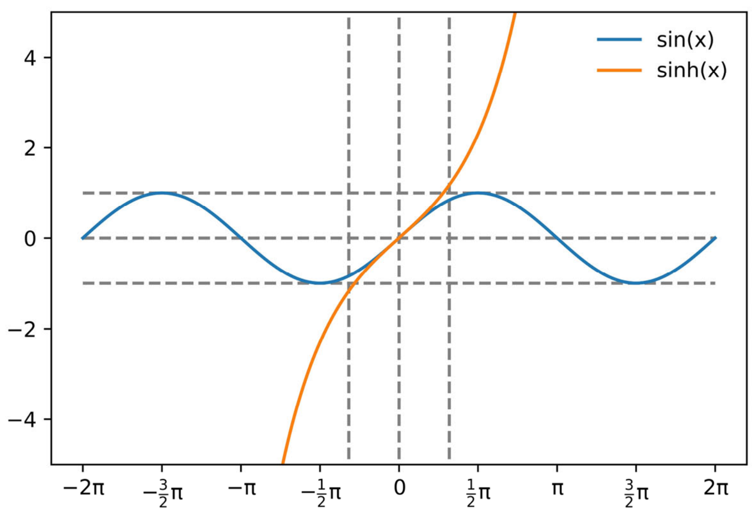
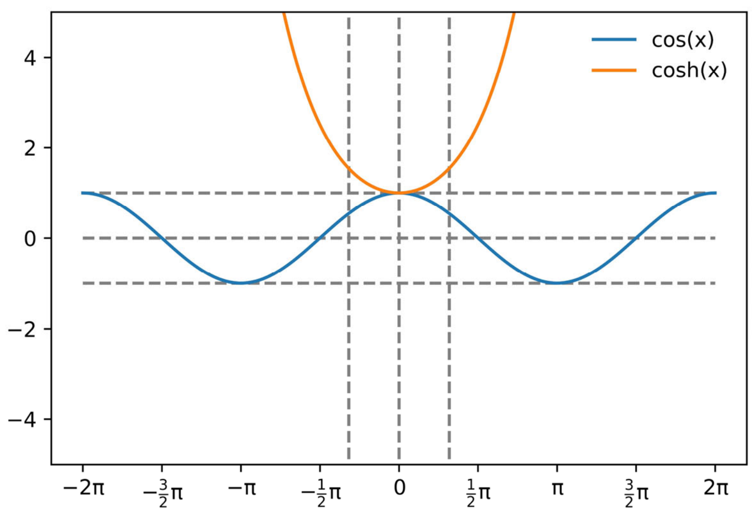
Have you ever wondered what's up with hyperbolic functions like $sinh(x)$, $cosh(x)$, or $tanh(x)$?

You’re not the only one. I’m a chemical engineer and only figured it out once I got to graduate school.

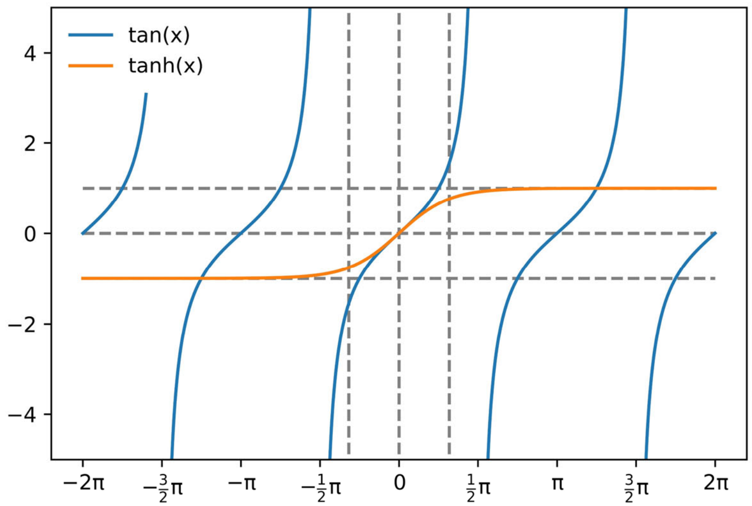
First, let’s compare the common sinusoid functions with their hyperbolic counterparts.



- $\sin(x)$
  - Similarities
    - Odd – symmetric about $y = x$
    - $\sin(0) = 0$
  - Differences
    - Repeating wave
- $\sinh(x)$
  - Similarities
    - Odd – symmetric about $y = x$
    - $\sinh(0) = 0$
  - Differences
    - Exponential nature



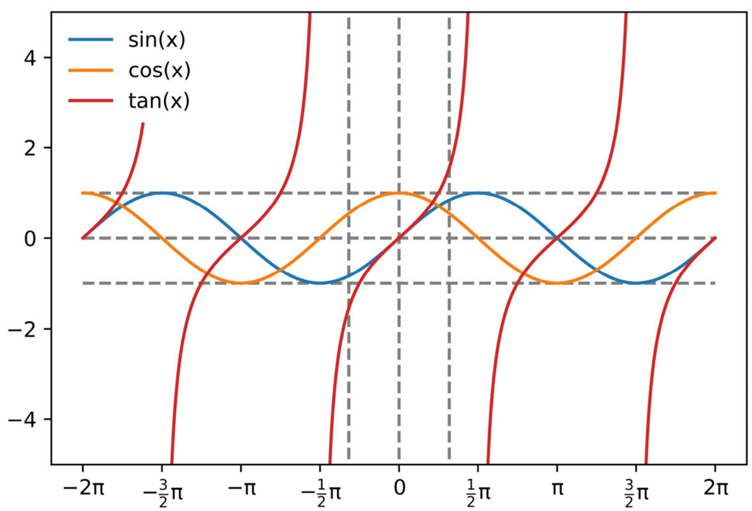
- $cos(x)$
  - Similarities
    - Even – symmetric about $x = 0$
    - $\cos(0) = 1$
  - Differences
    - Repeating wave
- $\cosh(x)$
  - Similarities
    - Even – symmetric about $x = 0$
    - $\cosh(0) = 1$
  - Differences
    - Exponential nature



- $\tan(x)$
  - Similarities
    - Odd – symmetric about $y = x$
    - $\tan(0) = 0$
  - Differences
    - Repeating asymptotes
- $\tanh(x)$
  - Similarities
    - Odd – symmetric about $y = x$
    - $\tanh(0) = 0$
  - Differences
    - No asymptotes

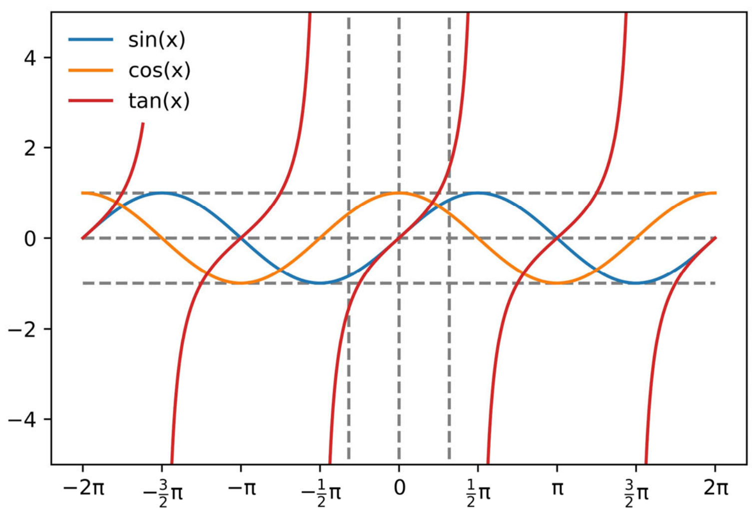
Side note, the past couple examples demonstrate 3 important principles. Let’s look at them quick.

1. Small denominators are powerful! When $\cos(x)$ is close to zero, $\tan(x)$ shoots off to either ∞ or $-\infty$.



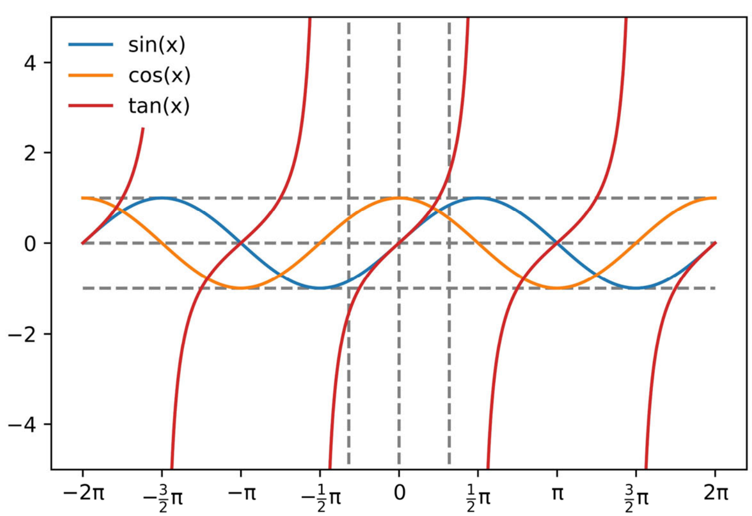
$$\tan(x) = \frac{\sin(x)}{\cos(x)}$$

2. You can't divide by zero. The answer just doesn't exist. You'll go from ∞ to $-\infty$ in an instant.



$$\tan(x) = \frac{\sin(x)}{\cos(x)}$$

3. The product of an odd function and an even function is another odd function. This property can be used to simplify complex problems.

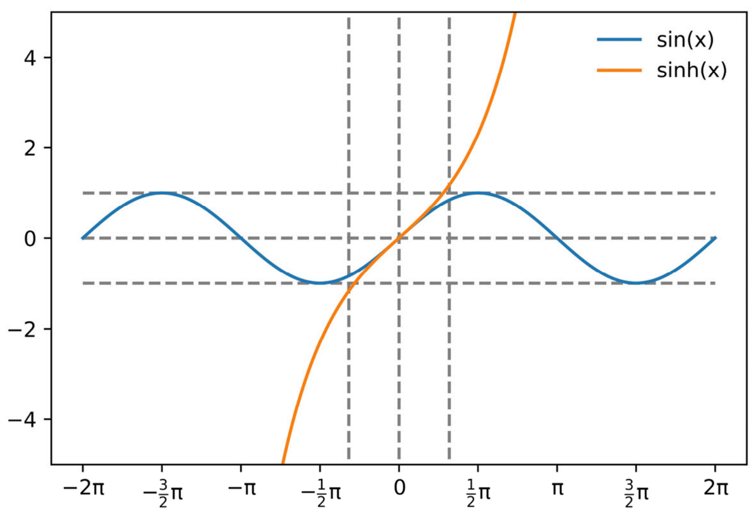


$$\tan(x) = \frac{\sin(x)}{\cos(x)}$$

- $\sin(x)$
  - Odd – symmetric about $y = x$
- $\cos(x)$
  - Even – symmetric about $x = 0$
- $\tan(x)$
  - Odd – symmetric about $y = x$

OK, back to hyperbolic functions. Let’s compare the definition of them versus their sinusoidal counterparts.

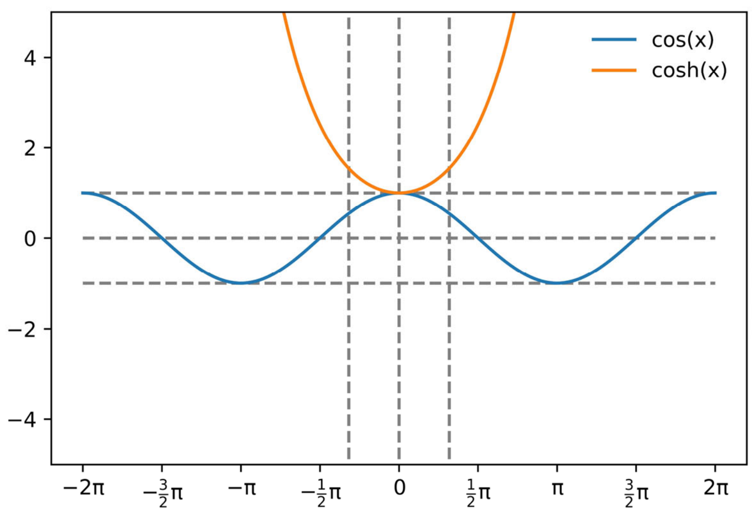
Now does the comparison of sinusoids and hyperbolics make more sense?



$$\sin(x) = \frac{e^{ix} - e^{-ix}}{2i}$$

$$\sinh(x) = \frac{e^{x} - e^{-x}}{2}$$

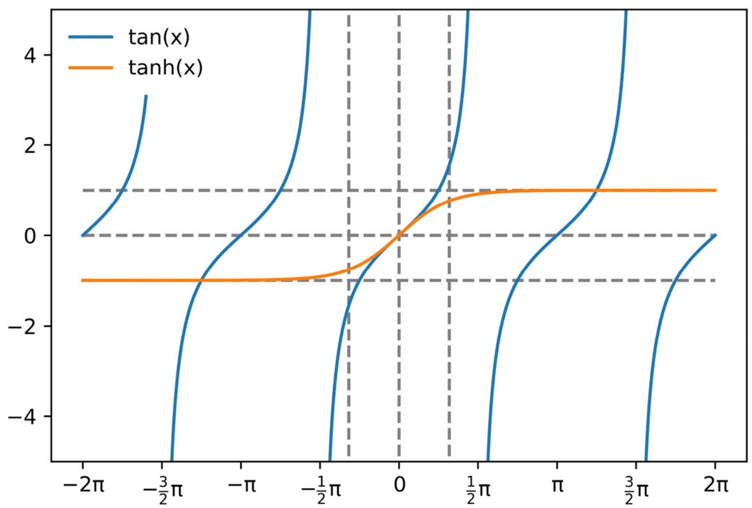
How about now?



$$\cos(x) = \frac{e^{ix} + e^{-ix}}{2i}$$

$$\cosh(x) = \frac{e^{x} + e^{-x}}{2}$$

Or now?

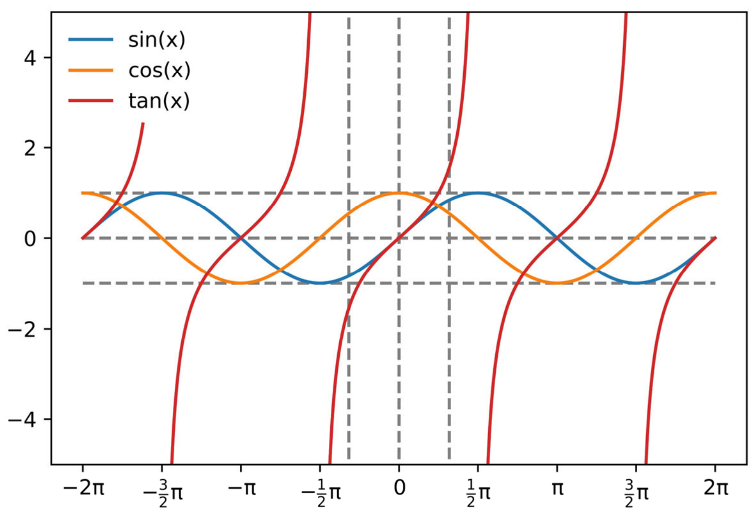


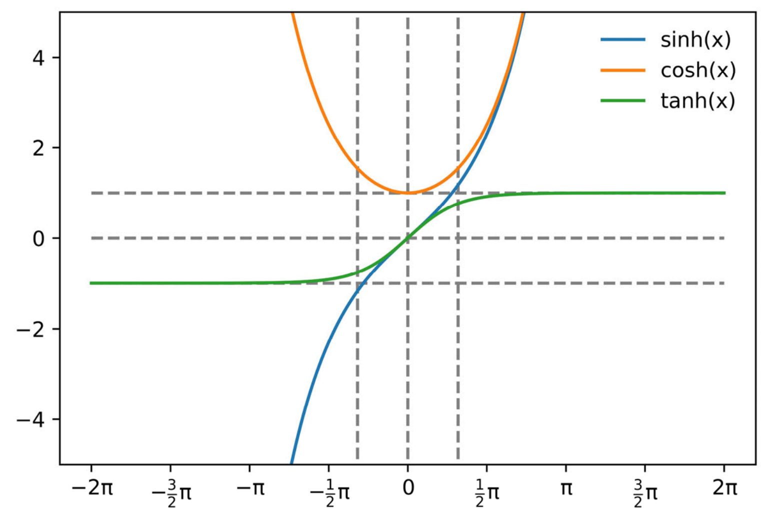
$$\tan(x) = \frac{\sin(x)}{\cos(x)} = \frac{e^{ix} - e^{-ix}}{e^{ix} + e^{-ix}}$$

$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)} = \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}}$$

But why aren’t there any asymptotes for $\mathbf{tanh}(x)$ when there are so many for $\mathbf{tan}(x)$?

Take a look for yourself. What made $\tan(x)$ have asymptotes? Why will that condition never be met for $\tanh(x)$?



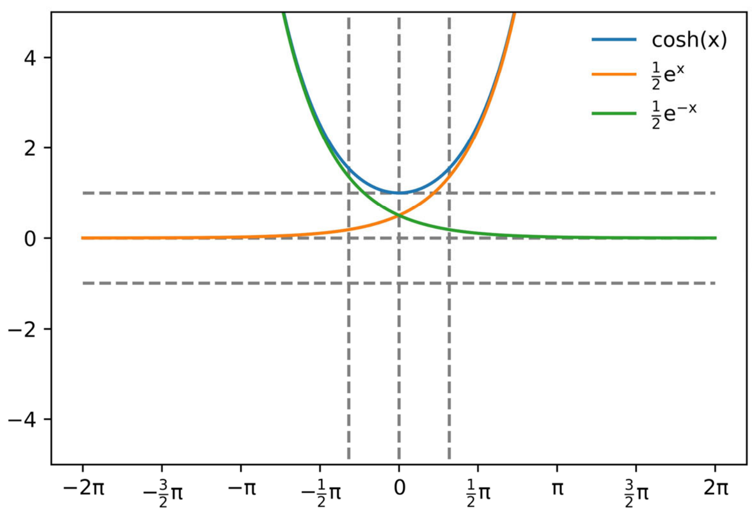


$$\tan(x) = \frac{\sin(x)}{\cos(x)}$$

$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)}$$

That wraps up the discussion, but you should stick around if you feel unsettled defining hyperbolics with exponential functions.

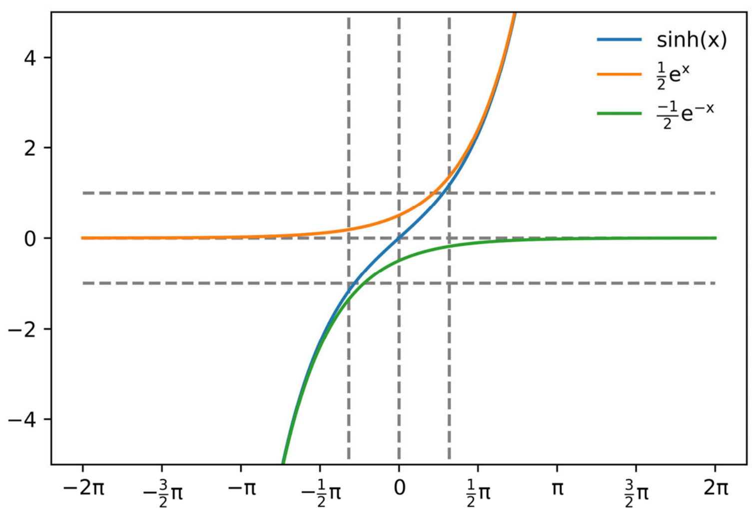
Take a look. Convince yourself that by combining the orange and green curves, you get the blue curve.



$$\cosh(x) = \frac{e^x + e^{-x}}{2}$$

$$\cosh(x) = \frac{1}{2}e^x + \frac{1}{2}e^{-x}$$

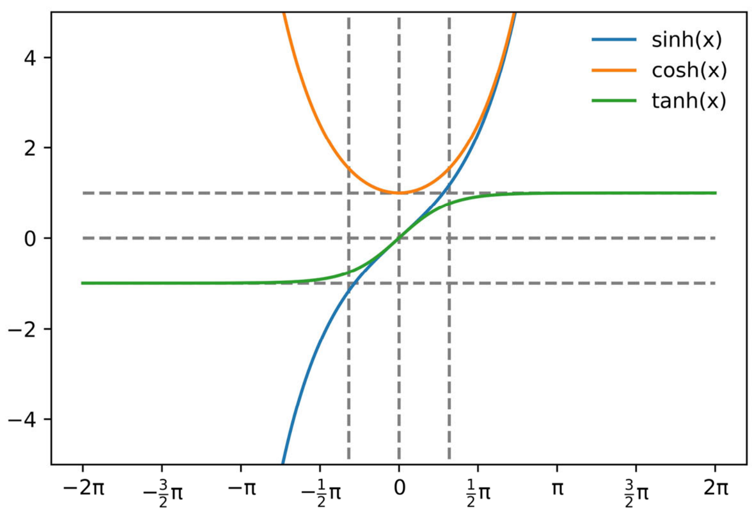
Do it again, but with these curves.



$$\sinh(x) = \frac{e^x - e^{-x}}{2}$$

$$\sinh(x) = \frac{1}{2}e^x + \frac{-1}{2}e^{-x}$$

You can now do the same with $\mathbf{tanh(x)}$.



$$\tanh(x) = \frac{\sinh(x)}{\cosh(x)}$$