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We need to make the following assumptions:

$$\begin{aligned}
\mathbb{E}\left(\boldsymbol{\varepsilon}\right) &= \mathbf{0}_{J\times 1} \\
\mathbb{V}\left(\boldsymbol{\varepsilon}\right) \equiv \mathbb{E}\left(\boldsymbol{\varepsilon}'\boldsymbol{\varepsilon}\right) &= \Omega_{J\times J} \\
\Omega_{[j,j]} &= \sigma_j^2 \\
\Omega_{[j,k]} &= 0 \\
\mathbb{E}\left(\boldsymbol{\theta}\right) &= \mathbf{0}_{2\times 1} \\
\mathbb{V}\left(\boldsymbol{\theta}\right) &= \Sigma_{\boldsymbol{\theta}}
\end{aligned}$$

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Then

$$\mathbb{E}(u) = \mathbf{0}_{J\times 1}$$
$$\mathbb{V}(u) = \Lambda\Sigma_{\boldsymbol{\theta}}\Lambda' + \Omega$$
$$\boldsymbol{\Sigma}_{\boldsymbol{\theta}} = \begin{bmatrix} \sigma^2_{\theta_1} & \sigma_{\theta_1\theta_2} \\ \sigma_{\theta_1\theta_2} & \sigma^2_{\theta_2} \end{bmatrix}$$

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- three elements of $\Sigma_{\boldsymbol{\theta}}$

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- In general, the model is not identified. Need to impose further assumptions

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So $J \geq 5$ is necessary (but not sufficient) for identification

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- If all 6 measurements come from a cog. test, can't identify a non-cog. factor
- Could possibly identify $\sigma_{\theta_1\theta_2}$ if overlap in measurements