Manifold Dynamics and their Applications to Low-Energy Transfers: Project Plan Report

Project Overview

Manifold Dynamics and their Applications to Low-Energy Transfers aims to study the dynamical phenomena of the circular-restricted three-body problem (CR3BP): an examination of the motion of a third, massless particle moving under the influence of two other particles (primaries) that orbit in circles about their barycenter. In particular, the CR3BP yields many dynamical structures – stable and unstable manifolds and bouding surfaces – that provide means for matter transfer into and from the primaries. Exploiting these dynamical phenomena can provide the construction of complex, non-Keplerian orbits to achieve mission objectives, as well as potential propellant savings (**?**).

Its' inception is believed to have begun with (**?**), who introduced a synodic coordinate system during his studies of Lunar motion. Attracting a range of well-known mathematicians to its' formulation, the three-body problem received significant contributions from (**?**), who discovered the equilibrium solutions now known as the Lagrangian points, and (**?**) who 're-discovered' the Jacobi integral, the only constant of motion in the CR3BP. (**?**) later used this integral in his formulation of forbidden regions and bounded motion. Perhaps most important for analysis of the CR3BP, (**?**)'s methods of section, phase-space and deterministic chaos ended the classical period of study for the three-body problem [1](**?**).

The use of this formulation of spacecraft motion was first used to construct a low-energy transfer by Edward Belbruno, to rescue the Japanese MUSES-A mission – now renamed Hiten – and place it into a Lunar orbit(**?**). Since then,the CR3BP has been studied heavily for its' ability to reveal mission plans about the Lagrangian points – now home to several scientific payloads – and dynamically-assisted trajectories, including those used by spacecraft such as SMART-1, Genesis, and GRAIL (**?**).

By studying the equations of motion of the CR3BP (Equations **??** through 4), along with a numerical computation suite, the construction and visualisation of these orbits is possible. Figure **??** reports a family of periodic Lyapunov – planar – orbits about a Lagrangian point, constructed utilising a software suite in MATLAB, and a numerical technique known as differential correction.

$$\ddot{x} - 2\dot{y} = \frac{\partial \Omega}{\partial x} \tag{1}$$

$$\ddot{y} + 2\dot{x} = \frac{\partial \Omega}{\partial y} \tag{2}$$

$$\ddot{z} = \frac{\partial \Omega}{\partial z} \tag{3}$$

$$\Omega = \frac{1}{2}(x^2 + y^2) + \frac{1-\mu}{r_{12}} + \frac{\mu}{r_{23}} \tag{4}$$

By determining and applying disturbances to these orbits in their stable and unstable directions, and integrating the equations of motion forwards or backwards, we discover trajectories

[1] For the development of these methods, Sweden's King Oscar II awarded Poincaré a prize to award him for being the first to solve the n-body problem. Poincaré had not, in fact, even solved the general case of the three-body problem.

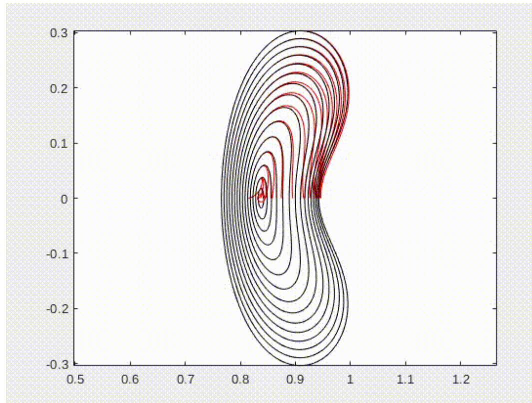

Figure 1: A family of Lyapunov orbits about Earth-Moon L_1 ($\mu = 0.0121$). The red lines indicate the interations of the differential correction subroutines.

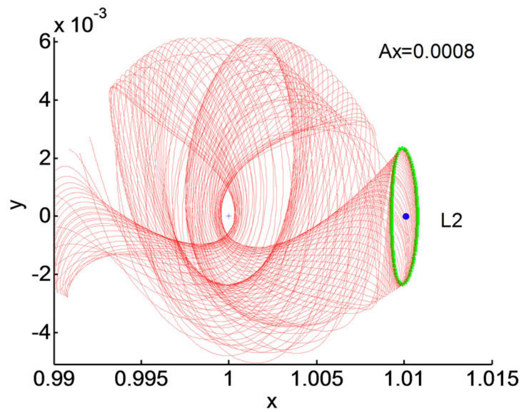

Figure 2: Unstable manifold computed from initial state x_0 using the relation $x_0\prime = x_0 \pm \varepsilon\frac{\lambda}{|\lambda|}$, where λ represents the unstable eigenvalue of the Jacobian matrix of the local subspace; ε = 10^{-6}

that exponentially approach or depart these periodic orbits. Discretizing and plotting these trajectories around the entirety of the orbit yields stable and unstable manifold tubes, respectively. It is these which are used to construct 'low-energy' transfers[2]; these manifolds are visualised in Figure **??**.

Multiple uses for these missions have been proposed: (**?**) has proposed their usage for asteroid retrieval missions, where the required impulse to capture is significantly reduced when utilising stable manifolds. Alternatively, (**?**) and (**?**) recommended them for use in interplanetary transfers, where the stable manifold acts as a route for quasi-ballistic capture, decreasing delta-V requirements in some cases, and increasing the launch periods. These uses will be evaluated in a full, sample mission case study to form the main deliverable for the project.

[2]The definition of low-energy is ambiguous. For this case, low-energy is taken to represent a transfer that exploits natural and/or dynamical phenomena to reduce spacecraft propellant usage.

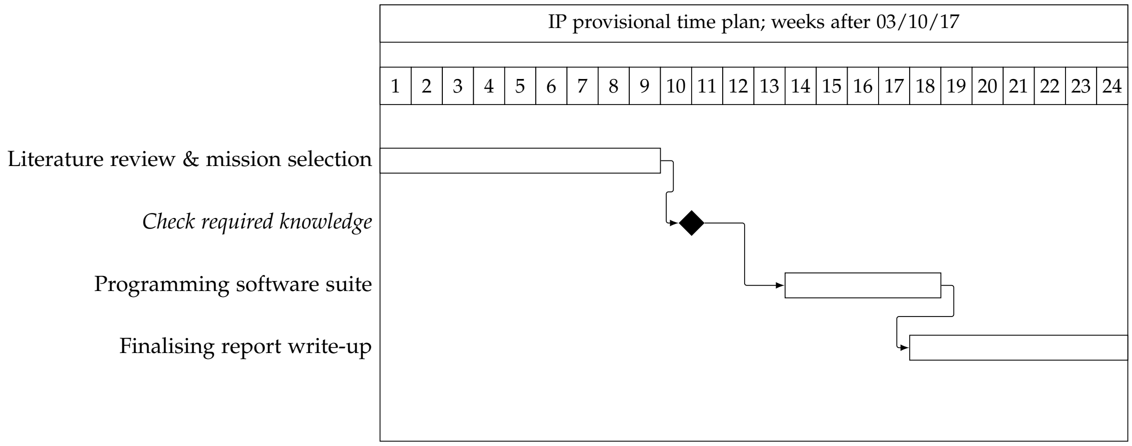

Figure 3: Provisional Gantt chart for the Part III individual project.

Project Outcomes, Aims and Scope

The project outcome is to deliver a full case study for a mission that exploits dynamical phenomena of the CR3BP. To this end, it sets the following aims:

- A comprehensive literature review on CR3BP theory, potential applications and numerical techniques for the contruction of orbits in the CR3BP;
- A suitable mission selection that will exploit a range of features of the CR3BP;
- The creation of software bundle to allow for the computation of the mission analyses and orbit construction.

with the following scope of work:

- The mathematics behind the CR3BP and its' dynamical phenomena;
- The theory behind, and implementation of, numerical techniques for orbit construction including numerical integration, differential correction and optimisation;
- Studies of mission trade-offs and requirements;
- Studies and implementation of software analytics: code coverage, MC/DC coverage, static/dynamic program analysis and parallel processing techniques for minimising runtimes.

Training needs analysis

An induction onto the Lyceum supercomputing cluster may be required, but is as yet unknown. No other training is required.

Gantt Chart

Figure **??** reports the provisional Gantt chart for the project.

Manifold Dynamics and their Applications to Low-Energy Transfers

Project Plan Presentation

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Mani-what?

- A particular case of spacecraft motion – the Circular-Restricted Three-body Problem (CR3BP) – studies the motion of a third body under the influence of two other much more massive bodies
- The CR3BP exhibits some *funky* behaviour as a dynamical system

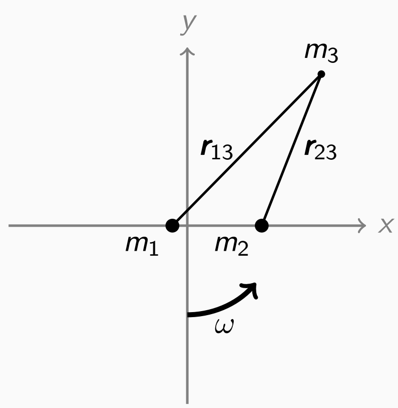


Figure 1: Normalised co-ordinate system for the CR3BP – ω and primary separation normalised to unity.

Introduction to Manifolds

One dynamical phenomenon, manifold tubes – orbits that approach and depart a periodic orbit in the CR3BP – can be exploited to give new ways of transferring matter in phase space

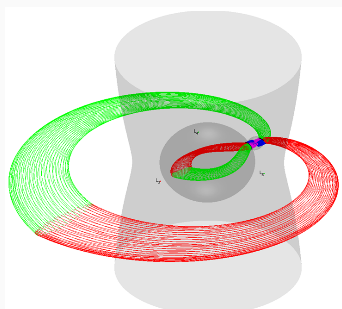


Figure 2: Stable (green) and unstable (red) invariant manifolds associated to Earth-Moon L1. Source: Renkli Seyler, `https://renklisheyler.wordpress.com/research/motions-in-cr3bp/invariant-manifold-tubes/`

Potential Applications

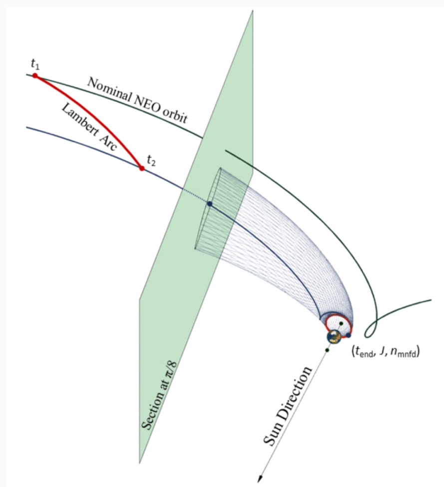


Figure 3: The use of stable invariant manifolds to perform an asteroid retrieval mission to Earth-Moon L2. [Sánchez and García Yárnoz, 2016]

Potential Applications - 2

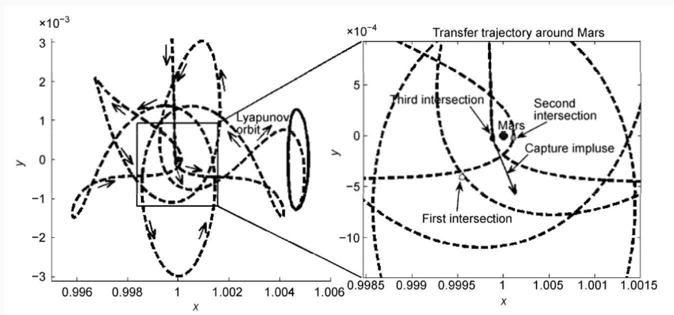


Figure 4: Earth Lagrangian point - Mars Lagrangian point interplanetary transfer via invariant manifolds and gravity assists. [Shuai et al., 2013]

Project aim and outcomes

- The project will aim to deliver a full case study for a mission in, and exploiting, the CR3BP
- To achieve this, the project sets the following aims:
  - A comprehensive literature review on CR3BP theory, potential applications and numerical techniques for the contruction of orbits in the CR3BP
  - A suitable mission selection that will exploit a range of features of the CR3BP
  - A software bundle to allow for the computation of the mission analyses and orbit construction

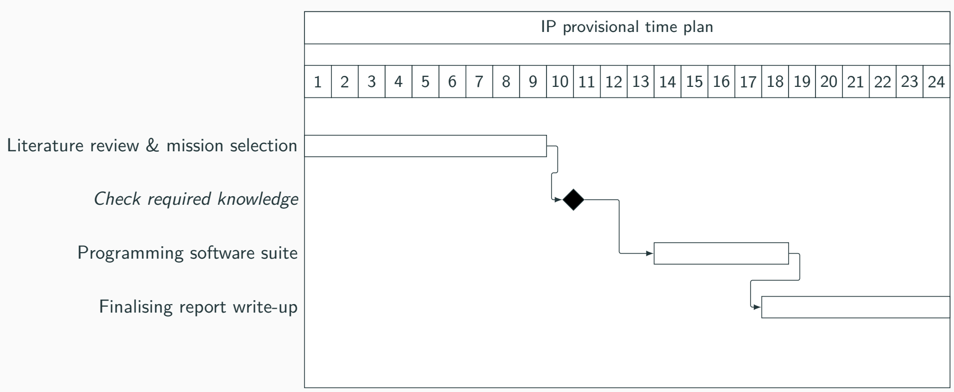
Project scope

- The mathematics behind the CR3BP and its' dynamical phenomena
- The theory behind, and implementation of, numerical techniques for orbit construction including numerical integration, differential correction and optimisation
- Mission trade-offs and requirements
- Software analytics: code coverage, MC/DC coverage, static/dynamic program analysis and parallel processing techniques for minimising runtimes and ensuring program robustness

Current Project Progress

- Self-proposal made it easy to do the literature review before Semester 1 started
- Mendeley library contains ~80 journal papers and books
- Already worked on (basic) underlying mathematics & written this up into a reference document (16k words)
- Written code (MATLAB, Fortran03) to single-shoot some closed orbits

(Very) provisional time plan



Any questions?

Figure 5: Integration of an unstable manifold emerging from state x_0 with unstable eigenvalue λ: $x_0\prime = x_0 \pm \epsilon \frac{\lambda}{||\lambda||}$

References

Sánchez, J. P. and García Yárnoz, D. (2016).
Asteroid retrieval missions enabled by invariant manifold dynamics.
Acta Astronautica, 127:667–677.

Shuai, W., Haibin, S., and Weiren, W. U. (2013).
Interplanetary transfers employing invariant manifolds and gravity assist between periodic orbits.
56(3):786–794.

Supplementary slides – zero velocity curves in the CR3BP