forests, Causal Forests (algorithm 2) whose estimated treatment effects are unbiased, consistent, and asymptotically Gaussian. With the introduction of these properties, one can conduct valid statistical inference with Random Forest, moreover, one may calculate impulse responses with Random Forests.

Wager and Athey (2018) define the leaf-level estimated treatment effect to be:

$$\frac{1}{|\{i : W_i = d, X_i \in L\}|} \sum_{\{i:W_i=d,X_i\in L\}}^{i} Y_i = E[Y|W = d, L] \tag{2}$$

So it follows that:

$$\tau_l(x) = E[Y|W = 1, L] - E[Y|W = 0, L] \tag{3}$$

and then averaging over all leafs in a tree:

$$\tau(x) = \frac{1}{|\{L \in T\}|} \sum_{j=1}^{|\{L\in T\}|} E[Y|W = 1, L_j] - E[Y|W = 0, L_j] \tag{4}$$

yields the unconditional marginal expected difference due to treatment W:

$$\tau(x) = E[Y|W = 1] - E[Y|W = 0] \tag{5}$$

Equation 5 is merely Equation 1 given $d = W = 1$. Therefore, a Causal Forest estimated treatment effects is also an impulse response for a single horizon.

However, the Causal Forest Wager and Athey (2018), similar to Jordà (2005), does not create a series of orthogonal shocks in conjunction with impulse response functions, rather, researchers are expected to produce shocks via a separate process. To use the same tree-based model to produce both orthogonal shocks and impulse response functions, Oprescu et al. (2019) extend the generalize Random Forest of Athey et al. (2019) and Wager and Athey (2018) with Neyman orthogonality (see Neyman (1979)) at the leaf level, allowing for a Double-ML instrumental variable approach à la Chernozhukov et al. (2017), using tree-based models in both steps.

Algorithm 2: Causal Forest using Double-Sample Trees

for $b = 1$ *to* B **do**
  Draw a bootstrap sample Z^* of size N from the training data;
  Split Z^* into two discrete groups I and J, where $|I| = |J| = \frac{N}{2}$;
  Grow a decision tree C_b to the bootstrapped data, by recursively repeating the following steps for each terminal node of the tree, until the minimum node size n_{min} is reached;
  while $n > n_{min}$ **do**
    Select m variables at random from the p variables;
    Pick the best variable/split point among the m based on any data in I, excluding outcome variable Y;
    Split the node into two daughter nodes;
  end
  In each terminal node, estimate the treatment effect T(X) based on data only from J;
end
Output the ensemble of trees ${T_b}_i^B$;

Source: Wager and Athey (2018)

4 Tree-based impulse response functions

We first state the proposition that Random Forest predictions are, consistent, unbiased, and normally distributed. Second, we define a local projection impulse response function. Third, we demonstrate how to place Random Forest in the context of local projection impulse responses and describe its proprieties.

4.1 Normality of Random Forest

A simple decision tree estimates the conditional mean of a variable given some set of data:

$$\widehat{Y} = T(\overline{\overline{X}}) \tag{6}$$

and a Random Forest, $RF(\cdot)$, estimates k simple decision trees using set $\overline{X}_k$ of k bagged subsamples of $\overline{\overline{X}}$:

$$\overline{X}_k = \{X_1, X_2, ...X_k\}$$

$$\widehat{Y} = \mathbb{E}[T_1(X_1), T_2(X_2), ...T_k(X_k)] = RF_k(\overline{X}_k) \tag{7}$$

Then, from Wager and Athey (2018) we know that causal forest predictions, $\hat{Y}(x)$, estimated on subsample size, s_n, are asymptotically unbiased and Gaussian:

$$\frac{\hat{Y}_n(x) - Y(x)}{\sigma_n(x)} \rightarrow \mathbb{N}(0,1)$$

for a sequence $\sigma_n(x) \rightarrow 0$. And the variance $\sigma_n(x)$ can be estimated using the infinitesimal jackknife of Wager et al. (2014):

$$\frac{\widehat{V}_{IJ}}{\sigma_n^2} \xrightarrow{P} 1$$

That is, we may estimate the conditional mean of Y with a causal forest and assert:

$$\widehat{Y} = \mathbb{E}[Y|\overline{X}] = RF(\overline{X}) \implies \widehat{Y} \sim \mathcal{N}(\mu_Y, \sigma_{IJ}^2) \tag{8}$$

given a sufficiently large set of subsamples $\overline{X}$.

4.2 Local projection impulse response functions

From Jordà (2005) we define a local projection as equation 1:

$$IR(t,s,d) = E[Y_{t+s}|X_{t-1}, v_t = d] - E[Y_t|X_{t-1}, v_t = 0]$$

then assuming the conditional distribution of $\widehat{Y}|X = x$ is independent of the conditional distribution $\widehat{Y}|X = 0$, and from basic probability theory:

$$IR(t,s,d) \sim \mathcal{P}\left(\mu_{IF(t,s,d)},\ var\widehat{Y}_{t+s}|X_{t-1}, v_t = d) + var(\widehat{Y}_{t+s}|X_{t-1}, v_t = 0)\right) \tag{9}$$

4.3 Random Forest impulse response functions

Given that impulse responses are only defined up to the expectation operator, and Jordà (2005) only requires consistency and well understood distributional proprieties of the estimator, we can use Causal Forest to estimate impulse response functions.
However, $\overline{X}$ is a set of randomly bagged sub-samples of $\overline{\overline{X}}$. Therefore, $IR(t,s,d)|\overline{X}$ will be stochastic with some unknown probability distribution. As a result, we focus on the sampled mean of impulse response functions:

$$\overline{IR} = \mathbb{E}[IR_1, IR_2, ...IR_n] \sim \mathcal{N}(\mu_{IF}, \sigma^2_{\overline{IF}}/n) \tag{10}$$

While variance will be the sum of variance due to subsampling— sampling statistic variance — and the variance due to model uncertainty — average impulse response function variance:

$$\sigma^2_{IF} = \sigma^2_{\overline{IF}}/n + \sigma^2(\widehat{Y}|X = x) + \sigma^2(\widehat{Y}|X = 0) \tag{11}$$

Thus, Random Forest impulse response functions are defined:

$$IR = \mathbb{E}[Y|X, d = v] - \mathbb{E}[Y|X, d = 0], \quad IR \sim \mathcal{N}(\mu_{IF}, \sigma^2_{IF}) \tag{12}$$

Where

$$\mu_{IF} = \overline{IR} = \mathbb{E}[IR_1, IR_2, ...IR_n]$$

given $\mathbb{E}[Y|X, d = v]$ estimated via RF and

$$\sigma^2_{IF} = \sigma^2_{\overline{IF}}/n + \sigma^2(\widehat{Y}|X = x) + \sigma^2(\widehat{Y}|X = 0)$$

We next use this definition of tree-based impulse response function in a series of application to demonstrate the non-linear and robustness properties of the method.

5 Application: Monetary policy

We first validate our tree-based impulse responses through a standard three-variable system with monetary policy, inflation, and the output gap, comparing our impulses to those of a standard VAR and local projection — in fact, as the local projection framework of Jordà (2005) enables the construction of tree-based impulse responses, it is only fitting that we first validate our technique on the same application to inflation-ouput trade-offs, using the same as data and econometric specifications, as Jordà (2005).

Studying the trade-off between inflation and output has long been a fascination economist and a fixation of central banks. Here we study the the relationship through a three variable system, including the output gap (Y), inflation (P), and the federal funds rate (FF). The output gap takes on its standard definition, the percentage gap between real GDP and potential GDP, measured by the Congressional Budget Office; inflation is the annualized quarterly GDP chain-weighted price index; while the federal funds rate is simply the average annualized quarterly federal funds rate. The sample, seen in Figure 1, emulating Jordà (2005), runs from 1955:Q1 through 2003:Q1.

The tree-based, VAR, and local projection impulse response functions are all calculated using four lags. Initial impacts are determined via Cholesky decomposition with the standard output gap, inflation, federal funds rate ordering.

Figure 2 shows the inflation-output system, where the thick red line is the tree-based impulse response mean point estimate and the blue shading is the tree-based 90 percent confidence intervals. The VAR and local projection estimated impulse responses are the navy dash-dotted and black dashed lines respectively. As one may note, the tree-based responses are never statistically significant than the VAR and local projection estimated impulse responses. Further, the three lines may be appropriately be called qualitative replications of one another. However, there are some notable deviations, chief among them is the increased magnitude of the autoregressive response demonstrated by either the output gap or the federal funds rate. The tree-based impulse responses suggests that in both cases, the output gap and the federal funds rate, the linear estimations of autoregressive response is biased downwards, and is reality slightly greater than one, not below.

In summary, by the inflation-ouput trade-offs application of Jordà (2005), tree-based impulse responses can replicate the results of their linear and well understood counterparts.

6 Application: Fiscal policy

We next demonstrate tree-based models non-linear flexibility, and its benefits in estimating impulse response functions. To this end, we turn to another long standing question, what is the fiscal multiplier in the United States, and it is state-dependent?

Auerbach and Gorodnichenko (2012) use a smooth-transition autoregressive (STAR) model to answer these questions, and determine the United States fiscal multiplier to be state-dependent on recessions and expansions. Specifically, the authors find that government spending has a positive return, as measured in dollars contributed to real GDP, while the economy is in recession, and a negative return while the economy is in expansion. However, Auerbach and Gorodnichenko (2012) explicitly search for a non-linearity and allow for its specific state-dependent structure through the model they use. The STAR model is the convex combination of two VARs, each estimated exclusively within a single state, creating a non-linear system dependent on a transition state variable used to weight both VARs. The model uses a three variable VAR comprised of government expenditures, government revenue, and real GDP, while calculating the transition state with the seven-quarter rolling average of the output growth rate. Conversely, a Random Forest approximating this data generating process will use these four variables and determine the presence of a state-dependent system without the need for explicitly calculating and imposing a state transition structure into the model. As a result, if the fiscal multiplier is in fact state-dependent, tree-based impulse responses will detect this non-linearity with fewer assumptions.

Following Auerbach and Gorodnichenko (2012), we estimate our tree-based impulse responses using government expenditures, government revenue, real GDP, and the output gap growth rate. Figure 3 shows the data quarterly from 1947:Q4 through 2009:Q1. However, unlike Auerbach and Gorodnichenko (2012), we do not impose a state-dependent structure and estimate our forests with all four variables in the design matrix — if the output gap growth rate determines an economic state, then the trees may split on this variable, and en masse these splits will create a continuous function over the series, resulting in the same smooth transition as the STAR model. Although, we will impose structure in the shocks applied to our forests, in accordance with the fiscal multiplier literature. The exercise will use the specification scheme of Blanchard and Perotti (2002). That is, we allow a quarterly delay in impact on output growth from a shock to government spending.

Figure 4 reports our results. When not conditioning on the state of the economy, the fiscal multiplier extends to be approximately 40 cents to every dollar spent. However, that positive return is only after 18 months after the initial government spending. From six to 18 months, a dollar of government spending results in either approximately one dollar of real GDP, or less. We also examine impulses responses, assuming the world is in an expansionary state or a recessionary state (the transformed output gap growth rate is above or below 0.5). First, we find the fiscal multiplier exhibits an upper bound at one dollar when the US is in an expansionary state. That is, when in an expansion, we find that government spending leads to a decrease in real GDP. Second, we find that when the US is in a recessionary state, the fiscal multiplier leads to an increase in real GDP, almost double every dollar of government spending twenty months after it occurred.

In summary, we confirm the results of Auerbach and Gorodnichenko (2012), the US fiscal multiplier is state-dependent. However, we did not have to presuppose a non-linearity existed, rather, the Random Forests detected the non-linearity and incorporated it into its approximation of the data generating process.

7 Application: Credit supply

We lastly consider tree-based models' ability to estimate impulse response functions in non-linear and high dimensional settings. In this example, we will take up an application to studying credit supply shocks and macroeconomic activity.

Recently, Brunnermeier et al. (2017) argue that literature describing an increase in credit precedes a decrease in output is correct. Hoewever, the authors go on to stipulate that this outcome may be attributed to monetary policy leaning against the wind after a run-up in credit, and it is ultimately this tightening monetary policy which drives decreases in output. Further, Brunnermeier et al. (2017) suggest that others have not found this relationship at because they have lacked at least one of three primary elements in their exercises: an endogenous monetary policy response, credit spreads, and a high dimensional non-linear setting. The authors argue that to appropriately consider credit shocks in a reduced from economy, one needs a rich set of variables. In response, Brunnermeier et al. (2017) create a structural VAR with 10 variables and heterskedacity non-normal errors.

Given the consideration that a large set of variables would be needed to appropriately study credit shocks in a reduced form exercise, we use this example to demonstrate trees ability to estimate impulse response functions in a high dimensional setting. Using the McCracken and Ng (2016) monthly dataset, with 128 variables from 1992:M2 through 2019:M9, we estimate tree-based impulse response functions to study shocks to the volume of commercial and consumer credit. The McCraken dataset is an ideal candidate for this exercise, as it spans eight major categories of economic variables: output and income, labor market, consumption and orders, orders and inventories, money and credit, interest rates and exchange rates, prices, and the stock market. These categories subsume monetary policy, credit spreads, and all eight other variables used in Brunnermeier et al. (2017). Further, as we use 12 lags of each variable, we end up using a design matrix of 1536 variables, but only 332 observations, demonstrating trees ability to estimate non-linear systems when $k > t$.

Figure 6 shows the tree-based impulse responses given a shock to commercial credit and a shock to consumer credit. A one standard deviation shock to commercial credit results in an immediate rise in the federal funds rate and no immediate rise in the unemployment rate, industrial production, or personal spending. Then, after six months the unemployment rate begins to rise, and industrial production and personal spending begin to decrease. The timing of the macro-movements appear to corroborate Brunnermeier et al. (2017) finding that a shock to credit first precipitates a lean-against-the-wind reaction in monetary policy, and that reaction in turn generates a deterioration in real activity.

However, Figure 6 also shows that a shock to the stock of commercial credit does not elicit the same monetary policy response. In fact, a sudden increase in consumer credit leads to a decrease in the federal funds rate, industrial production, and personal spending, as well as an increase in the unemployment rate — all to a statistically significant level. Although, it may be important to note that the magnitude of the impulse responses to a consumer credit shock is markedly smaller than responses to a commercial credit shock.

In summary, the tree-based models were able to fit a model with approximately 1500 variables and 300 observations, effectively approximating a data generate process for a rich reduced-form economy. In turn, while being able to control for a broad collection of economic variables, the tree-based model was able to estimate impulse response functions replicating the same credit shock and real economic behavior described by Brunnermeier et al. (2017).

8 Conclusion

We present a novel method to use a non-linear machine learning method, Random Forests, to estimate impulse response functions. As a result, we introduce a technique that improves upon the ordinary least squares local projection method in a number of ways. One, tree-based models may be non-linear, detecting variable interactions, non-linear forms, and state-dependence, without requiring explicit model specification ex ante. Two, tree-based models can incorporate a large number of coviarates, more than the number of observations available. Three, tree-based models are robust to model misspecifcation and overfitting.

Three empirical applications demonstrate tree-based impulse response functions non-linear advantages. First, the inflation-output trade-offs application of Jordà (2005) demonstrates that tree-based impulse response functions may generate the same results as linear VARs and local projections when approximating a linear process. Second, as in Auerbach and Gorodnichenko (2012), tree-based impulse responses demonstrate that the United State's fiscal multiplier is state-dependent. Third, our tree-based model approximates the data generating process of the United States economy through incorporating approximately 1500 variables, with only 300 observations, and in turn estimates an impulse response function which replicates the banking credit shocks and results of Brunnermeier et al. (2017).

Tree-based impulse response functions will allow researchers to exploit the non-linearities inherent in large-scale macroeconomic systems. Non-linearities of interest may include the effective zero lower bound of monetary policy, financial frictions, agent heterogeneity, and fiscal policy. However, we leave these applications to future research.

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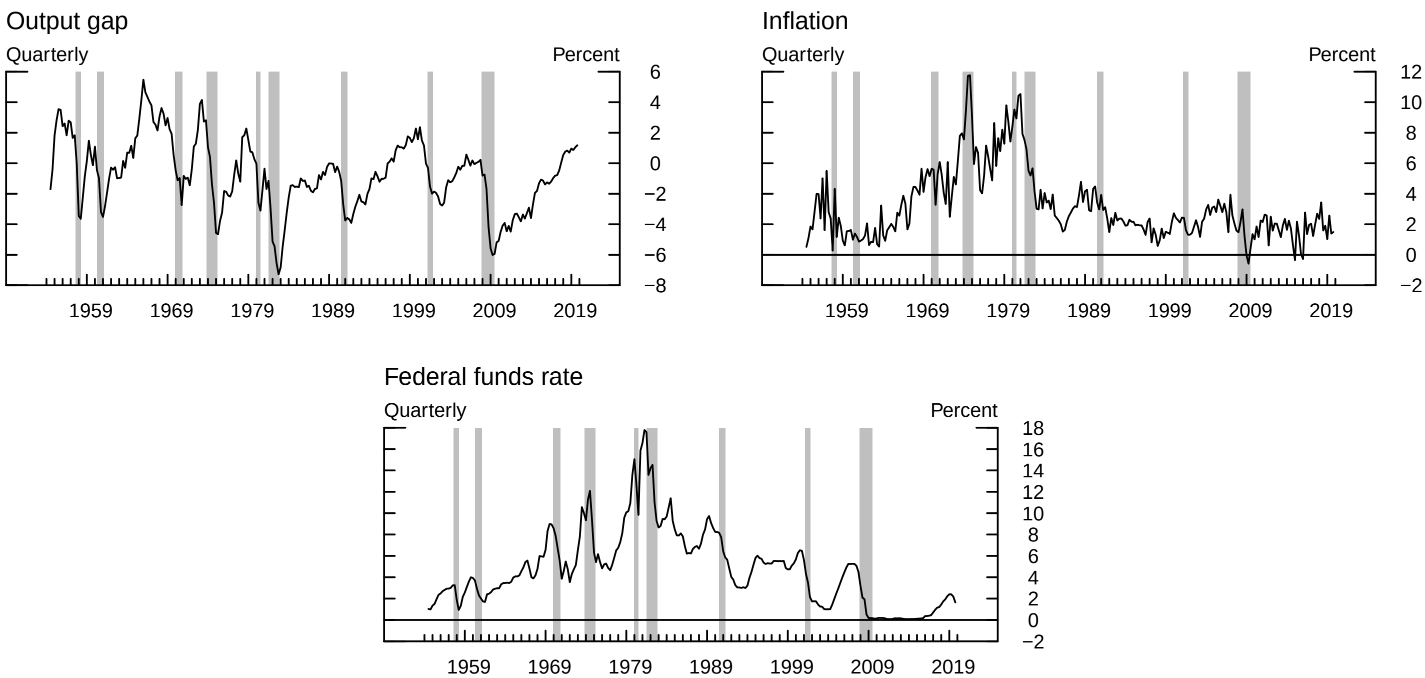

Figure 1: *Time series plots of the output gap, inflation, and federal funds rate*

Notes: Data are quarterly from 1954:Q1 through 2019:Q4. All series are presented in annual percent change.

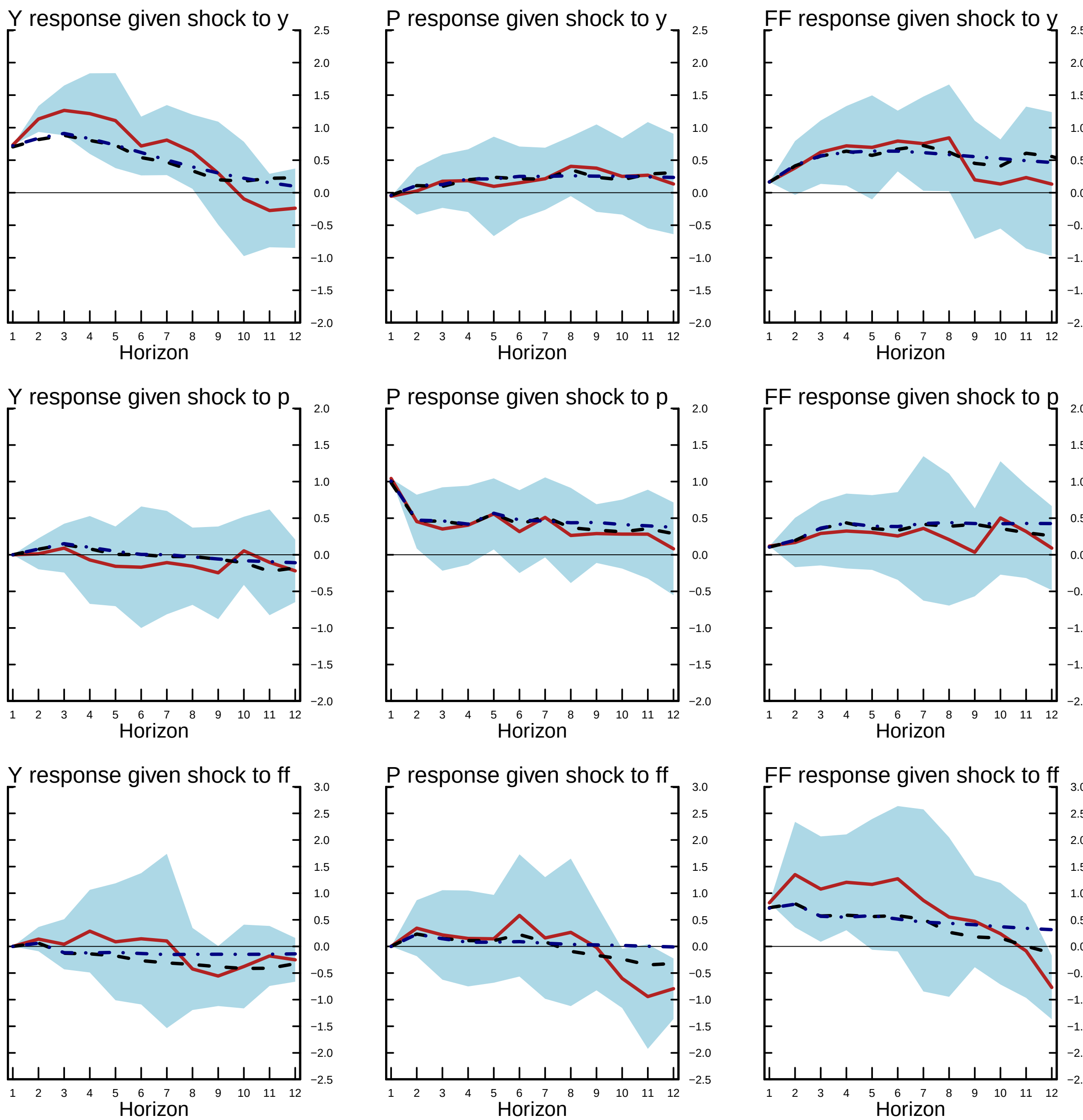

Figure 2: *Impulse Responses for the New Keynesian Model Based on a VAR, Linear, and Tree-based Projections*

Notes: The solid red line denotes the average point forecast estimated with Random Forests and blue bands denote the associated 90 percent confidence intervals; the black dashed line denote responses estimated by linear local projections, à la Jordà (2005); the navy dash-dotted lines denote responses estimated via standard VARs. Data are quarterly from 1954:Q1 through 2019:Q4. Responses are presented in standard deviations. The Random Forests are the orthogonal Random Forest of Oprescu et al. (2019), estimated via Microsoft's econML instrumental variable implementation, with 5 bagged forests. Structural shocks applied to the VAR and linear local projection are calculated via Cholesky decomposition.

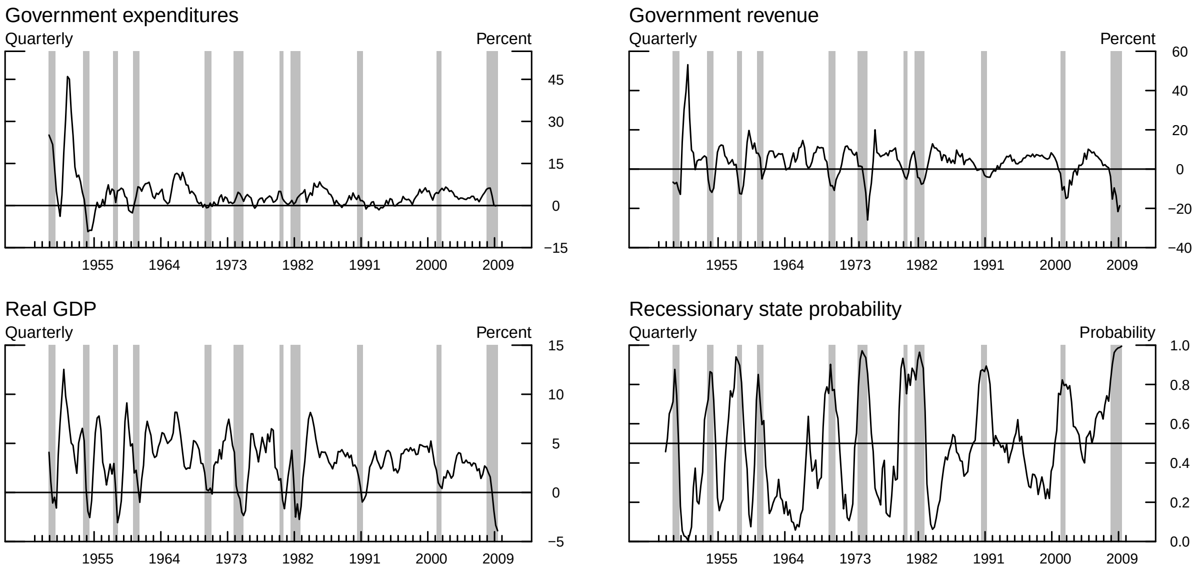

Figure 3: *Time series plots of the government expenditures and revenues, real GDP, and the output gap*

Notes: Data are quarterly from 1947:Q4 through 2009:Q1. Government expenditures, government revenue and real GDP are presented in annual percentage change. The recessionary state probability is the transformed seven-month moving average of the output gap, such that the median of the series is 0.5.

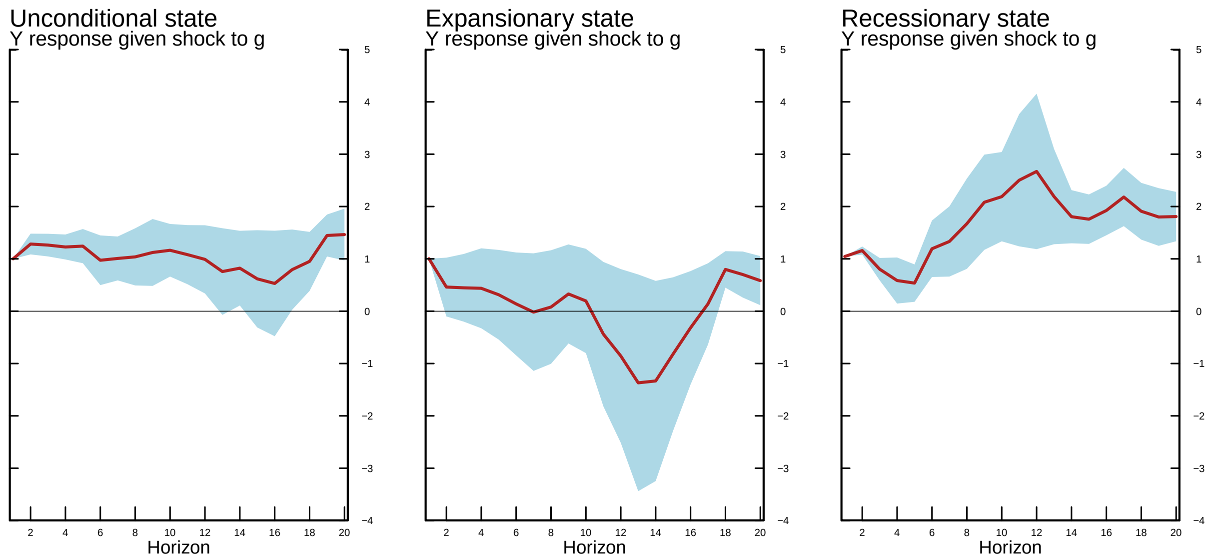

Figure 4: *Impulse responses for a fiscal policy VAR, estimated with tree-based local projections*

Notes: The solid red line denotes the average point forecast estimated with Random Forests and blue bands denote the associated 68 percent confidence intervals. Responses are presented as the change in one dollar of output given one dollar of government spending. Data are quarterly from 1947:Q4 through 2009:Q1. The left panel, the unconditional state, is estimated with all observations; the middle panel, the expansionary state, is estimated with observations when the recessionary state probability is less than 0.5; the right panel, the recessionary state, is estimated with observations when the recessionary state probability is greater than or equal to 0.5. Twenty quarterly horizons are estimated. The Random Forests are the orthogonal Random Forest of Oprescu et al. (2019), estimated via Microsoft's econML instrumental variable implementation, with 5 bagged forests.

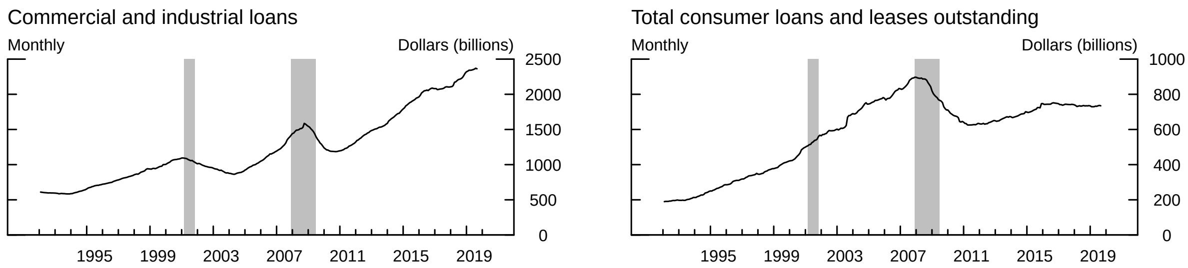

Figure 5: *Time series plots of the total commercial loans and total consumer loans and leases*

Notes: Data are quarterly from 1992:M2 through 2019:M9. Both commercial and consumer credit series are presented in billions of dollars and are not seasonally adjusted.

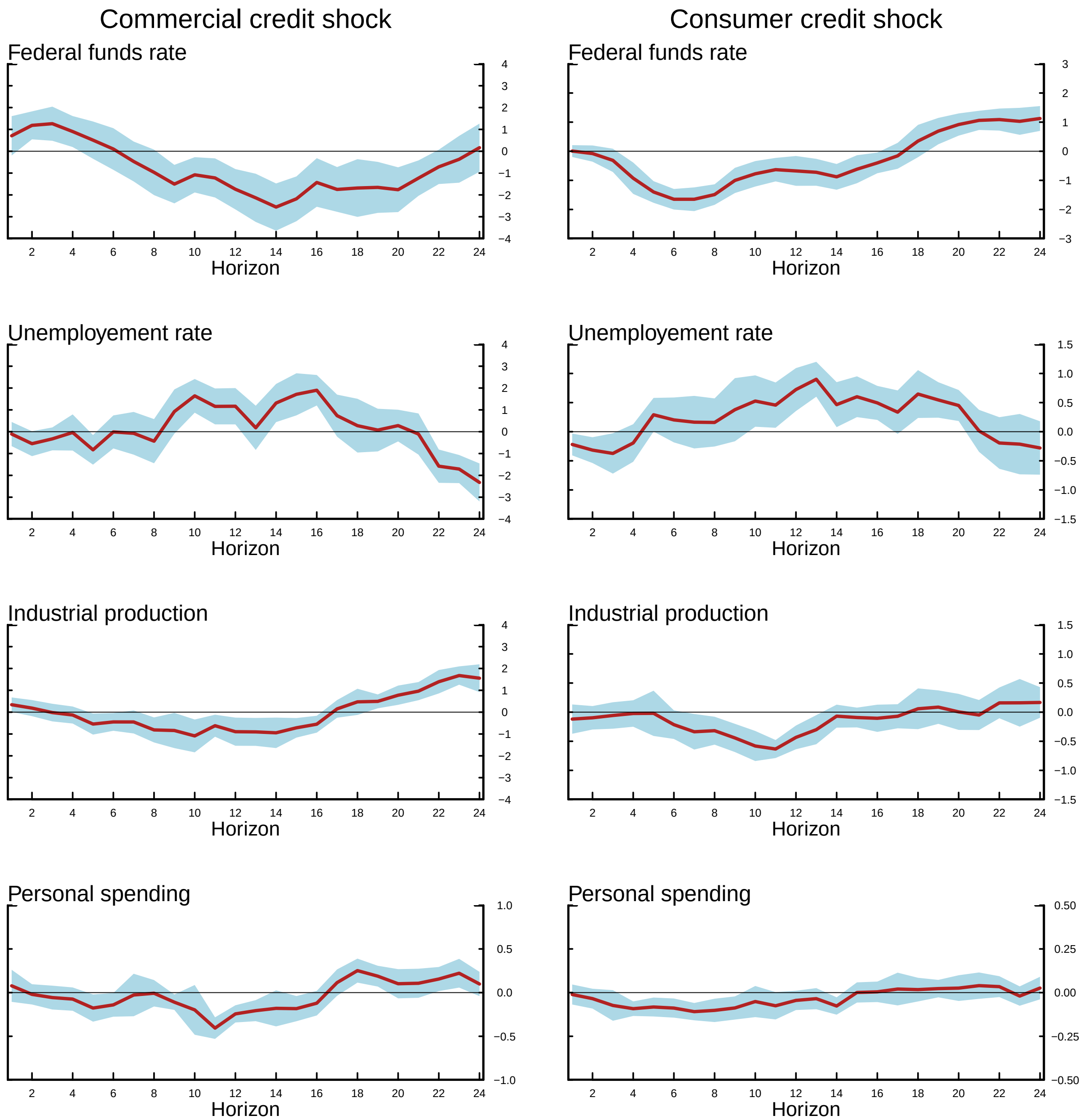

Figure 6: *Impulse responses to credit shocks in a large scale reduced-form economy, estimated with tree-based local projections*

Notes: The solid red line denotes the average point forecast estimated with Random Forests and blue bands denote the associated 68 percent confidence intervals. Responses are measured in standard deviations. Data are monthly from 1992:M2 through 2019:M9, 332 total observations. The data contains 128 variables and 12 lags of each variable, 1536 total variables. Twenty-four monthly horizons are estimated. The Random Forests are the orthogonal Random Forest of Oprescu et al. (2019), estimated via Microsoft's econML instrumental variable implementation, with 5 bagged forests.