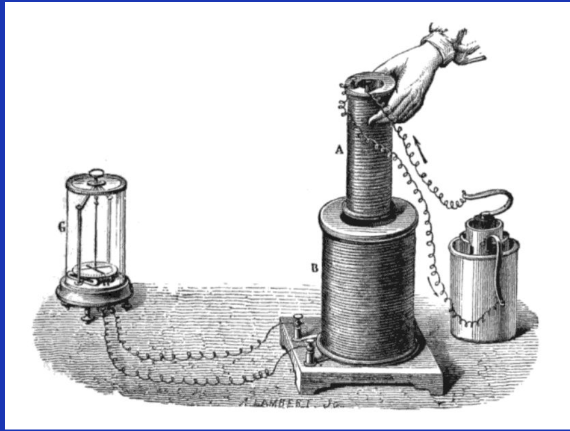
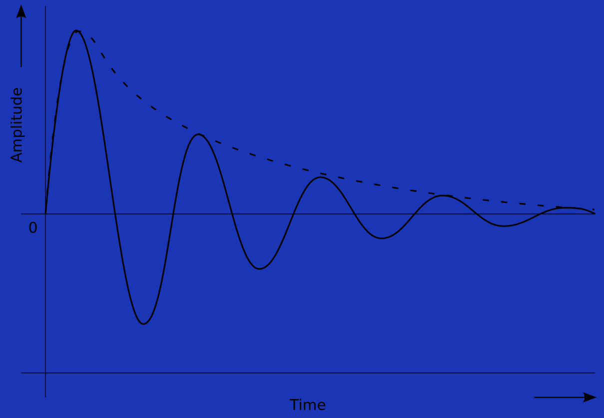
Investigating Fundamental Laws of Electromagnetism using a Magnetic Harmonic Oscillator





Yajur Preetham and Tyler Jung
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Abstract

- Our experiment combines Faraday’s Law, Lenz’s Law, Ohm’s Law, and damped harmonic oscillations
- Our physical setup includes a large coil connected to a closed circuit with a resistor, and an iOlab with magnets on one end connected to a spring on a dowel
- Oscillating the iOlab allows for electromagnetic induction to occur and results in damped motion due to the induced EMF/magnetic field in the coil, which theoretically depends on the resistance of the resistor

Motivation

- After learning about electromagnetic induction and circuits in 5B, we were interested in integrating it with our previous knowledge in damped harmonic oscillation, which we learn in 5A
- We hoped to learn about and get more experience in going through the whole intricate process of conducting a physics experiment relating to electromagnetism and oscillations
- Ultimately, we wanted to experimentally characterize the relationship between the resistance of the resistor and the damping coefficient of the oscillator, and compare it to theoretical predictions

Theory

Faraday's/Lenz's Law

$$V = -N\frac{d\Phi_B}{dt}$$

Ohm's Law

$$V = IR$$

Linear Velocity-Dependent Damped Oscillations

$$F_{damping} = \gamma v$$

Damping Coefficient Relation

$$\gamma = -\frac{LAN}{R}B_z\frac{dB_z}{dz}$$

Design and Methods

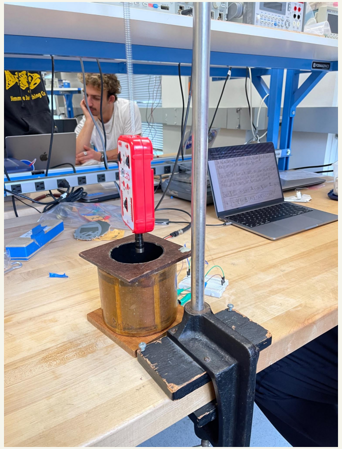
Our ***experiment*** is setup as follows:

- The iOlab is linked to a spring on one end and magnets on the other end, and hung on a dowel
- Directly below it, the large coil is placed so that the center is lined up with the magnets
- The ends of the coil are connected to a resistor (which will be our changing variable) and are supported by a breadboard

Link to video of experiment:
https://drive.google.com/file/d/1opZsZ0f3PU6uDIc2zqmu2ioB-BnWXcAI/view?usp=sharing

Our ***procedure*** is as follows:

- Prepare the iOlab for data taking (connect to computer)
- Pull iOlab down so that magnets are surrounded by the coil, then gently let go, ensuring movement only in the y axis
- Receive data for 30 sec, then switch to different resistances and repeat



Challenges

We faced a few challenges at many levels of our project:

- The oscillations (or movement overall), should only be in the z-axis and the theory works only if the oscillator is lined up in the center of the coil. Since we were releasing with our hands using our eyes, this was hard to perfectly achieve. To mitigate this error, we marked a line on the pole to line up the iOlab when pulling it down and made sure only one person (Tyler) was doing it.

- The experimental method/procedures assumed that B_z*dB_z/dz was roughly constant because of the consistency of the initial conditions of the oscillator and partially because the theory behind B_z was hard to develop within the timeframe. In reality, this value was not constant across all experiments, as is expected due to the field's dependence on z, which in turn varies differently in time across the experiments. We acknowledge that this will play a non-trivial role in the relationship between ɣ and R, which we won't account for. Still, we will be able to see the correct characteristic relationship between the damping and resistance through the analysis methods we developed!

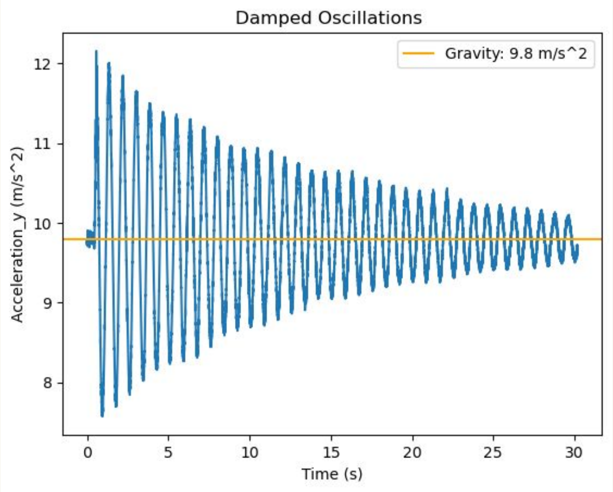
Data Reduction

The data:

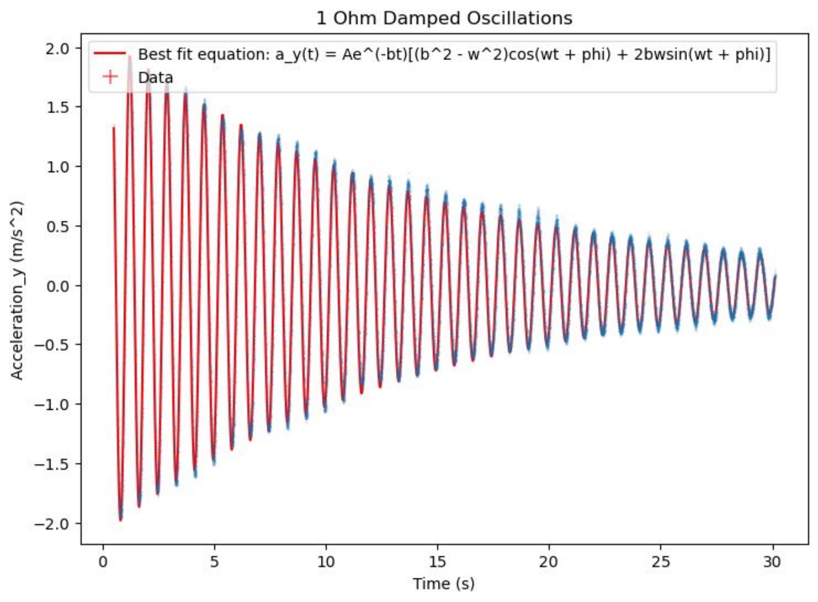
- We recorded acceleration in the y (z) direction of the iOlab over time in each experiment, corresponding to the oscillatory motion

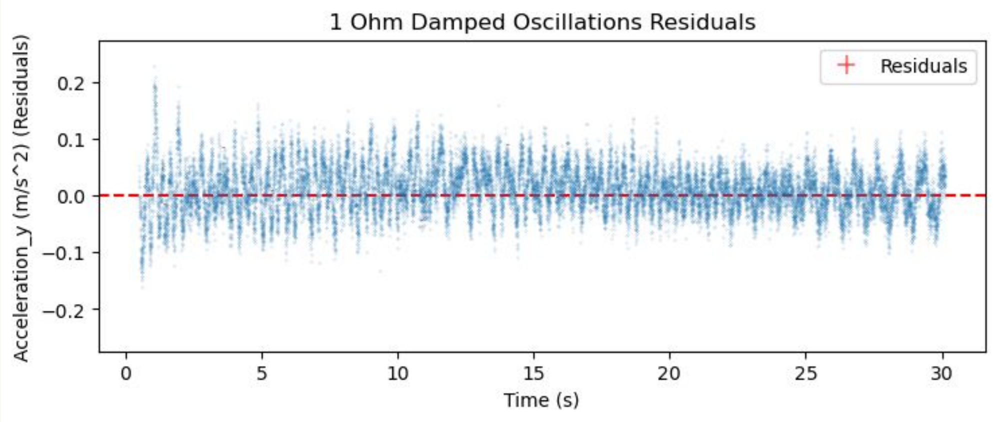
Use of the data:

- For each resistor value, we plotted the acceleration as a function of time, and then fit the relevant damped oscillation model to the data, obtaining a fitted value for ɣ. (Using scipy.optimize.curve_fit)
- In the end, we plot ɣ against R and analyze the resulting relationship characteristically (visually/intuitively) as well as by fitting a function of the form A/R where A is a constant and seeing how well the 1/R proportionality manifests itself.

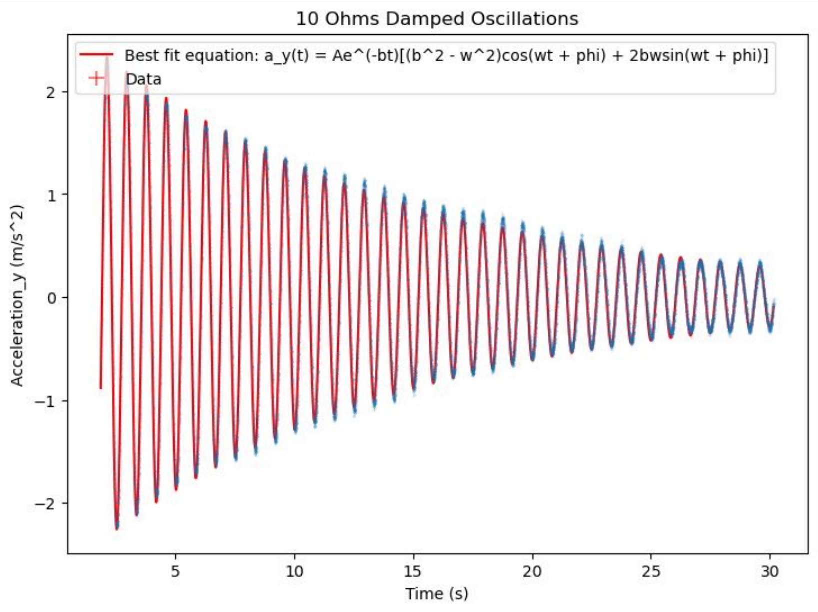


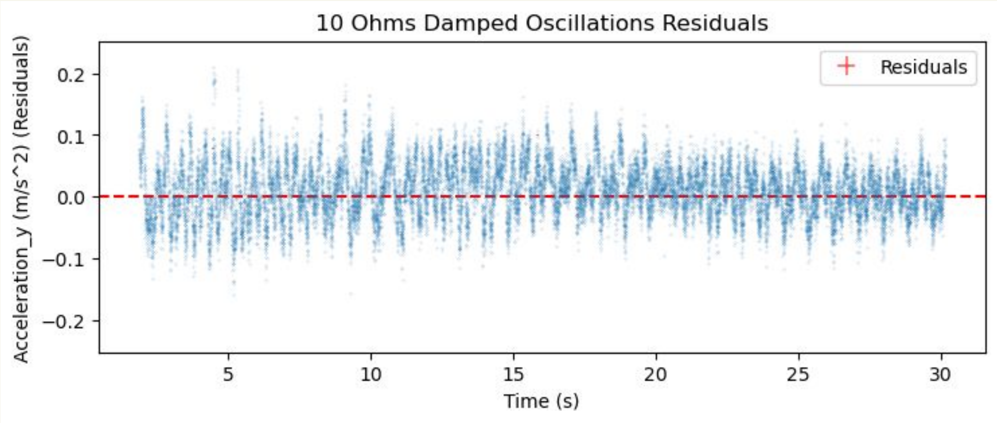
$\gamma = 0.0350 \pm 0.0001$ kg/s



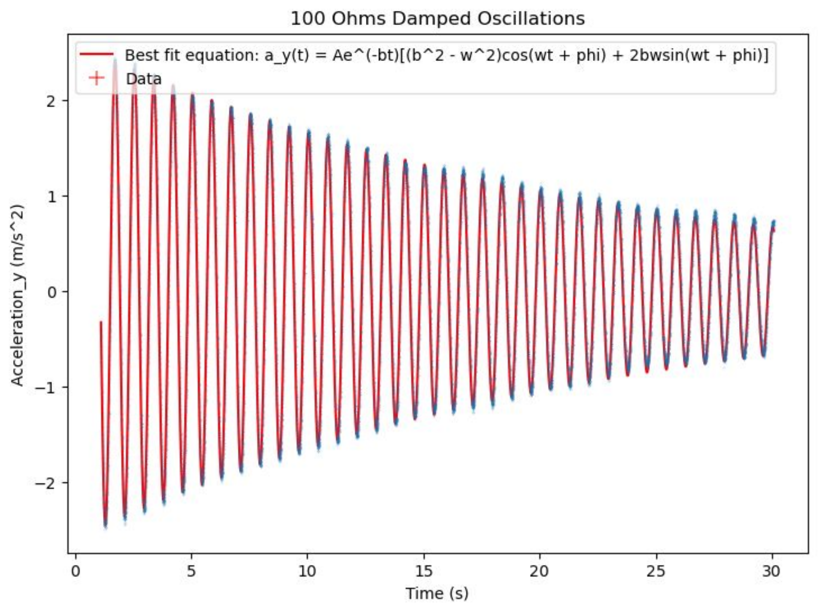


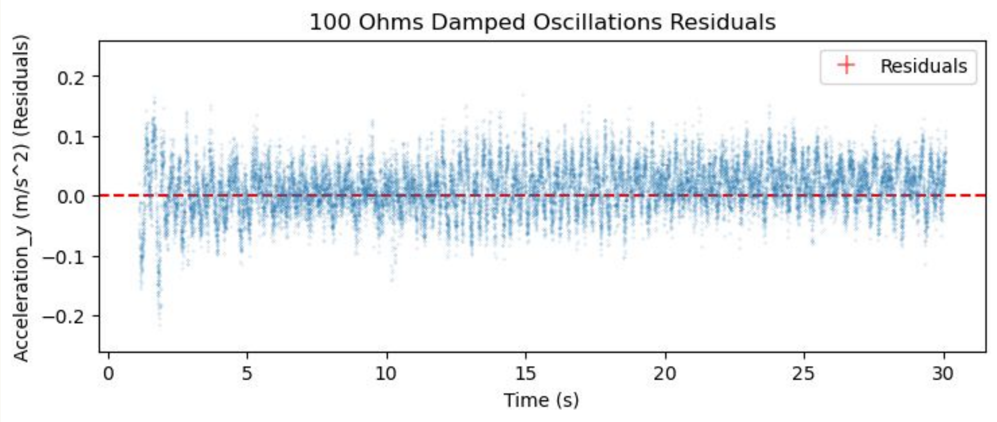
ɣ = 0.0363 ± 0.0001 kg/s





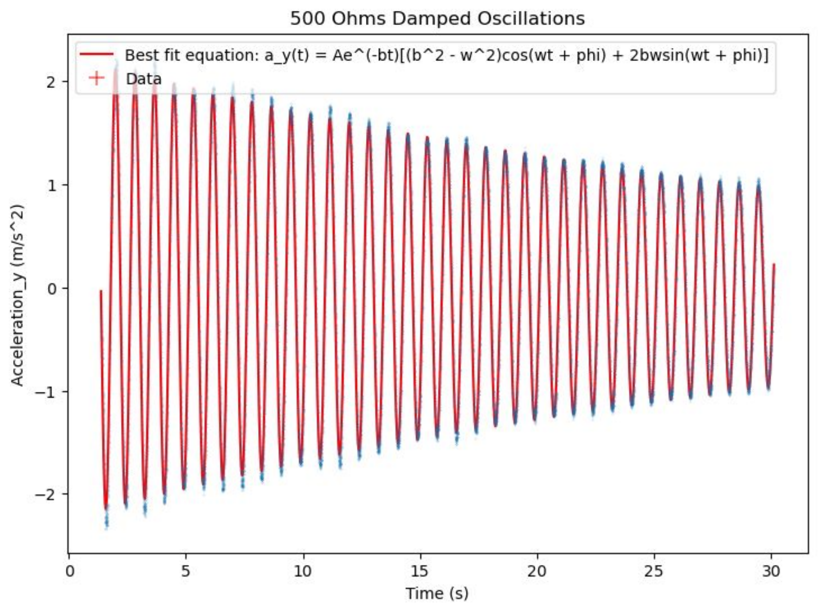
ɣ = 0.0221 ± 0.0001 kg/s

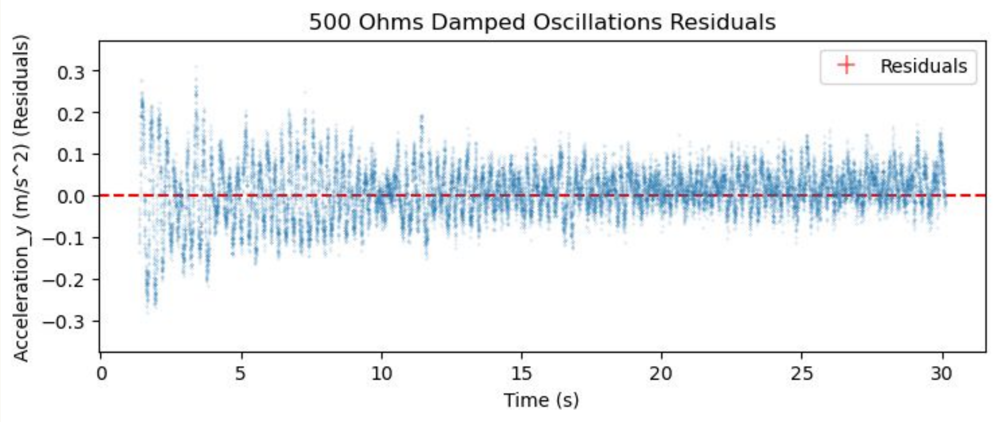




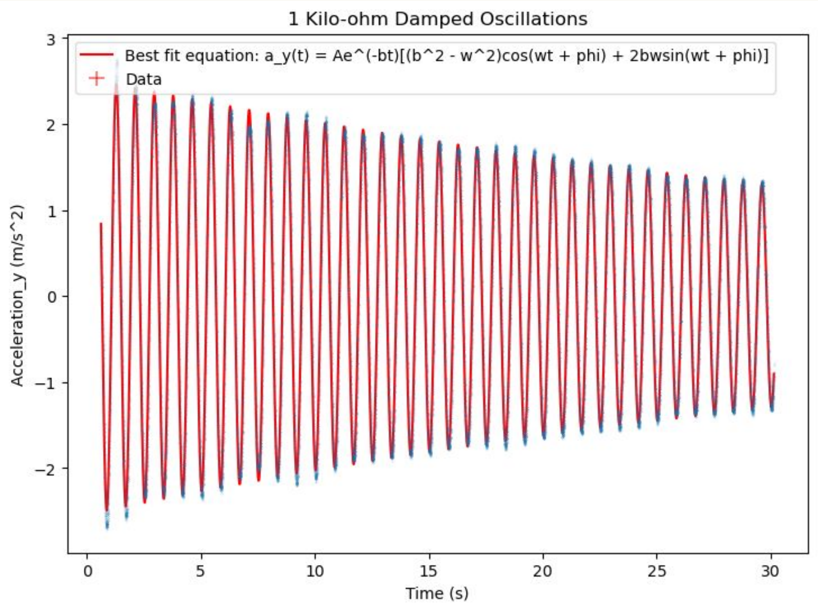
Confidential

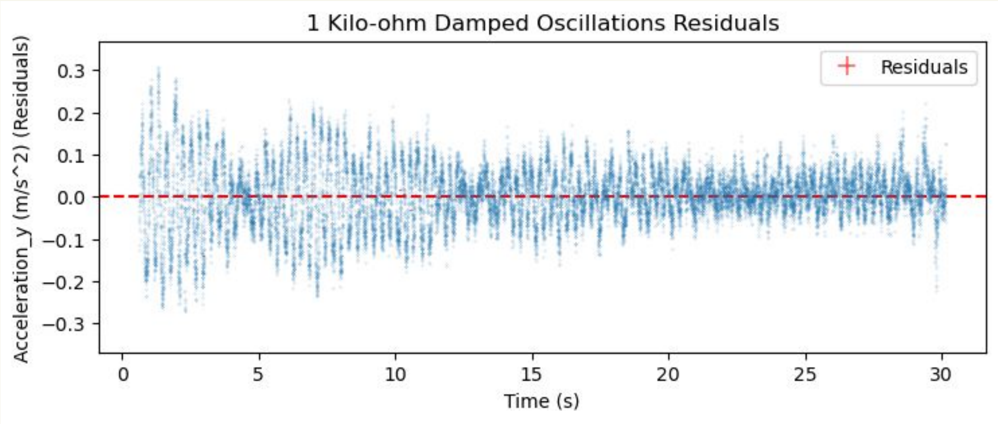
ɣ = 0.0137 ± 0.0001 kg/s



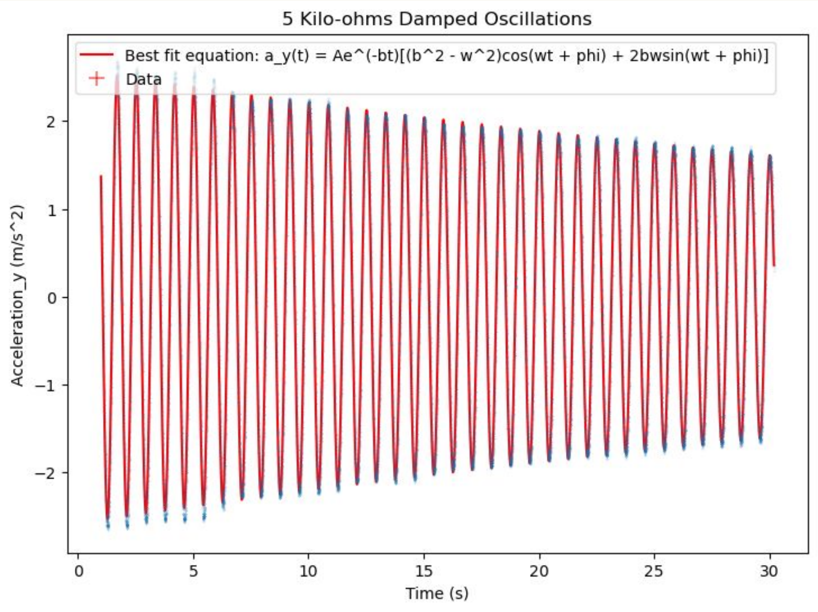


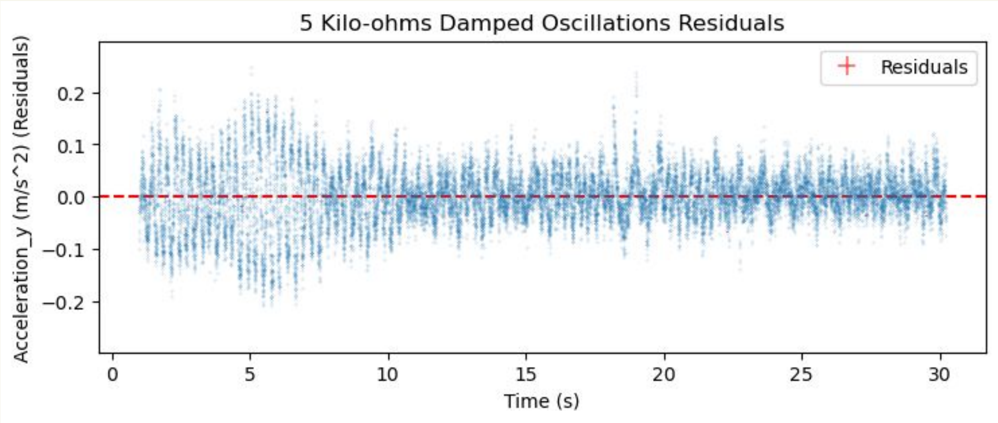
$\gamma = 0.0110 \pm 0.0001$ kg/s



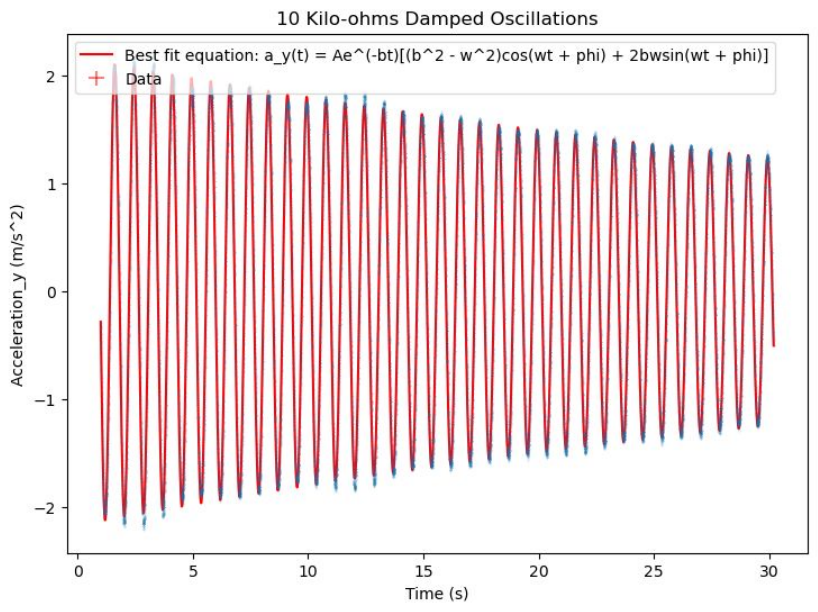


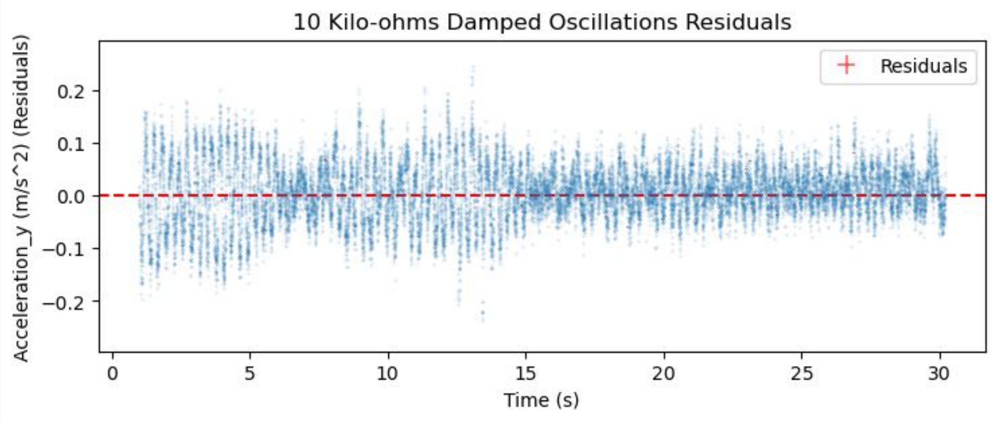
$\gamma = 0.0077 \pm 0.0001$ kg/s





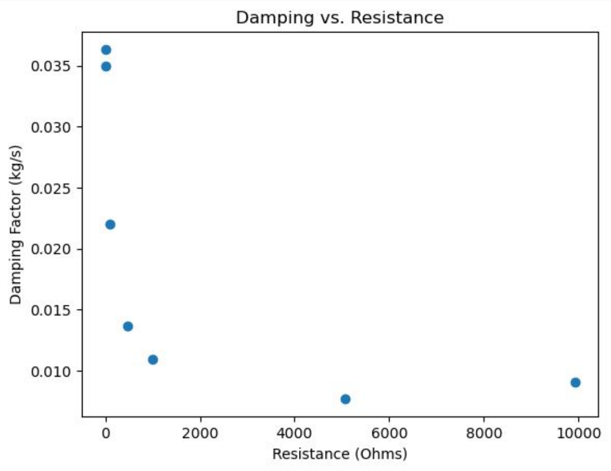
$\gamma = 0.0091 \pm 0.0001$ kg/s



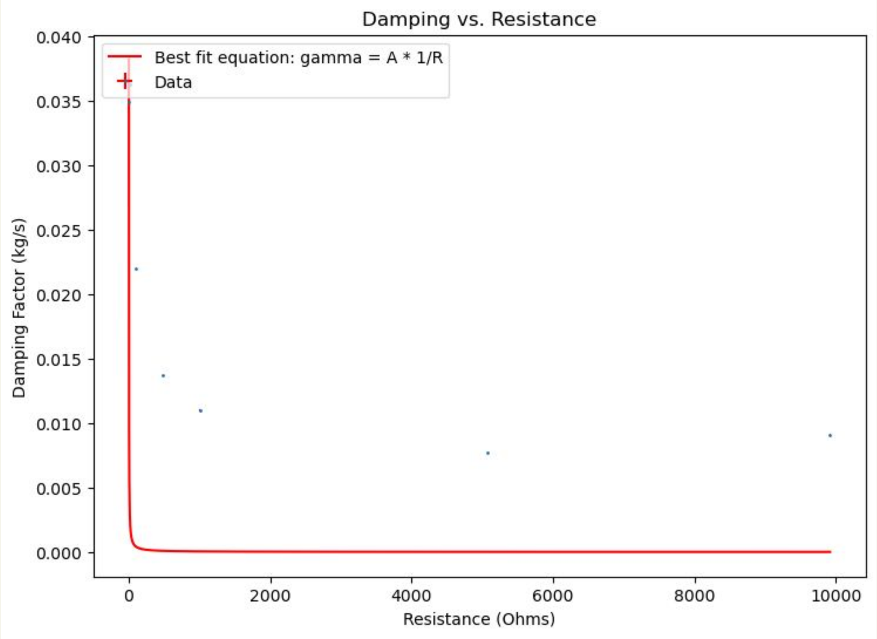


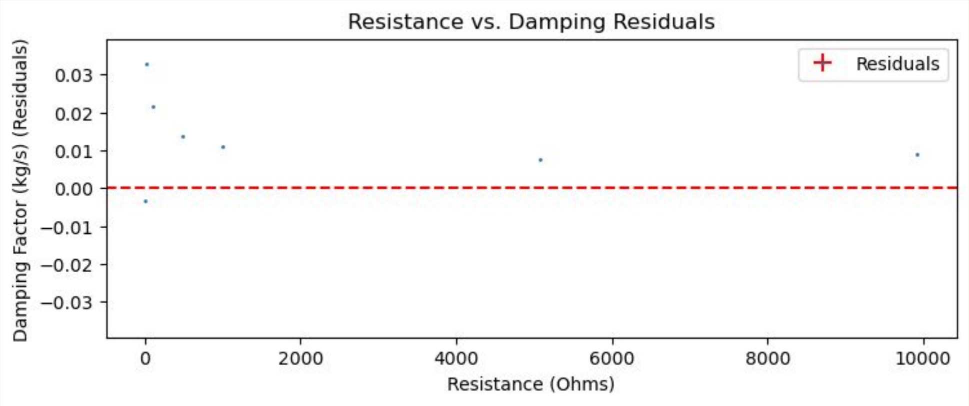
Analysis

- We see that characteristically, the theory is confirmed in that the ***damping decreases as the resistance increases***
- Furthermore, not only does ɣ decrease as R increases, but it does so in a sort of ***"sweeping" curve reminiscent of a 1/R curve***
- The actual fitting of a 1/R curve (times some constant factor) to the data is not that successful, which can be explained by a combo of:
  - Limitations of the theory and methodology developed
  - Errors in experimental procedure
  - No consideration of the non-ideal nature of the coil, breadboard, and connections



A = 0.0421 ± 0.0076 (ohms * kg/s)





Future Directions

Advice for future students:

- Build your own robust coils so that you can control aspects of it such as length, number of loops, and surface area, opening new doors for analysis
- Further develop the theory to account for the functional form of the magnetic field and its dependence on position/time

How to expand/improve this project:

- More attention can be paid to theoretically predicted values rather than relationships by careful control/measurement of coil properties
- Collect more data across a vaster range of resistors
- Find the function of best fit for OUR experimentally collected data, not necessarily 1/R, and try to study it

Summary and Conclusion

- Our experiment netted results expected by our theory
- We observed and analyzed the motion of our iOlab to be damped harmonic oscillations from our goodness of fit and consistent residuals following our model
- We also found that our damped coefficient is sort of inversely proportional to the resistance, which was further supported through multiple different resistances and developing a model. Although the model was not completely accurate to the 1/R graph.