Ast 8110 (LSS & GL)
Fall 2022
Problem Set #3 (due Tuesday October 25)

Finding positions of images using two methods, (a) and (b).

All quantities below are assumed to be projected on the plane of the sky.

The mass distribution in the lens is axisymmetric, centered on (0,0), and its radial density profile is given by

$$\kappa = \kappa_0 \left(\frac{|\vec{\theta}| + \theta_0}{\theta_0} \right)^m,$$

where $\kappa_0 = 30$, $\theta_0 = 1$, $m = -3.2$. The density profile is truncated at $\theta_{\rm max} = 5$, so $\kappa = 0$ outside this radius.

Let us place a source on the positive θ_x-axis, at a distance $\theta_x = \beta = 2$ from the origin. Because of the symmetry, all the images will be located on the θ_x-axis.

(a) Time delay surface. Given this density distribution numerically calculate the cross-section of the time delay surface along the θ_x-axis. Use

$$T(\theta_x) \propto \frac{1}{2}(\theta_x - \beta)^2 - \frac{1}{\pi} \int d\theta'_x \, d\theta'_y \, \kappa(\theta'_x, \theta'_y) \ln \sqrt{(\theta_x - \theta'_x)^2 + (\theta'_y)^2}.$$

Plot this cross-section of T as a function of θ_x. Locate the images as the local extrema. You don't need to calculate their θ_x values; just mark the images on the plot as accurately as you can. Label the images by the type of extrema: maxima, minima, or saddle points.

(b) Lens equation. Lens equation can be written as

$$\vec{\theta} - \vec{\beta} = \vec{\alpha}.$$

In an axisymmetric case the contribution to the deflection angle at any location $\vec{\theta}$ in the lens plane comes only from the mass interior to $|\vec{\theta}|$. Therefore the magnitude of the deflection angle can be calculated as

$$\alpha = \frac{M(\leq \theta)}{\pi \, \theta}.$$

Plot two lines on the same plot:
(i) $\theta - \beta$ vs. θ, and
(ii) α vs. θ.
According to the lens equation, images form where the two lines intersect. Indicate the images on your plot. Are the positions you found using the lens equation consistent with those found using the time delay surface?