The Magnetic Harmonic Oscillator

Yajur Preetham and Tyler Jung

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Abstract

This exploration into fundamental electromagnetic laws aims to support expected theoretical results of electromagnetic induction and utilizes a magnetic harmonic oscillator and large coil to do so. Through derivation, we combined Faraday's Law, the Lorentz Force, and Ohm's law, as well as the theoretical mechanics behind damped harmonic oscillation to show a naive proportional relationship between γ (the damping coefficient) and one divided by the resistance of the resistor integrated with the coil system. However, through further analysis, we found our expected proportionality to be $(\frac{1}{R})^p$, and experimentally found our p to be approximately 0.15. Our experiment uses an iOlab device with magnets on one end, connected to a spring on a dowel for oscillations and a large coil for it to go through, which is connected to a closed circuit with the resistor bridging the 2 ends of the coil (which we changed to show the proportional relationship). Our analysis netted results similar to the expected theoretical inverse relationship after model fitting. Just as our theory predicted, we can conclude that the damped motion of the oscillator can be attributed to the inverse resistances of the resistor for the coil circuit. Since this implementation extracts a multitude of theoretical bases, it supports the accepted general electromagnetic principles.

0. Introduction

Our project stems from our curiosity in electromagnetic induction and the theory behind oscillatory motions. As students who have taken/are taking Physics 5B and 5A, we wanted to take knowledge from these classes and apply it to a real world application with tangible results that can help us confirm the expected results behind Faraday's Law, Ohms' Law, etc.

Additionally, through this process, we hope to achieve a better grasp of our conceptual understanding on these topics. Although we have done problem sets and visualized drawings of examples of electromagnetic induction in action, we have rarely seen an isolated example where the effects are so significant and realizable. Another inspiration for this is our desire to combine mechanics with electromagnetism through amalgamating the damped harmonic motion theories with Faraday's Law. Although we see this more frequently, deriving it into a result that associated resistance and the damping coefficient was a very interesting idea that we wanted to explore, as it solidifies the existence and potency of both sides of physics, particularly as students.

As for what we hoped to gain from conducting this experiment, we mainly hoped to learn from the previously mentioned conceptual understanding, but also experience in data collection, analysis, and modelling. As our dataset is very large, we had many ways to interpret our findings and decided to measure particularly the damped coefficient as a function of resistances. We were able to create a model fit that could show what the expected data should be, and if it accurately represented our theory.

Ultimately, this project allowed us to bridge theoretical physics concepts with tangible real-world applications. By exploring the interplay between electromagnetic induction and damped harmonic motion, we deepened our understanding of Faraday's Law, Ohm's Law, and the relationship between resistance and damping. The experiment not only reinforced key concepts from our Physics courses but also provided valuable experience in data collection, analysis, and modeling. Our findings, particularly the model fit correlating resistance and the damping coefficient, validated our theoretical predictions and demonstrated the synergy between mechanics and electromagnetism. This hands-on exploration has enriched our conceptual grasp and inspired further curiosity in the subject.

1. Theory and Background

The theory governing this experiment will combine Faraday's/Lenz's law, the Lorentz force, the definition of magnetic flux, Ohm's law, and the description of damped harmonic oscillations produced by a linear velocity-dependent damping force

1.1 - Faraday's Law and Lenz's Law (Electromagnetic Induction)

Suppose you had a coil of conductive wire with cross-sectional area A and number of coil turns (or loops) N. Now, position the coil vertically up on the z-axis and oscillate a magnet towards and away from it along its symmetric vertical axis. What you are doing is effectively making a magnetic field move through the coil's surface through time, which is akin to subjecting the coil to a change in magnetic flux over time. According to Faraday's law, this induces a voltage, or EMF, within the coil according to this equation: $V = -N\frac{d\Phi_B}{dt}$, where the negative sign is a result of Lenz's law, which states that the induced voltage in turn produces a current through the coil, which in turn produces a magnetic field from the coil that opposes the original change in magnetic flux. The derivative in the equation is the derivative of the magnetic flux through the coil with respect to time.

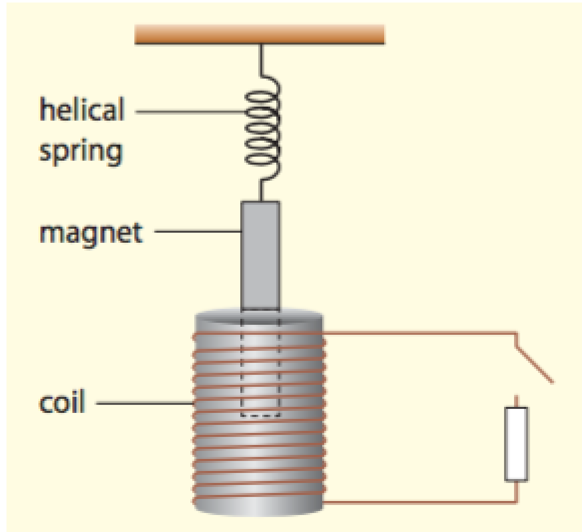

Figure 1.1.1: A visual representation of the vertically aligned coil and oscillating magnet.

Now, we consider the definition of magnetic flux, specifically for a loop of conductive wire: $\Phi_B = BA$. We can add onto this further by assuming that the magnetic field produced by the magnet, for our purposes, will be strictly a function of vertical position, which is a valid assumption given the symmetries of the system and magnets we will be using (cylindrical magnets). So the flux now becomes a function of z: $\Phi(z) = B(z)A$. We can now substitute this definition of the flux into the Faraday's law equation, and using the chain-rule we get that: $V = -N\frac{d}{dt}(B(z)A) = -NA\frac{dB}{dz}\frac{dz}{dt} = -NAv\frac{dB}{dz}$. Here, we made use of the fact that dz/dt is nothing but the vertical velocity of the oscillating magnet, v.

1.2 - The Lorentz Force

The Lorentz force is the force that acts on a charge as a result of its motion relative to an electromagnetic field. As we are only dealing with magnetic fields here, we consider the magnetic part of the Lorentz force: $F = qv \times B$. Where F is the force vector acting on the charge, v is the velocity vector of the charge, and B is the magnetic field vector. For a segment of wire of length L and current flowing through it I (in this case, a segment of wire in our coil), we can rewrite the force as: $F = IL \times B$, where I is a scalar magnitude and L is a vector pointing in the direction of the flow of current. This gives us an expression for the force acting on the coil from the magnet. According to Newton's 3rd law, if the magnet acts a force on the coil, the coil acts a force back on the magnet with equal magnitude and opposite direction. The mechanism by which the coil acts a force on the magnet is through generating an opposing magnetic field due to the induced EMF/current as discussed in the prior section. Once the symmetries of the geometry are accounted for, we get that the direction of the force acting on the magnet as it oscillates up and down is perfectly vertical due to the magnet being positioned in the center of the cylindrical coil. Furthermore, we can simplify the expression for the magnitude of the force on the coil (and consequently the force on the magnet) further by assuming that the induced current is perpendicular to the magnetic field, which makes sense given the geometry of the setup, and thus the cross product disappears: $|F| = |ILB(z)sin(90°)| = |ILB(z)|$.

<u>1.3 - Damped Harmonic Oscillations</u>

Now, we make the assumption that for the small oscillations of the magnet, the "friction-like" force acting on it, which is, as seen before, in opposition to its movement, can be expressed in the form: $|F| = \gamma|v|$. Here, γ is known as the damping coefficient and v is the velocity of the magnet. This is a valid assumption because, as we saw previously, the induced EMF/current in the coil is proportional to the velocity of the magnet, and hence we can infer that a similar proportionality should arise in the damping of the oscillations of the magnet.

If this is indeed the case, we know from the T1.M7 lab on Hooke's law and harmonic oscillators that the resulting closed-form solution for the magnet's vertical acceleration as a function of time is given by this equation:

$a(t) = Ae^{-\beta t}[(\beta^2 - \omega^2)cos(\omega t + \phi) + 2\beta\omega sin(\omega t + \phi)]$ $(Eq.\ 1.3.1)$, where

$\beta = \gamma/2m$ ($Eq.$ 1.3.2), where m is the mass of the magnet, and $\omega = \sqrt{\omega_0^2 - \beta^2}$, where $\omega_0 = \sqrt{k/m}$ is the frequency of oscillation, with k being the spring constant of the spring the magnet is oscillating on. We will use this model for the acceleration of the magnet in our data analysis section later to experimentally derive values for the magnet's damping coefficients using a non-linear least-squares regression fit from the Python library scipy.optimize.curve_fit.

1.4 - Ohm's Law

Finally, we will be able to tie everything together and obtain a useful representation of the damping coefficient by considering Ohm's Law. Before we do so, we will add one additional component to our general experimental setup which is outlined in Figure 1.1.1. Instead of the two ends of the coil being directly connected to each other, what we will be doing in our experiment is connecting the two ends with different resistors in between them. Thus, our experiment will primarily deal with the role the resistor plays in determining the damping of the magnetic harmonic oscillator.

Moving on, we can equate the magnitude of the force acting on the magnet as a consequence of Newton's 3rd law and the Lorentz force to the magnitude of the theorized damping force acting on the magnet. This yields the following equation: $\gamma|v| = |ILB(z)|$. Now, we consider Ohm's law, which states that the voltage across a resistor is equal to the current flowing through the resistor multiplied by its resistance: $V = IR$. Equivalently, $I = V/R$. Now, we can substitute this expression for I into the force equivalency from above, assuming that the current flowing through the coil is the same as the current flowing through the resistor, and that the EMF induced in the coil is the same as the voltage across the resistor, and that the resistance of the coil is negligible. After the substitution, we get that: $\gamma|v| = \frac{|V|}{R}|L||B(z)|$.

Finally, if we substitute in the equation we obtained in Section 1.1 for the induced voltage across the coil, V, then we get that: $\gamma|v| = \frac{NA|v|}{R}|\frac{dB}{dz}||L||B(z)|$. After cancelling out the velocities on both sides of the equation and rearranging, we obtain this final equation for the magnitude of the damping coefficient: $|\gamma| = \frac{|LAN|}{R}|B(z)\frac{dB}{dz}|$ $(Eq.\ 1.4.1)$.

Crucially, we notice that the damping coefficient is at first superficially proportional to the inverse of the resistance of the resistor. However, the magnetic field is a function of position, and the position of the oscillator will somehow depend on the resistance as well. Given the complexity of the theory and the timeframe of this experiment, we were unable to fully solve the closed-form expression for the magnetic field or its derivative. Instead, we will conduct our experiment assuming that everything in the expression for the magnitude of the damping coefficient above is constant except for the resistance of the resistor, which is the only variable we could control. In the end, while doing our data analysis, we will see how far off this assumption is from reality, and we will also try to experimentally find the proportionality $|B(z)\frac{dB}{dz}|$ must have with resistance up to some power in order for our experimentally collected data to make sense. What this will manifest as in the data is that the damping won't be proportional to 1/R per se, but rather 1/R^p, where p is some positive real number. All in all, the main thing we expect from the expression for the damping coefficient and our intuition is that as resistance increases, the damping must decrease. This is because there will be less current flowing through the coil, thus leading to a weaker induced opposing field, thus leading to less damping in the magnetic harmonic oscillator's motion.

2. Experimental Setup (Design and Methods)

Our experiment had multiple possibilities when it came to the physical setup and by working through and brainstorming practical ideas, we were able to flesh out the experiment we believed would be best for the data we were looking for.

Ultimately, our setup required a way for resistance to be interchangeable and a way for harmonic motion to be detected and visualized. As explained above, our initial theoretical assumption was that the damping coefficient γ is inversely proportional to resistance R. This means we want our experimental variables to be γ and R, where γ is a function of R, which we will manually change. However, it is important to note that our setup must include a closed loop that includes the resistance of which the magnetic field will operate on, as our theory suggests. Since there are many parts of the setup, we will break them down into 2 sections.

2.1 - The oscillator

We initially thought of the possibility of a horizontally moving oscillator, where it would move along the table, but that would require us to account for the friction in our damped harmonic oscillation calculations, which would be suboptimal. We decided an easier and more intuitive way to proceed was to have the oscillations vertical so it would be possible to put on a spring that would hang (Figure 1.1.1). This would also allow for the theoretical predictions to be more accurate. We also decided on using the iOlab's accelerometer to track our motion. However, we also required the production of a magnetic field, specifically one that would move with the oscillating iOlab, so we landed on magnets, which conveniently stuck onto the bottom of the iOlab. Our final setup for the oscillator was as follows:

1. Setup a clamp and dowel in which the spring can hang off of 45.5 centimeters up the pole.
2. Attach the magnets to the bottom of the iOlab and the spring to the top, then hang on the dowel. The whole oscillator was 243.9 ± 0.1 grams.
3. The iOlab's data was received by the computer, which can convert it into a csv file.

2.2 - The coil

The coil setup was more complicated and we ran into more challenges. Our initial idea was to create a coil ourselves in which the oscillator will go through. This coil had a diameter of 9.4 centimeters and 13 loops. This created some issues because there were not enough loops for a significant emf to be produced. Additionally, this setup was flimsy and needed tape to keep the coils together, a shoe box to hold the coils up, and longer wires to connect to the breadboard. This resulted in noisy and meager results that would be very difficult to work with. For a reasonable scale for our experiment, we required something with significantly more coils, larger, and more robust. Fortunately, the lab provided us with exactly what we needed: a large, thick coil that was supported by a wood frame. With this, we were able to create a large enough emf to work with significant data. This also created some issues however. Most importantly, our experiment was now exclusive to just the relationship between resistance and the damping coefficient because we now have to treat variables like area constant. Additionally, due to the infinitesimal nature of new coil, we are unable to find the number of loops, which is also a constant, and this makes our experiment look for the proportionality of the damping coefficient and resistance, and develop a predicted value for the remaining variables, which we also assume constant. Regardless, this made our data collection much easier and straightforward. The final setup for the coil is as follows:

1. Place the large coil directly under the oscillator so that the center of the magnets are directly above the center of the coil area. This ensures that the magnetic field produced fully affects the oscillator since the vector component of the field is vertical at that point (according to our derivations) and allows for avoiding deflections.
2. Connect wires to the coil on one end and into the breadboard on the other. Place a resistor between the two on the breadboard.
3. Connect another set of wires to the oscilloscope. This ensures that during our procedure and while the oscillator is moving, we are receiving expected results and to also compare with our iOlab data.

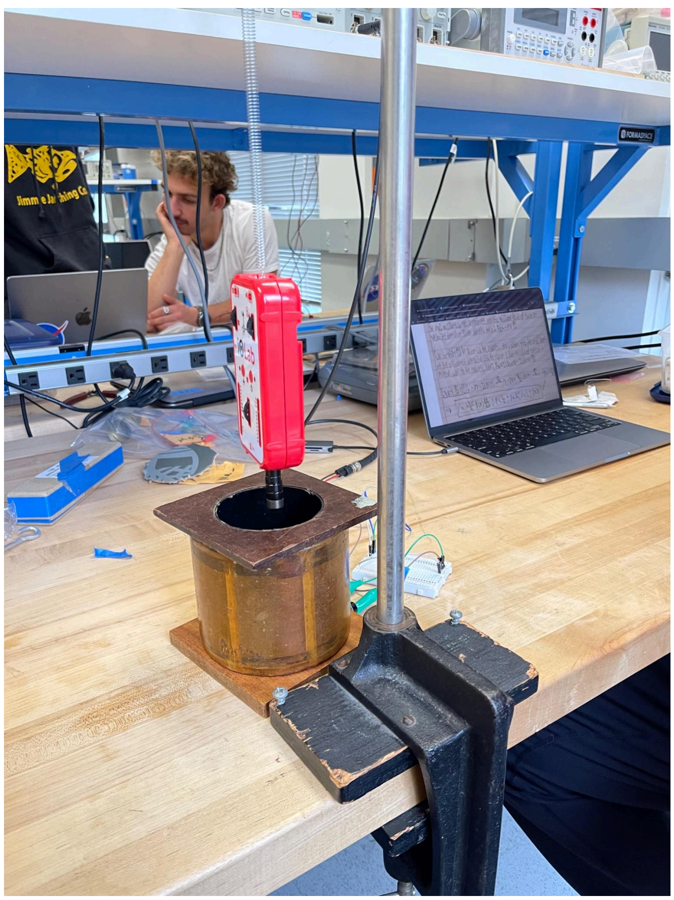

Figure 2.1.1: Our final experimental setup ready for data collection.

2.2 - Procedural Methods

Our procedure now mainly consists of changing out the resistors and pulling down the oscillator for its motion. However, we found many ways to reduce potential errors.

Many of the possible errors come from the human nature of the pull down. By this, we mean how the iOlab/magnets must first be displaced from equilibrium to move in an oscillating motion, but we had no consistent and mechanical system to do this. Therefore, we resorted to using our hands, which could cause some systematic errors. To reduce this, we ensured the following:

1. Placing a piece of tape on the pole in where the top of the iOIab can be referenced to before letting go. This was placed 19 centimeters above the base of the pole. This makes sure the release distance from equilibrium is the same every time. It also ensures that the magnet fully is surrounded by the coil at one point.
2. Pulling the iOlab down with two fingers on the top. This makes sure that there are no sideways oscillations that may occur due to the inconsistent pulling from the hand.
3. Having one person look from the top view to make sure the iOlab is lined up correctly. This reduces error in other axes of unwanted motion.
4. Having one person do all the pulling. If different pull, it could cause some potential variances in the data.

Ultimately, all of these were to reduce motion in axes in which we did not want motion (anything other than the vertical axis). Following the development of these rules, our final procedure was as follows:

1. Test a resistor using the DMM to check its resistance and precision error. Then insert this resistor into the breadboard. (The first one is without any resistor). Log this data.

2. Adjust the oscilloscope resolutions so the sine motion fits on the screen. Log this data.
3. Pull down the oscillator (following all the above rules), then release lightly.
4. Wait for 30 seconds, then end the data collection. Save this data as a csv file and name accordingly.

Another potential issue we may have been affected by was the overall resistance produced by the parts we used, such as wires. However, this impact was seemingly minimal and we were not able to obtain more advanced parts to account for this.

*Link to a video of our experiment in action:

https://drive.google.com/file/d/1UsrUWh7qh1txF6C0TYfJSPGv_ULBuB4a/view?usp=sharing

3. Data Collection

Following the procedures above, we obtained our dataset, which we made some preliminary inferences from. Overall, our data supported our theoretical findings. As resistance increases, we observed the general trend of the damping to decrease, which is the inverse relationship we expected. As mentioned above, we also adjusted the oscilloscope resolutions in order to see the sinusoidal motions more clearly and observe our expected damping. This is also important because we can't see the change in acceleration be visualized real time as our experiment is operating, we needed another method to confirm damping oscillatory decay. Also, throughout all the data we implemented a **60 kHz** noise filter. Our choices for resistance values were: **No resistance, 1 ohm, 10 ohms, 100 ohms, 500 ohms, 1 Kilo-ohm, 5 Kilo-ohms, 10 Kilo-ohms**. However, we tested our resistors with a DMM, and found the values to be slightly off in some cases. For our analysis we used the actual observed resistances and the errors (obtained from the DMM's precision) instead of the labeled resistance. Again, our data was collected through the iOlab instrument and was uploaded for analysis as csv files.

The oscilloscope and other important data are shown below, and when we mention “resolution” of the oscilloscope, it refers to the settings of the vertical and horizontal scale knobs.

No resistance (cleaner data)

Resistance (white wire): **0.8 ± 0.1 ohms**

Resolution: **5 mV and 2 seconds**

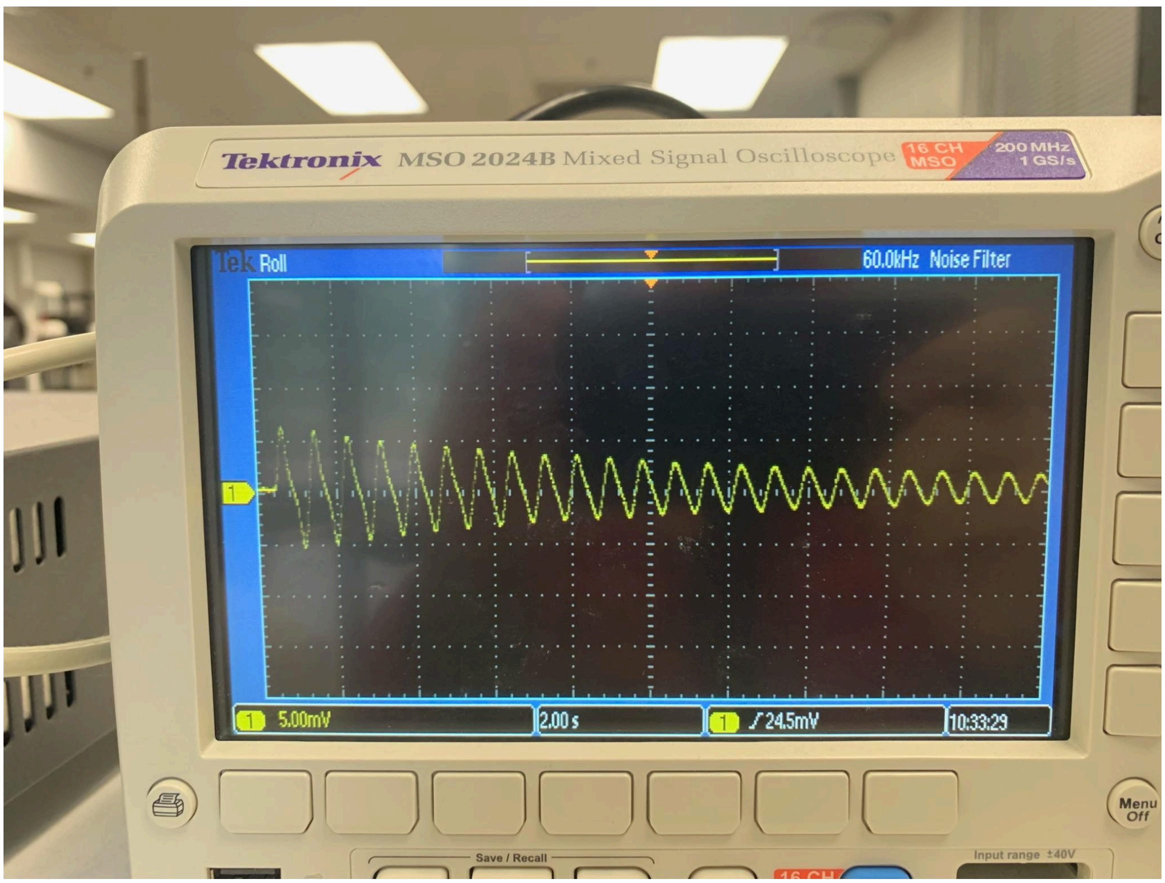

Figure 3.1.1: Oscilloscope graph for no resistance. Resolution information is above.

1 Ohm

Observed Resistance: **1.1 ± 0.1 ohms**

Resolution: **5 mV and 2 seconds**

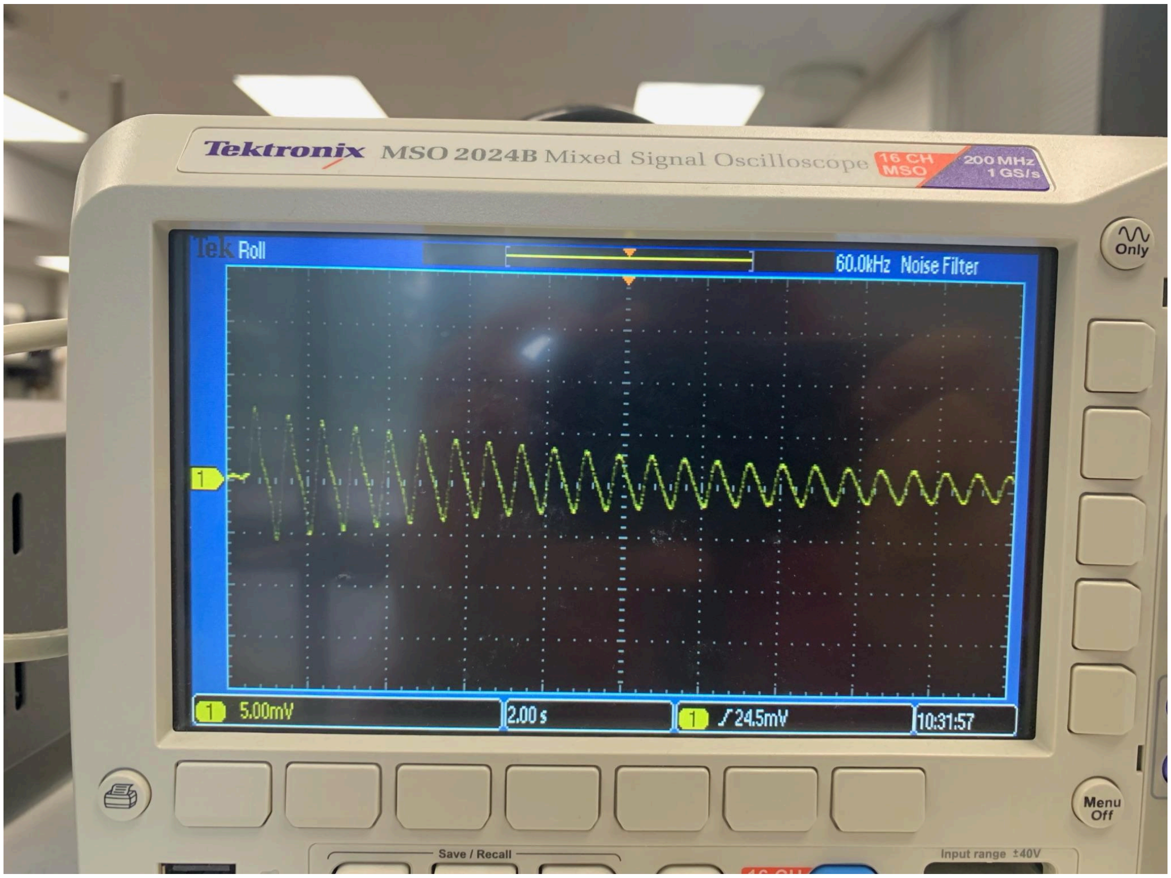

Figure 3.1.2: Oscilloscope graph for 1 ohm. Resolution information is above.

10 Ohms

Observed Resistance: **12.1 ± 0.1 ohms**

Resolution: **50 mV and 2 seconds**

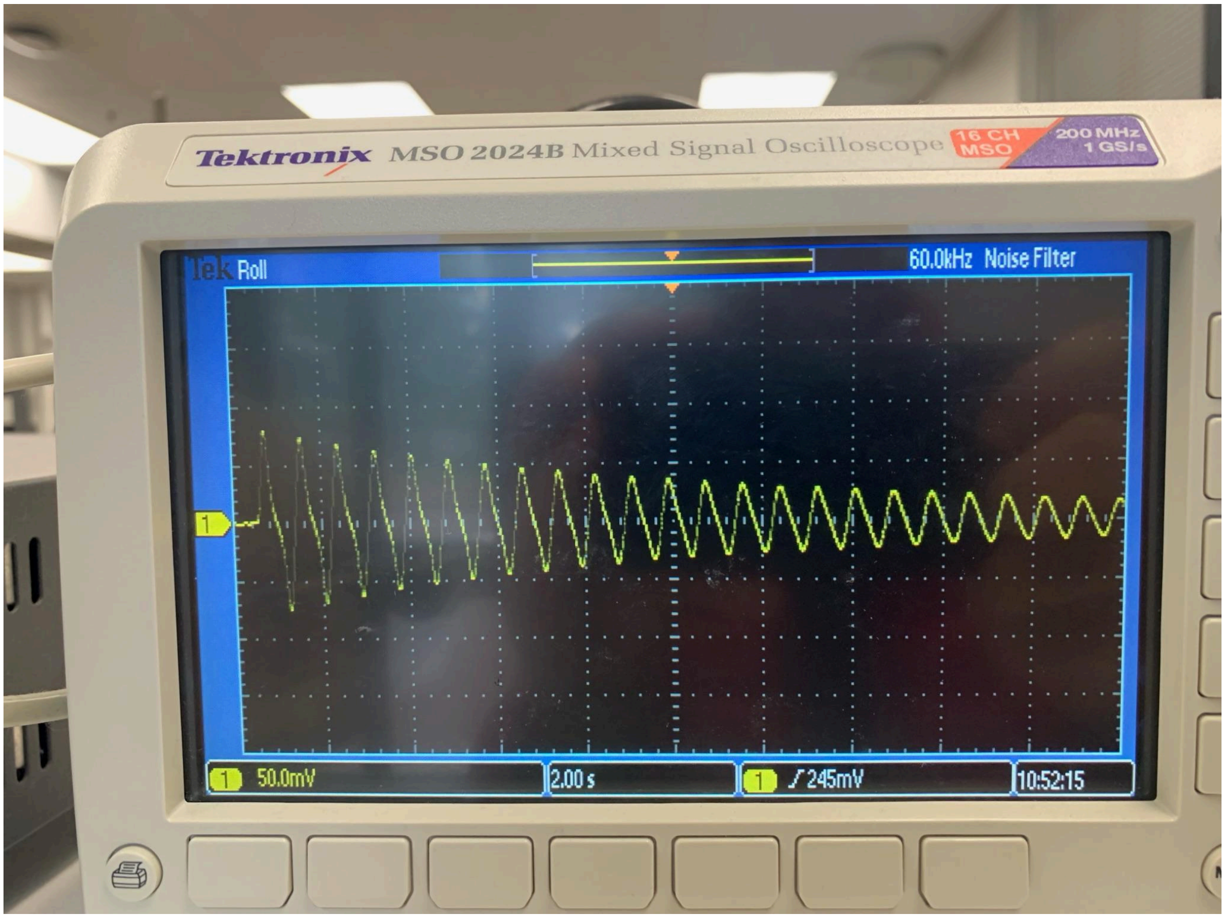

Figure 3.1.3: Oscilloscope graph for 10 ohms. Resolution information is above.

100 Ohms

Observed Resistance: **100.3 ± 0.1 ohms**

Resolution: **200 mV and 2 seconds**

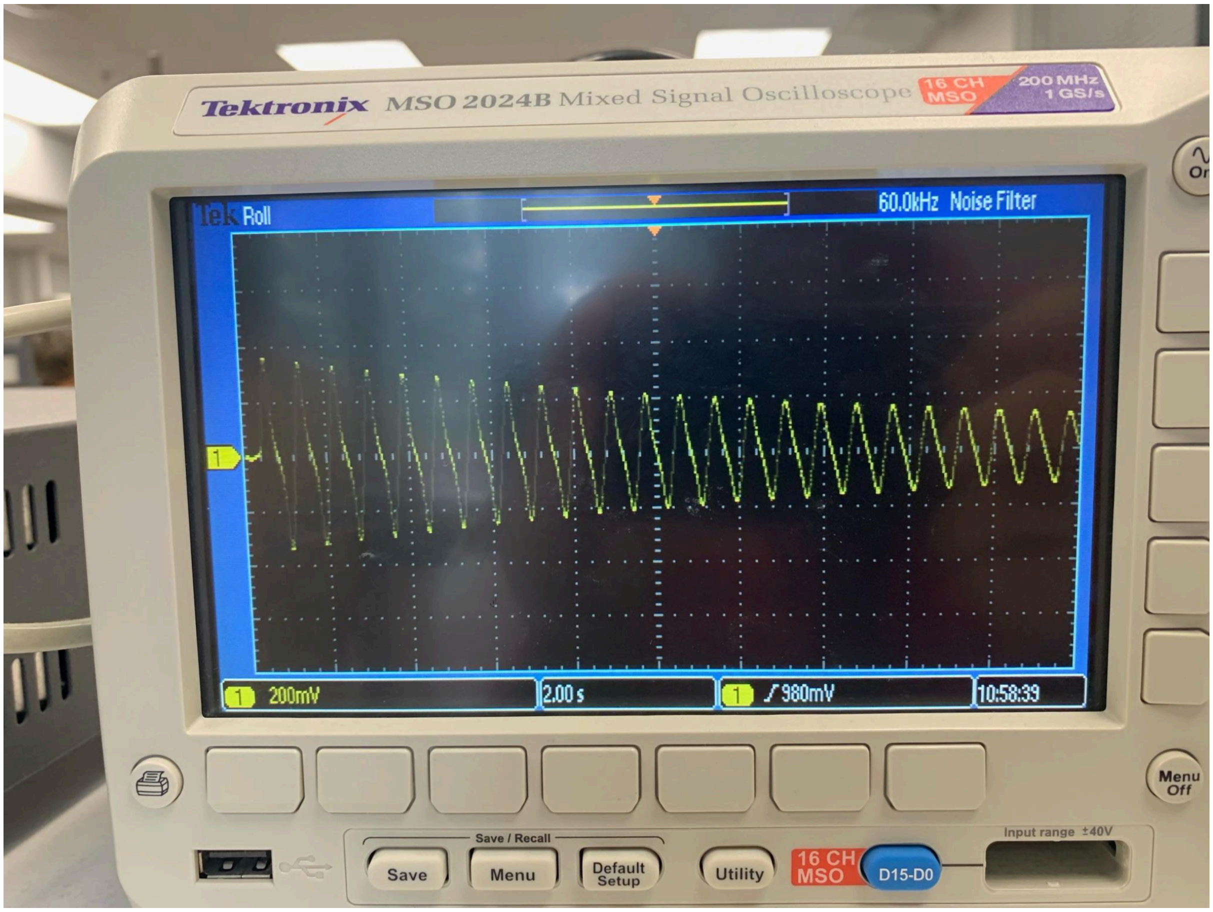

Figure 3.1.4: Oscilloscope graph for 100 ohms. Resolution information is above.

500 Ohms

Observed Resistance: **477.3 ± 0.1 ohms**

Resolution: **200 mV and 2 seconds**

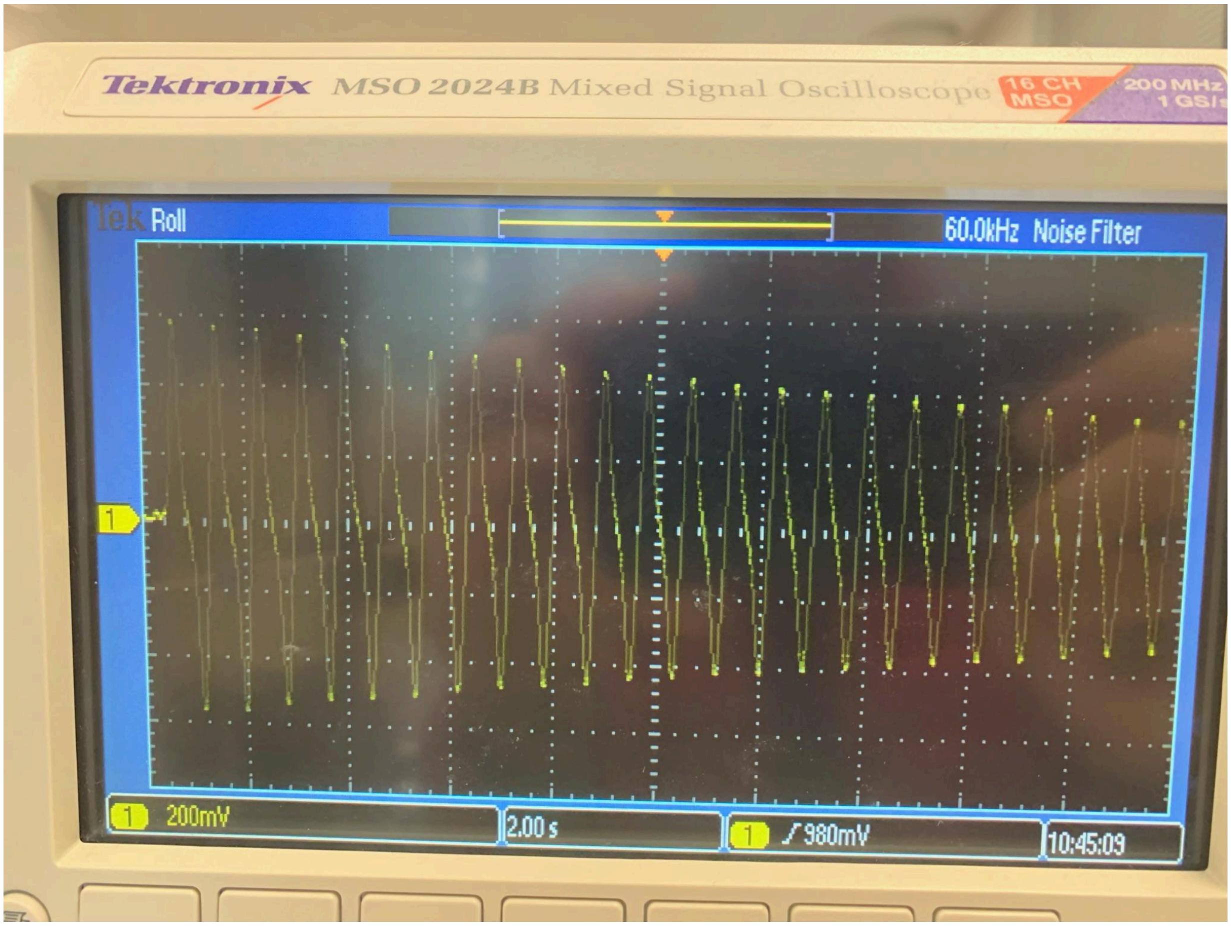

Figure 3.1.5: Oscilloscope graph for no resistance. Resolution information is above.

1000 Ohms

Observed Resistance: **1.003 ± 0.001 Kilo-ohms**

Resolution: **200 mV and 2 seconds**

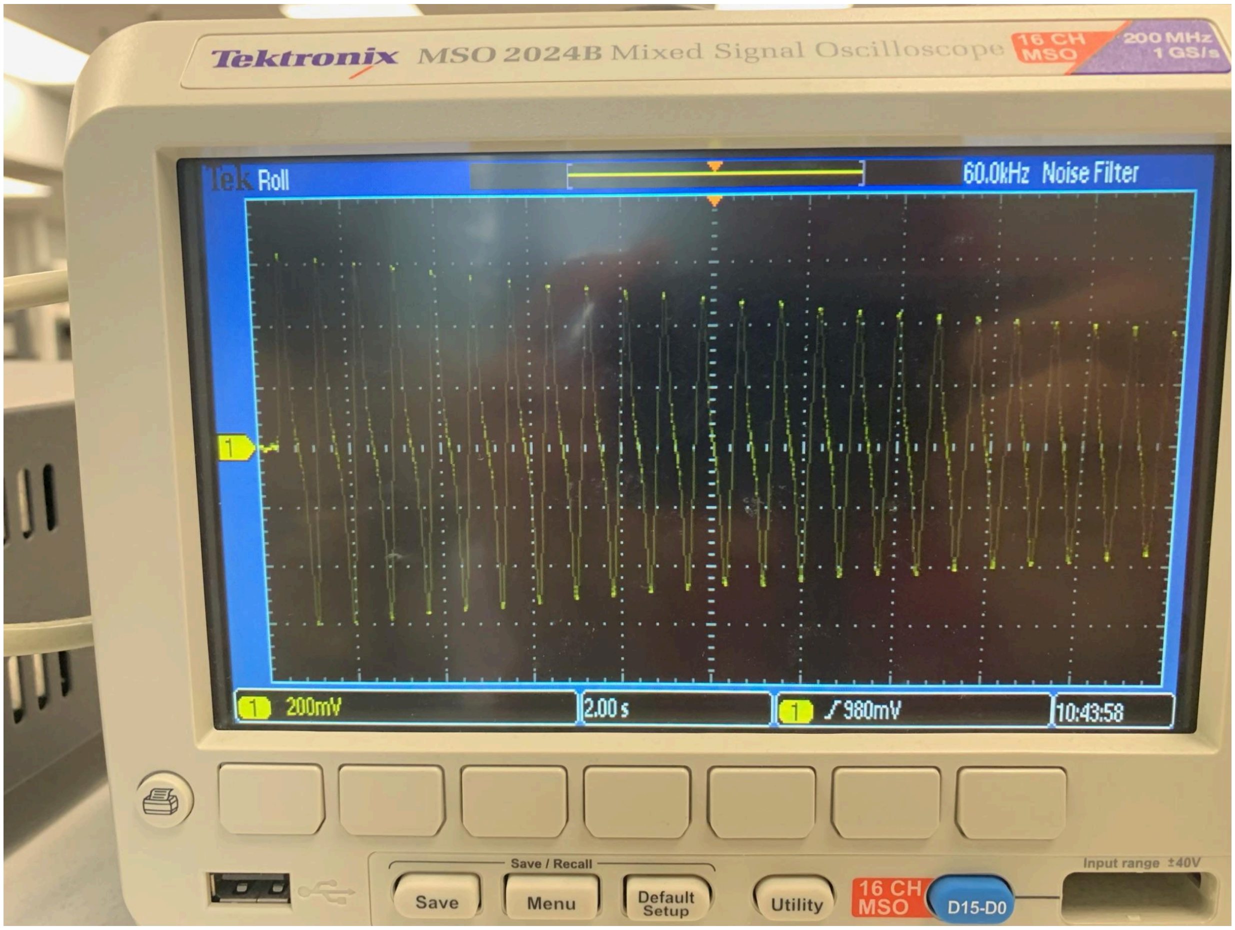

Figure 3.1.6: Oscilloscope graph for 1000 ohms. Resolution information is above.

5000 Ohms

Observed Resistance: **5.075 ± 0.001 Kilo-ohms**

Resolution: **200 mV and 2 seconds**

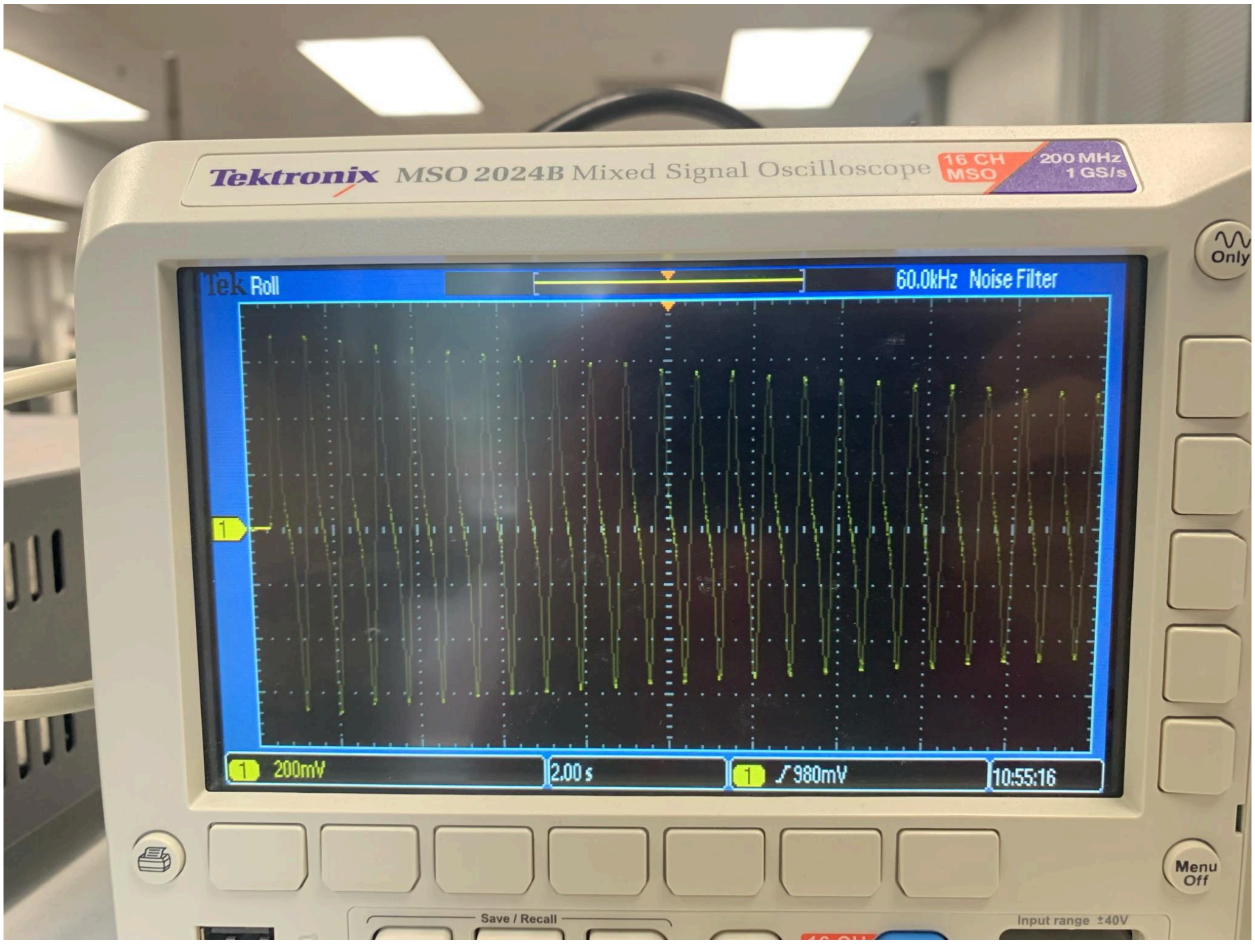

Figure 3.1.7: Oscilloscope graph 5000 ohms. Resolution information is above.

10000 Ohms

Observed Resistance: **9.92 ± 0.1 Kilo-ohms**

Resolution: **200 mV and 2 seconds**

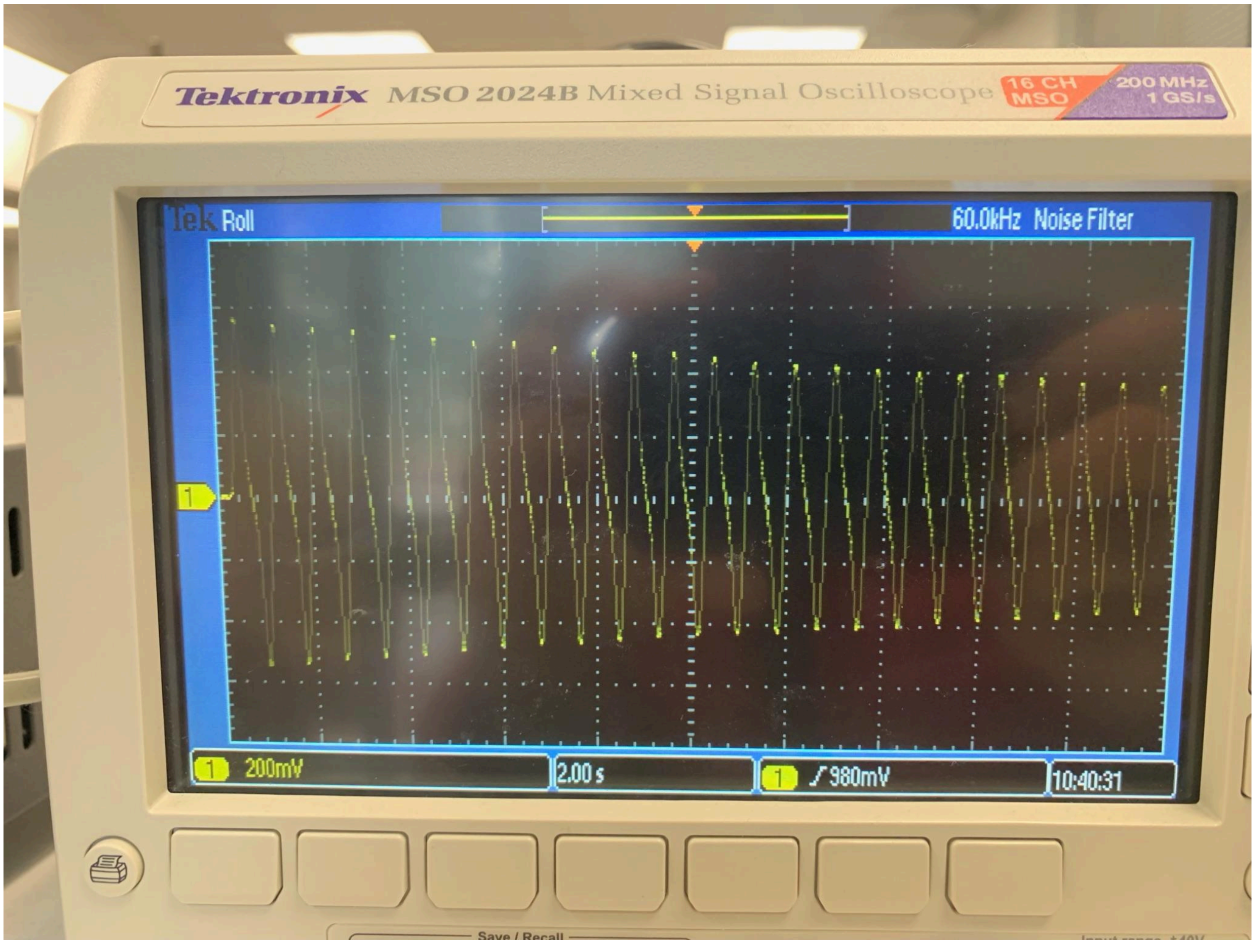

Figure 3.1.8: Oscilloscope graph for 10000 ohms. Resolution information is above.

4. Data Analysis (Key Findings)

Now that we have collected data about the magnetic oscillator's vertical acceleration over time for different resistor resistances, we can begin digging deeper into experimentally probing the theoretical framework governing this experiment.

4.1 - Data Analysis Procedure

1. Load all the data collected using the IOlab into a Python notebook.

2. For each dataset, corresponding to a unique resistor value, make a graph with time on the x-axis and acceleration on the y-axis.
3. Visually find starting times to cut the data at so as to create datasets with purely oscillatory characteristics, removing the messy data associated with the start of each experiment's data collection when the oscillator has just been let go.
4. Offset every new "cleaned" dataset by subtracting 9.8 m/s^2 from the vertical accelerations so as to account for the offset produced by standard gravitational acceleration.
5. Define a fitting function for the vertical acceleration of the magnetic oscillator as a function of time as described in Eq. 1.3.1.
6. For each dataset, corresponding to a distinct resistor value, make a scatter plot of the data and, using the Python library scipy.optimize.curve_fit, perform a non-linear least-squares regression fit on the data using the fitting function described in the previous step; then plot this fitted curve on the same plot.
    a. The uncertainties in the acceleration and time data are given by the precision error of the IOlab, 0.000001 m/s^2 and 0.000001 s each respectively.
7. Make a residual plot for each of the fits described in the previous step.
8. From the fits, you will have obtained a corresponding β value for each dataset, as described in Eq. 1.3.2. Now, multiply all the β values by 2m so as to get the corresponding γ values (damping coefficients).
    a. The mass of the oscillator is $m = 0.2439 \pm 0.0001$ kg, as measured by a digital scale.

b. The uncertainties, or standard errors, in the β values are given by the square-roots of the variances of β.

c. The uncertainties in γ are determined through error propagation, as seen in Figure 4.1.1 below.

9. Now, you will have a dataset with data points of the form (resistor value, damping coefficient). This is the key data that we will be interpreting, so first, fit an inverse proportionality function to the data and make scatter/best fit and residual plots similar to the fits above in steps 6 and 7. The fitting function is given by $\gamma = A/R$ where A is a fitted constant parameter.
10. Fit a power law function to the data and make similar plots as above in order to explore the complex proportional dependence of $|B(z)\frac{dB}{dz}|$ with the resistance, as is described in the last paragraph of Section 1. The fitting function is given by $\gamma = A/(R^p)$ where A and p are constant fitted parameters.

$$\beta = \frac{\gamma}{2m} \rightarrow \gamma = 2m\beta$$

$$\gamma_{err} = \sqrt{\left(\frac{d\gamma}{d\beta}\beta_{err}\right)^2 + \left(\frac{d\gamma}{dm}m_{err}\right)^2}$$

$$\gamma_{err} = \sqrt{4m^2\beta_{err}^2 + 4\beta^2 m_{err}^2}$$

$$\gamma_{err} = 2\sqrt{(0.2439)^2\beta_{err}^2 + \beta^2(0.0001)^2}$$

Figure 4.1.1: Error propagation calculations to derive the error in the damping coefficient as a function of the fitted β values and their associated errors.

4.2 - Damped Harmonic Oscillation Fit Plots

As a sanity check, we first ran the experiment once with no resistor bridging the gap between the two ends; the two ends were connected to each other, simply making sure that the coil was closed. What we observed in the data visually confirmed to us that the oscillations were indeed damped, and that the offset was indeed going to be 9.8 m/s^2 due to gravity.

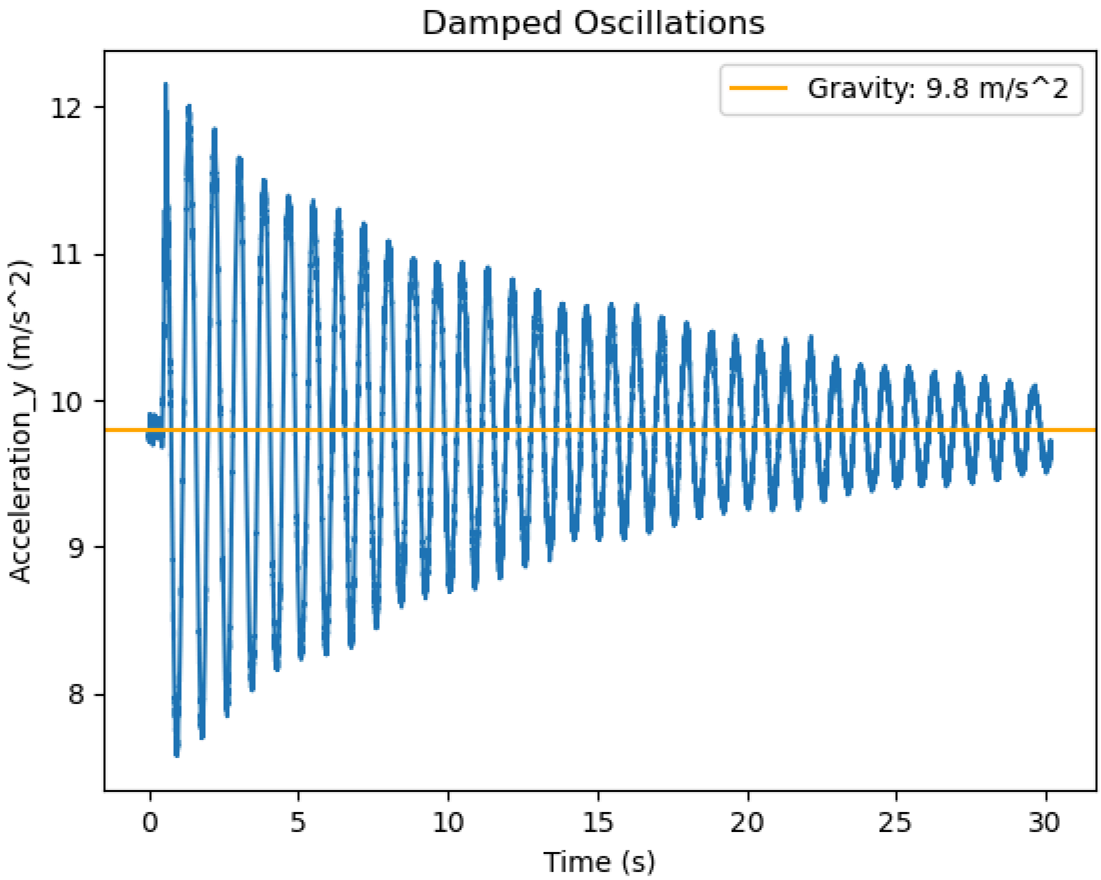

Figure 4.2.1: Vertical acceleration of the magnetic oscillator as a function of time with no resistor connecting the two ends of the coil. The oscillations look damped and are centered about 9.8 m/s^2 as expected.

Now, we will show all the plots relating to the fits as described in steps 6-7 of the data analysis procedure. In the captions of the best fit plots we will also list the fitted parameter values for the best fit curve.

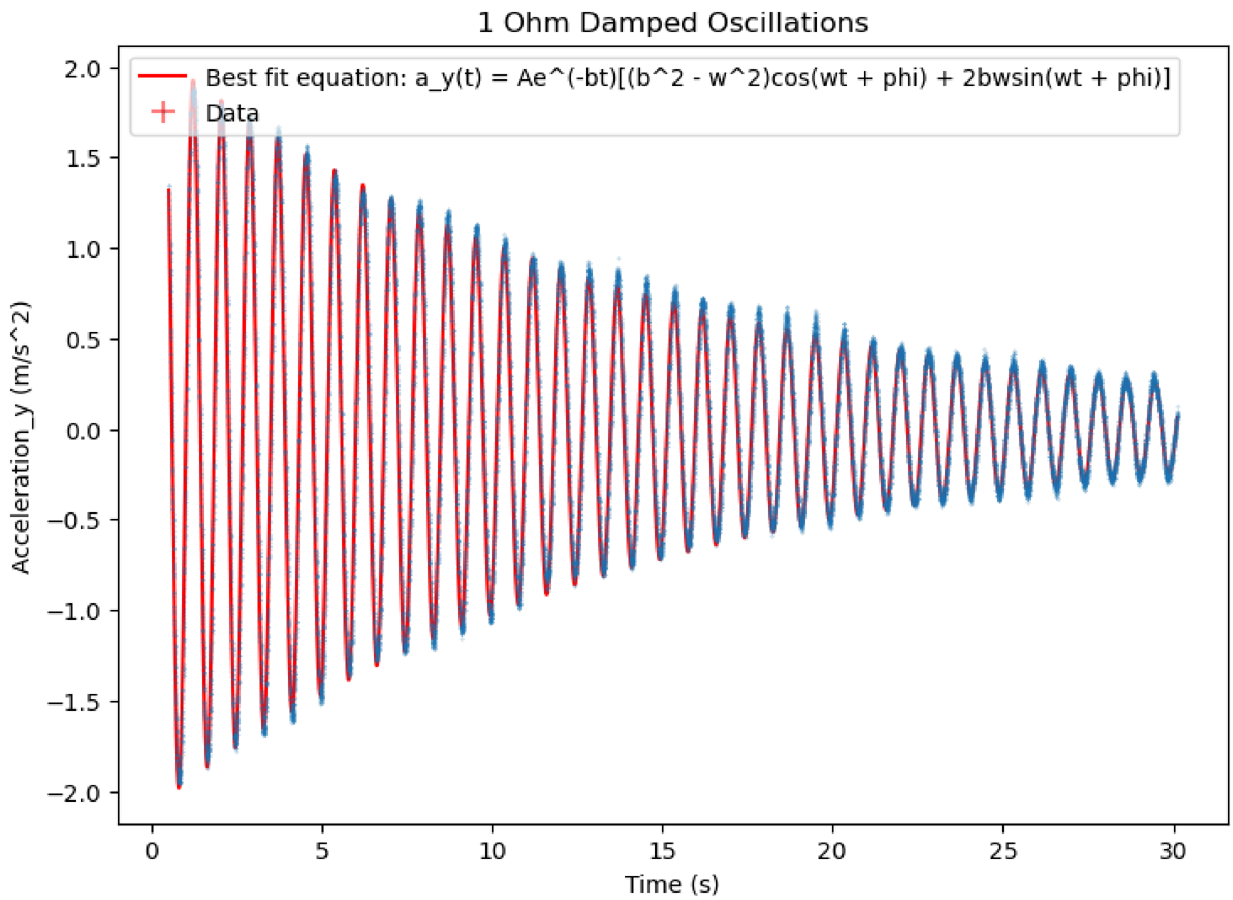

Figure 4.2.2: Best-fit plot for the 1 Ohm setup. A = 0.03671 ± 0.00002, β = 0.07168 ± 0.00007, ω = 7.5633 ± 0.0001, ϕ = -6.0880 ± 0.0006.

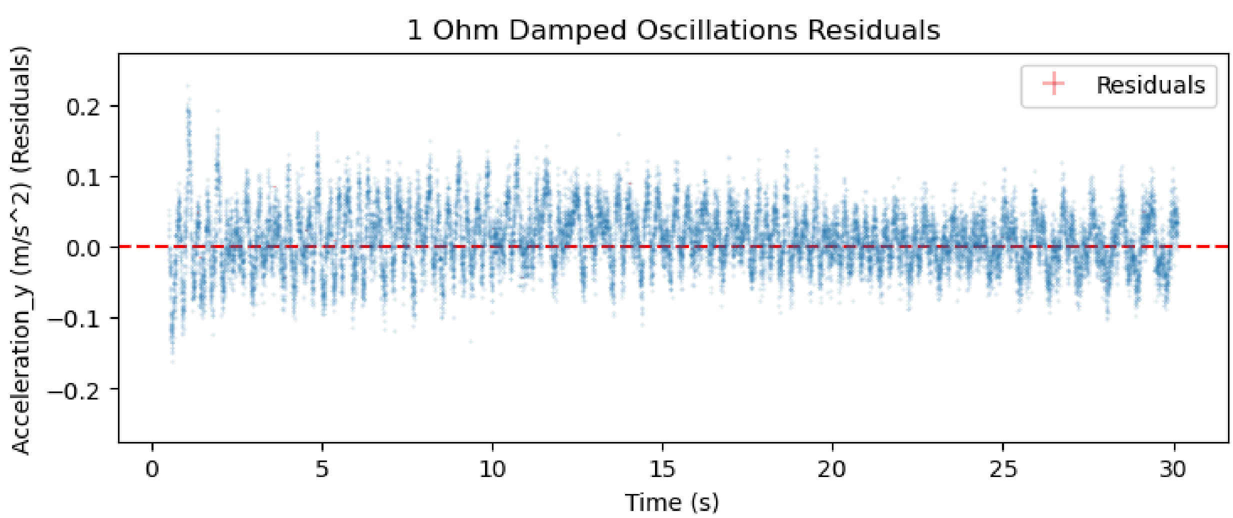

Figure 4.2.3: Residual plot for the 1 Ohm setup.

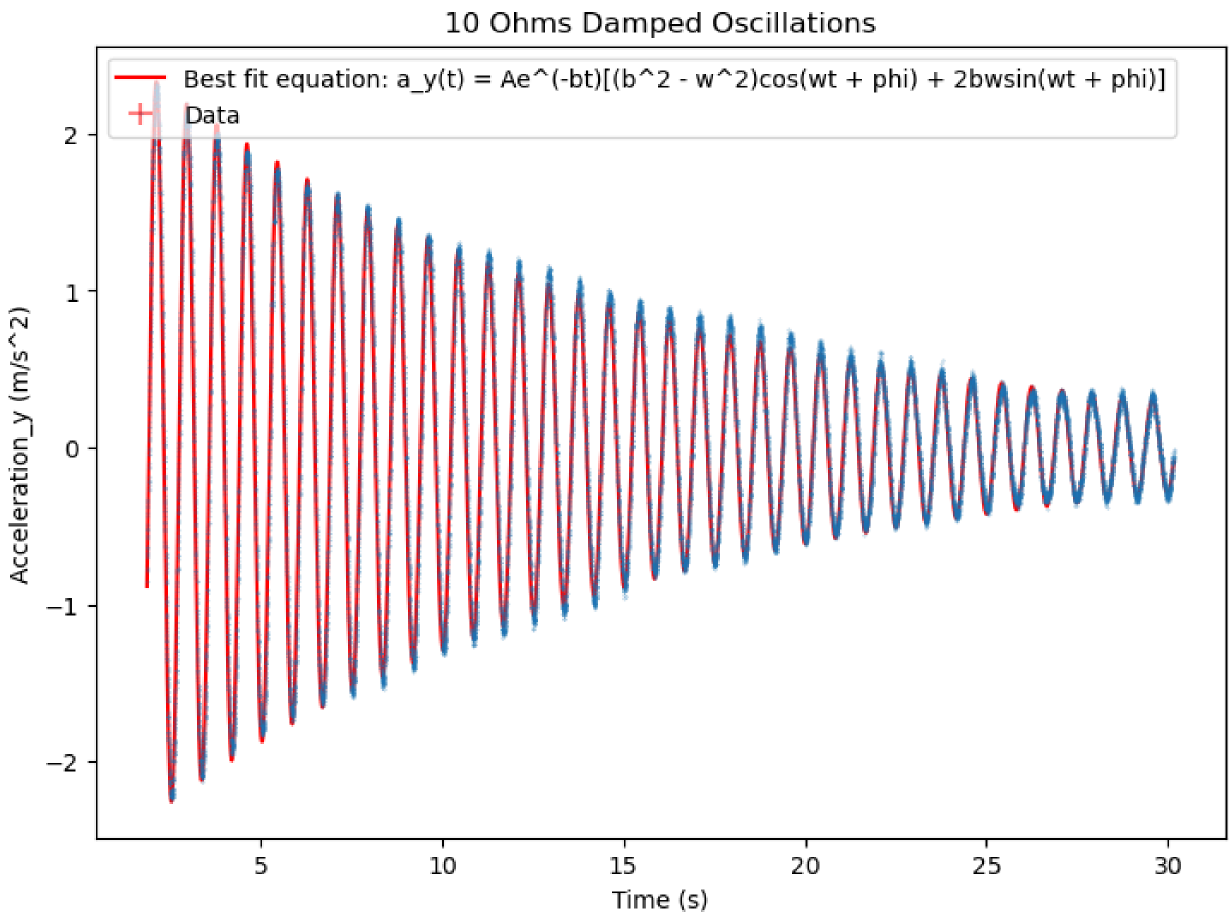

Figure 4.2.4: Best-fit plot for the 10 Ohm setup. A = 0.04783 ± 0.00003, β = 0.07450 ± 0.00006, ω = 7.5550 ± 0.0001, ϕ = -13.0058 ± 0.0006.

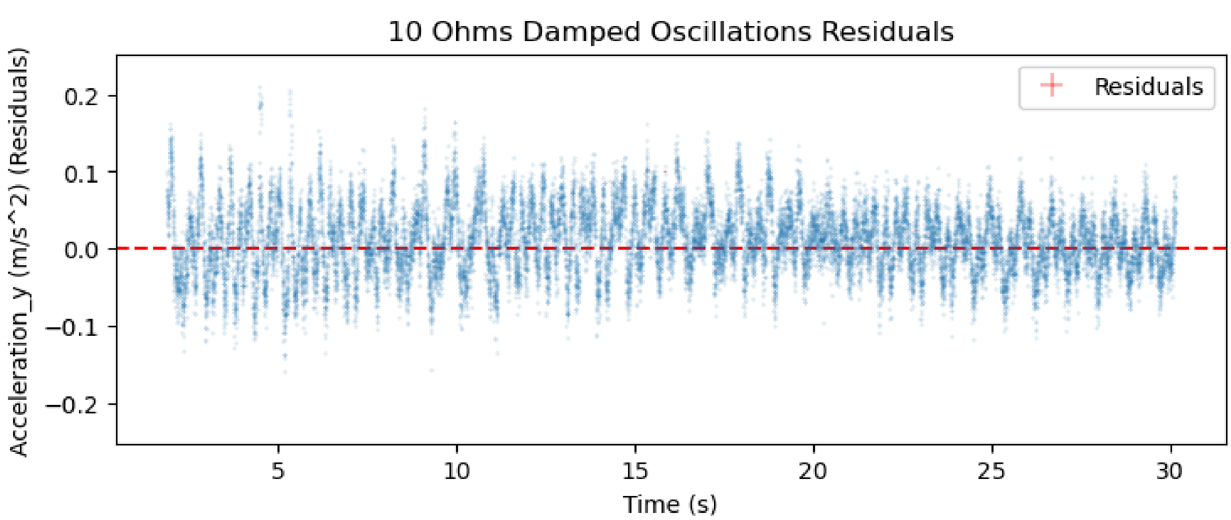

Figure 4.2.5: Residual plot for the 10 Ohm setup.

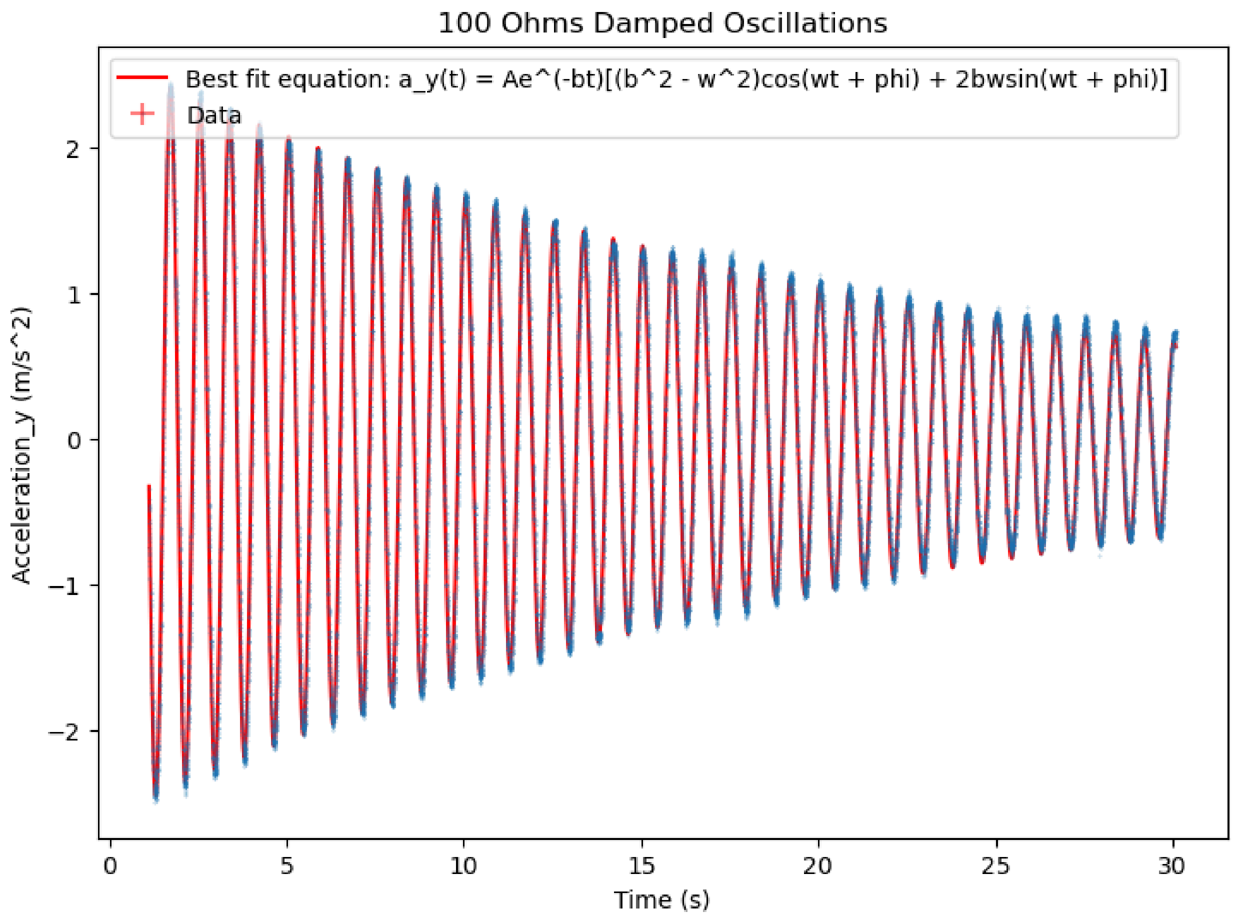

Figure 4.2.6: Best-fit plot for the 100 Ohm setup. A = 0.04566 ± 0.00002, β = 0.04520 ± 0.00004, ω = 7.5513 ± 0.0001, ϕ = -9.9535 ± 0.0005.

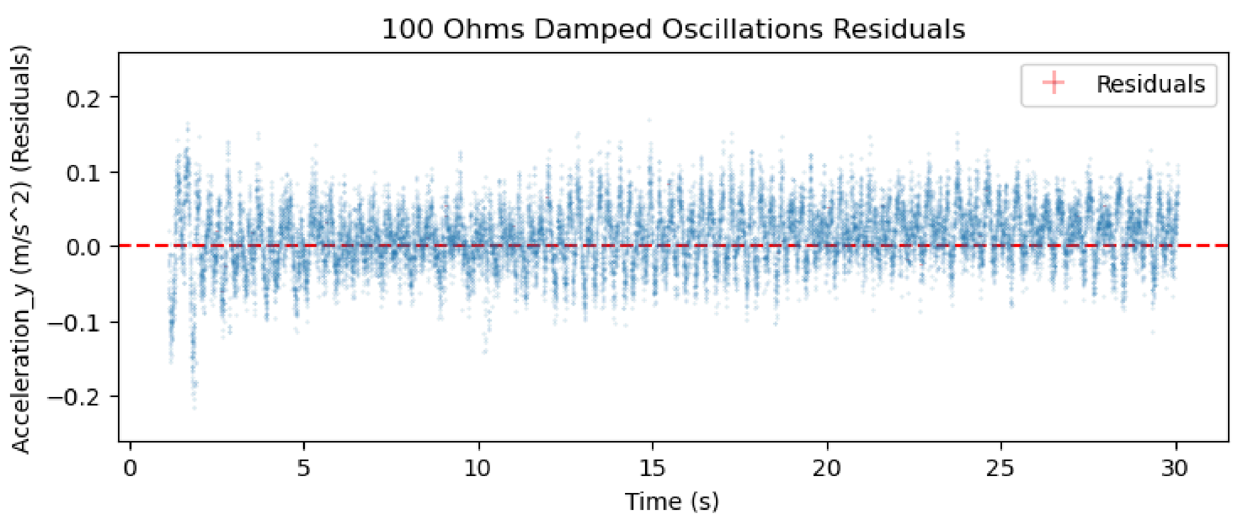

Figure 4.2.7: Residual plot for the 100 Ohm setup.

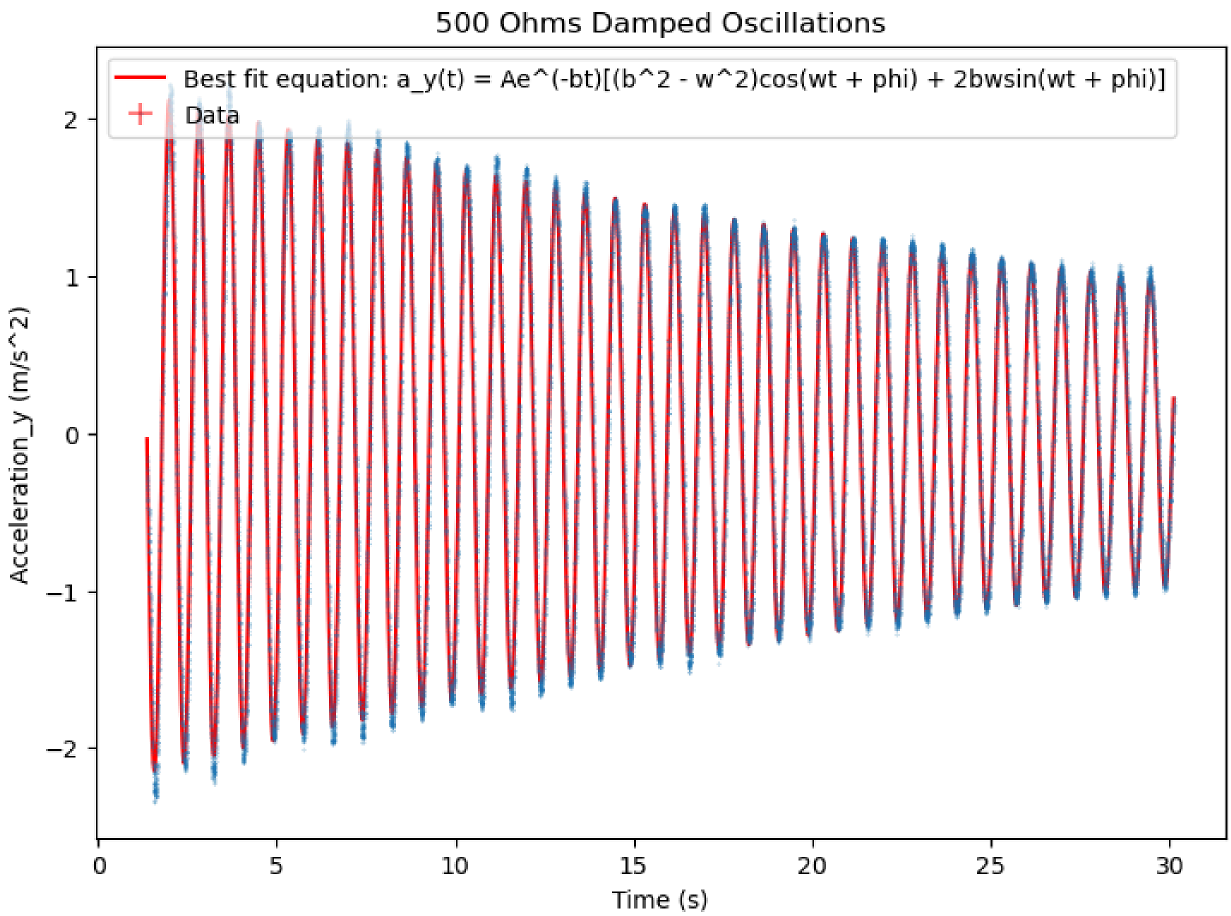

Figure 4.2.8: Best-fit plot for the 500 Ohm setup. A = 0.03936 ± 0.00003, β = 0.02808 ± 0.00005, ω = 7.5516 ± 0.0001, ϕ = -8.8128 ± 0.0007.

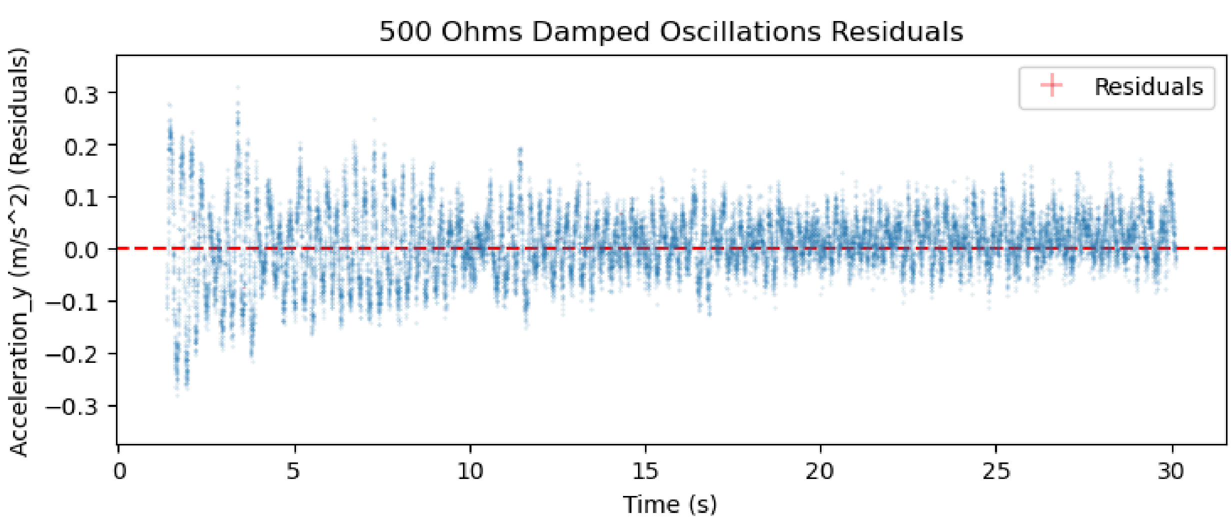

Figure 4.2.9: Residual plot for the 500 Ohm setup.

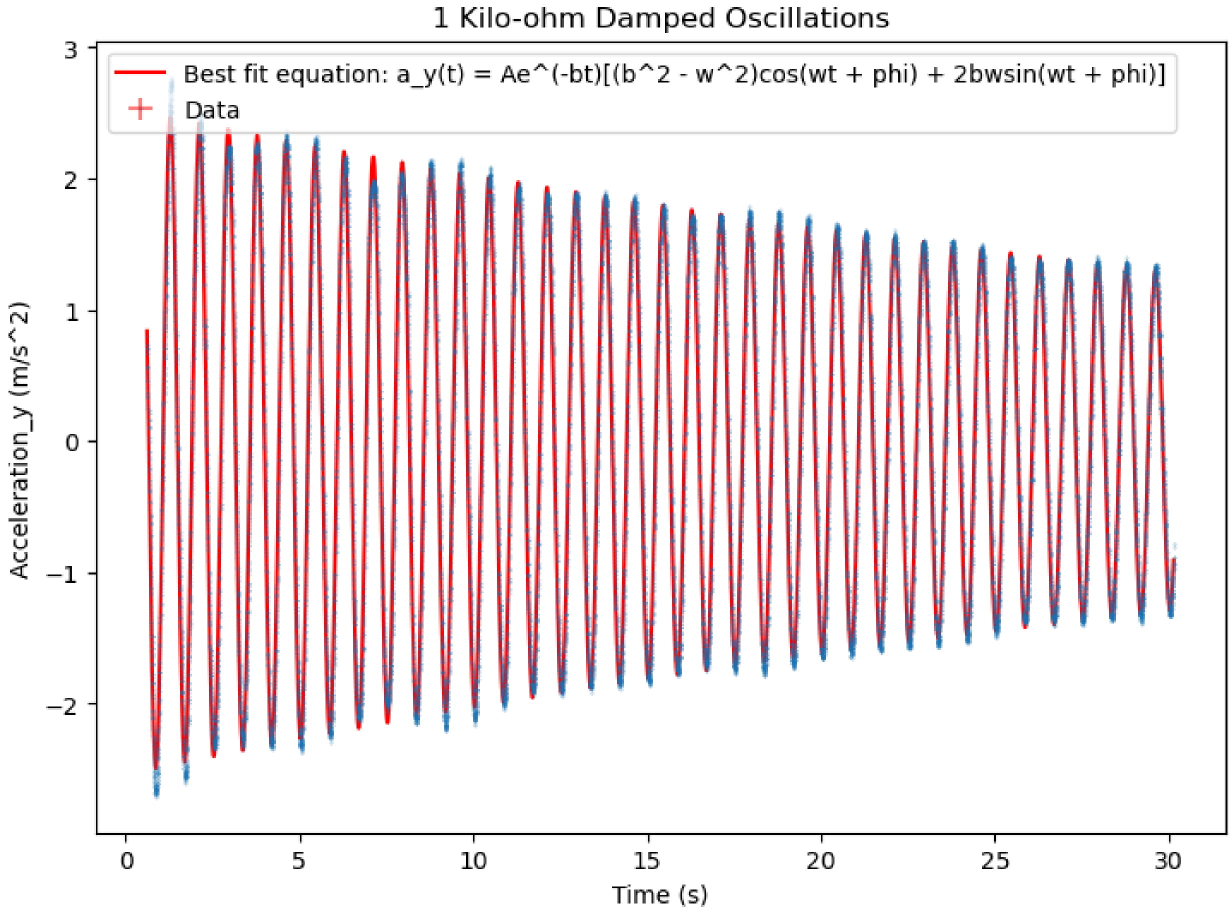

Figure 4.2.10: Best-fit plot for the 1 kOhm setup. A = 0.04467 ± 0.00003, β = 0.02248 ± 0.00003, ω = 7.5429 ± 0.0001, ϕ = -6.6426 ± 0.0006.

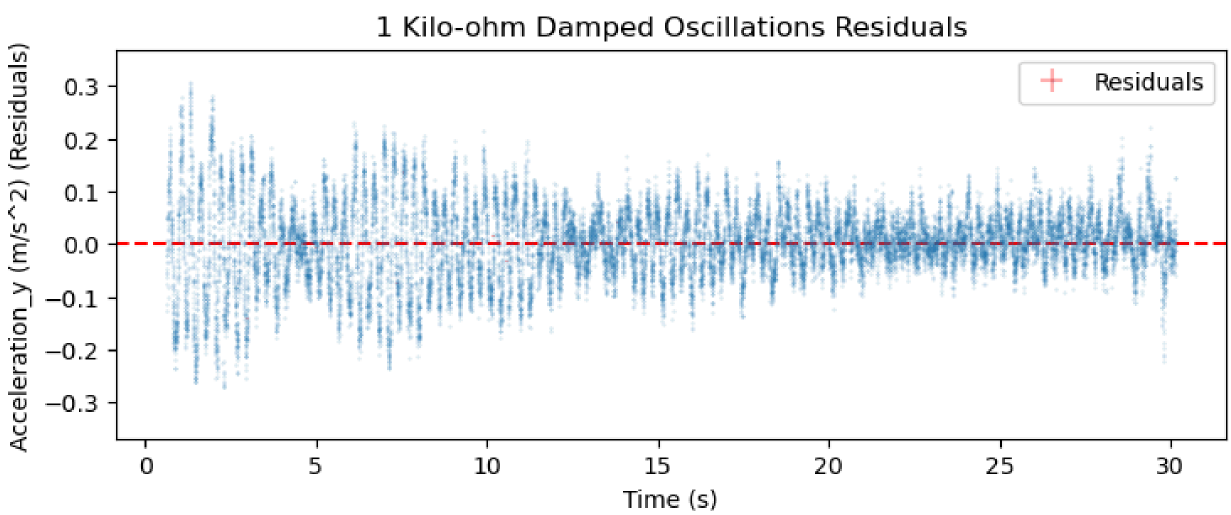

Figure 4.2.11: Residual plot for the 1 kOhm setup.

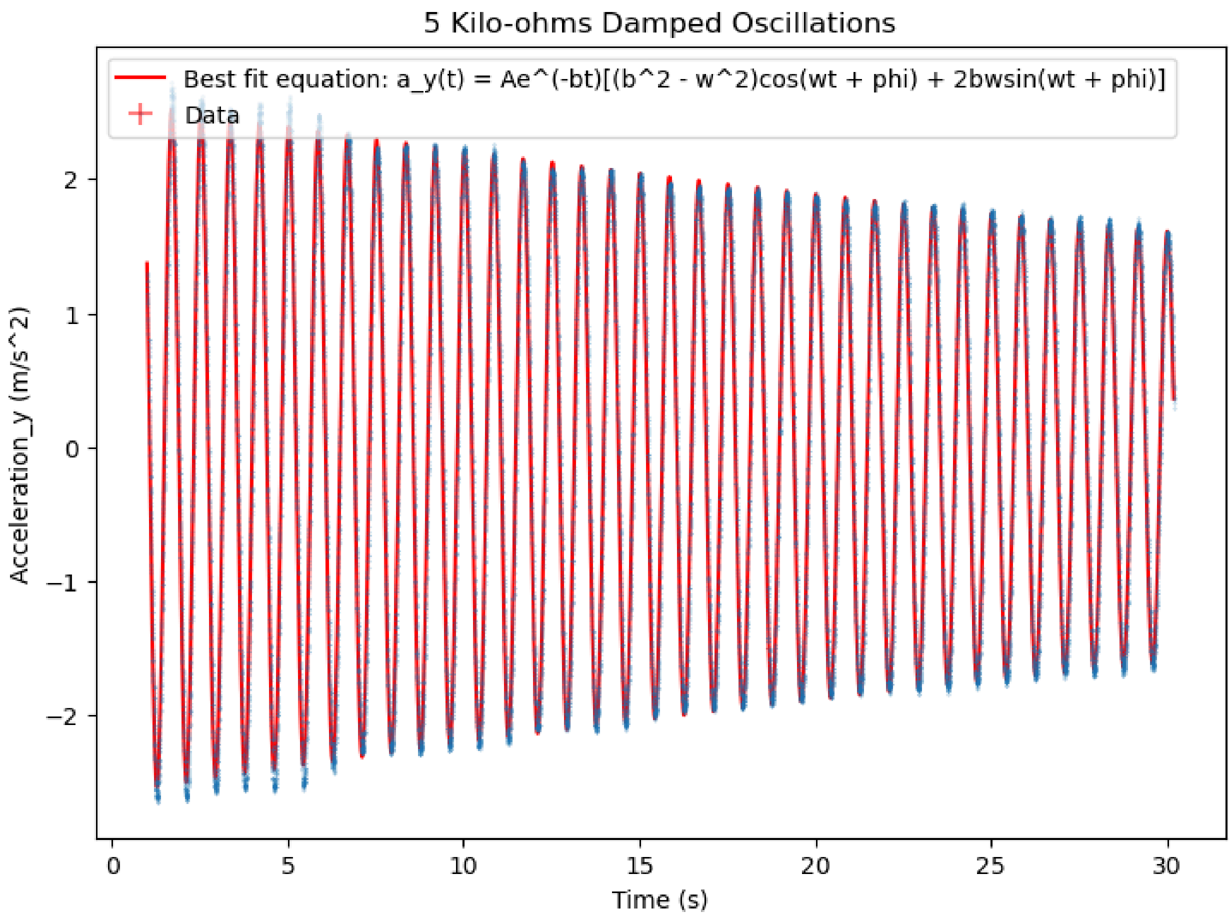

Figure 4.2.12: Best-fit plot for the 5 kOhm setup. A = 0.04550 ± 0.00002, β = 0.01580 ± 0.00003, ω = 7.5433 ± 0.0001, ϕ = -9.6951 ± 0.0005.

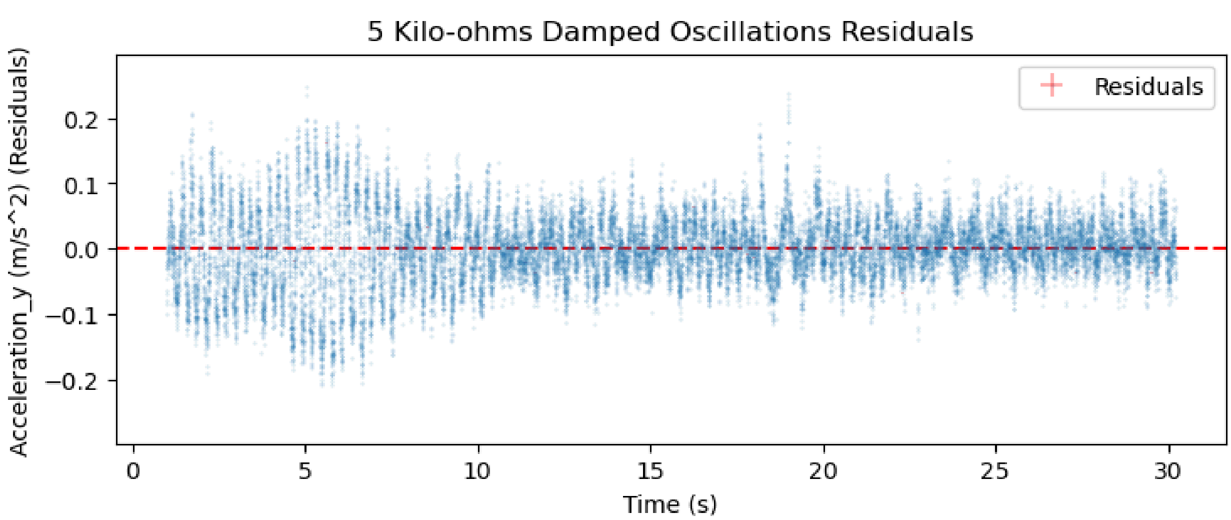

Figure 4.2.13: Residual plot for the 5 kOhm setup.

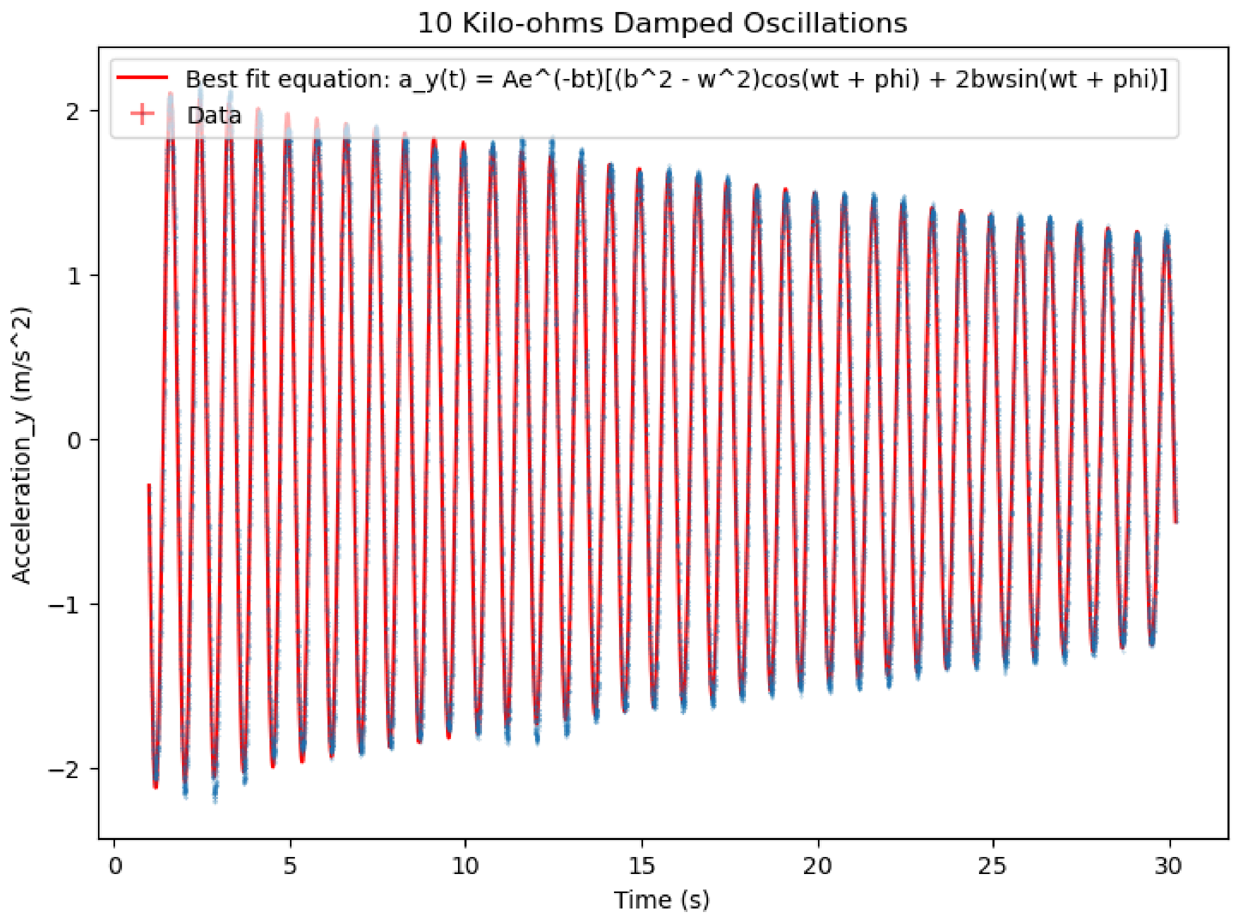

Figure 4.2.14: Best-fit plot for the 10 kOhm setup. A = 0.03813 ± 0.00002, β = 0.01860 ± 0.00004, ω = 7.5439 ± 0.0001, ϕ = -8.9962 ± 0.0006.

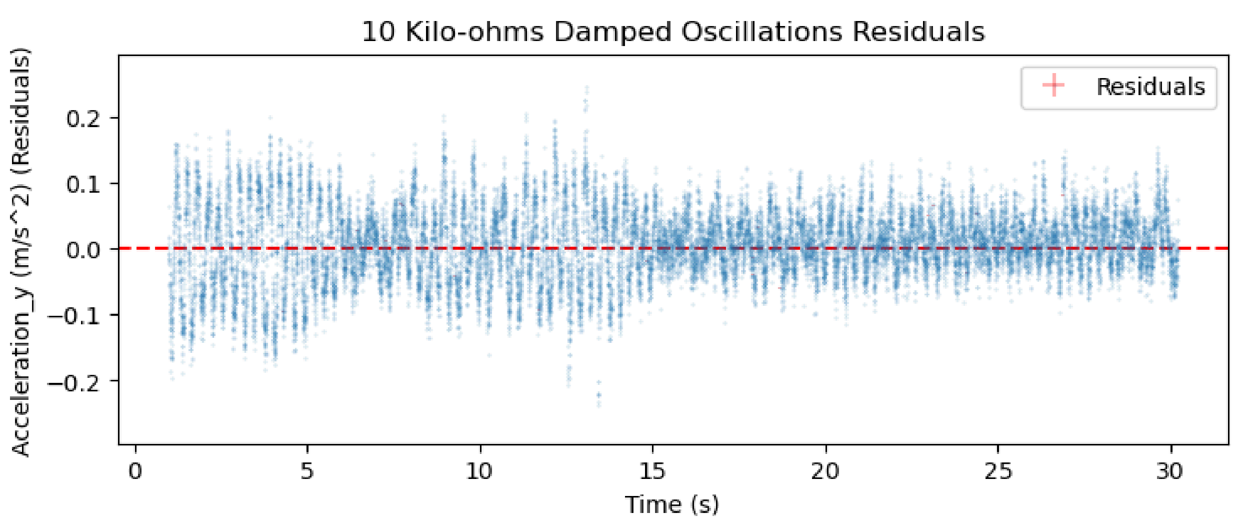

Figure 4.2.15: Residual plot for the 10 kOhm setup.

Clearly, the theoretical assumption that the motion of the magnetic oscillator would be damped according to a linear velocity-dependent damping force was a good one. All the plots show a very good fit of this model to the data visually, and when looking at the residual plots, we see that for the most part, they are small in magnitude and randomly distributed around the 0-value. This indicates that most of the error in our data collection and fit can be attributed to random errors and that overall, the theoretical assumption of damped harmonic oscillations has been validated.

4.3 - Analyzing the Damping vs. Resistance Data

Now that we have successfully performed all the fits and residual analysis described in steps 6-7 of the data analysis procedure, we will move onto completing steps 8-10 of the procedure. To begin, after performing the relevant algebra and error propagation described in step 8, we obtain the key data relating the damping coefficient to the resistance value, which is summarized in the table below:

Resistor Value (Ohms) – R	**Damping Coefficient (kg/s) – γ**
1.1 ± 0.1	0.03496 ± 0.00004
12.1 ± 0.1	0.03634 ± 0.00003
100.3 ± 0.1	0.02205 ± 0.00002
477.3 ± 0.1	0.01370 ± 0.00003
1003 ± 1	0.01097 ± 0.00002
5075 ± 1	0.00771 ± 0.00001
9920 ± 10	0.00907 ± 0.00002

Simply plotting the data, we see that the theoretical framework behind this experiment is confirmed, at least characteristically:

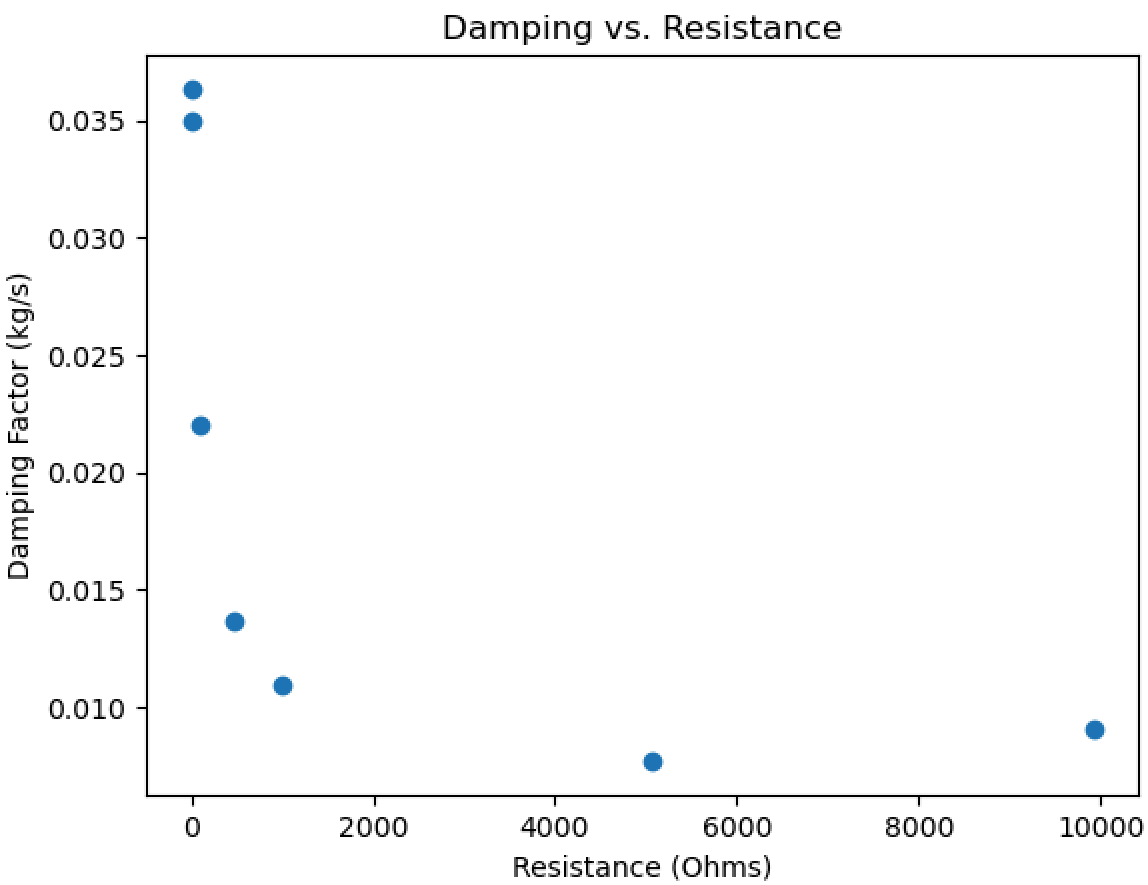

Figure 4.3.1: Scatter plot with the damping coefficient on the y-axis and the resistance on the x-axis.

Indeed, the damping decreases as the resistance increases, which is what we expect theoretically. Furthermore, the data seems to follow a curve shape reminiscent of a 1/R proportionality, or more generally, a power law proportionality of the form 1/R^p. This characteristically confirms the theoretical predictions made in Eq. 1.4.1, which states that the damping coefficient naively should be proportional to 1/R, or more truly, somehow proportional to 1/R^p if the intricacies of the $|B(z)\frac{dB}{dz}|$ term are considered. We will now complete steps 9 and 10 of the data analysis procedure, ultimately comparing the naive assumption that everything but the resistance is a constant to the complex assumption that the $|B(z)\frac{dB}{dz}|$ term is intricately proportional to the resistance up to some power. The relevant plots are shown below, with the fitted parameter values and uncertainties listed in the captions of the best-fit plots:

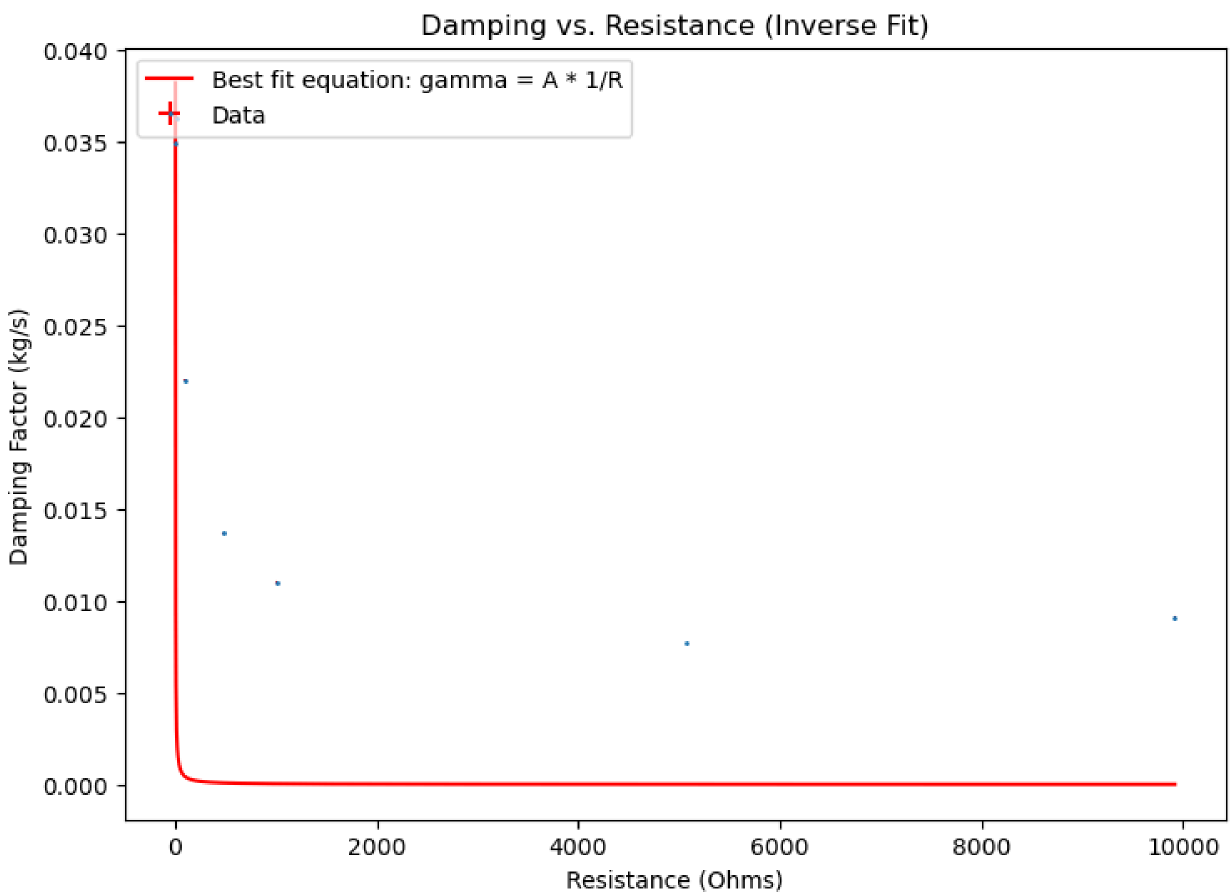

Figure 4.3.2: Best-fit plot for the damping as a function of resistance assuming an inverse proportionality. A = 0.04206 $\pm$ 0.02002.

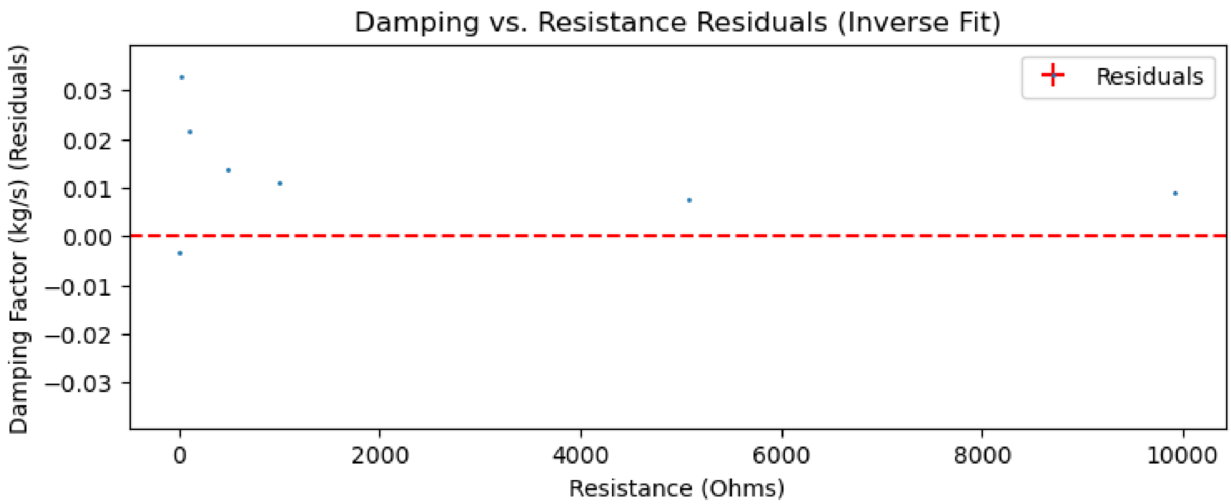

Figure 4.3.3: Residual plot for the curve fit assuming an inverse proportionality between damping and resistance.

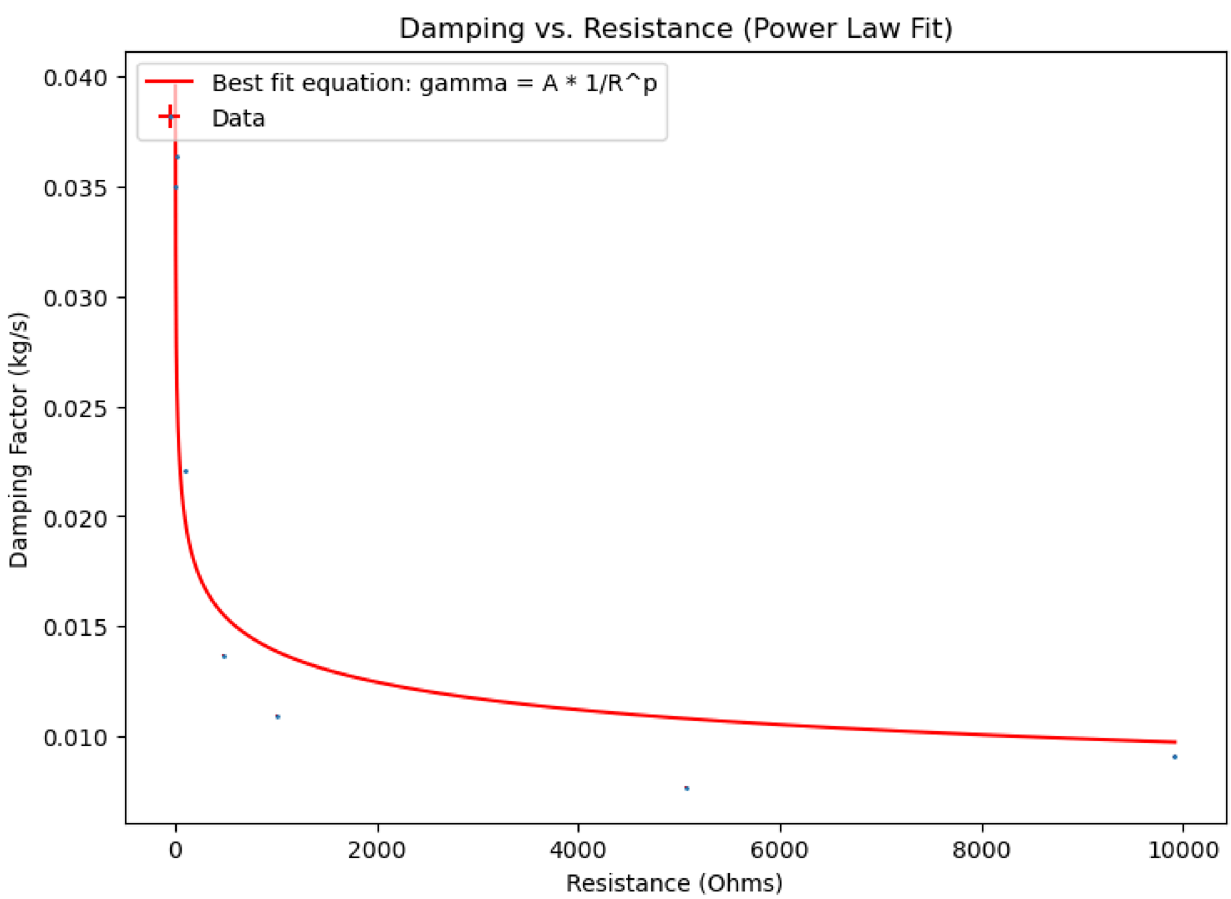

Figure 4.3.4: Best-fit plot for the damping as a function of resistance assuming a power-law proportionality. A = 0.04014 $\pm$ 0.00475, p = 0.15375 $\pm$ 0.03131.

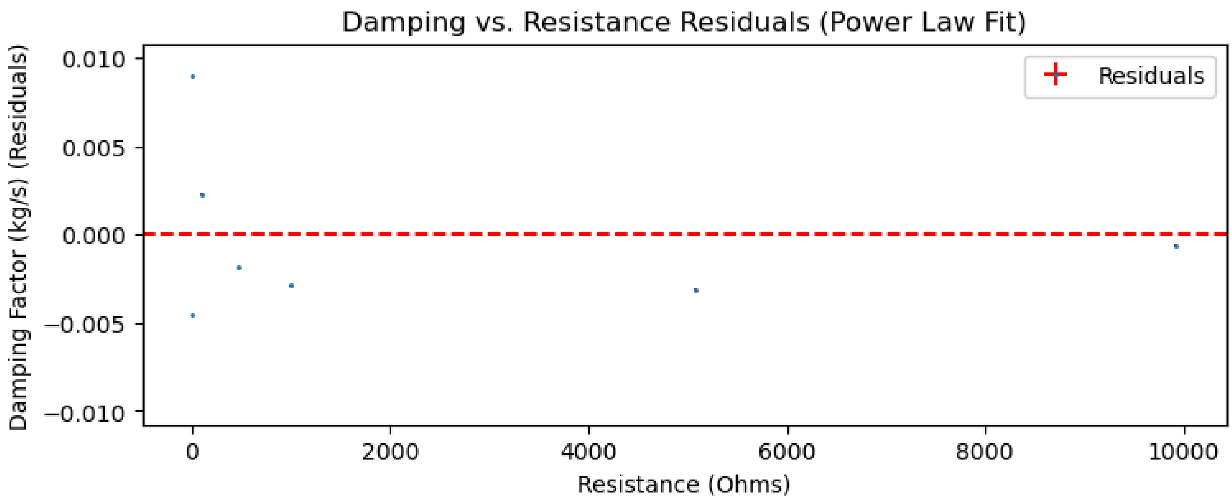

Figure 4.3.5: Residual plot for the curve fit assuming a power-law proportionality between damping and resistance.

Clearly, we see that the 1/R fit did not perform that well. Analyzing the residuals, we see that they are not randomly distributed about the 0-value, even with the limited amount of samples we have. This provides heavy evidence for the case that $|B(z)\frac{dB}{dz}|$ might be proportional to the resistance up to some power. Indeed, we can probe this further through the power-law fit, which performed decently well with the curve "hugging" the data relatively well. Analyzing the residuals, they do look randomly distributed about the 0-value, though it is hard to tell with the limited amount of data points. Also, we can partially attribute these residuals to inherent errors in the experimental data collection procedure, such as with the imperfect nature of the oscillations not being perfectly vertical, and with our theoretical assumptions of a perfectly vertical magnetic field and disregarding the resistive properties of the coil's wiring.

It is interesting to note that the fitted proportionality constant parameter A agrees to 1 significant-figure between both the inverse fit and power law fit. This seems to suggest that most of the "blame" for the difference between the two fits can be attributed to the power of the resistance in the denominator, thus placing significant importance on it. Moreover, the fitted parameter value for p in the second fit, p = 0.15375 $\pm$ 0.03131, allows us to probe the nature of the $|B(z)\frac{dB}{dz}|$ term in Eq. 1.4.1 further. The natural question to ask next is: "What power of resistance is $|B(z)\frac{dB}{dz}|$ proportional to?". We can answer this question using Eq. 1.4.1 because in the expression for the value of the damping coefficient, we have "LAN", which is a constant for sure throughout our experiment, and we also have 1/R and $|B(z)\frac{dB}{dz}|$. If we are experimentally observing that the data follows a 1/R^p curve, then $|B(z)\frac{dB}{dz}|$ must be proportional to R^x where x-1 = -p, or equivalently, x = -p+1. Ultimately, the experimental evidence seems to suggest that the $|B(z)\frac{dB}{dz}|$ term is roughly proportional to R^(0.84625) ~ R^(0.85). This makes sense because characteristically, as the resistance increases, we know that the damping decreases, which in turn means that the magnet spends more time in motion, allowing it to effectively have more of an influence on the space around it than if the resistance was small and the oscillations died down quickly. If the magnet oscillates slowly, or not at all, then the dB/dz term decreases drastically. Thus, it makes sense that as the resistance increases, $|B(z)\frac{dB}{dz}|$ also increases, roughly proportional to R^(0.85).

4.4 - Data Analysis Summary

All in all, after going through all the steps in the data analysis procedure and then interpreting the results, we realized that the theoretical predictions relating to the damped harmonic oscillations of the magnet-IOlab oscillator was confirmed experimentally. Furthermore, we confirmed the theory characteristically when we found that the damping decreases as the resistance increases, following a curve shape reminiscent of a 1/R or 1/R^p proportionality, for positive p. Our expectations were confirmed when we saw that the naive 1/R fit didn't perform too great but the 1/R^p fit performed much better. Finally, this allowed us to experimentally probe the nature of the $|B(z)\frac{dB}{dz}|$ term in Eq. 1.4.1 describing the relationship the damping coefficient has with the resistance of the resistor, magnetic field of the magnet, and coil properties. We found that the $|B(z)\frac{dB}{dz}|$ term is roughly proportional to R^(0.85). At least, this is what it needs to be proportional to in order to fit our data relatively well, giving us valuable insight into the variable nature of the magnetic field and its derivative across the different experiments we ran.

5. Conclusion

In conclusion, this capstone project saw us exploring the fundamentals of electromagnetism related to electromagnetic induction and resistors as well as the fundamentals of mechanics related to damped harmonic oscillations. From the relatively simple building blocks of a spring, a magnet, an IOlab, a coil, and a resistor, we were able to build an experimental setup that allowed us to study the intricate interplay between an oscillating permanent magnet and an induced magnetic field in the coil opposing these oscillations, with a strength determined by the resistance of the resistor. Ultimately, we wanted to probe the relationship between the resistance of the resistor and the damping of the oscillatory motion, and what we found agreed with the theory very well. As expected, the oscillation of the magnet could be explained by damped harmonic motion attributed to a linear velocity-dependent damping force, and the relationship between the resistance and the damping followed a characteristic inverse power law curve, with the damping decreasing as the resistance increased. While our theoretical framework was held back by complexities and time constraints, our experimental methods revealed to us that, to a good degree of approximation, the damping coefficient of the magnetic harmonic oscillator is roughly proportional to 1/R^(0.15). This teaches us that indeed, the resistor plays an important role in determining the damping characteristic of the oscillations, with a "strength" given by this exponent, $p \approx 0.15$.

Moreover, throughout this project we faced many challenges. The two biggest had to do with the experimental setup and data collection procedure as well as the development of the exact theory governing our experiment. This is because it was very difficult to get the oscillator to be perfectly lined up in the middle of the coil and to consistently oscillate it perfectly vertically from the same starting position, and also because the theory behind modelling the nature of the magnetic field as a function of position and time was complex. This showed us that we have room for improvement given the current state of the project, with the most important improvements needing to be made being:

1. Develop a more robust and reliable data collection procedure in terms of oscillating the magnetic oscillator relative to the coil.
2. Spend more time on developing the theory so that we can begin testing more than just predicted characteristic relationships, but actually being able to test and validate numerical predictions such as expected damping factor values for a given set of parameters.
3. Collect more data across a wider range of resistors so as to achieve greater statistical significance and to be able to more accurately judge the nature of fits.

Furthermore, if we had to give advice to future students attempting to redo and build upon this project, some of the most crucial next steps to expand this project would be as follows:

1. Actually build our own coils so that we can control characteristics of it such as the number of loops, the cross-sectional surface area, and the total coil length. This would itself open 3 additional degrees of freedom for us to experiment with and theoretically ponder on other than just the resistance of the resistor.

2. Experiment with more interesting circuit elements in between the 2 ends of the coil than just resistors. For example, we could try and use inductors and capacitors as well as resistors on the breadboard, and then study the theoretical and experimental implementations of doing so.
3. Experiment with the "inverse" setup of the project. By this, we mean having the same physical setup with the magnetic oscillator and the coil. However, we could use a signal generator to apply different voltage waveforms across the coil and study the subsequent effect on the motion of the oscillator, which would start from rest.

All in all, this capstone project was a truly remarkable experience. While it taught us of the hardships that come with designing and conducting an experiment from the ground up, it also gave us invaluable experience in utilizing our creativity and academic knowledge in an applied way, allowing us to build up a vast range of skills from mathematical derivations to setting up experiments to doing data analysis. In the end, this experiment, "The Magnetic Harmonic Oscillator", was a great success!