Natural Frequency and Quality Factor of a Parallel RLC Circuit

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***Abstract*—This research investigates the analysis of the natural frequency and Quality Factor (Q-Factor) of a parallel RLC (Resistor, Inductor, and Capacitor) circuit, emphasizing their significance in Electrical Engineering applications. Engineers can understand the fundamental principles governing its frequency response characteristics by deriving and examining the circuit's transfer function. The natural frequency is indicative of the circuit's oscillatory behavior in the absence of external damping. Additionally, the Q-Factor, which quantifies the sharpness of the resonance and the circuit's energy dissipation, is further talked about. The findings found in this paper provide valuable insights for the design and optimization of parallel RLC circuits in various electronic and communication systems, which ensures stability and efficiency.**

***Keywords*—Transfer function, natural frequency ω_n, damped natural frequency ω_d, quality factor Q, frequency response, magnitude response, phase response.**

I. Introduction

A. *Transfer Functions*

Transfer functions play a critical role in characterizing the behavior of systems in the frequency domain when it comes to the analysis of electrical circuits. A transfer function, denoted by *H(s),* is a mathematical representation that defines the relationship between the input and output of a system through a ratio of polynomials in the Laplace transform variable *s* [2]. For parallel RLC circuits, transfer functions are essential in determining the natural frequency and Q-Factor. The natural frequency indicates the rate at which the circuit oscillates in the absence of damping, while the Q-Factor measures the sharpness of the resonance peak. By analyzing transfer functions, engineers can predict how a circuit will respond to different frequencies, optimize its design for specific applications, and ensure stability and efficiency.

B. *Fundamentals of RLC Circuits*

RLC circuits, composed of a resistor (R), inductor (L), and capacitor (C), are fundamental components in electrical engineering, serving as building blocks for a wide range of applications. These circuits can be configured in series or parallel, each arrangement exhibiting distinct characteristics. In a parallel RLC circuit, the resistor, inductor, and capacitor are connected in parallel across the same voltage source [3]. This configuration is particularly significant due to its ability to exhibit resonance, where the inductive and capacitive reactance cancel each other out, leading to a resistive impedance at the resonant frequency. Understanding the fundamental principles of RLC circuits is crucial for designing and optimizing electronic systems that rely on precise frequency control and signal processing.

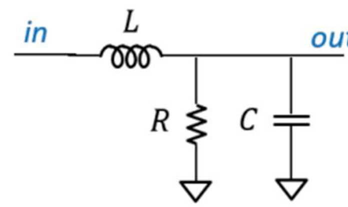

Figure 1: RLC low pass circuit. [1]

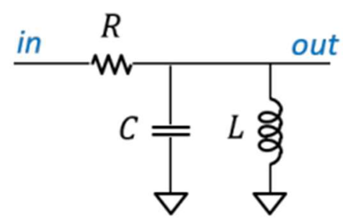

Figure 2: RLC band-pass circuit. [1]

C. *Importance of Natural Frequency and Q-Factor*

The natural frequency and Quality Factor (Q-factor) are critical parameters in the analysis and design of parallel RLC circuits. The natural frequency, often denoted as ω_0, represents the frequency at which the circuit naturally oscillates when not driven by an external source. It is a fundamental property that determines the circuit's response to alternating current (AC) signals. The Q-factor, on the other hand, quantifies the sharpness of the resonance peak and is defined as the ratio of the natural frequency to the bandwidth over which the circuit responds significantly. A high Q-factor indicates low energy loss relative to the stored energy, leading to a sharp resonance peak, which is desirable in applications like filters and oscillators where precise frequency selection is essential [4]. Conversely, a low Q-factor indicates higher energy dissipation and a broader resonance peak. Understanding and optimizing these parameters are vital for ensuring the efficient performance of RLC circuits in various electronic and communication systems. The analysis of the natural frequency and Q-factor provides insight into the energy dynamics and stability of the circuit, enabling the design of systems with enhanced performance and reliability.

D. Equations

The circuit shown in Figures 1 and 2 can be expressed by expressions (1) and (2) respectively.

$$H_{Fig.1}(s) = \frac{1}{s^2LC + s\frac{L}{R} + 1} \tag{1}$$

$$H_{Fig.2}(s) = \frac{s\frac{L}{R}}{s^2LC + s\frac{L}{R} + 1} \tag{2}$$

Alternatively, the transfer functions can be reformulated using the undamped natural frequency ω_0 and their quality factor Q; equations (3) and (4) present these expressions. The natural frequency, also known as the resonant frequency, is measured in radians per second (rad/sec), while the Q-Factor is unitless.

$$H_{Fig.1}(s) = \frac{{\omega_n}^2}{s^2 + \frac{\omega_n}{Q}s + {\omega_n}^2} \tag{3}$$

$$H_{Fig.2}(s) = \frac{\frac{{\omega_n}^2}{Q}s}{s^2 + \frac{\omega_n}{Q}s + {\omega_n}^2} \tag{4}$$

$$where\ \omega_n = \frac{1}{\sqrt{LC}}\left[\frac{rad}{sec}\right]\ and\ Q = \frac{R}{\sqrt{LC}} \tag{5}$$

Frequency response is another method to characterize the behavior of BIBO (bounded-input/bounded-output) stable Linear Time-Invariant (LTI) circuits. The frequency response is obtained by evaluating the transfer function *H(s)* at $s = j\omega$, where j represents $\sqrt{-1}$. Equations (6) and (7) define the frequency response for the circuits in Figures 1 and 2.

$$H_{Fig.1}(j\omega) = \frac{{\omega_n}^2}{({\omega_n}^2 - \omega^2) + j\frac{\omega_n}{Q}\omega} \tag{6}$$

$$H_{Fig.2}(j\omega) = \frac{j\frac{\omega_n}{Q}\ \omega}{({\omega_n}^2 - \omega^2) + j\frac{\omega_n}{Q}\omega} \tag{7}$$

The quality factor of a 2nd order band pass filter can be calculated from equation (8), where ω_L and ω_H are the frequencies for which $\angle H(j\omega)$ is -45 degrees and 45 degrees.

$$Q_{BP} = \frac{\omega_n}{\omega_H - \omega_L} = \frac{\sqrt{\omega_L\omega_H}}{\omega_H - \omega_L} \tag{8}$$

E. Materials Used

This experiment utilizes three resistors (1.5 kΩ, 510Ω, and 390Ω), a capacitor (10nF), an inductor (10mH), a Function Generator (Agilent 33120A), a Two-Input Oscilloscope, some Banana-to-Grabbers, and a Breadboard.

II. Data

A. RLC Low Pass Filter

The first step was to build the circuit in Figure 1 using the 10nF capacitor, 10mH inductor, and the 1.5 kΩ resistor.

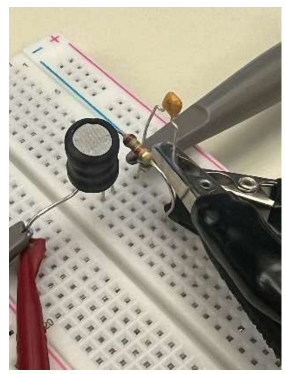

Figure 3: Circuit representation of an RLC Low Pass Filter.

Above is an image of a real-life circuit representation of Figure 1. A 0-to-1V step input was applied to this circuit and the result was captured in Figures 4, 5, and 6.

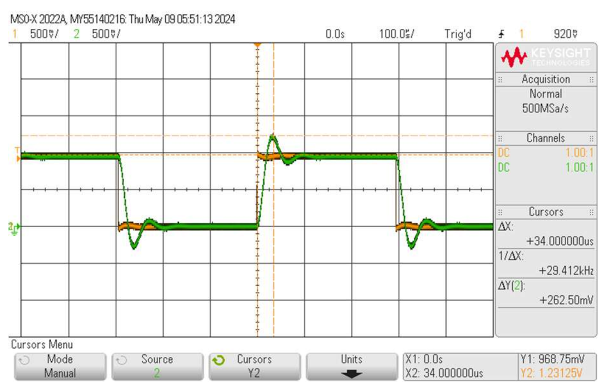

Figure 4: Underdamped and Overshot.

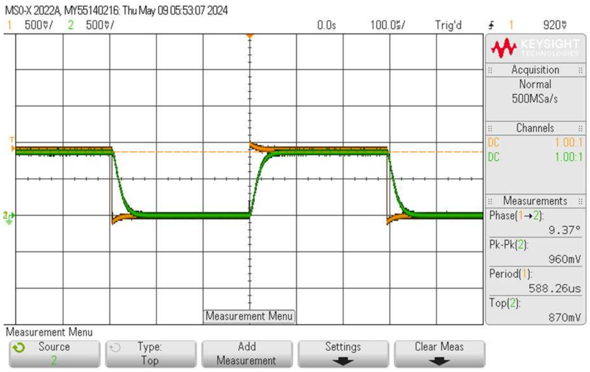

Figure 5: Critically Damped.

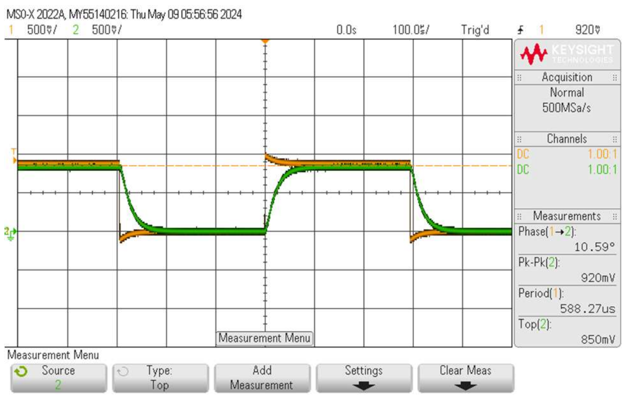

Figure 6: Overdamped.

Figure 4 used a 1.5 kΩ resistor, Figure 5 used a 510Ω resistor, and Figure 6 used a 390Ω resistor. The 1.5kΩ resistor caused an under-damp and overshot because the quality factor was above 1, figure 5 had a quality factor of 1, which resulted in a critically damped output, and figure 6 had a quality factor of less than one, which resulted in overdamping.

With the 1.5 kΩ resistor, there was an overshot of 0.262 Volts, which is about an overshot of 26.2%. There was also a 32 µsec time to the first peak, which resulted in a natural frequency of 100k rad/sec.

For this next part of the lab, the input signal was changed from a square wave to a sinusoidal wave with a magnitude of 1 Volt. When varying the frequency of the signal generator to get an input-output phase difference of 90 degrees, the table below shows the frequency, the peak-to-peak voltage of the output signal, and the Q of the network for the 390Ω, 510Ω, and the 1.5 kΩ resistors. Figures 7, 8, and 9 depict these scope captures.

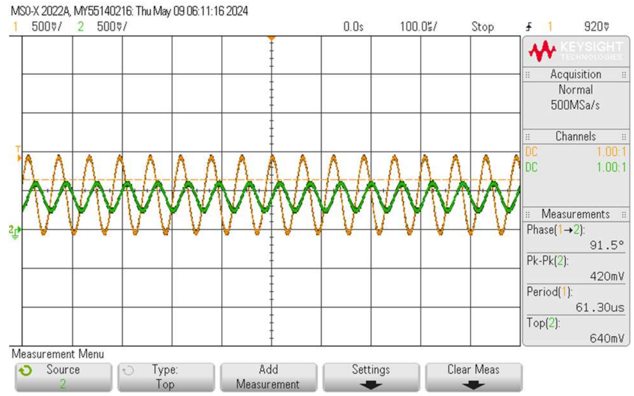

Figure 7: 390Ω Resistor Plot.

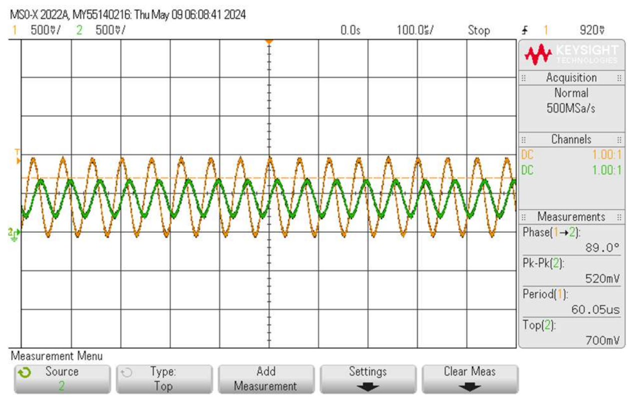

Figure 8: 510Ω Resistor Plot.

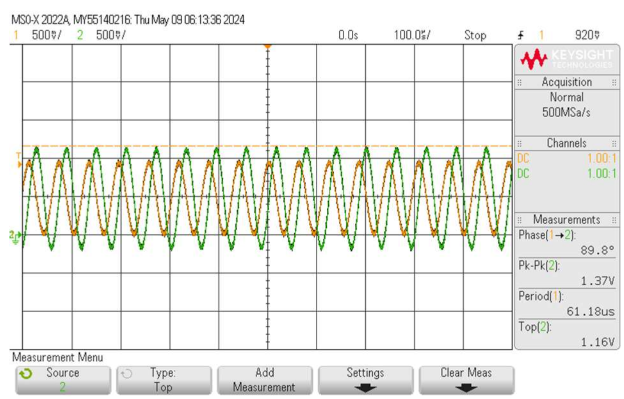

Figure 9: 1.5kΩ Resistor Plot.

Table I. Figure 1 Circuit Data Table

Resistance	Frequency	V_{pp}	Q
390Ω	16.3 kHz	0.42 V	0.381
510Ω	16.7 kHz	0.52 V	0.501
1.5kΩ	16.3 kHz	1.37 V	1.493

As expected, as there was an increase in resistance, so did the V_{pp} and Q of the network. The calculations for Q are based on equation (5).

B. RLC High Pass Filter

For this next part of the lab, figure 2 was assembled using a 10nF capacitor, 10mH inductor, and resistor that was varied from 390Ω to 510Ω to 1.5 kΩ. A square wave was then applied to the circuit with a magnitude of 1V. For each resistance, the phase difference was varied from -45 degrees to 45 degrees in increments of 45 degrees.

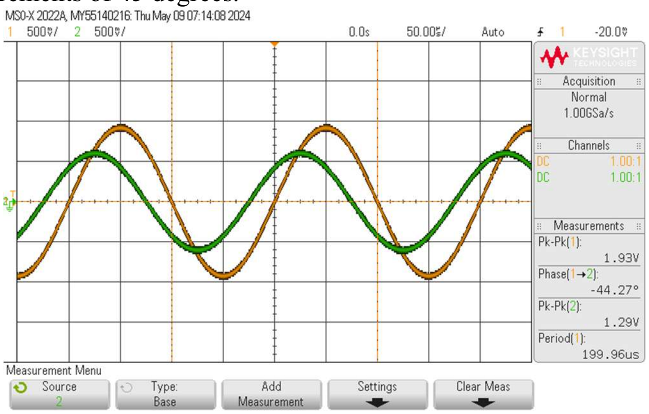

Figure 10:390Ω with a -45-degree phase difference.

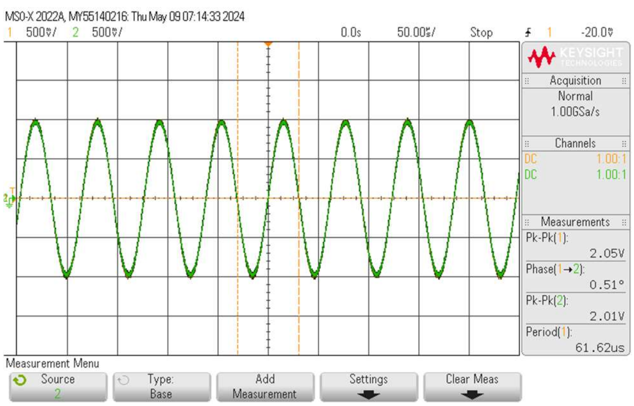

Figure 11: 390Ω with a 0-degree phase difference.

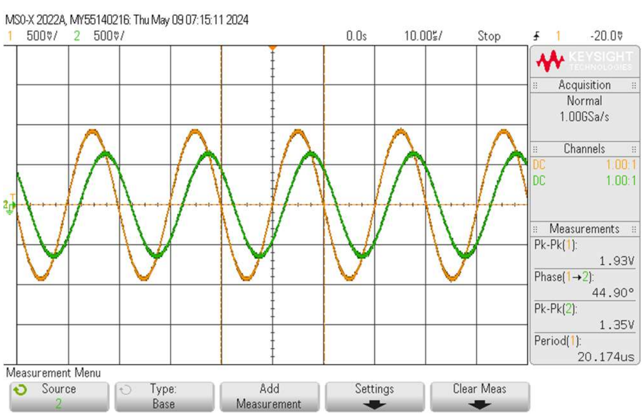

Figure 12: 390Ω with a 45-degree phase difference.

The first data collection was used with a resistance of 390Ω. For Figure 10, the frequency was 5kHz, for Figure 11, the frequency was 16.2kHz, and for Figure 12, the frequency was 49.6kHz. This resulted in a Q-Factor of 0.381 based on equation (8).

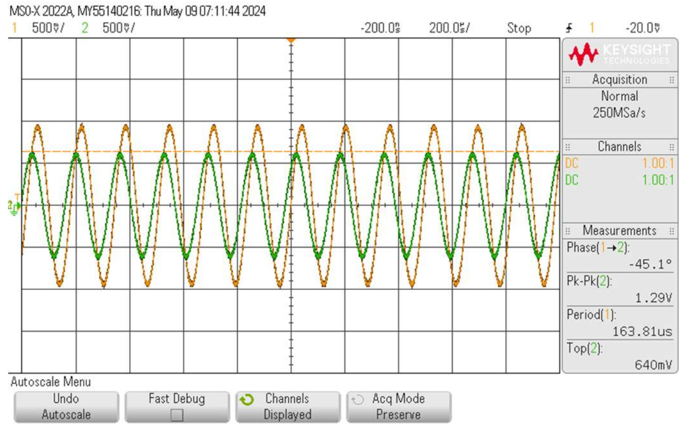

Figure 13: 510Ω with a -45-degree phase difference.

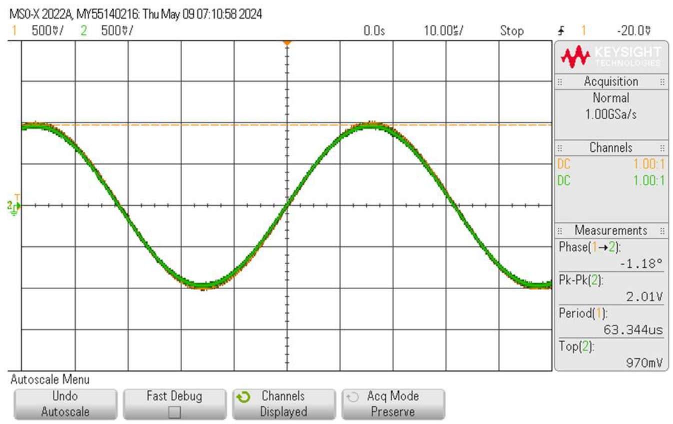

Figure 14: 510Ω with a 0-degree phase difference.

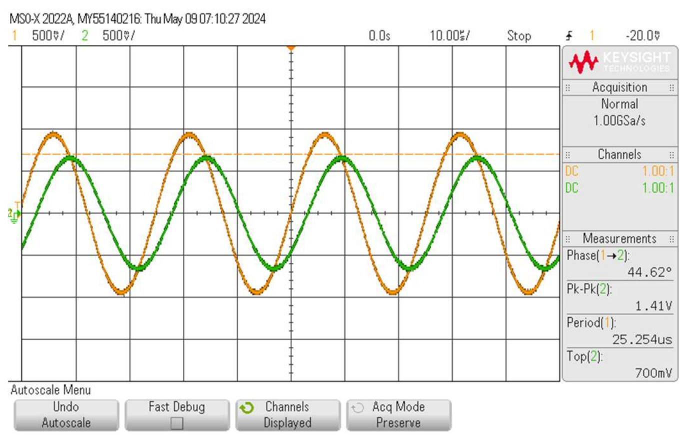

Figure 15: 510Ω with a 45-degree phase difference.

The second data collection was used with a resistance of 510Ω. For Figure 13, the frequency was 6.1kHz, for Figure 14, the frequency was 15.8kHz, and for Figure 15, the frequency was 39.6kHz. This resulted in a Q-Factor of 0.501 based on equation (8).

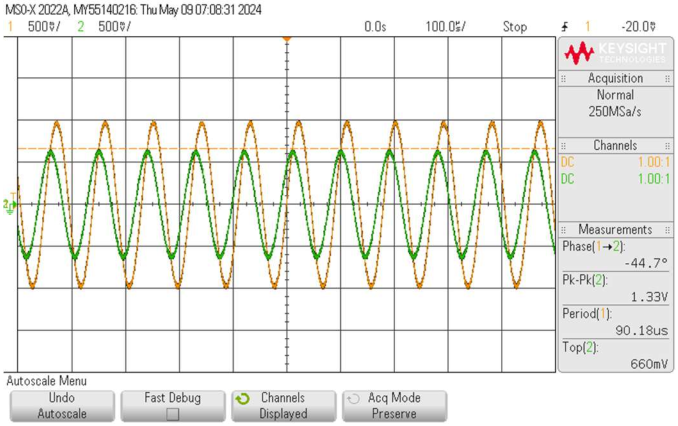

Figure 16: 1.5 kΩ with a -45-degree phase difference.

Figure 17: 1.5 kΩ with a 0-degree phase difference.

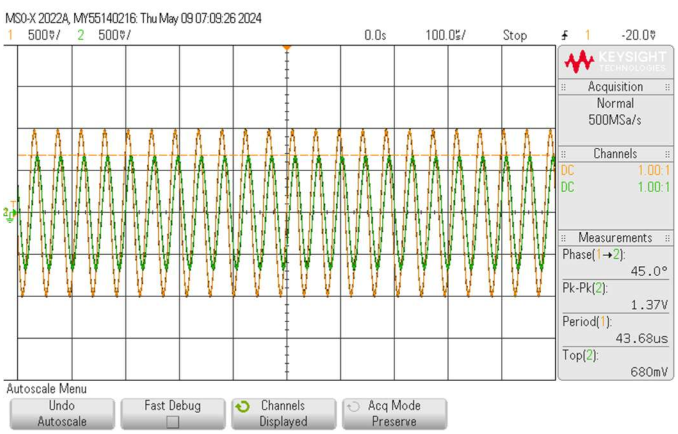

Figure 18: 1.5 kΩ with a 45-degree phase difference.

The third data collection was used with a resistance of 1.5kΩ. For Figure 16, the frequency was 11kHz, for Figure 17, the frequency was 16.1kHz, and for Figure 18, the frequency was 23kHz. This resulted in a Q-Factor of 1.493 based on equation (8).

III. Conclusion

The experimental data aligns with the theoretical predictions regarding the impact of RLC circuits on input and output signals, providing valuable insights into the operation of low-pass and band-pass filters. While there remains a need to further explore the theoretical foundations and applications of these circuits in practical scenarios, the accurate collection of experimental data has enhanced the understanding and utilities of these filters. This study serves as a foundational introduction to advanced signal processing, with the anticipation that comprehension will deepen with continued theoretical application and practice.

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