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- $\hat{\Delta}$ estimated from covariance of within-person mean residuals

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Nested EM algorithm (Two latent variables: $\mathbf{a}_i$ and unobserved type r)

- Outer: regular EM to get π_r's, q_{ir}'s, γ's, λ's
- Inner: specific EM to recover beliefs over $\mathbf{a}$'s and estimate $(\beta, \Delta, \sigma^2)$