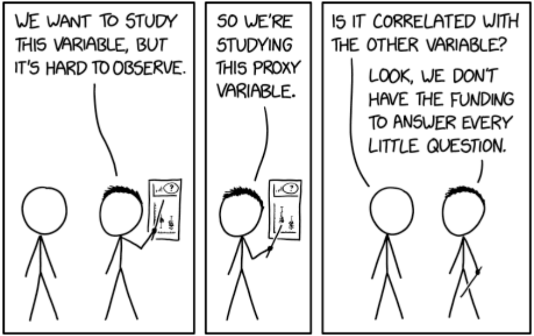
What are proxy variables?



Source: https://xkcd.com/2652/

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- e.g. IQ a proxy for unobserved ability that biases returns to schooling

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    - "non-classical measurement error"

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- So it seems we face a trade-off between OVB and attenuation bias

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- What if the test scores are highly correlated with each other?