Notes on F_2^n/F_2^p – Offshell Modification

Written by Efrain Segarra

1 Method

We would like to demonstrate the ability to constrain F_2^n/F_2^p by utilizing $A = 3$ nuclei within a model that accounts for nuclear effects such as binding, fermi-motion, and offshellness:

$$\begin{aligned} F_2^A(x,Q^2) &= \int_x^A \frac{d\alpha d^3\mathbf{p}}{\alpha}\Big[ZF_2^p(\frac{x}{\alpha},Q^2)\rho_p^A(\alpha,\mathbf{p})\mathcal{O}_p(\frac{x}{\alpha},\nu) + NF_2^n(\frac{x}{\alpha},Q^2)\rho_n^A(\alpha,\mathbf{p})\mathcal{O}_n(\frac{x}{\alpha},\nu)\Big] \\ &= \int_x^A \frac{d\alpha d^3\mathbf{p}}{\alpha}\ F_2^p(\frac{x}{\alpha},Q^2)\Big[Z\rho_p^A(\alpha,\mathbf{p}) + N\rho_n^A(\alpha,\mathbf{p})\frac{F_2^n(\frac{x}{\alpha},Q^2)}{F_2^p(\frac{x}{\alpha},Q^2)}\Big]\mathcal{O}(\frac{x}{\alpha},\nu) \end{aligned} \tag{1}$$

where F_2^p (F_2^n) are the free proton (neutron) structure functions, $\mathcal{O}(\frac{x}{\alpha},\nu)$ describes offshell nucleon modification (which we assume to be the same for neutrons and protons, and the same for all nuclei), and $\rho_p^A(\alpha,\mathbf{p})$ ($\rho_n^A(\alpha,\mathbf{p})$) are the nucleon light-cone momentum distributions for protons (neutrons) in nucleus A.

Then, we can constrain parameters that describe F_2^n/F_2^p and $\mathcal{O}(\frac{x}{\alpha},\nu)$ by calculating $F_2^A(x,Q^2)$ and comparing to experimental data. Ideally, we would like to show that when only considering ^{3}He, extracting F_2^n/F_2^p is highly correlated with $\mathcal{O}(\frac{x}{\alpha},\nu)$, and when using the leverage gained by ^{3}H, one say something, highlighting the need for the MARATHON data. In this case, we are using a model for $\rho^A_{(p,n)}(\alpha,\mathbf{p})$.

2 Details

Since we have spectral functions for ^{3}He readily available, we can start out with these non-relativistic distributions before utilizing light-cone distributions. However, these objects are in terms of $(\mathbf{p},E)$, not (α,ν). The E that we are given in the Kaptari spectral function is defined as:

$$\begin{aligned} E &= \sqrt{P_{A-1}^2} + m_N - m_A \\ E &= (m^*_{A-1}) + m_N - m_A \\ &\rightarrow (m^*_{A-1}) = E + m_A - m_N \end{aligned} \tag{2}$$

and a PWIA framework is used, so that $\mathbf{p} = \mathbf{p_{A-1}}$.

Thus, given $(\mathbf{p},E)$, we can relate this to ν:

$$\begin{aligned} \nu &= P^2 - m_N^2 = (P_A - P_{A-1})^2 - m_N^2 \\ &= \Big[(m_A - E_{A-1})^2 - \mathbf{p}^2\Big] - m_N^2 \\ &= \Big[\Big(m_A - \sqrt{m^{*2}_{A-1} + \mathbf{p}^2_{A-1}}\Big)^2 - \mathbf{p}^2\Big] - m_N^2 \\ &= \Big[\Big(m_A - \sqrt{(m_A - m_N + E)^2 + \mathbf{p}^2}\Big)^2 - \mathbf{p}^2\Big] - m_N^2 \end{aligned} \tag{3}$$

and similarly for α:

$$
\begin{aligned}
\alpha &= \frac{E_i - |\mathbf{p}|\cos\theta}{m_N} \\
&= \frac{(m_A - E_{A-1}) - |\mathbf{p}|\cos\theta}{m_N} \\
&= \frac{\left(m_A - \sqrt{m_{A-1}^{2*} + \mathbf{p}^2}\right) - |\mathbf{p}|\cos\theta}{m_N} \\
&= \frac{\left(m_A - \sqrt{(m_A - m_N + E)^2 + \mathbf{p}^2}\right) - |\mathbf{p}|\cos\theta}{m_N}
\end{aligned} \tag{4}
$$

We also have to evaluate a Jacobian to go from $d\alpha d^3\mathbf{p} \to dEd\theta d\phi dp$:

$$
\begin{aligned}
\frac{d\alpha d^3\mathbf{p}}{\alpha} &= \frac{1}{\alpha}\mathcal{J} dEd\theta d\phi dp \\
\frac{d\alpha d^3\mathbf{p}}{\alpha} &= \frac{1}{\alpha}\left[\frac{\mathbf{p}^2 \sin\theta\,(m_A - m_N + E)}{m_N\sqrt{(m_A - m_N + E)^2 + \mathbf{p^2}}}\right] dEd\theta d\phi dp
\end{aligned} \tag{5}
$$

Our evaluation of the integral for estimating $F_2^A(x, Q^2)$ is now as follows:

$$
\begin{aligned}
F_2^A(x, Q^2) = \frac{2\pi}{A} &\sum_{i=0}^{i_{end}} \Delta E \sum_{j=0}^{j_{end}} \Delta p \sum_{k=0}^{k_{end}} \Delta\theta \, \frac{1}{\alpha}\left[\frac{\mathbf{p_j}^2 \sin\theta\,(m_A - m_N + E_i)}{m_N\sqrt{(m_A - m_N + E_i)^2 + \mathbf{p_j}^{\mathbf{2}}}}\right] \\
&\cdot \left[Z S_p^A(\mathbf{p_j}, E_i) + N S_n^A(\mathbf{p_j}, E_i)\frac{F_2^n(\frac{x}{\alpha}, Q^2)}{F_2^p(\frac{x}{\alpha}, Q^2)}\right] \cdot F_2^p(\frac{x}{\alpha}, Q^2)\mathcal{O}(\frac{x}{\alpha}, \nu) \\
\alpha &= \frac{\left(m_A - \sqrt{(m_A - m_N + E_i)^2 + \mathbf{p_j}^2}\right) - |\mathbf{p_j}|\cos\theta_k}{m_N} \\
\nu &= \left[\left(m_A - \sqrt{(m_A - m_N + E_i)^2 + \mathbf{p_j}^2}\right)^2 - \mathbf{p_j}^2\right] - m_N^2
\end{aligned} \tag{6}
$$

where $\Delta E, \Delta p, E_0, E_{end}, p_0, p_{end}$ are determined by the discretization of the spectral function calculation, $\Delta\theta$ we choose weighing efficiency/accuracy, $\theta_{end} = \pi$, $S_{(p,n)}^A(\mathbf{p_j}, E_i)$ are the given spectral function calculations for nucleons in given nucleus A. I implemented the $F_2^p(x, Q^2)$ parameterization from ALLM (see here) and checked that it sufficiently described world data (see here). I parameterize $\frac{F_2^n(\frac{x}{\alpha}, Q^2)}{F_2^p(\frac{x}{\alpha}, Q^2)} = \alpha + \beta x + \gamma e^{\delta(1-x)}$ with $\alpha, \beta, \gamma, \delta$ to be minimized and initial parameter guesses from fitting this form to the F_2^n/F_2^p I extracted from nuclear DIS data. Finally, I take a simple paramterization of the off-shell modification to be $\mathcal{O}(\frac{x}{\alpha}, \nu) = \nu^2 + \epsilon$, with ϵ to be minimized.