

- Estimate multi-dimensional learning model

• Individuals have vector of abilities across J = 6 occupations

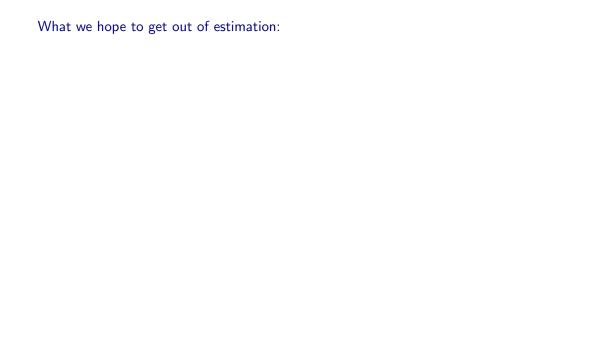
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- ullet $\hat{\Delta}$ estimated from covariance of within-person mean residuals

Problem: If $d_{it} = j$ depends on $\mathbb{E}_t \mathbf{a}_i$, then $\mathbb{E}[\varepsilon_{ijt} | d_{it} = j] \neq 0$

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Solution: Joint model of wages and choice, add permanent unobserved type *r*

Wage:
$$\log w_{ijt} = X_{itr}\beta_j + a_{ij} + \varepsilon_{ijt}$$

Choice: $P(d_{it} = j | \mathbf{a}_i, X_{it}, r) = \frac{\exp(X_{itr}\gamma_j + \lambda_j \mathbb{E}_t a_{ij})}{\sum_k \exp(X_{itr}\gamma_k + \lambda_k \mathbb{E}_t a_{ik})}$

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Nested EM algorithm (Two latent variables: a_i and unobserved type r)

- Outer: regular EM to get π_r 's, q_{ir} 's, γ 's, λ 's
- Inner: specific EM to recover beliefs over **a**'s and estimate $(\beta, \Delta, \sigma^2)$