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If $X \sim \text{Fréchet}(\alpha)$, then $Y = \log X$ has a Gumbel-like distribution

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Economic interpretation:

- Relative comparisons matter more than absolute levels
- Natural for productivity, trade costs, firm/city size distributions

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Contrast with Gumbel: moderate heterogeneity

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- Trade (Eaton-Kortum Model)
- Firm productivity (Melitz Model)
- Income and wealth inequality (Piketty-Saez)
- Tail risk in financial losses or natural disaster damages