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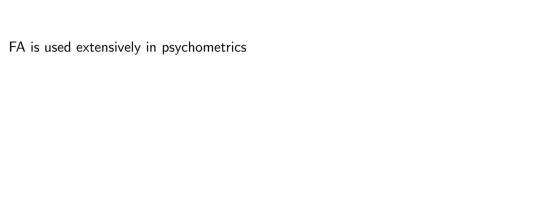
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EFA: see what factors might be in the data

• CFA: write down a model and use the data to test it

In structural econometrics, we pretty much only do CFA



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• Tests tend to measure the same set of skills, so they are correlated

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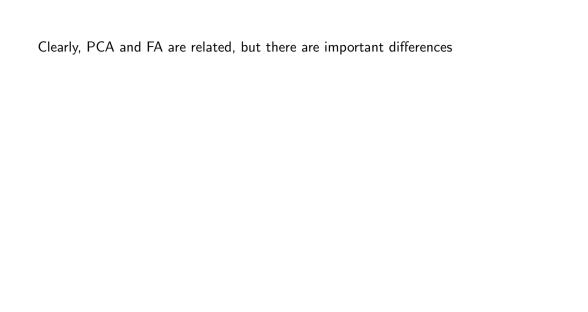
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In FA, we call  $\theta$  factors, and we call  $\Lambda$  factor loadings and  $\varepsilon$  uniquenesses



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- Key challenge: We need additional assumptions for identification