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We need to make the following assumptions:

$$\mathbb{E}(\boldsymbol{\varepsilon}) = \mathbf{0}_{J \times 1}$$

$$\mathbb{V}(\boldsymbol{\varepsilon}) \equiv \mathbb{E}(\boldsymbol{\varepsilon}' \boldsymbol{\varepsilon}) = \Omega_{J \times J}$$

$$\Omega_{[j,j]} = \sigma_j^2$$

$$\Omega_{[j,k]} = 0$$

$$\mathbb{E}(\boldsymbol{\theta}) = \mathbf{0}_{2 \times 1}$$

$$\mathbb{V}(\boldsymbol{\theta}) = \Sigma_{\boldsymbol{\theta}}$$

Recall our factor model with observable characteristics:

$$M = X\beta + \underbrace{\theta \wedge + \varepsilon}_{i}$$

Then

$$\mathbb{E}\left(u\right) = \mathbf{0}_{J imes 1}$$
 $\mathbb{V}\left(u\right) = \Lambda \Sigma_{oldsymbol{ heta}} \Lambda' + \Omega$
 $\Sigma_{oldsymbol{ heta}} = \left[egin{array}{cc} \sigma_{ heta_1}^2 & \sigma_{ heta_1} heta_2 \ \sigma_{ heta_1 heta_2} & \sigma_{ heta_2}^2 \end{array}
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- The J diagonal elements of Ω (i.e. the σ_i^2 's)
- 2J elements of Λ
- 25 elements of 7

Our only data source to estimate Λ and Σ_{θ} is $\mathbb{V}\left(M-X\beta\right)\equiv\mathbb{V}\left(u\right)$

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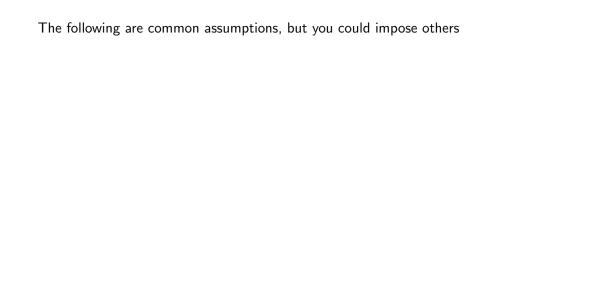
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• In general, the model is not identified. Need to impose further assumptions



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So $J \ge 5$ is necessary (but not sufficient) for identification

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- Likewise, the second row of Λ should be 0 for the cognitive measures
- If all 6 measurements come from a cog. test, can't identify a non-cog. factor
- Could possibly identify $\sigma_{\theta_1\theta_2}$ if overlap in measurements