EMEC 303
Spring 2025
Project 1: Cabin Project

Background:
You are planning a weekend in the wilderness. It's supposed to be very cold out – great for skiing! The cabin is way out in the woods and doesn't have electricity. The wood stove is your only way to stay warm. Your friends know you're an engineering student and overall highly competent planner and elect YOU to decide how to manage the fireplace. Of course, you enthusiastically accept this task and embark on a modeling project before heading out into the woods.

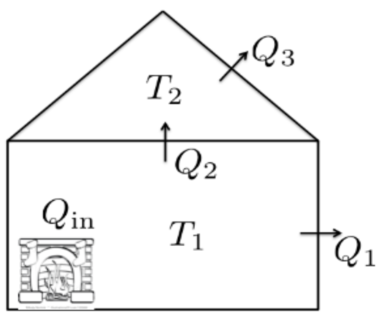

Figure 1: Heat transfer for cabin.

Figure 1 shows a schematic of the cabin. The $\dot{Q}$'s represent heat fluxes in W/m^2. $\dot{Q}_1$ is the heat transfer from the downstairs level through the walls of the cabin. $\dot{Q}_2$ is the heat transfer from the downstairs to upstairs. $\dot{Q}_3$ is the heat transfer from the upstairs level through the walls and ceiling. The heat fluxes from the cabin to the outside are related to the temperatures and the amount of insulation by:

$$\dot{Q}_1 = k_1(T_1 - T_{out}) \qquad Eq. 1$$
$$\dot{Q}_3 = k_3(T_2 - T_{out}) \qquad Eq. 2$$

The k's are the coefficients of thermal resistance which you assume for this cabin are k_1 = 0.2 W/(m^2 K) and k_3 = 0.5 W/(m^2 K). The higher the heat transfer coefficient, the worse the insulation and the larger the heat flux. You know from experience that this cabin loses more heat through the roof compared to the walls. The heat flux from downstairs to upstairs is modeled using:

$$\dot{Q}_2 = k_2(T_1 - T_2) \qquad Eq. 3$$

Importantly, k_2 is adjustable since you can control a door that separates the downstairs from the upstairs. By closing or opening the door you can vary k_2 from 1 to 10 W/(m^2 K).

$Q_{in}(t)$ is the heat flux from the fireplace which you can also control to maintain a comfortable temperature in the cabin. The area of the fireplace is A_{in} = 0.5 m^2. You know that the amount of energy needed to change the temperature of the cabin depends on the cabin's size, the heat capacity of the air (C_{air} = 1.005 kJ/(kg K), and the density of the air (ρ_{air} =1.2 kg/m^3) within the cabin. Figure 2 shows the dimensions of the cabin.

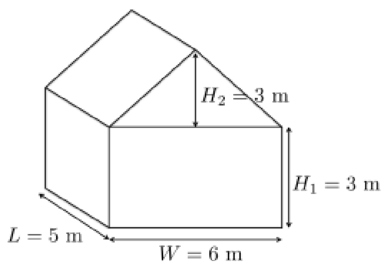

Figure 2: Dimensions of cabin.

After studying the forecast for the weekend, you decide to model the outside temperature using:

$$T_{out}(t) = -10 * \sin\left(\frac{2\pi t}{86400}\right) \qquad Eq. 4$$

In Eq. 4., t is measured in seconds (86400 is the number of seconds in a day). T_{out} is measured in Celsius. You know from a previous (very chilly) experience that the cabin should be very cold when you arrive, with $T_1(0) = 7$ C and $T_2(0) = 5$ C. You are going to arrive at the cabin at 6 pm on Friday and leave at 6 pm on Sunday.

Process:

1) Construct the ODEs. Using the information provided, write first-order ODEs for T1 and T2. Use Newton's law of cooling. $\dot{Q}_i$ is the ith heat flux through an area A_i with direction into the volume V. Fluxes out of the room should have a negative sign. Hint: be careful with units.

$$\frac{dT}{dt} = \frac{\sum_i A_i \dot{Q}_i}{C_{air}\rho_{air}V_i}$$

2) Write a Python code to correctly solve the ODEs using RK methods. You may use Euler's, Heun's, RK4, or solve_ivp methods. (Hint: start with Euler's method and add complexity, if you want, after it is working well). To start, use $Q_{in} = 0$ W/m^2 and k2 = 10.

3) Now, determine a sophisticated control schedule (Q_{in} vs time) for keeping the temperature comfortable in the cabin. You can change k_2 by opening or closing the door to the upstairs. You can also add logs to the fire (up to 5 at a time), but you cannot remove them. You can't add part of a log. Adding one log to the fire adds 500 W/m^2 initially, but this decays over time (Eq. 5). Your friends insist that both the upstairs and downstairs stay within 12.5 and 27 C for the entire stay; they will otherwise incessantly complain. Keep k2 = 5 for this part.

   Keep in mind that your method should be physically possible and reasonable. In other words, you will not want to spend the entire weekend adjusting the fireplace. So – don't open/close the door or add logs more than once an hour. Don't adjust anything between midnight and 6 am so that people can sleep in peace.

$$\dot{Q}(t) = \dot{Q}_0(1 + kt)^{-n} \qquad Eq. 5$$

   For Eq. 5, you can choose pine (k = 0.5 h^{-1}, n = 2.1) or oak (k = 0.2 h^{-1}, n = 1.4), or some combination of those. Both pine and oak initially add $\dot{Q}_0 =$ 500 W/m^2.

4) Extra credit (optional). Update your method to also include k2 within your control. Don't adjust it between midnight and 6 am. Were you able to tighten the temperature range of the cabin?

<u>Deliverables:</u>

Your team needs to write a report that contains the following labeled sections. Your team should turn in ONE report with all teammates' names on it. Include your .py code as an appendix to the report and turn the full .py code in on D2L.

Intro:
- What problem are you trying to solve?

Methods:
- Clearly and concisely describe your numerical approach.
- Clearly and concisely describe the design of your control.

Results:
- Clearly and concisely describe your key results of the numerical method, both for the situation of Qin = 0 and for when you implemented your temperature control scheme.
- Include your figures in this section. Make sure they are of excellent quality (high resolution, easy to read axis labels / legends / titles, sensible color schemes) and that all figures have captions.

Discussion:
- Did you control scheme meet your goals? Why or why not?
- What are sources of error in your modeling? These could pertain to your numerical method, the model assumptions, or potentially other things.
- What about your model could be improved for the future? How would you implement those improvements?

What your code needs to achieve

Plot T_{out} vs T in Figure 1. (10 points)

↓

Plot T1, T2 vs time in the cabin for $Qin = 0\ W/m^2$ n^2 in Figure 2. (40 points)

↓

Plot a reasonable control function for Qin vs time in Figure 3. (10 points)

↓

Implement your control function to update your model of T1, T2 vs time, in Figure 4. (20 points)

Extra credit: Improve your control to also vary k2 (up to 5 points)

What your report needs to achieve

Report contains all required parts and they are complete (intro, method, results, discussion) (10 points).

Professionalism. Free of typos and grammatical errors. (4 points)

Plots are excellent quality (reasonable axis labels and legends). (4 points)

Turned in correctly (assigned sections on Gradescope; turned in code on D2L). 2 points