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- How do we know a restaurant we visited for the first time is actually good?
- How do we know we didn’t just happen to get their best dish on a good night?

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- People can't see the future
- Instead, have expectations about their future states and preference shocks
- We compute individuals' expectations according to the  $\mathbb{E}$  max formula
- For tractability, we impose a strong assumption on the distribution of  $\epsilon$

Consider a setting where an agent is trying to learn about something, call it  $a_i$

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- The agent updates his beliefs to  $\mathbb{E}_{t+1}[a_i]$  by incorporating new information in  $S_{it}$
- This process repeats itself in each period where  $S_{it}$  is received

If  $\varepsilon_{it}$  is pure noise, then it is independent of  $a_i$  for all  $t$

Assume WLOG that  $\mathbb{E}(a_i) = 0$  and  $\mathbb{E}(\varepsilon_{it}) = 0$  for all  $t$

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Then we can decompose the variance of the signal  $S_{it}$

$$\begin{aligned}\mathbb{V}(S_{it}) &= \mathbb{V}(a_i) + \mathbb{V}(\varepsilon_{it}) \\ &= \sigma_a^2 + \sigma_\varepsilon^2\end{aligned}$$

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This ratio measures the quality of the signal (bigger is better)

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This is used in common formulas for updating beliefs