Manifold Dynamics and their Applications to Low-Energy Transfers

Project Plan Presentation

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October 10, 2017

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Introduction

Mani-what?

- A particular case of spacecraft motion – the Circular-Restricted Three-body Problem (CR3BP) – studies the mass of a third body under the influence of two other much more massive bodies
- The CR3BP exhibits some *funky* behaviour as a dynamical system

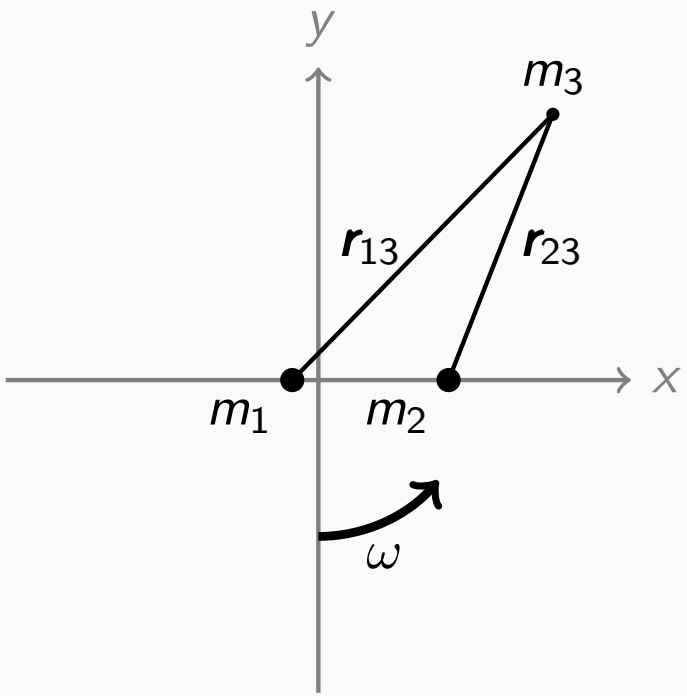


Figure 1: Normalised co-ordinate system for the CR3BP – ω and primary separation normalised to unity.

Quirks of the CR3BP

- Lagrangian points are points where the gravity of the two primaries 'cancel out'
- L-points (particularly Sun-Earth L_1) offer interesting characteristics for some science applications
- Computing these (see right) *periodic orbits* requires the use of numerical techniques, but is well understood

Introduction to Manifolds

If we integrate orbits that exponentially approach and depart a certain periodic orbit in the CR3BP, we discover manifold tubes...

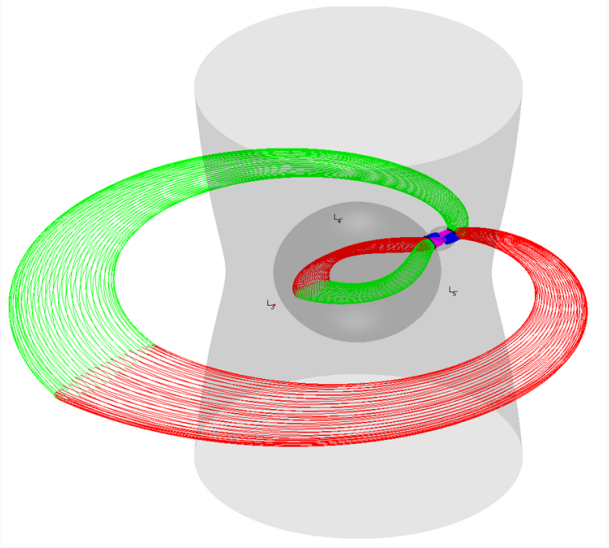


Figure 2: Stable (green) and unstable (red) invariant manifolds associated to Earth-Moon L1. Source: Renkli Seyler, `https://renklisheyler.wordpress.com/research/motions-in-cr3bp/invariant-manifold-tubes/`

...exploiting these dynamical phenomena can give rise to interesting applications

Potential Applications

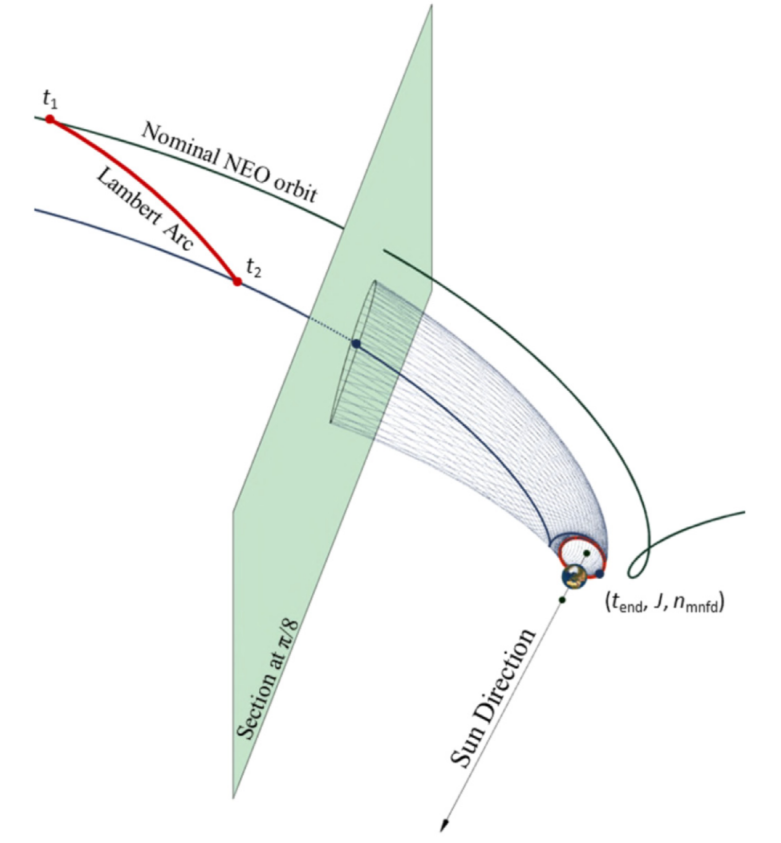


Figure 3: The use of stable invariant manifolds to perform an asteroid retrieval mission to Earth-Moon L2. Source: Sánchez, J. P., & García Yárnoz, D. (2016). Asteroid retrieval missions enabled by invariant manifold dynamics/

Potential Applications - 2

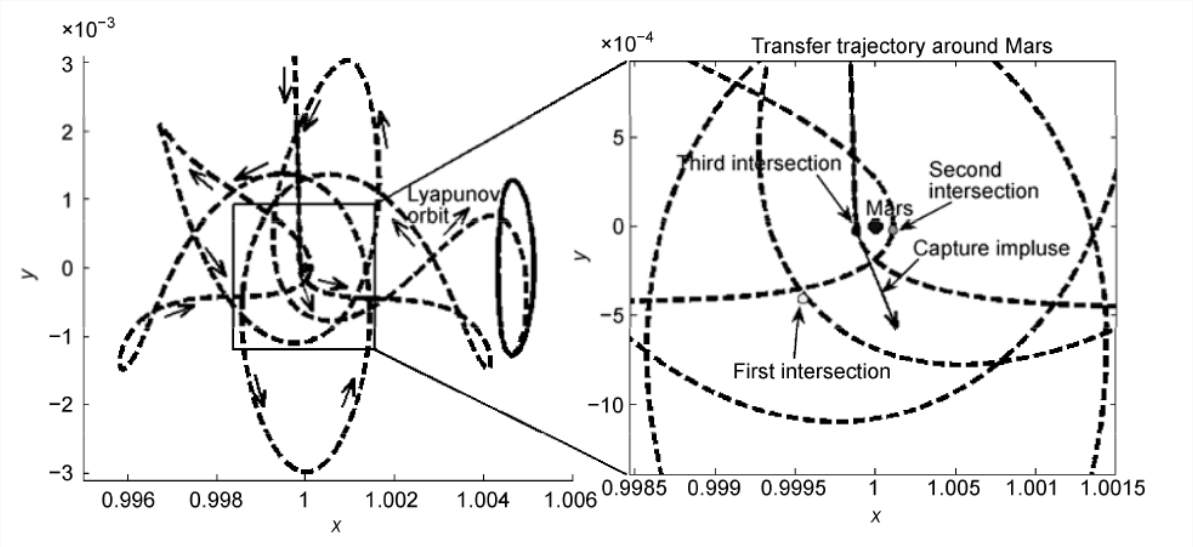


Figure 4: Earth Lagrangian point - Mars Lagrangian point interplanetary transfer via invariant manifolds and gravity assists. Shuai, W., Haibin, S., & Weiren, W. U. (2013). Interplanetary transfers employing invariant manifolds and gravity assist between periodic orbits, 56(3), 786–794.

Formal project proposal

"The application of dynamical systems theory to space mission design has attracted attention for its' ability to construct new and unusual orbits that cannot be found through the classical approach to spacecraft trajectory design. In particular, the three-body problem yields many dynamical structures - stable and unstable manifolds and bounding surfaces - that provide phase-space structures for the transfer of objects into and from the smaller primary body. These dynamical structures can be exploited to construct 'low-energy transfers', where the path of the third body exists as a harmony of gravitation, and can allow for the use of complex, non-Keplerian orbits to achieve mission objectives, as well as potential propellant savings.

Throughout this project, *the underlying mathematics behind the three-body problem will be studied and appropriate analytical and numerical techniques will be implemented, which enable the systematic design of low-energy spacecraft trajectories by exploiting manifold dynamics around libration points*. A particular focus will be placed on the use of these dynamical phenomena to assemble spacecraft trajectories that exploit the characteristics of the Sun-Earth-Moon system."

Current Project Progress

- Self-proposal made it easy to do the literature review before Semester 1 started
- Mendeley library contains ~80 journal papers and books
- Already worked on (basic) underlying mathematics & written this up into a reference document (16k words)
- Written code (MATLAB, F03) to single-shoot some closed orbits

Any questions?

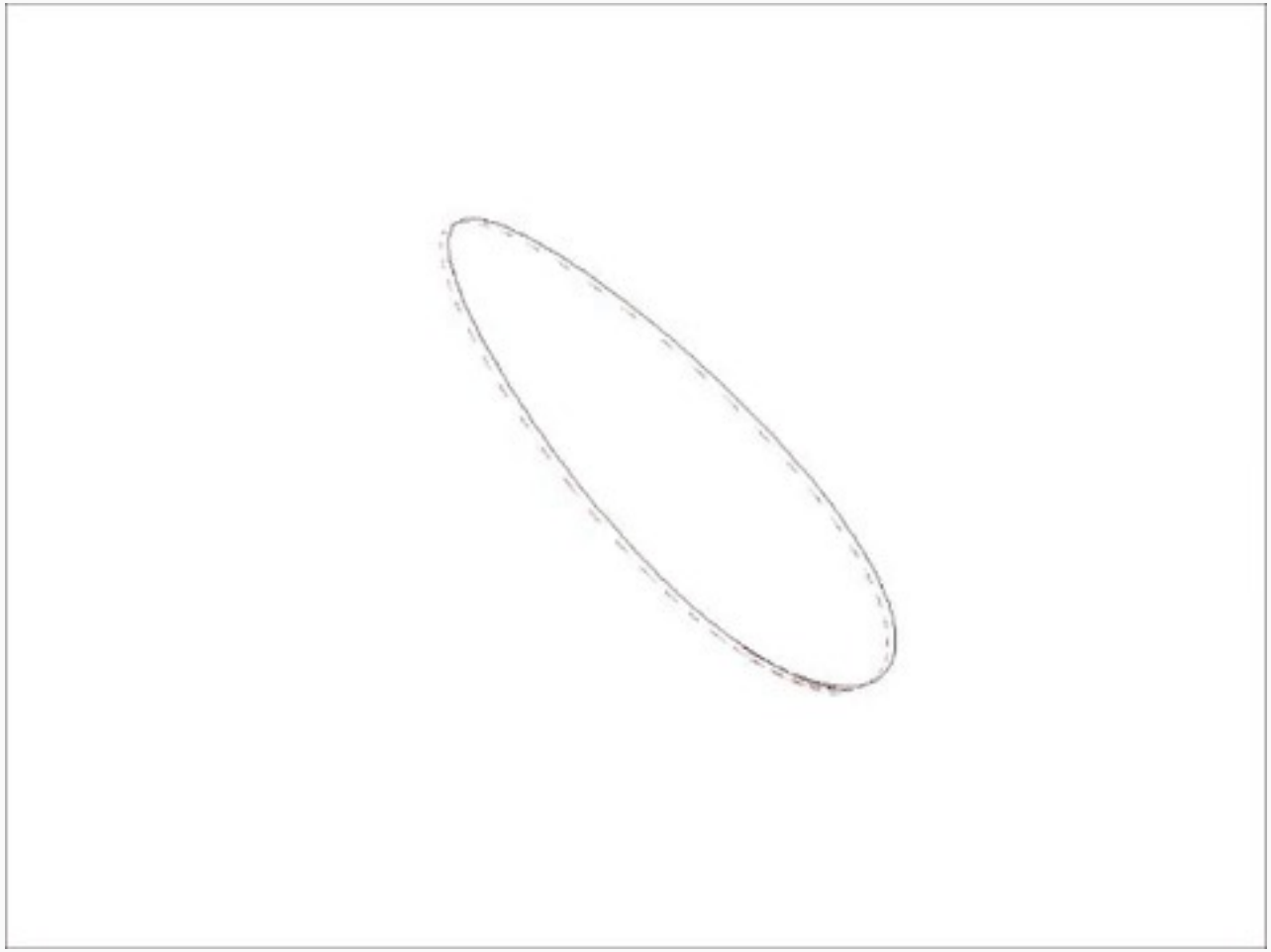
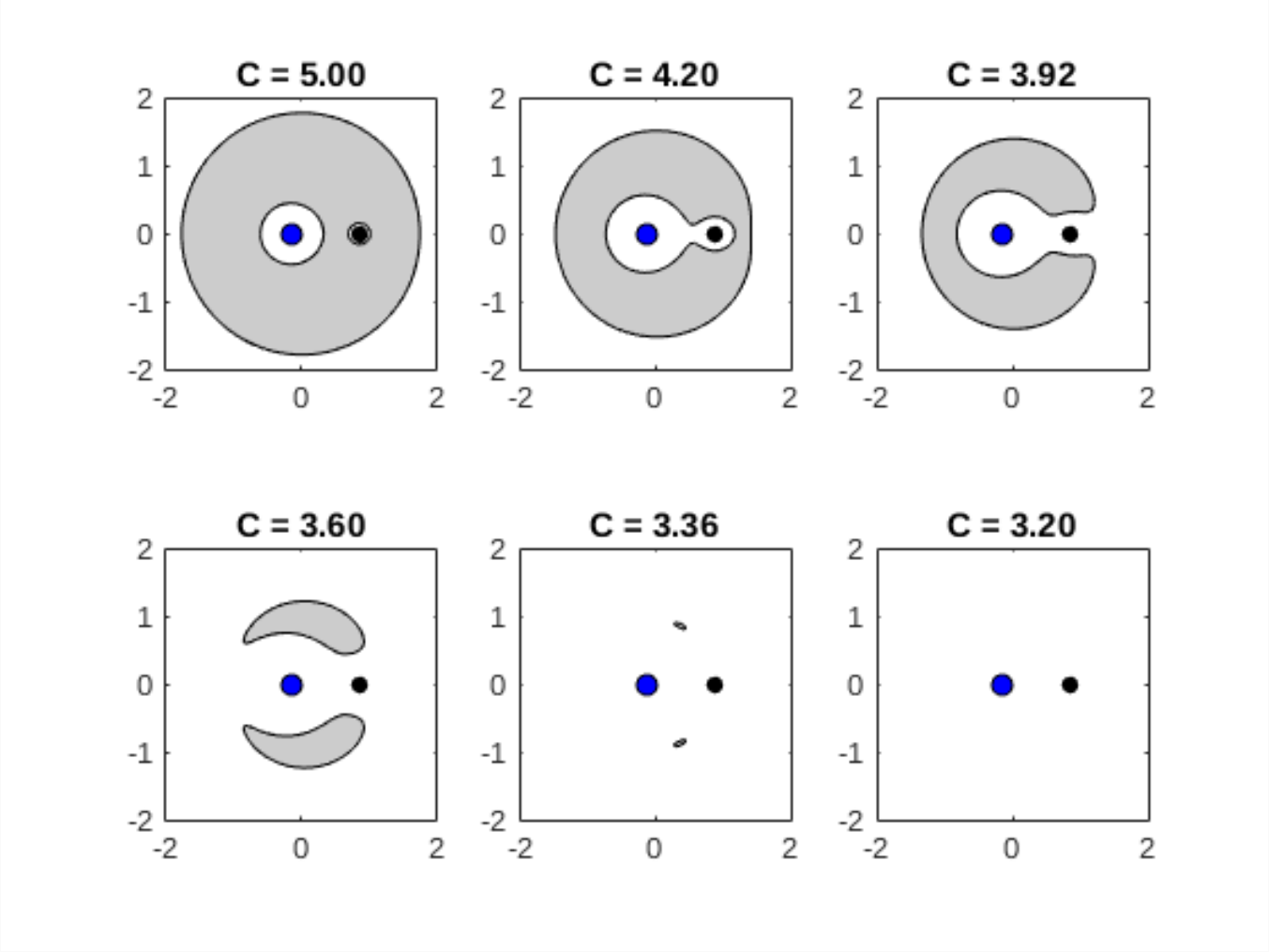


Figure 5: Integration of an unstable manifold emerging from state x_0 with unstable eigenvalue λ: $x_0\prime = x_0 \pm \epsilon \frac{\lambda}{||\lambda||}$

Supplementary slides – zero velocity curves in the CR3BP



Project aims, outcomes and scope

- This project aims to lay out the underlying mathematics behind the existence and manipulation of manifold structures, and use numerical techniques to construct transfers involving them
- The main project 'deliverable' is a software suite that provides the computation of several case studies of these low-energy transfers
- BUT the project was proposed to be open-ended, and gives a lot of wiggle room for taking a different path further down the line