Manifold Dynamics and their Applications to Low-Energy Transfers: Project Plan Report

Project Overview

Manifold Dynamics and their Applications to Low-Energy Transfers aims to study the dynamical phenomena of the circular-restricted three-body problem (CR3BP): an examination of the motion of a third, massless particle moving under the influence of two other particles (primaries) that orbit in circles about their barycenter. In particular, the CR3BP yields many dynamical structures – stable and unstable manifolds and bouding surfaces – that provide means for matter transfer into and from the primaries. Exploiting these dynamical phenomena can provide the construction of complex, non-Keplerian orbits to achieve mission objectives, as well as potential propellant savings (**?**).

Its' inception is believed to have begun with (**?**), who introduced a synodic coordinate system during his studies of Lunar motion. Attracting a range of well-known mathematicians to its' formulation, the three-body problem received significant contributions from (**?**), who discovered the equilibrium solutions now known as the Lagrangian points, and (**?**) who 're-discovered' the Jacobi integral, the only constant of motion in the CR3BP. (**?**) later used this integral in his formulation of forbidden regions and bounded motion. Perhaps most important for analysis of the CR3BP, (**?**)'s methods of section, phase-space and deterministic chaos ended the classical period of study for the three-body problem [1](**?**).

The use of this formulation of spacecraft motion was first used to construct a low-energy transfer by Edward Belbruno, to rescue the Japanese MUSES-A mission – now renamed Hiten – and place it into a Lunar orbit(**?**). Since then,the CR3BP has been studied heavily for its' ability to reveal mission plans about the Lagrangian points – now home to several scientific payloads – and dynamically-assisted trajectories, including those used by spacecraft such as SMART-1, Genesis, and GRAIL (**?**).

By studying the equations of motion of the CR3BP (Equations 1 through 4), along with a numerical computation suite, the construction and visualisation of these orbits is possible. Figure **??** reports a family of periodic Lyapunov – planar – orbits about a Lagrangian point, constructed utilising a software suite in MATLAB, and a numerical technique known as differential correction.

$$\ddot{x} - 2\dot{y} = \frac{\partial \Omega}{\partial x} \tag{1}$$

$$\ddot{y} + 2\dot{x} = \frac{\partial \Omega}{\partial y} \tag{2}$$

$$\ddot{z} = \frac{\partial \Omega}{\partial z} \tag{3}$$

$$\Omega = \frac{1}{2}(x^2 + y^2) + \frac{1-\mu}{r_{12}} + \frac{\mu}{r_{23}} \tag{4}$$

By determining and applying disturbances to these orbits in their stable and unstable directions, and integrating the equations of motion forwards or backwards, we discover trajectories

[1] For the development of these methods, Sweden's King Oscar II awarded Poincaré a prize to award him for being the first to solve the n-body problem. Poincaré had not, in fact, even solved the general case of the three-body problem.