Assignment #1: MATLAB Review

Due Date: Wednesday, January 19 at 5:00 PM

Type of Assignment: Individual

Submission Procedure: Please submit your assignment via Canvas. We request you send your files in a single zip archive.

Instructions: This assignment is to be completed entirely in MATLAB. Note that writing a "program" requires function and subroutine files. Organize these in the order they are used. All files (program, figures, description) for each problem need to be labeled so the instructor can find your work.

Complete the following problems.

1. If a projectile is fired into the air at an angle θ (in terms of degrees) and with an initial airspeed of V (in terms of m/s), the horizontal and vertical displacements of the projectile as a function of time will be given by the equations

$$
\begin{aligned}
x(t) &= V\cos(\theta)t \\
y(t) &= \frac{1}{2}gt^2 + V\sin(\theta)t
\end{aligned}
$$

   where g is the acceleration due to gravity in terms of m/s^2 (that is, $g = -9.81\ m/s^2$), x is the horizontal displacement of the projectile in terms of meters, y is the vertical displacement of the projectile in terms of meters, and t is the time in terms of seconds. Write a MATLAB script that prompts a user for the angle θ in degrees and initial airspeed V in m/s, and plots the horizontal and vertical displacements as functions of time. Use array arithmetic to perform the necessary computations, and be sure to include proper labels in your plots.

   Hint: For a termination time, choose the time at which the projectile returns to its original vertical position.

2. In analyzing experimental data, we often wish to fit a curve to data which may contain experimental error. The method of least-squares allows us to find the "best-fit" line $y = mx+b$ which approximates the data. Given a set of N data values (x_i, y_i), the method of least squares yields the values of m and b which minimize the total error

$$
E = \sum_{i=1}^{N}(y_i - (mx_i + b))^2.
$$

   The corresponding equations for m and b are given by

$$
m = \frac{AB - NC}{A^2 - ND} \qquad \text{and} \qquad b = \frac{AC - BD}{A^2 - ND}
$$

   where

$$
A = \sum_{i=1}^{N} x_i\,, \quad B = \sum_{i=1}^{N} y_i\,, \quad C = \sum_{i=1}^{N} x_i y_i\,, \quad D = \sum_{i=1}^{N} x_i^2\,.
$$

   Write a MATLAB function which calculates the best-fit line ($y = mx + b$) for a given set of experimental data. Your function should have two inputs (the vectors x and y) and two outputs (m and b). For example, if you ran your function using the inputs defined as follows:

x	-3	12.1	20	0	8	3.7	-5.6	0.5	5.8	10
y	-11.06	58.95	109.73	3.15	44.83	21.29	-27.29	5.11	34.01	43.25

then your function should return $m = 5.0860$ and $b = 2.0040$.

Test your code using the following scenario. A 1 : 32 scale model of a F-117 Nighthawk was placed in a wind tunnel. The following coefficient of lift, C_l, data was recorded for various angles of attack, α.

α	-5	-2	0	2	3	5	7	10	14
C_l	-0.008	-0.003	0.001	0.005	0.007	0.006	0.009	0.017	0.019

Determine the best-fit line for the data using your MATLAB function and plot the data and best-fit line on the same figure. The figure should have appropriate labels and a legend.

3. A lighter than air balloon will rise if it weighs less than the air it displaces. Conversely, the balloon will drop if it weighs more. If we define the total weight of the balloon (in kilograms[1]) as W_{total} and the weight of the displaced air as W_{air}, then the balloon will rise if $W_{air} > W_{total}$ and drop if $W_{air} < W_{total}$. The total weight of the ballon, W_{total}, is given by the sum

$$W_{total} = W_{gas} + W_{payload} + W_{balloon}$$

where W_{gas} is the weight of the gas in the balloon, $W_{payload}$ is the weight of the payload, and $W_{balloon}$ is the weight of the balloon when empty and unloaded. The weight of the gas in the balloon is given by the expression

$$W_{gas} = \frac{4\pi\rho_0 r^3}{3}\left(\frac{MW}{28.966}\right)$$

where r is the radius of the balloon (in meters), $\rho_0 = 1.225$ kg/m^3 is the density of air at sea level, and MW is the molecular weight of the gas being used. The weight of the air the balloon displaces, W_{air}, is given by the expression

$$W_{air} = \frac{4\pi\rho r^3}{3}$$

where ρ is the density of air (in terms of kg/m^3) at the current altitude of the balloon. To evaluate ρ, we use the following expression which gives the density in terms of the temperature T (in degrees Celcius) and pressure P (in kiloPascals) at the given altitude:

$$\rho(T, P) = \frac{P}{0.2869(T + 273.1)}$$

Finally, the temperature and pressure are given by the following equations where h is the given altitude in meters:

$$\begin{cases} T = 15.04 - 0.00649h & \\ P = 101.29\left(\frac{T+273.1}{288.08}\right)^{5.256} & 0 < h \leq 11000 \\ & \\ T = -56.46 & \\ P = 22.65e^{1.73-0.000157h} & 11000 < h \leq 25000 \\ & \\ T = -131.21 + 0.00299h & \\ P = 2.488\left(\frac{T+273.1}{216.6}\right)^{-11.388} & h > 25000 \end{cases}$$

[1]Technically, W_{total} refers to the total mass of the balloon, but the colloquial term weight is used in place of mass.

Based on the preceding discussion, complete the following tasks.

(a) Write a MATLAB function which calculates the total weight of a balloon given the radius of the balloon, the weight of the payload, the weight of the balloon when empty, and the molecular weight of the gas in the balloon. This function should have four inputs and one output.

(b) Write a MATLAB function which calculates the weight of the air a balloon displaces given the radius and altitude of the balloon. This function should have two inputs and one output. You may find it useful to write one or two additional MATLAB functions to find the temperature and pressure at a given altitude.

(c) Write a MATLAB function which computes the maximum attainable altitude of a balloon given its radius, payload weight, and empty weight as well as the molecular weight of the gas in the balloon. To find the maximum attainable altitude, the function should calculate the weight of the displaced air for an initial altitude of $h = 0$ m and then for altitude increments of $\Delta h = 10$ m (**Hint:** use a while loop). Once the total weight of the balloon exceeds the weight of the displaced air, the balloon has reached its maximum altitude. To calculate the weight of the balloon and displaced air, use the functions from parts (a) and (b). This function should have four inputs and one output.

(d) Use the function you created in part (c) to find the maximum attainable altitude of a helium-filled weather balloon with radius $r = 3.0$ m, payload weight 5 kg, and empty weight 0.6 kg. The molecular weight of helium is $MW = 4.02$.