Calorimetric Efficiency of the Thermoelectric Peltier Device

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Contributions

Jaeha: Introduction, Data, Analysis, Theory

Tyler: Theory, Design/Methodology/Challenges, Data

Abstract
This thermoelectric experiment investigates and analyzes the Coefficient of Refrigerative Performance of a Peltier device using calorimetric analysis. This experiment was motivated by our interest in thermodynamic systems and their applications. We designed a setup involving a copper plate, insulated styrofoam container, and a known mass of water to capture heat extracted from the device's cold side. By measuring temperature changes over time and calculating electrical input through circuit theory, we derived COP values for various trials under different current and voltage conditions. Significant design considerations were required to mitigate error, such as submerging the heat sink to reduce backflow of heat and sealing the apparatus to prevent water leakage. Through statistical analysis and error propagation, we identified key factors affecting accuracy, including thermal loss through insulation, imperfect thermal contact, and measurement fluctuations. Linear model fits and reduced chi-squared analyses were used to assess model validity. Our results indicate that COP generally decreases with higher power input, aligning with theoretical expectations. The final submerged trials showed improved efficiency and linearity, validating the refined experimental design. This project provided a hands-on application of thermodynamics and basic circuit theory, while also highlighting the importance of careful experimental control in calorimetric efficiency measurements.

Motivation/objectives

This project was mainly inspired by both of our enjoyment of the fundamental thermodynamic laws we learned in Physics 5C, particularly the sections of thermal efficiency. Learning about the metrics and derivations behind engines, heat transfer, and heat cycles was very interesting to us and we wanted to go further by conducting this experiment. Additionally, we wanted to create and test an apparatus that was applicative in nature. We decided to determine the efficiency of the Peltier Device because it relates to common everyday applications, which was very relatable. We also did not go too in depth into the Coefficient of Refrigerative Performance or the Peltier Effect much in class and believed this would be a great opportunity to solidify our knowledge in these fields. This experiment also brought challenges, which we were excited to face, particularly with dealing with errors, which was probably the most difficult part of this report. However, by pushing through this, we gained skills in learning how to create a setup that minimizes systematic errors to the best of our ability based on the resources we had. Additionally, it gave us much more familiarity on how to deal with errors that couldn't be controlled and allowed us to conduct a more comprehensive approach than the other labs in the 5CL gave us.

Our main objective of this lab was to test the efficiency of the Peliter device that we were supplied with, of course, but also to learn how to deal with factors that play a major part in contributing to errors in a seemingly straightforward experiment. Another sub-objective would also be to combine theory from Electromagnetism and apply it, which was a great exercise in working with theory from different fields. We also hoped to expand our skills in statistical analysis and interpreting our results, which was a major part of our conclusions.

Theory

The main theory governing our experiment lies in the Thermoelectric effect, particularly the Peltier effect. We will utilize this phenomenon in order to achieve a setup that allows us to obtain the Coefficient of Refrigerative Performance (COP), which is our main objective. The general structure of our theory begins with the Peltier effect, making use of circuit theory and electric work based on known voltages and currents, and the calorimetric heat capacity to compute the COP.

The Peltier Effect essentially states that when an electric current is made to flow through a junction between two conductors, heat may be transferred across the junction.

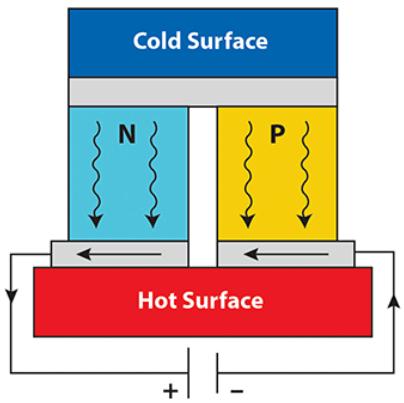


Figure 1.1.1: This figure shows a visual representation of the Peltier Effect in action (Current flow inducing heat removal). The Peliter device requires charge carriers (N/P-type semiconductors) for direction control, however this is not required to be accounted for in our calculations, just an interesting property of Peltier devices and is beyond the scope of this lab.

Generally, the heat transferred by a Peltier device would be calculated by:

$$Q = \Pi I t,$$

where Π denotes the Peltier coefficients, I is current, and t is time. In our limited setup, we did not have access to the Peltier coefficients so we devised a different way of obtaining the heat. Instead we utilized a setup that allows us to use heat capacity, particularly of water, which was the most efficient option we had in terms of cost and convenience (the other one being air). Although more on this in our setup/design challenges, water is able to distribute and hold heat much better than air, which is why we chose it. As for how we used it, we developed a setup that includes a copper plate the same size as our Peltier device surrounded by styrofoam walls, and filling it with a known mass of water. This is built on the assumption that all the cooling on the surface of the plate is fully transferred through the copper plate into the water and measuring the temperature (initial and final) of the water will give us the necessary information to compute heat. Although this experimental design or concept is prone to errors (which will be addressed in later sections), we believe it was a viable setup given our resources and time. To summarize this in an equation, we use calorimetry heat capacity:

$$Q = mc\Delta T \text{ (eq. 1)}$$

where m is the mass of the water, c is the specific heat capacity of water, and ΔT is the total change in temperature of the water (T_{final} - $T_{initial}$).

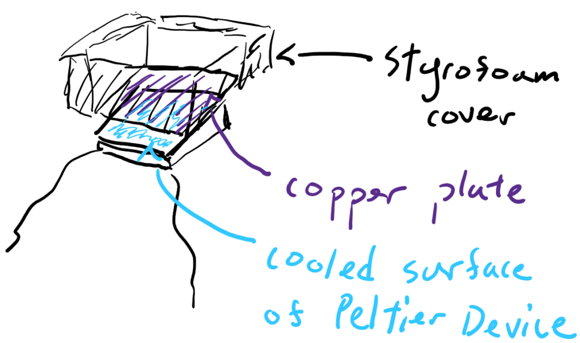


Figure 1.2.1 and 1.2.2: Details how the copper plate is placed onto the surface of the Peltier Device (directly) with matched dimensions. Next figure shows the actual copper plate + styrofoam cover itself.

We must also calculate the power inputted into our Peltier device and ultimately the electric work inputted. This can be done using circuit theory:

$$P = IV \text{ (eq. 2)}$$

where P is power, I is current, and V is voltage. Since P is defined as

$$P \equiv \frac{W}{\Delta t} \text{ (eq. 3),}$$

we can calculate the electric work as the following:

$$W = P\Delta T = IV\Delta T \text{ (eq. 4)}$$

Given all of this information, we can determine the Coefficient of Refrigerative Performance, which is defined as:

$$COP = \frac{Q_c}{W} \text{ (eq. 4.5)}$$

where Q_c is the heat extracted from the cold surface and W is the electrical work input in Joules/Watts. Using eq. 1 and 4,

$$COP = \frac{mc\Delta T}{IV\Delta t} \text{ (eq. 5)}$$

Our experiment has many opportunities for error, particularly heat dissipated through the sides of the Peltier Device and also through the sides of the styrofoam wall, which we initially assumed to be fully insulated, but obviously is not the case. We can account for this quantitatively with the Fourier Heat Law:

$$dQ/dt = -kA\frac{(T_2 - T_1)}{\Delta x} \text{ (eq. 6)}$$

where k is the thermal conductivity of the material, A is the surface area of contact, Δx is the thickness of the container, and T_2/T_1 refer to the temperature on the inside/outside of the container. The additional heat will then be accounted for before calculating the final COP. More insight on errors can be found later in the design and analysis sections.

Later in our analysis we also look to create a linear model to fit values for the COP. Substituting T_f and T_i for ΔT and rearranging eq. 5, we can create a linear model:

$$T_f = \frac{(COP)IV\Delta t}{mc} + T_i \text{ (eq. 7)}$$

Eq. 6 follows the form $y = ax + b$, where $a = \frac{(COP)IV}{mc}$, y = T_f , and x = Δt.

Solving for COP,

$$COP = \frac{amc}{IV} \text{ (eq. 8)}$$

Agreement test (eq. 9)

$|A - B| < 2\sqrt{((\Delta A)^2 + (\Delta B)^2)} \rightarrow$ Agrees

$|A - B| > 2\sqrt{((\Delta A)^2 + (\Delta B)^2)} \rightarrow$ Does not agree

Challenges, Design, & Methodology

Our Capstone Project's main challenges were in the design and setup. We iterated through multiple different ideas and attempted to implement many new setups, all in efforts to minimize error. It was apparent from the start that our experiment would be quite prone to errors, since we were using old and limited equipment and because it was difficult to accurately measure the true heat transfer without the influence of confounding factors.

The first design considerations were the materials of our container. The bottom surface needed to conduct heat efficiently (higher thermal conductivity), so we chose copper, which was also in stock in the 5L lab material storage. The walls needed to insulate heat so that the heat through the bottom went directly into the water that we measured, and styrofoam was ideal for this purpose (lower thermal conductivity). We ensured that the copper plate had the same area as the Peltier Device's surface as we did not want it to dissipate elsewhere that is not into the plate.

Our first setup initially used a *Triplett DC Power Supply* connected to the Peltier device on a table (shown below).

Figure 2.1.1 and 2.1.2: Our Peliter Device connected to the power supply, with our container next to it.

However, the issue with the setup above is that the power supply doesn't actually give us a set, non-changing voltage/current. We noticed high amounts of fluctuation in both V and I, which would cause a great amount of error. Due to this, we decided to move to the 5BL room and use the power supply there, which had constant current and less fluctuations in voltage.

We then conducted our first trials and recorded our results. We noticed that our COP values were quite low and attempted to reason why. We realized that there was a significant amount of heat on the other side of the plate (and the black metal holder), and that the heat sink re-transfers the heat back into the cold side (particularly potent at higher voltages). To combat this, we submerged the bottom of the heat sink into a water bath to prevent it from heating up and conducted another set of trials.

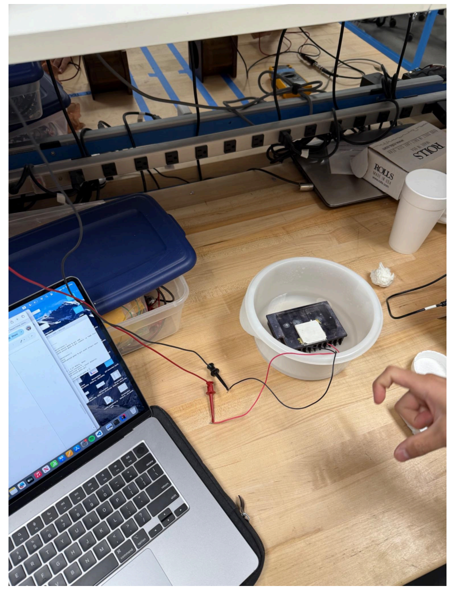


Figure 2.2.1 and 2.2.2: First image shows the bath without water next to the device, second image shows the submerged experiment right before putting the copper plate on top.

The results from this experiment were logged and prepared for analysis as well. As discussed later on in this report, the data netted closer results to the expected value.

Some additional design concepts to mention include the dimensions of the styrofoam walls and the styrofoam lid. The walls were cut out based on the dimensions of the copper plate to snuggly fit the perimeter and be thick enough to (ideally) insulate. As we can continue our analysis later on, we will actually account for the small amount of dissipated heat here. We also designed the top styrofoam cap as well, which we got from the base of a styrofoam cup.

Procedure

With the design detailed, here is the procedure we followed to collect our data.

1. Prepare water in apparatus (and place peltier device base in the tub of water for submerged experiments)
    a. Measure mass of water with digital scale
2. Configure power bank to desired I, V
3. Run known power through Peltier device
    a. Calculate input work across time interval through eq. 4
4. Place container of water on Peltier device
5. Use Loggerpro to measure the temperature of the water for 120 seconds. Make sure to hold steady.

- Measure change in temperature in water across time interval

6. Repeat for different input currents/amplitudes
    - V and I will be known, since we will use a power supply

Additional Challenges

Our design and methodology, however, had some additional challenges that we addressed through design reform or through accounting for it in our error calculations.

The first issue was keeping our copper plate watertight with our styrofoam outer wall. This is important because getting water in the Peltier device would cause it to short circuit, since it utilizes semiconductors. It was resolved by obtaining insulating glue which could create a seal between the two pieces. The material of choice was thermally insulating silicone adhesive. Additionally, as mentioned earlier, there was still heat that dissipated through the styrofoam walls, cap, and through the small sides of the device. Although this couldn't be resolved through a physical setup, we accounted for this as much as possible in our final calculations.

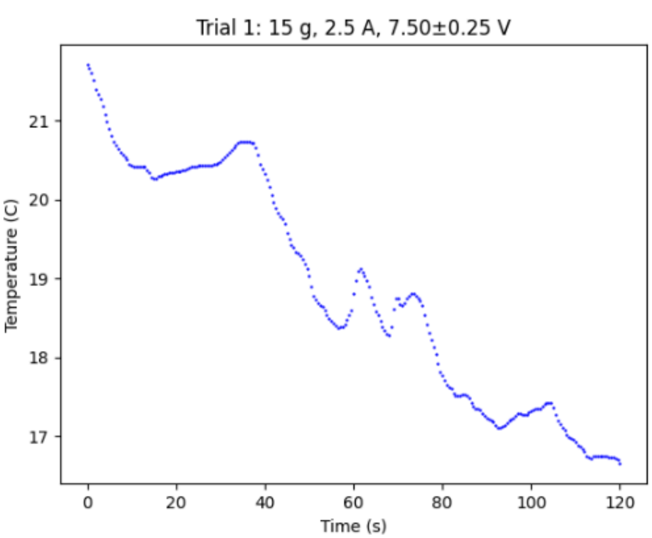
Figure 2.3.1, 2.3.2, 2.3.3: Our final styrofoam wall + copper plate setup

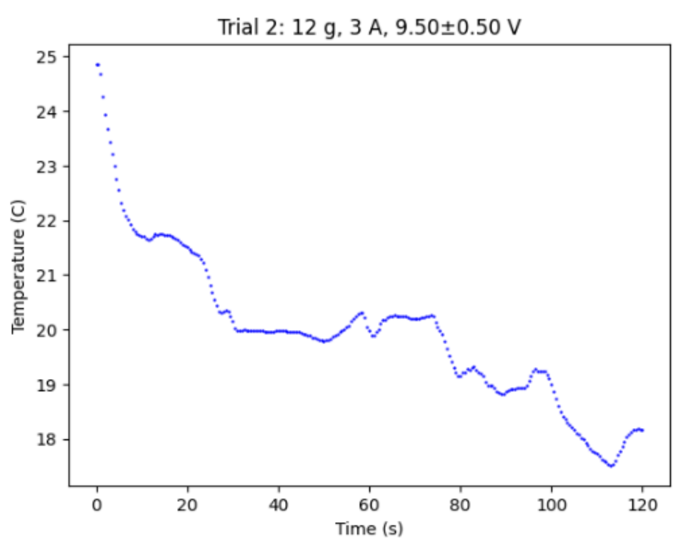
Data

Figure 3.1.1: Our **submerged** experiment in action; the LoggerPro device is held steady to measure the water's temperature.

Full LoggerPro datasets in CSV format can be accessed here:
https://drive.google.com/drive/folders/1uSw49so8rar8-D4wDN1LsHfiHL8M2HQ8?usp=sharing

Plots of the water's measured temperature over time for each trial are shown below. The mass of the water, the input current, and the input voltage for each trial are included in the titles. As mentioned before, the voltage changed slightly over the course of each trial, so we selected the midpoint of this range as our value and listed half of this range as the uncertainty. Trials before our adjusted setup (heat sink not yet submerged) are plotted in blue, while trials after our adjustment (heat sink submerged in water) are plotted in red.





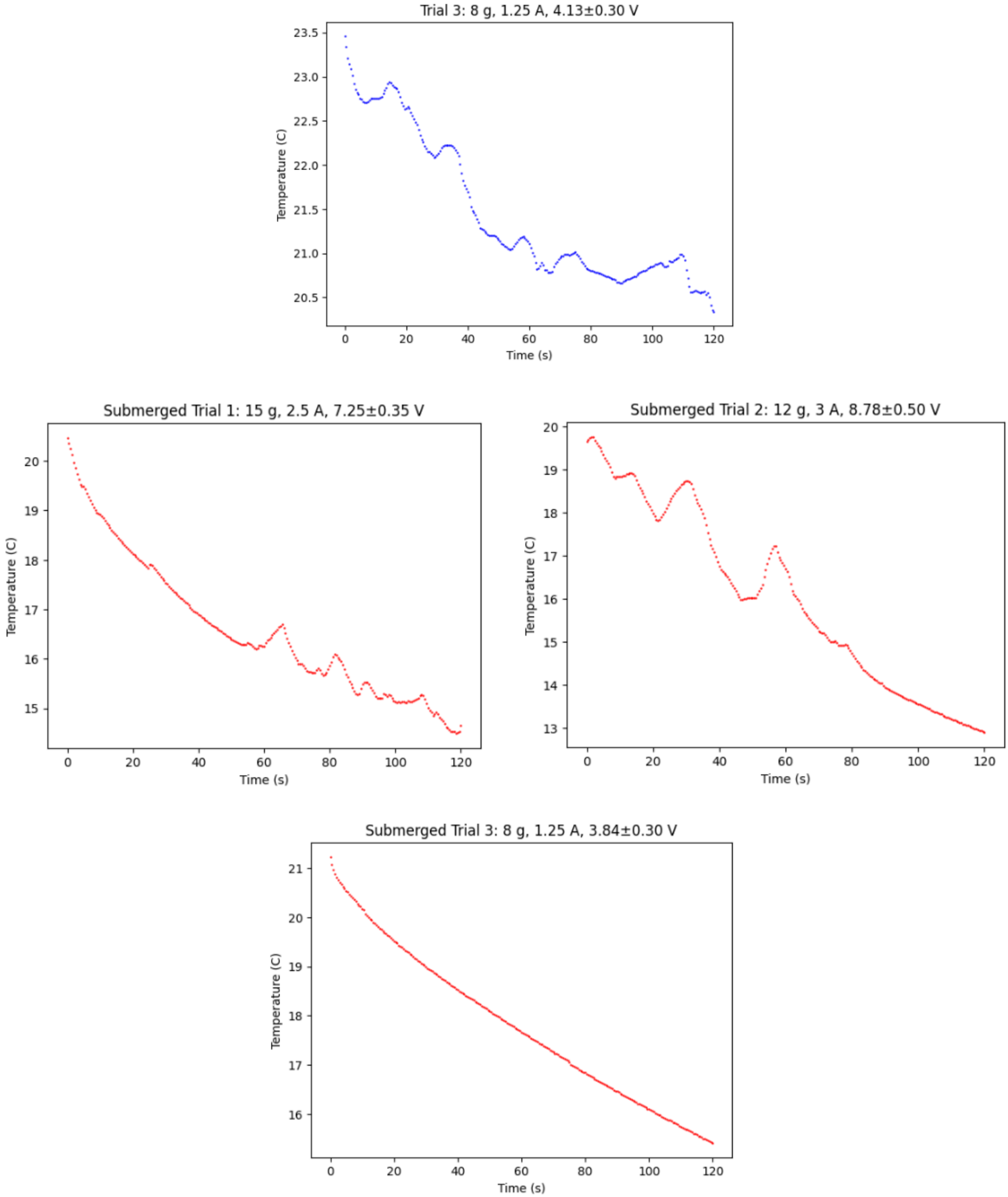


Figure 3.2.1 - 3.2.6: Plots of Temperature (C) over time

Before in depth analysis, we can see that the submerged trials are slightly more linear than the initial trials. Preventing the heat from flowing back into the water by submerging the heat sink may have had a stabilizing effect on the decreasing temperature of the water, which makes sense.

Trial 1
Calculated COP: 0.139 ± 0.010

Trial 2
Calculated COP: 0.098 ± 0.010

Trial 3
Calculated COP: 0.173 ± 0.025

Submerged Trial 1
Calculated COP: 0.167 ± 0.014

Submerged Trial 2
Calculated COP: 0.108 ± 0.011

Submerged Trial 3
Calculated COP: 0.337 ± 0.050

Analysis

Nominal COP Calculation

Before calculating any values using our data, we must determine the nominal COP of our Peltier device. According to the graphs provided by the manufacturer (see below), the COP fluctuates based on the input voltage and current.

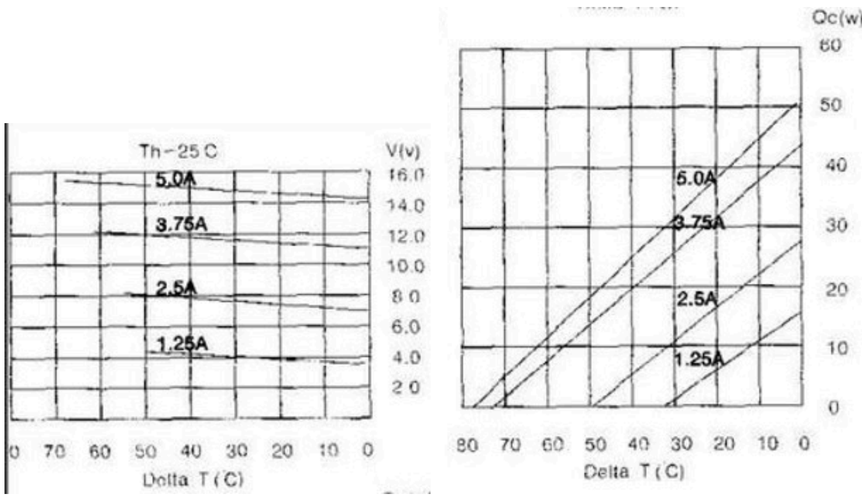


Figure 4.1.1 (L) & 4.1.2 (R): Temperature differentials caused by I/V pairings & Heat transfer rates based on I
Images from http://sciplus.com/1-9-16-square-peltier-junction-thermo-electro/

First, the temperature differential caused in the Peltier device by the input I/V pairing is located using figure 4.1.1. (For Trial 2 and Submerged Trial 2, which input 3 A, we estimate the line to be between the 2.5 A and 3.75 A lines). Then, the rate of heat transfer is determined by finding the intersection of the line and our desired temperature differential on figure 4.1.2. Next, the COP is calculated by dividing the heat transfer by the electrical work (I*V). Finally, the recorded fluctuations in voltage and uncertainty of the graph's units are propagated to find the nominal error. Using the appropriate input I/V for each trial, the results are as follows:

Trial #	**Nominal COP**
Trial 1	0.64 ± 0.06
Trial 2	0.53 ± 0.04
Trial 3	0.58 ± 0.20
Submerged Trial 1	0.72 ± 0.07
Submerged Trial 2	0.68 ± 0.05
Submerged Trial 3	0.63 ± 0.21

Experimental COP Calculation

Now our data must be analyzed to find experimental COP values. We begin with the most straightforward approach, which is simply to plug in the appropriate values into eq. 5. Using the corresponding mass of water, input I/V, and the recorded final/initial temperatures of the water, the following results are obtained:

Trial # (Input current)	**Experimental COP**
Trial 1 (2.5 A)	0.139 ± 0.010
Trial 2 (3 A)	0.098 ± 0.010
Trial 3 (1.25 A)	0.173 ± 0.025
Submerged Trial 1 (2.5 A)	0.167 ± 0.014
Submerged Trial 2 (3 A)	0.108 ± 0.011
Submerged Trial 3 (1.25 A)	0.337 ± 0.050

After the first 3 trials, it was clear that our experimental COP values do not come close to the listed manufacturer values. Like mentioned in the challenges section, we noticed that our heat sink rose to significant temperatures, which could retransfer heat into the Peltier plate and reduce the COP. Our solution was to submerge the heat sink partially in water to prevent it from heating up and repeat the trials.

Now looking at the COPs, there are two trends that must be noted. Firstly, every single COP for our submerged trial (with identical water mass and input I/V to non-submerged counterparts) is higher than the COP of corresponding initial trials. Although the submerged COPs are still not near the nominal values and heat retransfer can not be eliminated, our solution does seem to have the intended effect.

Secondly, the COP consistently decreases with higher input current for both initial and submerged trials. This is because higher input current causes more extreme temperature differentials, and leads to a greater amount of heat retransferred into the water from the hot side. This trend is visualized below.

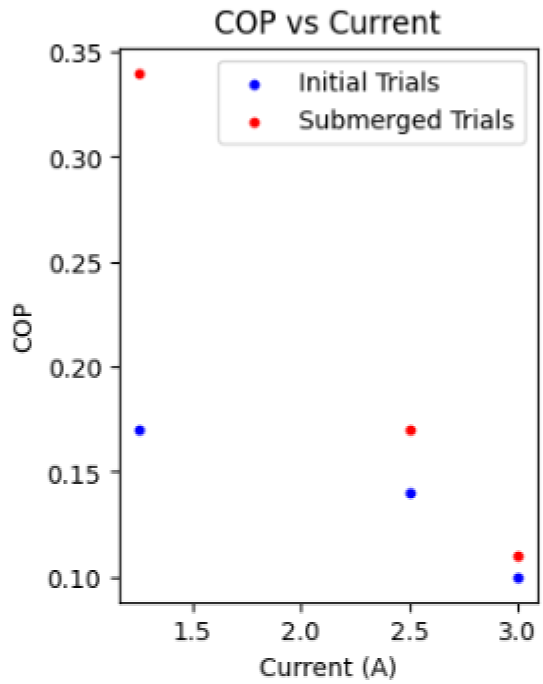


Figure 4.2: COP trends w.r.t. current

Experimental COP Adjustment: Thermal Conduction

For further analysis, the experimental COP will be adjusted to account for heat loss through the styrofoam walls. The rate of heat transfer for a given inner/outer temperature across a container can be determined using eq. 6. Since each data point corresponds to a different temperature, each point has a different rate of heat transfer. Using the fact that the time interval between each data point is 0.5 seconds, the total amount of heat lost during each data point can be calculated. These values are then summed to find the total heat loss throughout the trial, which will then be added to Q_c when calculating the COP through eq. 4.5.

To propagate error for the adjusted COP, there are multiple steps involved. First, the appropriate uncertainty in heat loss for each data point is propagated. All of these errors for each point are then added in quadrature to calculate the uncertainty in total heat loss. Lastly, the uncertainty in the adjusted COP is calculated using the total heat loss uncertainty and the electrical work uncertainty.

The results are summarized in the table below.

Trial #	**Adjusted COP**
Trial 1	0.140 ± 0.003
Trial 2	0.098 ± 0.003

Trial 3	0.173 ± 0.008
Submerged Trial 1	0.168 ± 0.004
Submerged Trial 2	0.108 ± 0.003
Submerged Trial 3	0.338 ± 0.015

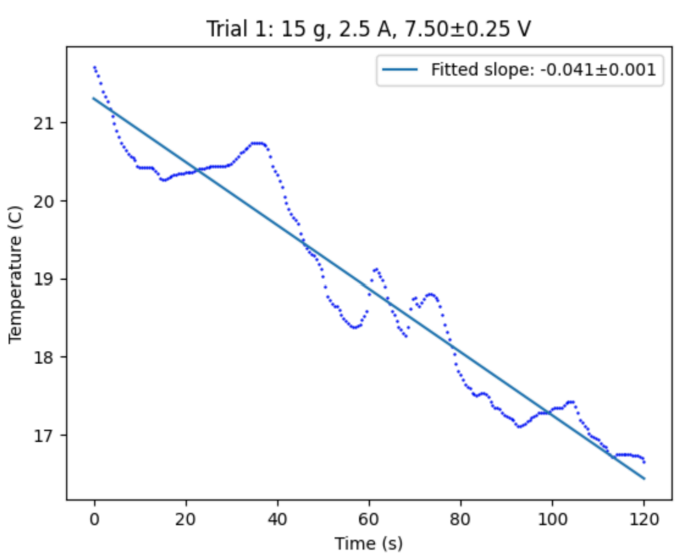
According to the agreement test (eq 9.), none of our adjusted COP values agree with nominal values.

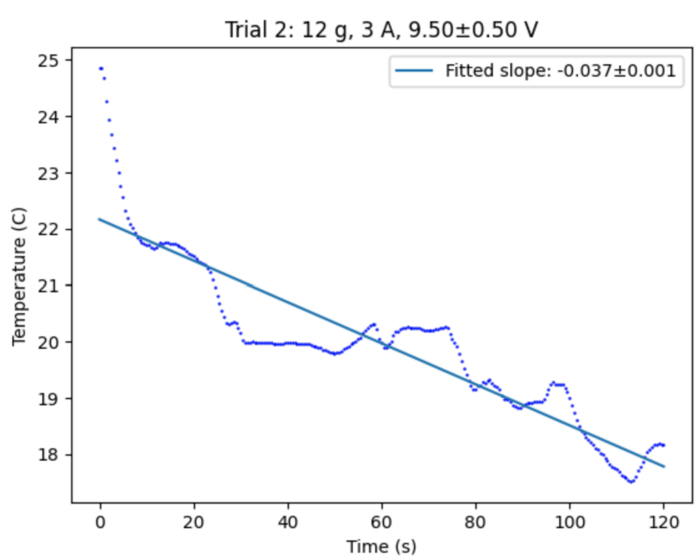
The adjusted COP values are nearly identical to the original experimental COP values. This indicates that the heat lost through the styrofoam walls is not large enough to significantly impact our measured COP. Considering the tiny surface area where the water actually touches the styrofoam, as well as a relatively small temperature differential (all trials less than 10℃ differential), insignificant heat loss seems reasonable.

Furthermore, the adjusted COP uncertainties are nearly an order of magnitude smaller than the initial COP uncertainties, which arises from our method of calculation. Since we sum the results of 120 slightly different heat transfer rates, the error in our sum gets much smaller due to a large number of data points.

<u>Alternative COP Calculation: Linear Model Fit</u>

In addition to our error analysis, we found it to be appropriate to develop a model to fit our COP using the temperature values in accordance with time, given by our LoggerPro device. Using these changing values, we can create a linear fit with the COP calculated through obtaining the slope and "unravelling" it (eq. 8). Plots of the fit labeled with fit parameters and uncertainties are shown below.





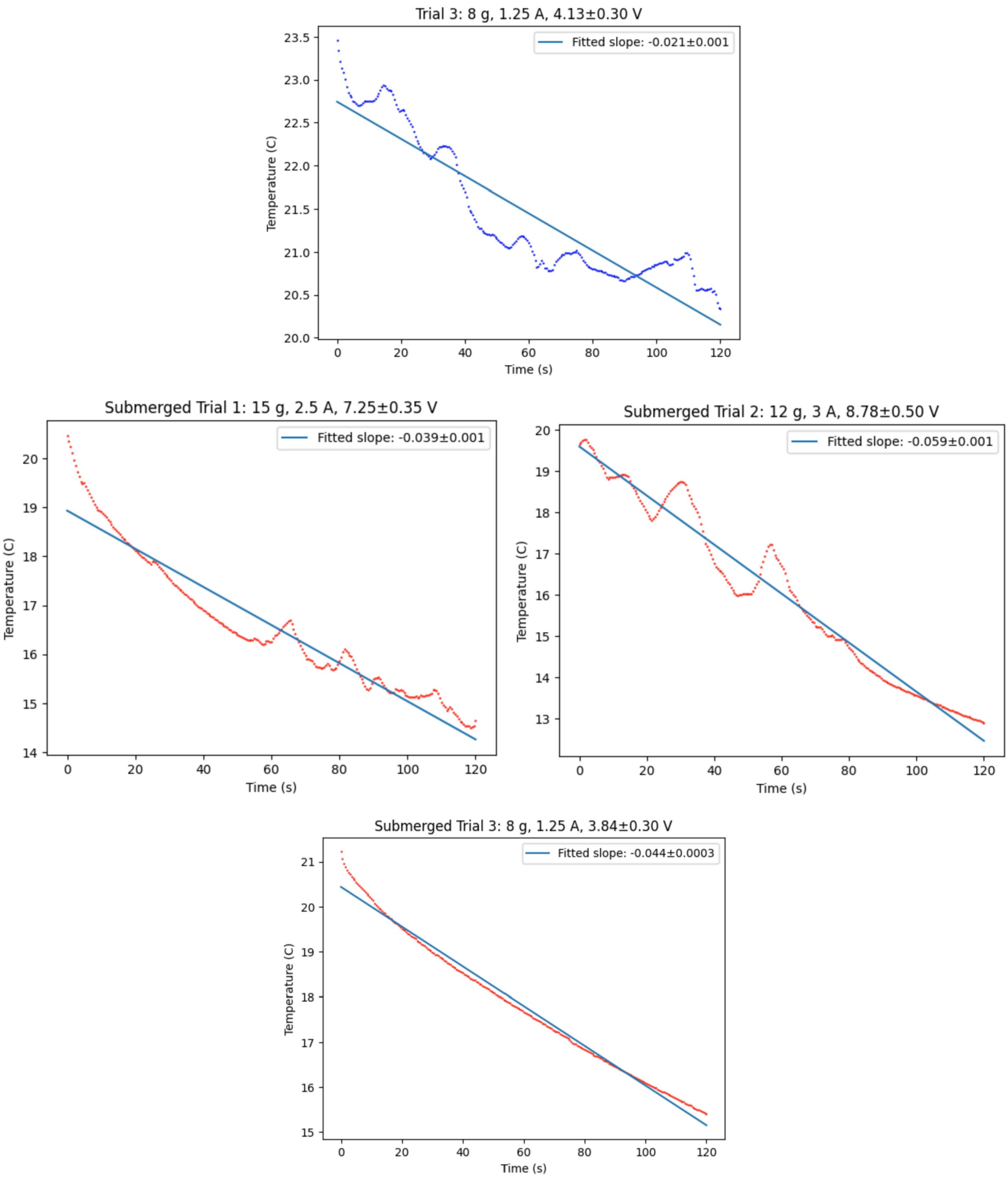


Figure 4.3.1 - 5.3.6

Results from the fit parameters after unraveling are shown below.

Trial #	**Fit parameter COP**
Trial 1	0.136 ± 0.005
Trial 2	0.064 ± 0.004
Trial 3	0.140 ± 0.011
Submerged Trial 1	0.135 ± 0.007
Submerged Trial 2	0.103 ± 0.007
Submerged Trial 3	0.307 ± 0.024

To assess our model fits, we can conduct a reduced chi-squared goodness of fit test.

$\chi^2_{\nu} = \frac{1}{\nu}\sum_{i=1}^{n}\left(\frac{y_i - f(x_i)}{\sigma_i}\right)^2$, where $\nu = n - p$ = DOF, y_i is the observed data point, $f(x_i)$ is the model's data point, and σ_i is the STD of each measurement. The chi-squared values are as follows:

Trial #	**Reduced chi-squared**
Trial 1	12.29
Trial 2	32.64
Trial 3	10.15
Submerged Trial 1	14.04
Submerged Trial 2	14.08
Submerged Trial 3	2.387

Summary

Each of our analytical approaches gave us a clear view on specific errors and how they impacted our data. Although many of the errors impact all of our approaches, our progressional analysis was very helpful in noticing the most significant and relevant ones.

A side by side comparison between our approaches and the nominal COP are shown in the table below.

Trial #	**Nominal COP**	**Adjusted COP**	**Fit parameter COP**

Trial 1	0.64 ± 0.06	0.140 ± 0.003	0.136 ± 0.005
Trial 2	0.53 ± 0.04	0.098 ± 0.003	0.064 ± 0.004
Trial 3	0.58 ± 0.20	0.173 ± 0.008	0.140 ± 0.011
Submerged Trial 1	0.72 ± 0.07	0.168 ± 0.004	0.135 ± 0.007
Submerged Trial 2	0.68 ± 0.05	0.108 ± 0.003	0.103 ± 0.007
Submerged Trial 3	0.63 ± 0.21	0.338 ± 0.015	0.307 ± 0.024

According to the agreement test (eq. 9), none of our results agree with nominal values.

Initial/Fourier Adjusted COP Results

The major realization from the initial approach was that our apparatus' errors had an opportunity to be accounted for quantitatively since we knew the dimensions of the places in which the heat was not perfectly insulated as first assumed. Referring to equation 5, we implemented our new total heat transfer, which is reflected in our COP values being marginally higher than our initial calculations.

However, there remain more qualitative sources of error (errors we can't quantify) that should be recognized. The first is that although our new power source doesn't have largely fluctuating voltages, it does fluctuate a very small amount. This does affect our results to a small degree and although we can't quantify it, it should be stated. Another source would be the imperfect contact between the plate and the Peltier Device. The copper plate isn't perfectly flat (it was supplied to us) and although it is approximately the same area as the device surface, it isn't perfect. This gives opportunity for heat from the device to not be conducted through the device into the plate, which can further explain our commonly lower Coefficient values. We can also consider that the heat in the water may not be fully uniform or doesn't have enough time to dissipate uniformly, which can lead to potentially higher temperature readings, which in turn decreases our observed COP.

Linear Regression COP Results

The linear regression model approach resulted in lower values compared to our previous calculations, but it also gave us an interesting insight into both the marginal and systematic errors not necessarily noticeable before.

Our reduced chi squared analysis gave us great insight into human error, particularly the shaking of the hand when we were holding our LoggerPro. Although we thought this may not affect much at first, we noticed some strangely large reduced chi-squared values in our models. With further look into our graphs and accounting for the exponential nature of the chi-squared analysis, we realized the fluctuations in the data (most likely caused by uncontrollable shaking movement, especially in the small space of the apparatus) caused significant contributions to the value. However, in the 3rd trial our data is smoother, which was most likely due to us realizing we can rest our wrists on a surface which stabilized the LoggerPro to be almost perfectly still. This marginal error can affect our fitted COP results in a major way and although they can't be easily quantified, they were insightful to acknowledge.

On the flip side, we noticed one last source of error, which we think was the highest contributor to the disagreement between our data and the nominal values. This was the fact that the Peltier Device we were using is quite old and has been used and experimented on many times. This means it could have gone through potential experiences in which the efficiency significantly dropped, such as electrical overload or physical damage, which we may never know. Based on the looks of it, the device seemed to have lived through many experiments in the past and this factor should be highly acknowledged.

Conclusion

Overall, our experiment allowed us to explore the fundamental concepts of Thermoelectric theory in practice and also the opportunity to learn how to account for and minimize sources of experimental error with limited resources. Our ultimate goal was to calculate the COP and compare with accepted values to the best of our ability, and throughout this lab, we were able to do so. Additionally we were able to learn how to sort out setup related nuisances, statistical challenges, and other unexpected features that arose from our seemingly straightforward experiment. Our general consensus from our experiments is that our nominal values did not match too well with our results, but through robust analysis with multiple different approaches, we were able to build reasoning for many sources of uncontrollable errors and applied techniques and acknowledged to accommodate/explain them. Our initial attempts had opportunities to reduce additional unexpected errors, such as the pre-submerged experiments, which allowed us to conduct better experiments with more accurate results. Additionally, we also conducted a linear regression model in addition to our comparative agreement tests to collect more COP's. With such a deep analysis, and obtaining results that were consistently lower than the nominal values, we were able to further infer the main sources of our systematic and human errors. We made the distinction between qualitative and quantitative errors, the former being things like the Peltier Device being old and used a lot and having imperfect contact with the copper plate, and the latter being nonperfect insulation, which we incorporated into our calculations in the analysis. Ultimately, this experiment was a major learning experience and allowed us to work through significant and relevant approaches and challenges that are faced in experimental physics.