Ast 8110 (LSS & GL)
Fall 2022
Problem Set #2 (due Tuesday October 4)

1. Popular cosmological models have $\Omega_m + \Omega_\Lambda = 1$, i.e., curvature is zero, and the contribution of radiation is negligible in today's Universe. For these models the angular diameter distance between the observer and source at z reduces to a simpler expression,

$$D_A(z) = \frac{c}{H_0}\, a(z)\, \chi = \frac{c}{H_0} \frac{1}{(1+z)} \int_0^z \frac{dz}{[\Omega_m (1+z)^3 + \Omega_\Lambda]^{1/2}} \tag{1}$$

Angular diameter distance between lens at z_l and source at z_s is

$$D_A(z_l, z_s) = \frac{c}{H_0}\, a(z_s)\, \chi_{ls} = \frac{c}{H_0} \frac{1}{(1+z_s)} \int_{z_l}^{z_s} \frac{dz}{[\Omega_m (1+z)^3 + \Omega_\Lambda]^{1/2}} \tag{2}$$

(a) Write a code to calculate D_A, in units of c/H_0, as a function of Ω_m, Ω_Λ, and z. (You don't need to turn in the source code.) Generate a curve of $D_A(z)/(c/H_0)$ using $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$, for observer at $z = 0$, and redshift ranging from just above 0, and up to $z = 5$.

(b) For a fixed lens redshift, $z_l = 0.4$, calculate and plot how Σ_{crit} varies with z_s, for z_s values between 0.41 and 7. Use units of g/cm^2, and this value, $c^2/(4\pi G)/(c/H_0) = 0.115$ g/cm^2.

(c) For a fixed source redshift, $z_s = 2$, calculate and plot how Σ_{crit} varies with z_l. From your plot, or the data generated by your code, determine z_l at whch Σ_{crit} attains a minimum. What is that value of Σ_{crit}? This z_l is where a lens of a fixed g/cm^2 surface mass density will be most effective as a lens for source at $z_s = 2$, typical of lensed sources.

(d) For a fixed source redshift, $z_s = 1000$, calculate and plot how Σ_{crit} varies with z_l. From your plot, or the data generated by your code, determine z_l at which Σ_{crit} attains a minimum. What is that value of Σ_{crit}? This z_l is where a lens of a fixed g/cm^2 surface mass density, will be most effective as a lens for the Cosmic Microwave Background as the source.