Ast 8110
Fall 2022
Problem Set #6 (due Thursday Dec 8)

1. Weak shear lensing.

In this problem we will construct a shear map of idealized background cosmic wallpaper consisting of a grid of regularly spaced, initially circular sources, of radius 0.01. Let the grid span $0 \to 1$ in both x and y directions on the sky. Place your sources on a grid, spaced 0.05 apart, so that there are 21 sources along each axis. Ignore the deflection of the images w.r.t. the source positions, i.e., assume that images appear at the locations of corresponding sources.

Let the foreground mass distribution be represented by a sum of 3 singular isothermal spheres at these locations: (0.44, 0.44), (0.51, 0.51), (0.58, 0.58). The combination of these 3 circular lenses approximates an elongated lens. Recall that lensing potentials from different mass distributions add up linearly.

We derived all the necessary lensing quantities for a singular isothermal sphere in class:

$$\Psi = b[x^2 + y^2]^{1/2}, \qquad \text{and} \qquad \kappa = \frac{b}{2R}, \qquad \gamma_1 = \frac{b[y^2 - x^2]}{2R^3}, \qquad \gamma_2 = \frac{-bxy}{R^3},$$

where $R = \sqrt{(x^2 + y^2)}$ is the distance from the lens, on the plane of the sky. Assume $b = 0.07$, and use expressions for reduced shear derived in class.

Plot a grid of lensed source shapes (sheared ellipses) due to this mass distibution. Note that in the central region of your image plane some shear ellipses may become very elongated.

2. Weak magnification lensing.

Weak magnification by a factor μ stretches the whole area behind the lens by μ. As we discussed in class, this has two effects on a population of background sources: it magnifies each source by μ, and at the same time stretches the area behind the lens by the same factor, thereby diluting the sky projected number density of sources.

Let the intrinsic (unlensed) luminosity function (LF) of sources, all located at some redshift behind the lens, be given by

$$n_U(f) = (f^{s_1} + f^{s_2})^{-1},$$

where $n_U(f)$ is the unlensed number density of sources with fluxes in a small interval around f, and s_1 and s_2 the slopes of the faint and bright end of the LF. Assume $s_1 = 0.3$ and $s_2 = 3.0$, and the LF applies between $f = 0.05$ and $f = 10.0$.

Suppose these sources are behind a lens of magnification probability distribution of a Gaussian form:

$$p(\mu)\, d\mu = \frac{1}{\sqrt{\pi \sigma_\mu^2}} \exp(-[\mu - \mu_0]^2/\sigma_\mu^2)\, d\mu.$$

The lensed LF is a convolution of the unlensed LF, $n_U(f)$ with magnification PDF, $p(\mu)$, and also includes the area dilution effect:

$$n_L(f) = \int \frac{n_U(f/\mu)\, p(\mu)}{\mu}\, d\mu.$$

Numerically calculate the lensed LFs and plot them on the same plot with the corresponding unlensed LF:
(a) $\mu_0 = 2.5$, $\sigma_\mu = 0.5$;
(b) $\mu_0 = 0.7$, $\sigma_\mu = 0.3$.
Make sure that the break in the LF is roughly in the middle of the plotted flux range, and the flux range extends over at least 2 decades in f. Please make your plots log-log.