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- Factor Analysis

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- If M were orthogonal to begin with, $\Lambda = I$ and $M = \boldsymbol{\theta}$

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- We say that we "select the first K principal components of M"

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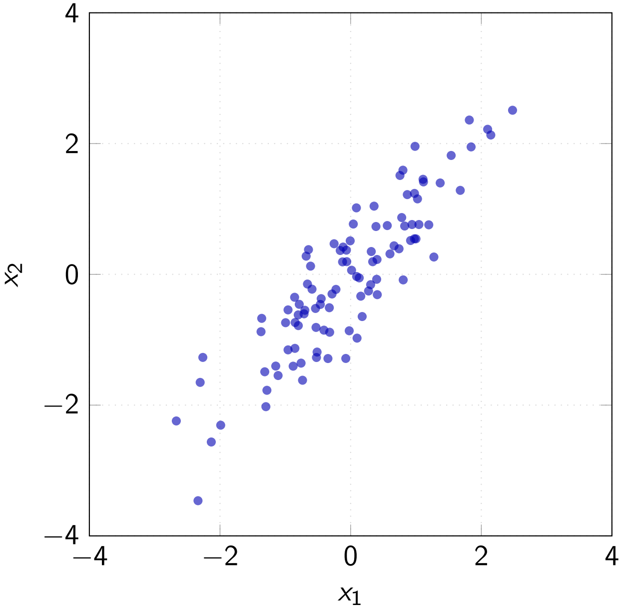
where $\boldsymbol{\varepsilon} \equiv M - \widetilde{M}$ is a $N \times J$ matrix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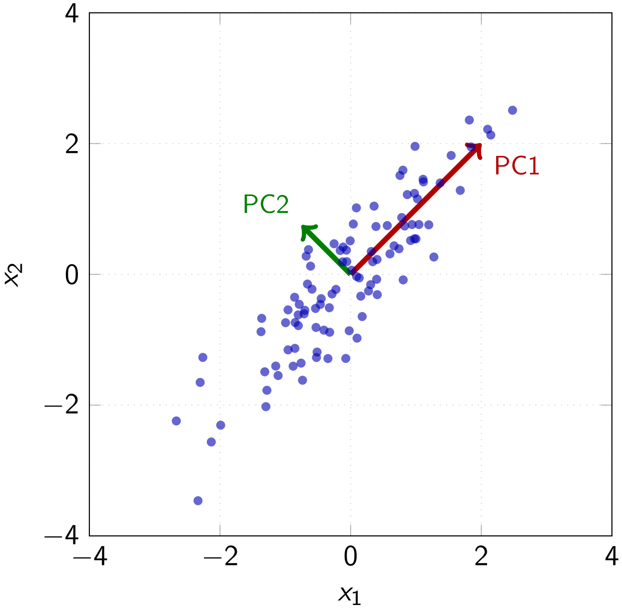
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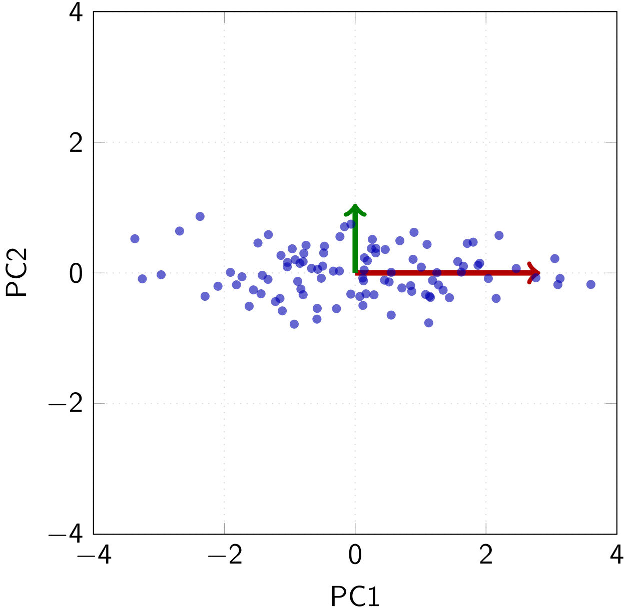

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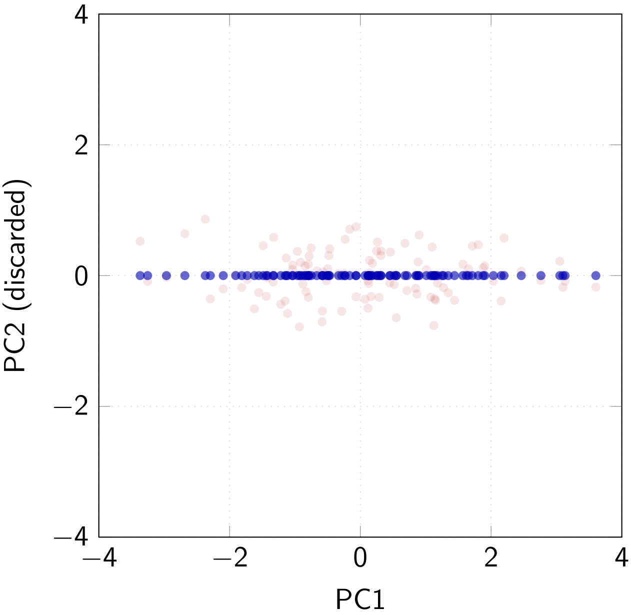

Use $PC1$ for all further analysis