Exercises from *Algebraic Topology* by Allen Hatcher

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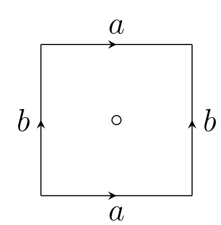
Contents

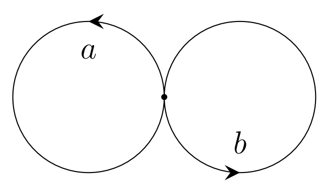
Chapter 0

Some Underlying Geometric Notions

Exercise 0.1 [Complete] Construct an explicit deformation retraction of the torus with one point deleted onto a graph of two circles intersecting at a point, namely, longitude and meridian circles of the torus.
Solution. Consider the fundamental square of the torus with a point removed:



where the deformation retract of the removed point retaracts to the boundary of the square; identifying the sides gives a bouquet of two circles:



where the point at the intersection of the circles is the identification of the four corners.

Alternatively, let $p \in T$ be the removed point. Choose your favorite open set U about p. Retract U to the meridian, so that T is now missing a longitudinal strip, contracted to a line spanning the meridial width of U. Then there are two open copies of S^1 seperated by this width; contract them along the meridian in the opposite direction of eachother. Then, exactly two copies of S^1 are left, connected by a single point. $\square$

Exercise 0.2 [TODO] Construct an explicit deformation retract from $\mathbb{R}^n - \{0\}$ to S^{n-1}.
Solution. Define a map

$$h : (\mathbb{R}^n - \{0\}) \times I \to \mathbb{R}^n - \{0\}$$

via

$$h(x,t) := (1-t)x + \frac{tx}{||x||}$$

where $||\cdot||$ is the Euclidean norm in $\mathbb{R}^n$. Then, $h(x,0) = x$ and $h(1) = x/||x||$, retracting each point to a point on S^{n-1}.

$\square$

Chapter 1

Fundamental Group

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