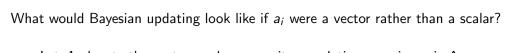
What would	Bayesian	updating	look lik	e if $a_i$	were a v	ector ra	ther than a	a scalar?	



• Let  $A_i$  denote the vector, and suppose its population covariance is  $\Delta$ 

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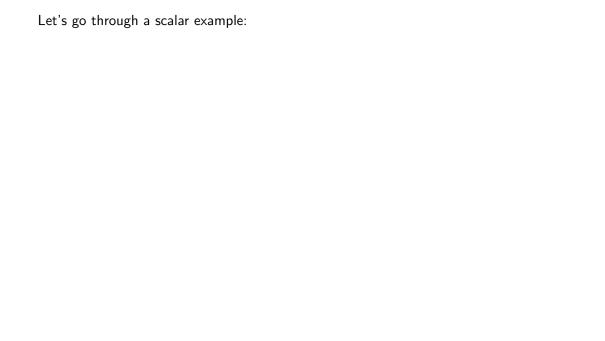
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- Elements of  $\Omega$  and **S** are set to 0 for signals that aren't received
  - (i.e., not all signals need to be received in every period)



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- Then, a signal  $S_{i1}$  is received and beliefs are updated according to the formulas:

$$\mathbb{E}_{2}[a_{i}] = \underbrace{\mathbb{E}_{1}[a_{i}]}_{0} \frac{\sigma_{\varepsilon}^{2}}{\sigma_{\varepsilon}^{2} + \underbrace{\mathbb{V}_{1}[a_{i}]}_{\sigma_{a}^{2}}} + S_{i1} \underbrace{\frac{\sigma_{a}^{2}}{\mathbb{V}_{1}[a_{i}]}}_{\mathbb{V}_{1}[a_{i}]}$$
$$= \frac{S_{i1}\sigma_{a}^{2}}{\sigma_{\varepsilon}^{2} + \sigma_{a}^{2}}$$

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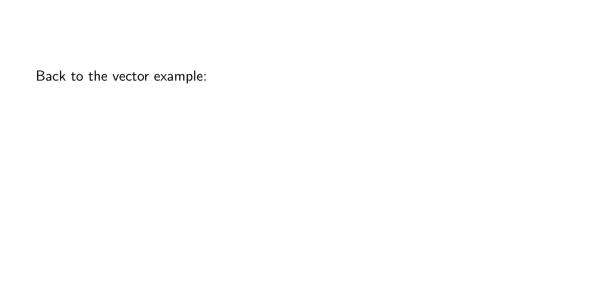
$$\mathbb{V}_2[a_i] = \underbrace{\mathbb{V}_1[a_i]}_{\sigma_a^2} \frac{\sigma_arepsilon^2}{\sigma_arepsilon^2 + \underbrace{\mathbb{V}_1[a_i]}_{\sigma_a^2}}$$

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$$= \frac{\sigma_{a}^{2}\sigma_{\varepsilon}^{2}}{\sigma_{\varepsilon}^{2} + \sigma_{a}^{2}}$$

It is straightforward to show that  $\mathbb{V}_2[a_i] < \mathbb{V}_1[a_i]$  when  $\sigma_a^2 > 0$ 



Back to the vector example:
Division in scalar form is replaced with matrix inversion

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Back	to	the	vector	exampl	e:
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- Division in scalar form is replaced with matrix inversion
- Multiplication is replaced with matrix multiplication
- Other matrices (e.g.  $\Omega$ ) defined for conformability