Physics 343 – Spring 2020
Exercise # 2: Useful observational relations

1. Redshift is defined by the relations

$$1 + z = \frac{\nu_0}{\nu} = \frac{\lambda}{\lambda_0} \tag{1}$$

where a 0 subscript indicates rest (emitted) frequency or wavelength. In contrast to the confusing inconsistency between radio and optical "velocities," which you explored earlier in the semester, there are unambiguous and exact relations between the *rest-frame velocity interval* Δv_0 and various observed-frame quantities and intervals:

$$\frac{\Delta v_0}{c} = \frac{\Delta \nu}{\nu} = \frac{\Delta \lambda}{\lambda} = \frac{\Delta z}{1+z} \tag{2}$$

Considering these relations, calculate the rest-frame velocity resolutions of

- A spectrum of the CO($J = 2 \rightarrow 1$) rotational line (rest frequency 230 GHz) in a galaxy at $z = 0$, obtained with a spectrometer with 10 MHz wide channels.
- A spectrum of the CO($J = 2 \rightarrow 1$) rotational line (rest frequency 230 GHz) in a galaxy at $z = 1$, obtained with a spectrometer with 10 MHz wide channels.
- A spectrum of the [O II] emission-line doublet (rest wavelength 3727 Å) in a quasar at $z = 0$, obtained with the Sloan Digital Sky Survey spectrograph, which had resolving power $R \equiv \lambda/\Delta\lambda = 1500$ at $\lambda \approx 3800$ Å and $R = 2500$ at $\lambda \approx 9000$ Å.
- A spectrum of the [O II] emission-line doublet (rest wavelength 3727 Å) in a quasar at $z = 1$, obtained with the Sloan Digital Sky Survey spectrograph, which had resolving power $R \equiv \lambda/\Delta\lambda = 1500$ at $\lambda \approx 3800$ Å and $R = 2500$ at $\lambda \approx 9000$ Å.

2. A reasonable approximation for the total dynamical mass $M_{\rm dyn}(< R)$ enclosed within radius R for a rotating disk with velocity $v(R)$ is

$$M_{\rm dyn}(< R) = \frac{Rv^2}{G} \tag{3}$$

where $G = 6.67 \times 10^{-11}\,{\rm m^3\,kg^{-1}\,s^{-2}}$ is the gravitational constant. If this equation is written with mass expressed in solar masses ($1\,M_\odot = 1.99 \times 10^{33}$ g), distance expressed in kpc ($1\,{\rm pc} = 3.09 \times 10^{18}$ cm), and velocity is expressed in ${\rm km\,s^{-1}}$, what is the value of the constant k in

$$\frac{M_{\rm dyn}(< R)}{M_\odot} = k \times \Big(\frac{R}{\rm kpc}\Big)\Big(\frac{v}{\rm km\,s^{-1}}\Big)^2 \tag{4}$$

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