

Exercises from *Matrix Groups for Undergraduates*
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Chapter 1

Matrices

Exercise 1.1. Describe a natural 1-to-1 correspondence between elements of $\mathrm{SO}(3)$ and elements of

$$T^1S^2 = \{(p, v) \in \mathbb{R}^3 \times \mathbb{R}^3 : |p| = |v| = 1 \text{ and } p \perp v\}$$

Solution. Using the globe analogy from Question 1.2, fix a point r to be the north pole, and a point e that lies on the equator induced by the choice of r , and assert this as the arbitrary ‘identity’.

Next, given some $A \in \mathrm{SO}(3)$, identify an element in T^1S^2 via $A \mapsto (Ar, Av)$, as in first where A maps the north pole r , and then how A rotates the globe about the axis induced by r and its antipodal point. \square

Chapter 2

All matrix groups are real matrix groups

2.1 Exercises

Exercise 2.1. Another exercise goes here.

Solution. Placeholder for your solution.

□

Chapter 3

The orthogonal groups

3.1 Exercises

Exercise 3.1. Another exercise goes here.

Solution. Placeholder for your solution.

□

Chapter 4

The topology of matrix groups

4.1 Exercises

Exercise 4.1. Another exercise goes here.

Solution. Placeholder for your solution.

□

Chapter 5

Lie algebras

5.1 Exercises

Exercise 5.1. Another exercise goes here.

Solution. Placeholder for your solution.

□

Chapter 6

Matrix exponentiation

6.1 Exercises

Exercise 6.1. Another exercise goes here.

Solution. Placeholder for your solution.

□

Chapter 7

Matrix groups are manifolds

7.1 Exercises

Exercise 7.1. Another exercise goes here.

Solution. Placeholder for your solution.

□

Chapter 8

The Lie bracket

8.1 Exercises

Exercise 8.1. Another exercise goes here.

Solution. Placeholder for your solution.

□

Chapter 9

Maximal tori

9.1 Exercises

Exercise 9.1. Another exercise goes here.

Solution. Placeholder for your solution.

□

Chapter 10

Homogeneous manifolds

10.1 Exercises

Exercise 10.1. Another exercise goes here.

Solution. Placeholder for your solution.

□

Chapter 11

Roots

11.1 Exercises

Exercise 11.1. Another exercise goes here.

Solution. Placeholder for your solution.

□