Extensions to Q-Learning

Reinforcement Learning
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Last updated: February 22, 2025

Agenda

> Double Q-Learning

> Prioritized Experience Replay

> Dueling Deep Q-Network

Double Q-Learning

Main Idea

DQN requires target estimates of this form:

$$Y_t^{\mathrm{Q}} \equiv R_{t+1} + \gamma \max_a Q(S_{t+1}, a; \boldsymbol{\theta}_t)$$

The max() operation is used to estimate value

There may be noise in the system

Tends to produce a bias: overestimating value of Q

Paper: Deep Reinforcement Learning with Double Q-learning

Hado van Hasselt, Arthur Guez, David Silver. Google DeepMind

Main Idea, contd.

Double DQN

The idea of Double Q-learning is to reduce overestimations by decomposing the max operation in the target into action selection and action evaluation. Although not fully decoupled, the target network in the DQN architecture provides a natural candidate for the second value function, without having to introduce additional networks. We therefore propose to evaluate the greedy policy according to the online network, but using the target network to estimate its value. In reference to both Double Q-learning and DQN, we refer to the resulting algorithm as Double DQN. Its update is the same as for DQN, but replacing the target Y_t^{DQN} with

$$Y_t^{\mathrm{DoubleQ}} \equiv R_{t+1} + \gamma Q(S_{t+1}, \underset{a}{\mathrm{argmax}}\, Q(S_{t+1}, a; \boldsymbol{\theta}_t); \boldsymbol{\theta}'_t)$$

Decompose max() operation into action selection, action evaluation

Notice there are two Qs

- One determines greedy policy using online network $\boldsymbol{\theta}_t$
- Another fairly evaluates the policy using the target network $\boldsymbol{\theta}'_t$

Double Q-Learning Algorithm

Algorithm 1 : Double Q-learning (Hasselt et al., 2015)

Initialize primary network Q_θ, target network $Q_{\theta'}$, replay buffer $\mathcal{D}$, $\tau << 1$
for each iteration **do**
  for each environment step **do**
    Observe state s_t and select $a_t \sim \pi(a_t, s_t)$
    Execute a_t and observe next state s_{t+1} and reward $r_t = R(s_t, a_t)$
    Store (s_t, a_t, r_t, s_{t+1}) in replay buffer $\mathcal{D}$
  for each update step **do**
    sample $e_t = (s_t, a_t, r_t, s_{t+1}) \sim \mathcal{D}$
    Compute target Q value:
      $Q^*(s_t, a_t) \approx r_t + \gamma\ Q_\theta(s_{t+1}, argmax_{a'} Q_{\theta'}(s_{t+1}, a'))$
    Perform gradient descent step on $(Q^*(s_t, a_t) - Q_\theta(s_t, a_t))^2$
    Update target network parameters:
      $\theta' \leftarrow \tau * \theta + (1 - \tau) * \theta'$

Bias Estimates

Theorem 1. *Consider a state s in which all the true optimal action values are equal at $Q_*(s,a) = V_*(s)$ for some $V_*(s)$. Let Q_t be arbitrary value estimates that are on the whole unbiased in the sense that $\sum_a (Q_t(s,a) - V_*(s)) = 0$, but that are not all correct, such that $\frac{1}{m}\sum_a (Q_t(s,a) - V_*(s))^2 = C$ for some $C > 0$, where $m \geq 2$ is the number of actions in s. Under these conditions, $\max_a Q_t(s,a) \geq V_*(s) + \sqrt{\frac{C}{m-1}}$. This lower bound is tight. Under the same conditions, the lower bound on the absolute error of the Double Q-learning estimate is zero. (Proof in appendix.)*

=> For larger #actions *m*, the lower bound decreases

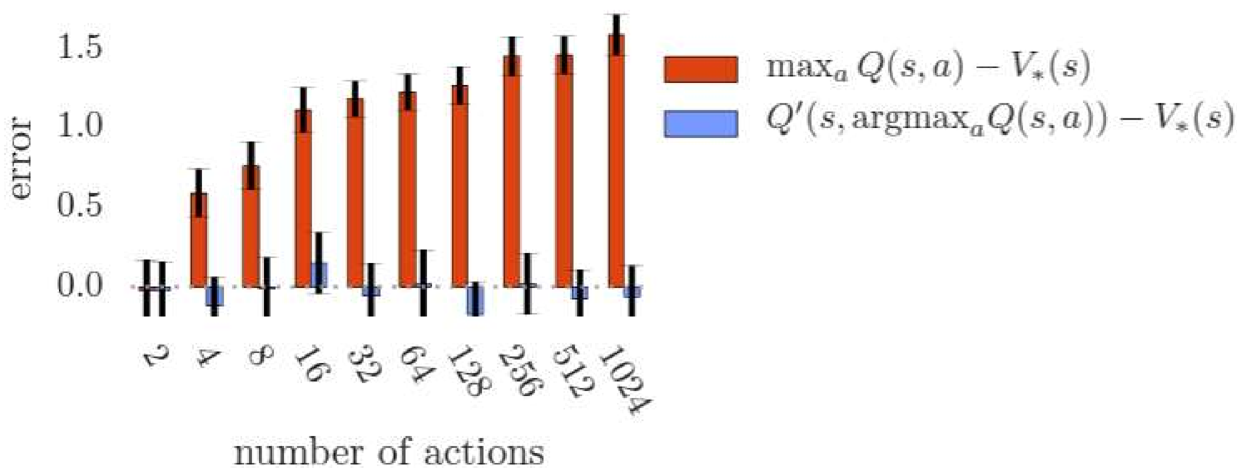
Hence, overoptimism increases for larger *m*

Bias Estimates, contd.

Red bars quantify this overoptimism

The gap increases for increasing #actions

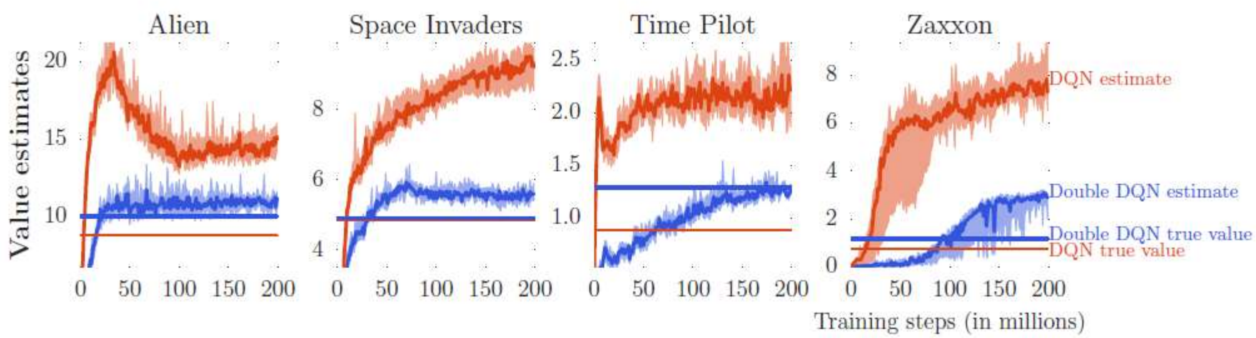
However, for Double Q-Learning, bias remains small



Bias Estimates - Atari

Red = DQN   Blue = Double DQN

DQN is overestimating values more than Double DQN (compare the curves)



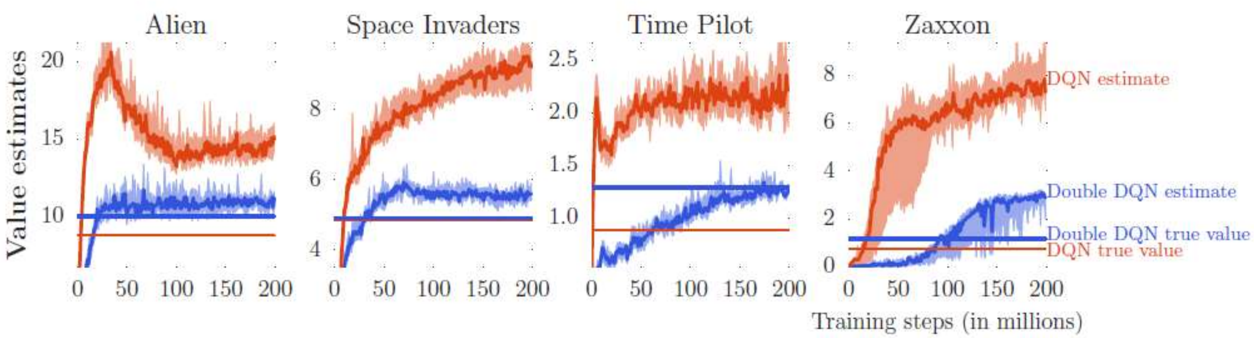
Bias Estimates - Atari

Red = DQN   Blue = Double DQN

DQN is overestimating values more than Double DQN (compare the curves)

Horizontal lines are **actual discounted value** of best learned policy (unbiased)
Computed by running learned policy and averaging actual discounted returns

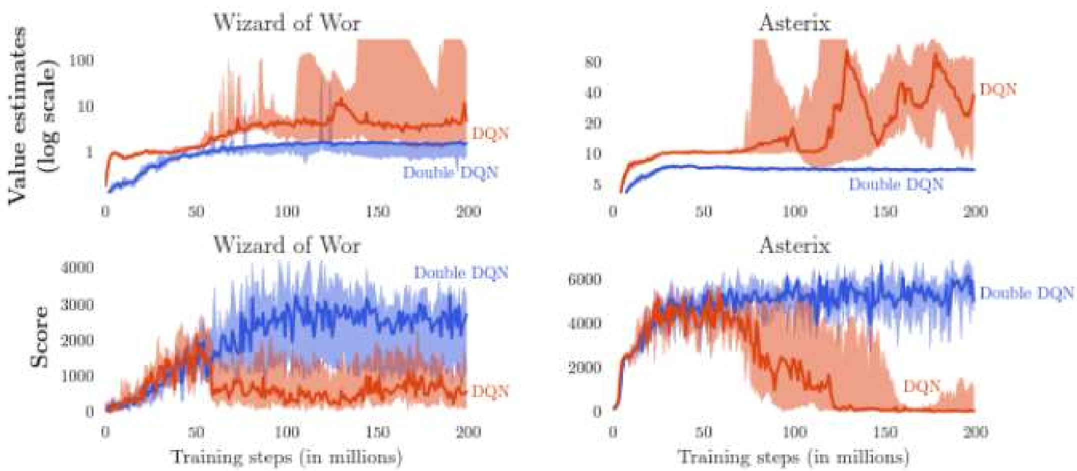
Double DQN best policy (blue horiz. line) outperforms DQN best policy (red horiz. line)



Bias Estimates – Atari, Extreme Cases

In these cases, overoptimism in DQN is extreme (red curve)

Where red separates from blue, bias increases



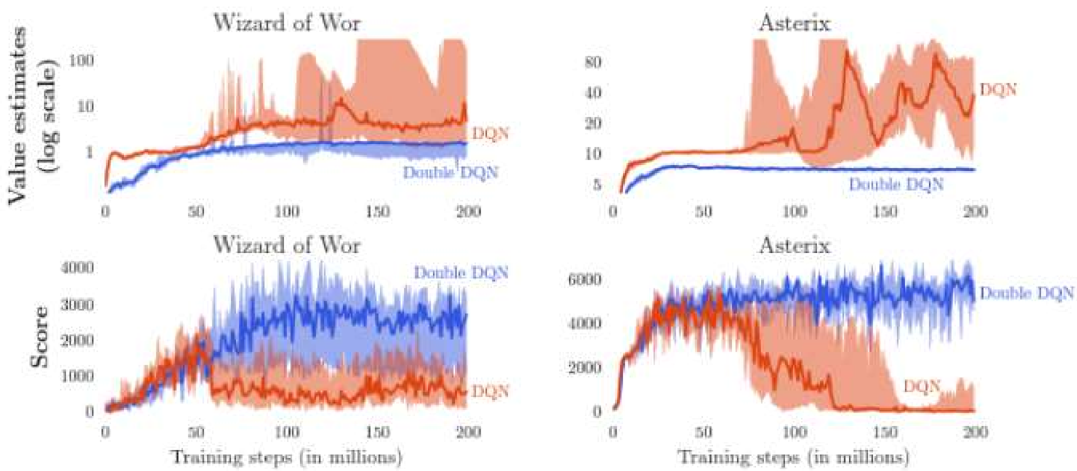
Bias Estimates – Atari, Extreme Cases

In these cases, overoptimism in DQN is extreme (red curve)

Where red separates from blue, bias increases

This has detrimental effect on score (Double DQN outperforms)

Double DQN is also more stable



Double DQN - Implementation Example

RL Applied to Sepsis Case – DQN Agent

```python
class DQNAgent:
    def __init__(self, state_size, action_size):
        self.state_size = state_size
        self.action_size = action_size
        self.memory = deque(maxlen=2000)
        self.gamma = 0.95    # discount rate
        self.epsilon = 1.0  # exploration rate
        self.epsilon_min = 0.01
        self.epsilon_decay = 0.99
        self.learning_rate = 0.001
        self.model = self._build_model()
        self.target_model = self._build_model()
```

Instantiate online and target models

Source: https://github.com/keon/deep-q-learning

RL Applied to Sepsis Case – Act & Evaluate

```
def act(self, state):
    if np.random.rand() <= self.epsilon:
        return random.randrange(self.action_size)
    act_values = self.model.predict(state)
    return np.argmax(act_values[0])  # returns action
```

Action selection using online network

```
def replay(self, batch_size):
    minibatch = random.sample(self.memory, batch_size)
    for state, action, reward, next_state, done in minibatch:
        target = self.model.predict(state)
        if done:
            target[0][action] = reward
        else:
            # a = self.model.predict(next_state)[0]
            t = self.target_model.predict(next_state)[0]
            target[0][action] = reward + self.gamma * np.amax(t)
```

Evaluation using target network

Source: https://github.com/keon/deep-q-learning

Findings

The bias resulting from max() in DQN can be large, **and it matters**

Double DQN can be more stable and reliable than DQN

Double DQN can find better policies than DQN

The code change to implement Double DQN is small

Prioritized Experience Replay

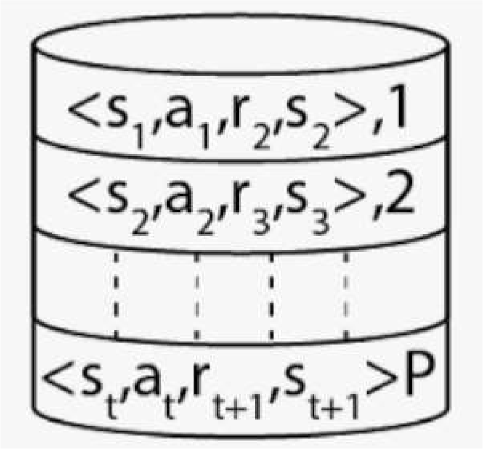
Buffer

DQN uses a buffer to store, retrieve, and replay past transitions (s_t, a_t, r_{t+1}, s_{t+1})

This makes DQN more efficient than Q-Learning

The transitions are sampled uniformly at random

Prioritized Experience Replay (PER)
tries to be more strategic about sampling



Paper: Prioritized Experience Replay. Schaul et. al.

Priority

PER uses absolute TD error $|\delta_i|$ as the priority

Could select transition with max abs TD error, but greedy approach may backfire

Instead, paper uses stochastic prioritization with two methods:

1) Proportional

2) Rank-based

Proportional Priority

Define proportional priority where $p_i = |\delta_i| + \epsilon$

Epsilon prevents transitions from not being visited when error is zero

Define probability of sampling transition

$$P(i) = \frac{p_i^\alpha}{\sum_k p_k^\alpha}$$

Alpha controls amount of prioritization; alpha = 0 is uniform case.

Rank-Based Priority

Rank is used for robustness

Define probability of sampling transition

$$p_i = \frac{1}{\text{rank}(i)}$$

Ranks are based on absolute TD error

Importance Sampling

PER induces bias by changing probability of transition selection

Can adjust for bias with importance sampling

$$w_i = \left(\frac{1}{N} \cdot \frac{1}{P(i)}\right)^{\beta}$$

Case where beta = 1 fully compensates for non-uniform probabilities

For Q-learning updates, use $|\delta_i|$ weighted by w_i

For stability, scale weights by 1 / max w_i

Algorithm

Algorithm 1 Double DQN with proportional prioritization

Input: minibatch k, step-size η, replay period K and size N, exponents α and β, budget T.
Initialize replay memory $\mathcal{H} = \emptyset$, $\Delta = 0$, $p_1 = 1$
Observe S_0 and choose $A_0 \sim \pi_\theta(S_0)$
for $t = 1$ **to** T **do**
  Observe S_t, R_t, γ_t
  Store transition $(S_{t-1}, A_{t-1}, R_t, \gamma_t, S_t)$ in $\mathcal{H}$ with maximal priority $p_t = \max_{i<t} p_i$
  if $t \equiv 0 \mod K$ **then**
    for $j = 1$ **to** k **do**
      Sample transition $j \sim P(j) = p_j^\alpha / \sum_i p_i^\alpha$
      Compute importance-sampling weight $w_j = (N \cdot P(j))^{-\beta} / \max_i w_i$
      Compute TD-error $\delta_j = R_j + \gamma_j Q_{\text{target}}(S_j, \arg\max_a Q(S_j, a)) - Q(S_{j-1}, A_{j-1})$
      Update transition priority $p_j \leftarrow |\delta_j|$
      Accumulate weight-change $\Delta \leftarrow \Delta + w_j \cdot \delta_j \cdot \nabla_\theta Q(S_{j-1}, A_{j-1})$
    end for
    Update weights $\theta \leftarrow \theta + \eta \cdot \Delta$, reset $\Delta = 0$
    From time to time copy weights into target network $\theta_{\text{target}} \leftarrow \theta$
  end if
  Choose action $A_t \sim \pi_\theta(S_t)$
end for

Results: Summary of Normalized Scores

	DQN		Double DQN (tuned)		
	baseline	rank-based	baseline	rank-based	proportional
Median	48%	106%	111%	113%	128%
Mean	122%	355%	418%	454%	551%
> baseline	–	41	–	38	42
> human	15	25	30	33	33
# games	49	49	57	57	57

DQN with rank-based priority results in higher scores for 41 out of 49 games

Double DQN is similarly helped by PER

Dueling Deep Q-Network

Dueling Networks

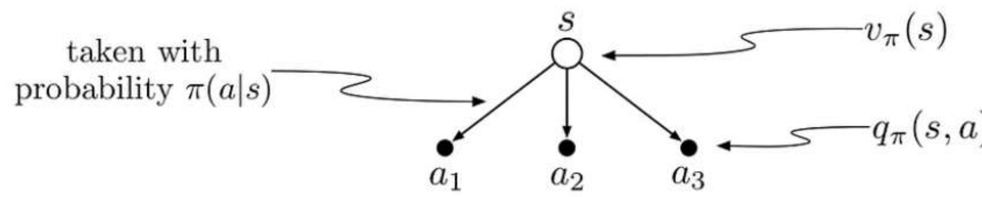
Deep RL before this paper used conventional architectures (CNN, LSTM)

Focus here is new architecture better suited to model-free RL

Dueling architecture separates state values and action advantages

Paper: *Dueling Network Architectures for Deep Reinforcement Learning.* Wang et. al.

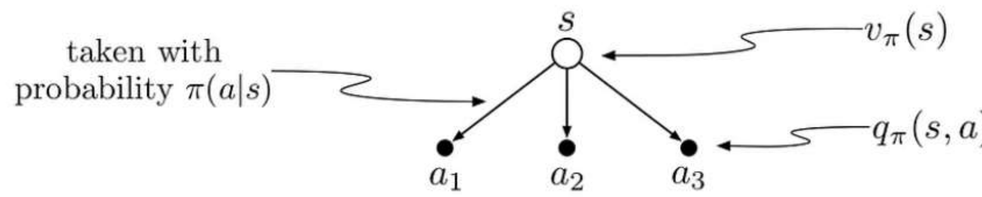
Value and Advantage Function Definitions



Relationship of value functions:

$$
\begin{aligned}
Q^\pi(s,a) &= \mathbb{E}\left[R_t \,|\, s_t = s, a_t = a, \pi \right] \\
V^\pi(s) &= \mathbb{E}_{a \sim \pi(s)} \left[Q^\pi(s,a) \right].
\end{aligned}
$$

Value and Advantage Function Definitions



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V^\pi(s) &= \mathbb{E}_{a\sim\pi(s)}\left[Q^\pi(s,a)\right].
\end{aligned}
$$

The advantage function isolates effect of action taken

$$A^\pi(s,a) = Q^\pi(s,a) - V^\pi(s)$$

V(*s*) measures how good it is to be in state *s*

A(s,a) measures relative importance of each action

Sometimes, the action doesn't matter

Dueling Architecture

Separate streams for value and advantage functions

Common convolution feature

Top figure is Q-network

Bottom figure is dueling Q-network

Value and advantage streams are combined

For each network, outputs are Q(s,a)

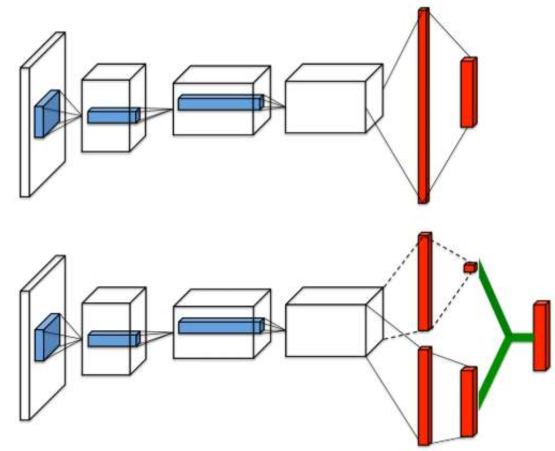
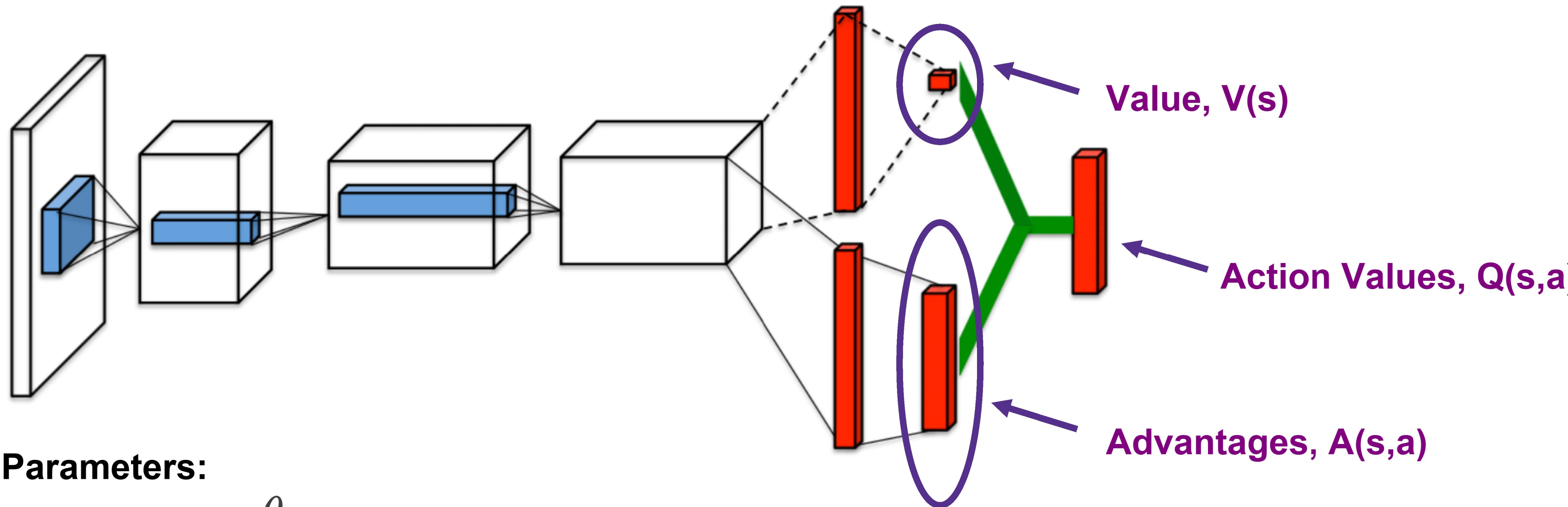


Figure 1. A popular single stream Q-network (**top**) and the dueling Q-network (**bottom**). The dueling network has two streams to separately estimate (scalar) state-value and the advantages for each action; the green output module implements equation (9) to combine them. Both networks output Q-values for each action.

Dueling Architecture, contd.



Parameters:

Shared weights θ

Value stream weights β

Advantage stream weights α

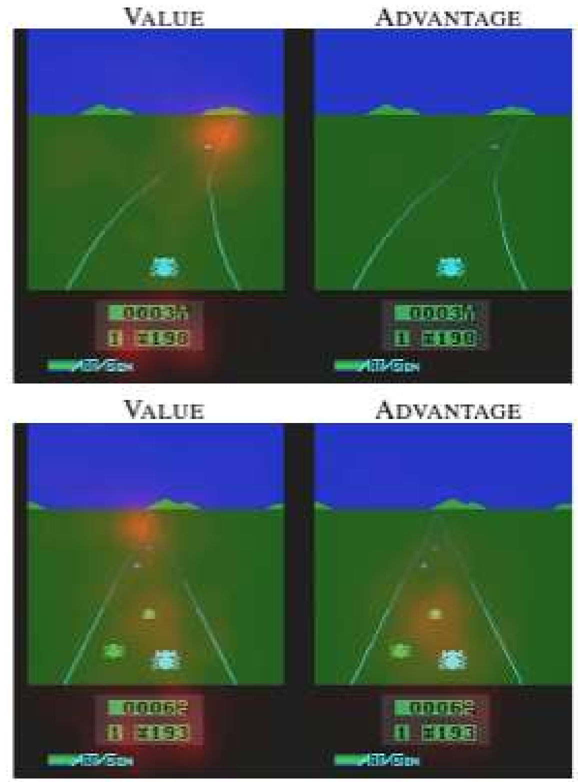
Value and Advantage Function Saliency

Figure shows saliency map for two time steps

Value stream pays attention to horizon and score

Advantage stream: when no cars are on road, action doesn't matter (no attention paid)

When cars are on road, advantage stream pays attention to car in front (bottom right)



Identifiability

Given Q = V + A, we cannot recover V and A uniquely
(adding any *c* to V, and subtracting *c* from A, leaves Q unchanged)

Add a constraint: subtract the average advantage from A:

$$Q(s,a;\theta,\alpha,\beta) = V(s;\theta,\beta) + \left(A(s,a;\theta,\alpha) - \frac{1}{|\mathcal{A}|} \sum_{a'} A(s,a';\theta,\alpha) \right)$$

For actions with above average advantage, the second term in (*) will be positive
For action with greatest advantage, (*) term will be largest across all actions

Review of Dueling Architecture Code

We can see an implementation of Dueling Q-Network here:

Paper: *Reinforcement Learning for optimal sepsis treatment policies* (2017)

Authors: Raghu, Komorowski, Ahmed, Celi, Szolovits, Ghassemi

GitHub repo:
https://github.com/aniruddhraghu/sepsisrl/blob/master/continuous/q_network.ipynb

See section:

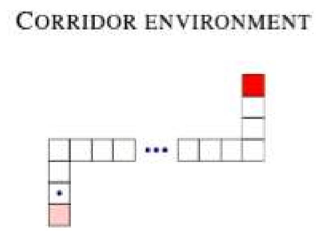
advantage and value streams

Implementation

Our two streams (V, A) for Q are part of the model architecture

Training step runs the same as a standard Q-network: backpropagation

Evaluation



Sets up simple environment called *corridor*.
Redness of state signifies reward

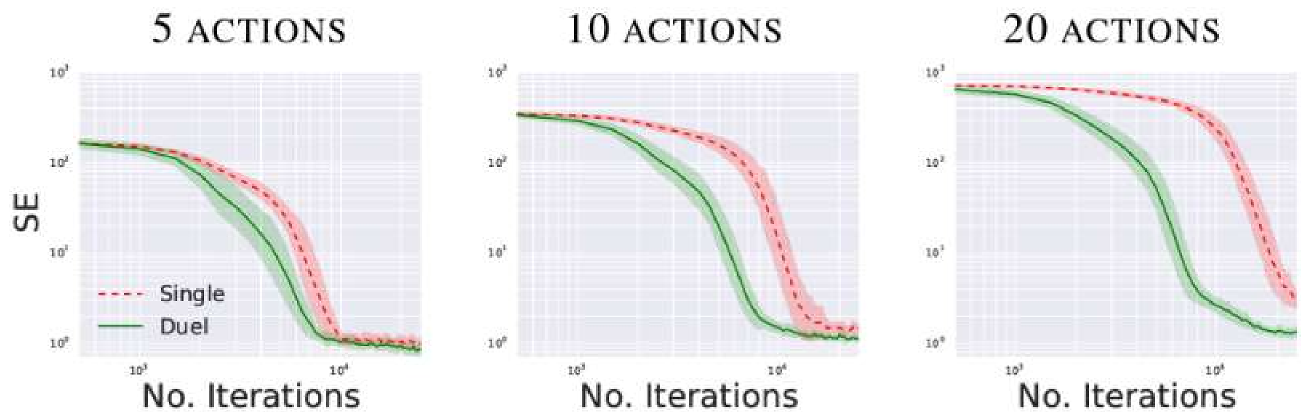
Can move up, down, left, right, or no move.

ε-greedy policy, measure performance by squared error (SE) vs. true action values

Single and Duel networks use MLP with three layers

For duel network, after first hidden layer, network branches off to two streams

Evaluation, contd.



Dueling network converges faster than Single

More pronounced as |A| increases

As |A| increases, SE is lower for Dueling network. It performs better.

Summary

We studied a novel architecture for Q function

Advantage function measures incremental importance over V(s):

Decomposes Q into value V and action advantage A streams

$$Q^\pi(s,a) = V^\pi(s) + A^\pi(s,a)$$

(we subtract average advantage for identifiability)

Advantage function measures incremental importance over V(s):

$$A^\pi(s,a) = Q^\pi(s,a) - V^\pi(s)$$

Dueling network converges faster than usual DQN, Double Q-Network

Errors shown to be lower for Atari games