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    - (i.e., not all signals need to be received in every period)

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$$\begin{aligned}
\mathbb{E}_2[a_i] &= \underbrace{\mathbb{E}_1[a_i]}_{0} \frac{\sigma_\varepsilon^2}{\sigma_\varepsilon^2 + \underbrace{\mathbb{V}_1[a_i]}_{\sigma_a^2}} + S_{i1} \frac{\overbrace{\mathbb{V}_1[a_i]}^{\sigma_a^2}}{\sigma_\varepsilon^2 + \underbrace{\mathbb{V}_1[a_i]}_{\sigma_a^2}} \\
&= \frac{S_{i1}\sigma_a^2}{\sigma_\varepsilon^2 + \sigma_a^2}
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It is straightforward to show that $\mathbb{V}_2[a_i] < \mathbb{V}_1[a_i]$ when $\sigma_a^2 > 0$

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- Other matrices (e.g. $\Omega$) defined for conformability