Selected Solutions from Tutorials

Adapted from **Shigley, Problem 13-16** (8th Ed., 13-14).

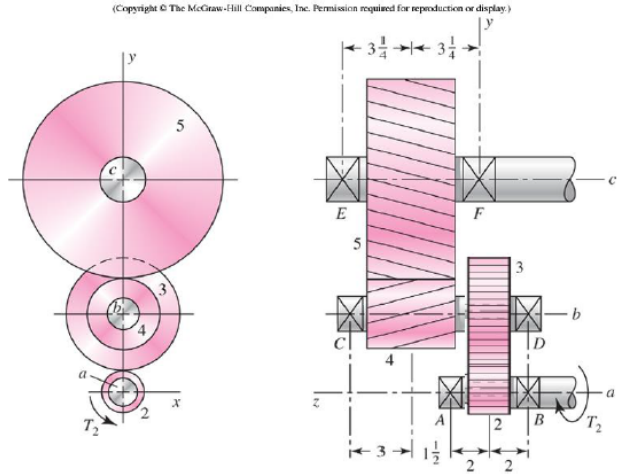
The double-reduction gearset shown in the figure is driven through shaft *a* at a speed of 900 rev/min. Gears 2 and 3 are spur gears and have a diametral pitch of 10 teeth/in and a pressure angle of 20°. The second pair of gears in the train, gears 4 and 5, are helical gears with a normal diametral pitch of 6 teeth/in, a 25° helix angle, and a normal pressure angle of 20° (see Figure 13-22). The tooth numbers for the four gears are: $N_2 = 14$, $N_3 = 54$, $N_4 = 16$, and $N_5 = 36$.

Part 1 (To be demonstrated by the TA). For the first stage (the spur gears) find:

d. The speed and direction of shaft *b*
e. The centre distance between shafts *a* and *b*

Part 2 (To be done by students). For the second stage (the helical gears) find:

f. The speed and direction of shaft *c*
g. The centre distance between shafts *b* and *c*
h. The directions of the thrust forces exerted by gears 4 and 5 on shafts *b* and *c*, respectively



2. Adapted from Shigley, Problem 13-16.

a) Find the speed and direction of shaft *b* (same as speed and direction of gear 3)

$$n_3 = e_{23} n_2 = -\frac{\text{driving teeth}}{\text{driven teeth}} n_2$$
$$= -\frac{N_2}{N_3} n_2 = -\frac{14}{54}(-900)$$
$$= 233 \text{ rev/min (CW)}$$

b) Find the centre to centre distance between the shafts *a* and *b*.

When properly spaced, pitch circles of gears 2 and 3 are in contact. The shaft to shaft distance is half of the sum of the two pitch circle diameters.

$$P = \frac{N}{d} \qquad \text{(Equation 13-1)}$$

where

P = diametral pitch (number of teeth per inch)
N = number of teeth
d = diameter (inch)

$$d_2 = \frac{N_2}{P} = \frac{14 \text{ teeth}}{10 \text{ teeth/in}} = 1.4 \text{ inch}$$
$$d_3 = \frac{N_3}{P} = \frac{54 \text{ teeth}}{10 \text{ teeth/in}} = 5.4 \text{ inch}$$

Shaft-to-shaft distance, y_{23}, = $\dfrac{d_2 + d_3}{2} = \dfrac{1.4 + 5.4}{2} = 3.4 \text{ inch}$

c) Find the speed and direction of shaft *c* (same as speed and direction of gear 5).

Note: gear 3 and gear 4 are on common shaft → $n_3 = n_4$

$$\begin{aligned} n_5 = e_{45} n_4 &= -\frac{\text{driving teeth}}{\text{driven teeth}} n_4 \\ &= -\frac{N_4}{N_5} n_4 = -\frac{16}{36}(-233) \\ &= 103.6 \text{ rev/min(CCW)} \end{aligned}$$

d) Find the centre to centre distance between the shafts *b* and *c*.

We are given normal diametral pitch (measured perpendicular to teeth). For a helical gear, this needs to be converted to transverse diametral pitch (measured around the circumference). See Fig 13-22 and Eq. 13-18 in text.

$$\begin{aligned} P_{transverse} &= P_{normal} \cos\psi \\ &= 6 \cos 25° \\ &= 5.44 \text{ teeth/in} \end{aligned}$$

We expect to cross fewer teeth per inch in the transverse direction so this makes sense.

The rest of the solution is the same as above.

$$P_t = \frac{N}{d}$$

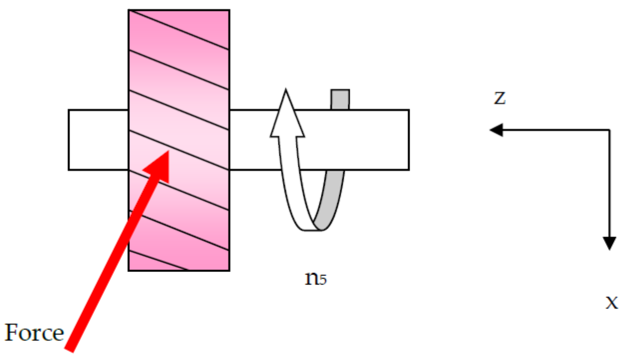
$$d_4 = \frac{N_4}{P_t} = \frac{16 \text{ teeth}}{5.44 \text{ teeth/in}} = 2.94 \text{ inch}$$

$$d_5 = \frac{N_5}{P_t} = \frac{36 \text{ teeth}}{5.44 \text{ teeth/in}} = 6.61 \text{ inch}$$

Shaft-to-shaft distance, y_{45}, $= \frac{d_4 + d_5}{2} = \frac{2.94 + 6.61}{2} = 4.78 \text{ inch}$

e) Find the direction of the thrust force exerted by gears 4 and 5 on shafts b and c, respectively.

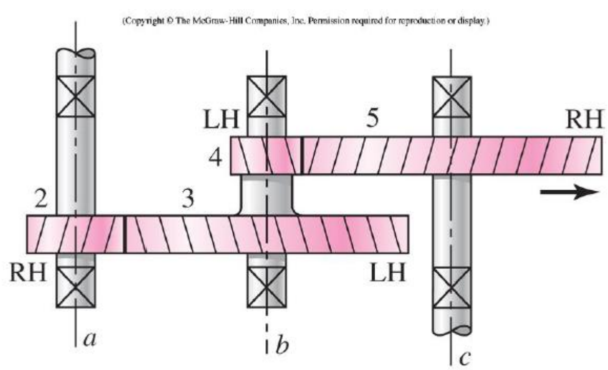
Consider looking at gear 5 and shaft *c* from below. The direction of rotation is as shown. In order to rotate this way, the contact force of gear 4 on gear 5 must be directed as indicated (note: the force is normal to the teeth). This force has a transverse component in the –z direction. Similar arguments can be used for shaft *b*.



Force on shaft *b* is in +z direction; Force on shaft *c* is in –z direction.

Shigley, Problem 13-47 (8th Ed., 13-33). (To be done by students.)

A gear train is composed of four helical gears with the three shaft axes in a single plane, as shown in the figure. The gears have a normal pressure angle of 20° and a 30° helix angle. Shaft *b* is an idler and the transmitted load acting on gear 3 is 500 lbf. The gears on shaft *b* both have a normal diametral pitch of 7 teeth/in and have 54 and 14 teeth, respectively. Find the forces exerted by gears 3 and 4 on shaft *b* (don't worry about determining moments)



3. **Shigley, Problem 13-47. (To be done by students.)**

Find the forces exerted by gears 3 and 4 on shaft *b*.

Refer to Fig 13-37 (pg. 704)

The forces from the gears look like:

W_3^t = 500 lb (given)

From Eq. 13-39 (pg. 704)

$$W_r = W \sin\phi_n$$
$$W_t = W\cos\phi_n \cos\psi$$
$$W_a = W\cos\phi_n \sin\psi$$

Where ψ = 30° (given) and φ_n = 20° (given)

Solving for W, W_r, and W_a we find

W_3 = 614.4 lb
W_3^r = 210.2 lb
W_3^a = 288.7 lb

Since gears 3 and 4 are on a common shaft (assumed rotating at constant velocity) the torques must balance. Therefore

$W_3^t d_3 = W_4^t d_4$

W_4^t $= W_3^t d_3 / d_4$
$= W_3^t (N_3 / P_{3t}) / (N_4 / P_{4t})$
$= W_3^t N_3 / N_4$ (since $P_{3t} = P_{4t}$)
= (500 lb)(54) / (14)
= 1929 lb

Solving for W_r and W_a on gear 4, as above, we find

W_4^r = 811 lb
W_4^a = 1114 lb

The total force on shaft *b* is the sum of the forces from both gears. Referring to the diagram at the start for directions:

$$\vec{W} = \left((210-811)\hat{i} + (-289+1114)\hat{j} + (-500-1929)\hat{k}\right) lb$$
$$= \left(-601\hat{i} + 825\hat{j} - 2429\hat{k}\right) lb$$

Adapted from **Shigley, Problem 14-19**.

A commercial enclosed gear drive consists of a 20° pinion having 16 teeth driving a 48-tooth gear. The pinion speed is 300 rev/min, the face width is 2 in, and the diametral pitch is 6 teeth/in. The gears are grade 1 steel, through-hardened at 200 Brinell, made to No. 6 quality standards, uncrowned, and are to be accurately and rigidly mounted. Assume a pinion life of 10^8 cycles and a reliability of 0.90. The power to be transmitted is 5 hp.

Part 1: To be demonstrated by the TA.

a. Determine the AGMA bending stress in the pinion
b. Determine the corresponding safety factor for the pinion bending stress

Part 2: To be done by students.

c. Determine the AGMA contact stress in the pinion
d. Determine the corresponding safety factor for the pinion contact stress

Part 3: Extra practice, to be done by students at home.

e. Repeat the above bending and contact stress analysis for the gear and show that the stresses are 9.59 ksi and 102.1 ksi, respectively, and that the safety factors in bending and contact are 3.29 and 1.02, respectively.

Part 1: To be demonstrated by the TA.

a. Determine the AGMA bending stress in the pinion

Working from the "roadmap" in Fig 14-17 (p. 758),

$$d_p = \frac{N_p}{P_d} = \frac{16 \text{ teeth}}{6 \text{ teeth/in}} = 2.667 \text{ in} \qquad \text{(pinion diameter)}$$

$$V = \frac{\pi d n}{12} = \frac{\pi(2.667 \text{ in})(300 \text{ rev/min})}{12} = 209 \text{ ft/min} \qquad \text{(pitch line velocity, note the units)}$$

$$W^t = \frac{33{,}000H}{V} = \frac{33{,}000(5 \text{ hp})}{209 \text{ ft/min}} = 789 \text{ lb} \qquad \text{(transmitted load)}$$

the gear bending stress equation is given by

$$\sigma = W^t K_o K_v K_s \frac{P_d}{F}\frac{K_m K_b}{J} \qquad \text{(14-15, p. 738)}$$

We know W^t from above, we are given P_d = 6 teeth/in, and we are given the face width, F = 2in. To compute σ, we need to solve for all of the coefficients.

K_o: overload factor

Since we don't have any other information, assume the power source and the driven machine have uniform loading. From the table on the bottom of p. 758,

K_o = 1

K_v: velocity factor

Although we could use Figure 14-9 (p. 750) to find K_v, it would be difficult to get an accurate value due to the steepness of the curve and low resolution of the V values on the x-axis.

Instead, use the expressions in eqns. 14-27 and 14-28 on p. 748.

$$K_v = \left(\frac{A+\sqrt{V}}{A}\right)^B$$
$$A = 50 + 56(1-B)$$
$$B = 0.25(12-Q_v)^{2/3}$$

With V = 209 ft/min (found above) and Q_v = 6 (given in question) we have

$$B = 0.8255$$
$$A = 59.77$$
$$K_v = 1.196$$

K_v = 1.196

K_s: size factor

From eq. (a) on p. 751

$$K_s = 1.192\left(\frac{F\sqrt{Y}}{P_d}\right)^{0.0535}$$

F = 2in (given), Y = 0.296 for the pinion (see Table 14-2, p. 730), P_d = 6 (given)

$$K_s = 1.192\left(\frac{(2\text{ in})\sqrt{0.296}}{6\text{ teeth/in}}\right)^{0.0535} = 1.088$$

K_s = 1.088

K_m: Load-distribution factor

From eq. 14-30 (p. 751)

$$K_m = 1 + C_{mc}\left(C_{pf}C_{pm} + C_{ma}C_e\right)$$

The "C" terms are all given on pp. 752-3.

$C_{mc} = 1$ (for uncrowned teeth)

$$C_{pf} = \frac{F}{10d} - 0.0375 + 0.0125F \quad \text{for } 1" < F \le 17"$$

$$= \frac{2 \text{ in}}{10(2.667 \text{ in})} - 0.0375 + 0.0125(2 \text{ in})$$

$$= 0.0625$$

$C_{pm} = 1$ (we don't know the pinion mounting but have no reason to assume it isn't centered)

$C_{ma} = 0.16$ (from Fig 14-11; could also use eq. 14-34 & Table 14-9) (note the error in Shigley's solutions; taken for precision enclosed units, not commercial enclosed units)

$C_e = 1$ (eq. 14-35, no information to suggest it should be 0.8)

Putting it all together,

$$K_m = 1 + 1(0.0625 \cdot 1 + 0.16 \cdot 1)$$

$$= 1.223$$

K_m = 1.223

K_b: rim thickness factor

From eq. 14-40 and Fig. 14-16 (p. 756), no reason to believe $m_b < 1.2$, so use

K_b = 1

J: Bending-Strength Geometry Factor

From Fig. 14-6 with N_p = 16 = "number of teeth for which geometry factor is desired" and load applied at highest point of single-tooth contact

J = 0.27

Finally, we can insert all of the coefficients into the stress expression

$$\sigma = W^t K_o K_v K_s \frac{P_d}{F} \frac{K_m K_b}{J}$$

$$= (789 \text{ lb})(1)(1.20)(1.088) \frac{6 \text{ teeth/in}}{2 \text{ in}} \frac{(1.223)(1)}{0.27}$$

$$= 14.00 \text{ ksi}$$

The bending stress on the pinion is 14.00 ksi.

b. Determine the corresponding safety factor for the pinion in bending

To find the safety factor in pinion bending, use the allowable stress in eq. 14-41 (p. 757)

$$S_F = \frac{S_t}{\sigma}\frac{Y_N}{K_T K_R}$$

The σ is the stress calculated in part a. The other values are found in a similar manner to above – by looking up the data in the tables and charts as referenced on p. 757).

<u>S_t: bending strength (Fig. 14-2, p. 739)</u>

$$\begin{aligned} S_t &= 77.3H_B + 12,800 \text{ psi} \\ &= 77.3(200) + 12,800 \text{ psi} \\ &= 28.3 \text{ ksi} \end{aligned}$$

(H_B = 200 is the Brinell hardness)

<u>Y_N: repeated bending stress cycle factor</u>

With 10^8 cycles, $Y_N \approx 0.95$ (read from Fig 14-14, pg. 755)
(Shigley uses a less conservative value)

<u>K_T: Temperature factor</u>

Since no reason to expect T > 250°F, $K_T = 1$ (Section 14-15, p. 756)

<u>K_R: Reliability factor</u>

With reliability of 0.90, $K_R = 0.85$ (from Table 14-10, p. 756)

Combining all terms,

$$S_F = \frac{28.3 \text{ ksi}}{14.0 \text{ ksi}}\frac{0.95}{1 \cdot 0.85} = 2.26$$

The safety factor for the pinion in bending is 2.26.

c. Determine the AGMA contact stress in the pinion

The process is the same as above. The required equation is eq. 14-16 (p. 759)

$$\sigma_c = C_P \sqrt{W^t K_o K_v K_s \frac{K_m}{d_p F} \frac{C_f}{I}}$$

The terms W^t = 789 lb, K_o = 1, K_v = 1.196, K_s = 1.088, K_m = 1.223, d_p = 2.667″, and F = 2″ were either given or solved for in the previous part of the problem. It remains to determine C_P, C_f, and I.

C_P: Elastic coefficient

C_P = 2300 psi$^{1/2}$ (from steel pinion-steel gear in Table 14-8, p. 749)

C_f: Surface condition coefficient

C_f = 1 (as noted on p. 759)

I: Surface-Strength Geometry Factor

$$I = \frac{\cos\phi_t \sin\phi_t}{2m_N} \frac{m_G}{m_G + 1}$$ (eq. 14-23, p. 747, internal gears)

As noted below eq. 14-23 (p. 747) m_N = 1 for spur gears.

m_G is the speed ratio as given in eq. 14-22 (p. 746)

$$m_N = \frac{N_G}{N_P} = \frac{48}{16}$$
$$= 3$$

For a spur gear, φ_t is just φ, the pressure angle. As given in the problem description, φ = 20° in this case.

Substituting to the expression for I above, we get

$$I = \frac{\cos 20° \sin 20°}{2 \cdot 1} \frac{3}{3+1}$$
$$= 0.1205$$

Plugging everything into the expression for σ_c we get

$$\sigma_c = 2300\sqrt{\left(789\ \text{psi}^{1/2}\right)(1)(1.196)(1.088)\frac{1.223}{(2.667\ \text{in})(2\ \text{in})}\frac{1}{0.1205}}$$
$$= 101.7\ \text{ksi}$$

The contact stress is 101.7 ksi

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d. Determine the corresponding safety factor for the pinion in terms of contact stress

The safety factor is given by eq. 14-42 (p. 759)

$$S_H = \frac{S_c Z_N C_H}{K_T K_R}\frac{1}{\sigma_c}$$

The same process as before is used. In part a., it was already determined that K_T = 1 and K_R = 0.85. For σ_c we use our result from part c., σ_c = 101.7 ksi.

<u>S_c: Contact-fatigue strength</u>

$$S_c = 322 H_B + 29{,}100\ \text{psi}$$
$$= 322(200) + 29{,}100$$

(Fig 14-5, p. 742)

S_c = 93,500 psi

<u>Z_N: Stress Cycle Factor</u>

From Figure 14-15 (p. 755) with 10^8 cycles, choose the centre of the shaded region (more conservative than Shigley)

Z_N = 0.90

<u>C_H: hardness ratio factor</u>

As noted in Sec 14-12 (p. 753), C_H applies to the gear only. We can either neglect the term or use C_H = 1, with the same effect.

C_H = 1

The final expression is

$$S_H = \frac{(93.5 \text{ ksi})(0.90)(1)}{(1)(0.85)} \frac{1}{101.7 \text{ ksi}}$$

$S_H = 0.97$

With a safety factor less than unity, pinion wear is a concern so hardness should be increased. (It won't disintegrate immediately, but it will not last 10^8 cycles with 0.90 reliability.)

Adapted from **Shigley, Problem 17-2** (8th and 9th Eds., 17-1).

A 6-in-wide polyamide F-1 flat belt is used to connect a 2-in-diameter pulley to drive a larger pulley with an angular velocity ratio of 0.5. The centre-to-centre distance is 9 ft. The angular speed of the small pulley is 1750 rev/min as it delivers 2 hp. The application is such that a service factor K_s of 1.25 is appropriate.

Part 1: To be demonstrated by the TA.

a. Find F_c, F_i, F_{1a}, and F_2

d = 2 in	(drive pulley diameter, given)
D = 2 in / 0.5 = 4 in	(driven pulley diameter based on 0.5 velocity ratio)
b = 6in	(belt width, given)

From Table 17-2 (p. 869) for a polyamide F-1 flat belt

t = 0.05 in	(thickness)
γ = 0.035 lbf/in^3	(specific weight)
f = 0.5	(coefficient of friction)
F_a = 35 lb/in	(allowable tension per width at 600 ft/min)

F_c = hoop tension due to centrifugal force

$$F_c = \frac{w}{g}\left(\frac{V}{60}\right)^2 = \frac{w}{32.17}\left(\frac{V}{60}\right)^2$$ (eq. 17-8e, p. 885)

$$\begin{aligned} V &= \pi dn/12 \\ &= \pi(2 \text{ in})(1750 \text{ rev/min})/12 \\ &= 916.3 \text{ ft/min} \end{aligned}$$ (belt speed in ft/min, p. 885)

$$
\begin{aligned}
w &= 12\gamma bt \\
&= 12\left(0.035\ \text{lb/in}^3\right)(6\ \text{in})(0.05\ \text{in}) \qquad \text{(belt weight per foot, p. 885)} \\
&= 0.126\ \text{lb/ft}
\end{aligned}
$$

$$
\begin{aligned}
F_c &= \frac{0.126\ \text{lb/ft}}{32.17}\left(\frac{916.3\ \text{ft/s}}{60}\right)^2 \\
&= 0.913\ \text{lb}
\end{aligned}
$$

F_c = 0.913 lb

<u>F_{1a} = allowable largest tension</u>

$F_{1a} = F_a b C_p C_v$ (eq. 17-12, p. 888)

F_a = 35 lb/in (allowable tension per width, Table 17-2, p. 889)
b = 6in (belt width, given)

C_p = 0.70 (Table 17-4, p. 890, with 2in small pulley diameter)

C_v = 1 (velocity correction factor, =1 for polyamide and urethane belts, see note on p. 887)

$$
\begin{aligned}
F_{1a} &= (35\ \text{lb/in})(6\ \text{in})(0.70)(1.0) \\
&= 147\ \text{lb}
\end{aligned}
$$

F_{1a} = 147 lb

<u>F_2 = loose side tension</u>

$$
\begin{aligned}
F_2 &= F_{1a} - \Delta F \\
&= F_{1a} - 2T/d
\end{aligned}
$$

Need to find T

$T = H_d/\omega$ (torque transmitted = power supplied / angular velocity)

$H_d = H_{nom} K_s n_d$ (see note on p. 888
H_d = power delivered
H_{nom} = nominal power delivered = 2 hp in this case, given
K_s = service factor (for deviations from normal load)
= 1.25 in this case, given
n_d = design safety factor = 1 in this case, implied)

$T = H_d/\omega$ is in SI units, in units of hp, lb·in, and rev/min the expression is

$$
\begin{aligned}
T &= \frac{63,025 H_d}{n} \\
&= \frac{63,025 H_{nom} K_s n_d}{n} \\
&= \frac{63,025 (2 \text{ hp})(1.25)(1)}{1750 \text{ rev/min}} \\
&= 90.0 \text{ lb} \cdot \text{in}
\end{aligned}
$$

F_2 = F_{1a} - 2T/d
= 147 lb - 2(90.0 lb·in)/(2 in)
= 57 lb

F_2 = 57 lb

<u>F_i = initial tension</u>

F_i = 0.5 (F_1 + F_2) - F_c (eq. 17-8j, p. 886)
= 0.5 (147 lb + 57 lb) - 0.916 lb
= 101.1 lb

F_i = 101.1 lb

Part 2: To be done by students.

b. Find the allowable power, H_a

H_a = transmitted power with maximum allowable tension used
H_a = (F_{1a} - F_2)·V / 33,000 (from eq. (j), p. 887, note: the 33,000 is so units work in hp, lb, and ft/min)

= (147 lb - 57 lb)·(916.3 ft/min)/33,000
= 2.5 hp

H_a = 2.5 hp

c. Find the factor of safety, n_{fs}

Note: you may need to explain in words how to compute the factor of safety

n_{fs} = (allowable power) / [(nominal power)·(service factor)]
= H_a / [H_{nom}·K_s]
= (2.5 hp) / [(2 hp)(1.25)] (transmitted power found above,
= 1

n_{fs} = 1

d. Find the belt length, L

$$L=\sqrt{4C^2-(D-d)^2}+\frac{1}{2}(D\theta_D+d\theta_d)$$ (eq. 17-2, p. 883)

C = centre distance = 9 ft (given)
d = small pulley diameter = 2in (given)
D = large pulley diameter = 4 in (found above, d/0.05)

$$\begin{aligned}\theta_d &= \pi-2\sin^{-1}\frac{D-d}{2C}\\ &= \pi-2\sin^{-1}\left(\frac{4\text{ in}-2\text{ in}}{2\cdot 9\text{ ft}\cdot 12\text{ in/ft}}\right)\\ &= 3.132\text{ rad}\end{aligned}$$ (eq. 17-1, p. 883, must use radians)

$$\begin{aligned}L &= \sqrt{4(9\text{ ft}\cdot 12\text{ in/ft})^2-(4\text{ in}-2\text{ in})^2}+\frac{1}{2}\left[(4\text{ in})(3.151)+(2\text{ in})(3.132)\right]\\ &=225.4\text{ in}\end{aligned}$$

L = 225.4 in

e. Find the dip

$$\begin{aligned}dip &= \frac{C^2 w}{96F_i}\\ &= \frac{(9\text{ ft}\cdot 12\text{ in/ft})^2(0.126\text{ lb/ft})}{96(101.1\text{ lb})}\\ &= 0.151\text{ in}\end{aligned}$$ (e.q. 17-13, p. 892)

(Note: the mismatched units are accounted for in the coefficient 96; see p. 892 for details.)

Dip = 0.151 in

A polyamide A-3 flat belt 6 in wide is used to transmit 15 hp under light shock conditions where $K_s = 1.25$, and a factor of safety equal to or greater than 1.1 is appropriate. The pulley rotational axes are parallel and in the horizontal plane. The shafts are 8 ft apart. The 6-in driving pulley rotates at 1750 rev/min in such a way that the loose side is on top. The driven pulley is 18 in in diameter. See Fig. 17–10. The factor of safety is for unquantifiable exigencies.

(*a*) Estimate the centrifugal tension F_c and the torque T.

(*b*) Estimate the allowable F_1, F_2, F_i and allowable power H_a.

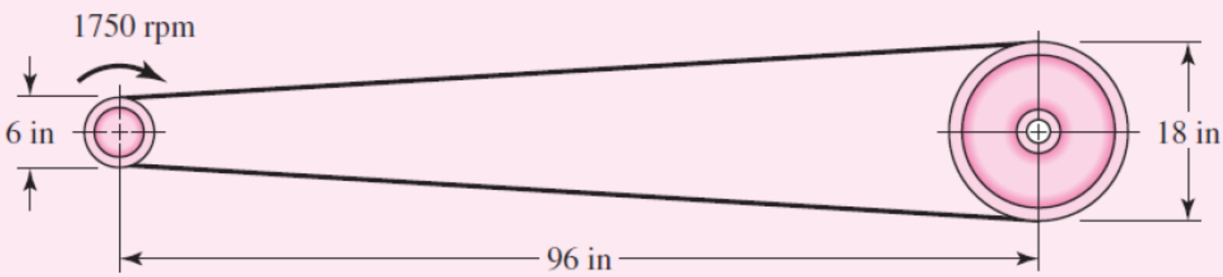
(*c*) Estimate the factor of safety. Is it satisfactory?

Belt 6 in × 0.130 in

15 hp

$\gamma = 0.042 \ \frac{\text{lbf}}{\text{in}^3}$

$d = 6$ in, $D = 18$ in



(*a*) Eq. (17–1): $$\phi = \theta_d = \pi - 2\sin^{-1}\left[\frac{18-6}{2(8)12}\right] = 3.0165 \text{ rad}$$

Table 17–2: $$\gamma = 0.042 \text{ lbf/in}^3 \qquad f = 0.8 \qquad F_a = 100 \text{ lbf/in}$$

$$\exp(f\phi) = \exp[0.8(3.0165)] = 11.17$$

$$V = \pi(6)1750/12 = 2749 \text{ ft/min}$$

$$w = 12\gamma bt = 12(0.042)6(0.130) = 0.393 \text{ lbf/ft}$$

Eq. (*e*): $$F_c = \frac{w}{g}\left(\frac{V}{60}\right)^2 = \frac{0.393}{32.17}\left(\frac{2749}{60}\right)^2 = 25.6 \text{ lbf}$$

$$T = \frac{63\,025 H_{\text{nom}} K_s n_d}{n} = \frac{63\,025(15)1.25(1.1)}{1750}$$

$$= 742.8 \text{ lbf} \cdot \text{in}$$

(b) The necessary $(F_1)_a - F_2$ to transmit the torque T, from Eq. (h), is

$$(F_1)_a - F_2 = \frac{2T}{d} = \frac{2(742.8)}{6} = 247.6 \text{ lbf}$$

For polyamide belts $C_v = 1$, and from Table 17–4 $C_p = 0.70$. From Eq. (17–12) the allowable largest belt tension $(F_1)_a$ is

$$(F_1)_a = bF_aC_pC_v = 6(100)0.70(1) = 420 \text{ lbf}$$

then

$$F_2 = (F_1)_a - [(F_1)_a - F_2] = 420 - 247.6 = 172.4 \text{ lbf}$$

and from Eq. (i)

$$F_i = \frac{(F_1)_a + F_2}{2} - F_c = \frac{420 + 172.4}{2} - 25.6 = 270.6 \text{ lbf}$$

The combination $(F_1)_a$, F_2, and F_i will transmit the design power of $H_a = H_{\text{nom}}K_sn_d = 15(1.25)(1.1) = 20.6$ hp and protect the belt. We check the friction development by solving Eq. (17–7) for f':

$$f' = \frac{1}{\phi}\ln\frac{(F_1)_a - F_c}{F_2 - F_c} = \frac{1}{3.0165}\ln\frac{420 - 25.6}{172.4 - 25.6} = 0.328$$

As determined earlier, $f = 0.8$. Since $f' < f$, there is no danger of slipping.

(c) From step 9 on p. 880,

$$n_{fs} = \frac{H_a}{H_{\text{nom}}K_s} = \frac{20.6}{15(1.25)} = 1.1 \qquad \text{(as expected)}$$

The belt is satisfactory and the maximum allowable belt tension exists. If the initial tension is maintained, the capacity is the design power of 20.6 hp.

A mine hoist uses a 2-inch 6 × 19 monitor-steel wire rope. The rope is used to haul maximum loads of 4 tons from a 480 ft. shaft. The drum has a diameter of 6 feet and the sheaves are of good-quality cast steel, and the smallest is 3 feet in diameter.

Part 1: To be done by the TA.

a. Using a maximum hoisting speed of 1200 feet/minute and a maximum acceleration of 2 ft/s^2, estimate the stresses in the wire rope.

Part 2: To be done by students.

b. Estimate the various factors of safety

a. Using a maximum hoisting speed of 1200 feet/minute and a maximum acceleration of 2 ft/s^2, estimate the stresses in the wire rope.

The objective of the problem is to explore factors of safety in wire rope. We will express strengths as tensions.

Given: Monitor steel 2-in 6 × 19 rope, 480 ft long.

From Table 17-27:
Minimum diameter of a sheave is 30*d*. To allow a greater factor of safety we can use the "better sheave diameter," 45*d*. preferably 45(2) = 90 in. The hoist abuses the wire when it is bent around a sheave.

Also, from Table 17-24:
The nominal tensile strength as 106 kpsi.

The ultimate load is

$$F_u = (S_u)_{\text{nom}} A_{\text{nom}} = 106\left[\frac{\pi(2)^2}{4}\right] = 333 \text{ kip} \quad Ans.$$

The tensile loading of the wire is given by Eq. (17-47)

$$F_t = \left(\frac{W}{m} + wl\right)\left(1 + \frac{a}{g}\right)$$
$$W = 4(2) = 8 \text{ kip}, \quad m = 1$$

From Table (17-24), the weight/foot × length gives us:

$$wl = 1.60d^2 l = 1.60(2^2)(480) = 3072 \text{ lbf} = 3.072 \text{ kip}$$

Therefore,

$$F_t = (8 + 3.072)\left(1 + \frac{2}{32.2}\right) = 11.76 \text{ kip} \quad Ans.$$

From Eq. (17-41) and Table 17-27:

$$F_b = \frac{E_r d_w A_m}{D}$$

With a 72-in drum, we get:

$$F_b = 12(10^6)(0.067\text{d})(0.40\text{d}^2)/\text{D}$$

$$F_b = 12(10^6)(0.067(2))(0.40(2^2)/72 = 35.7 \text{ kip} \quad Ans.$$

For use in Eq. (17-44), from Fig. 17-21, S_u = 240kpsi from page 912

$$(p / S_u) = 0.0014$$
$$S_u = 240 \text{ kpsi}, \quad \text{p. } 920$$
$$F_f = \frac{0.0014(240)(2)(72)}{2} = 24.2 \text{ kip} \quad Ans.$$

Tension Load = 11,760 lbs., Bending Load = 35700 lbs, and Fatique Load = 24200 lbs.
The Ultimate Load is

Part 2: To be done by the students.

b. Estimate the various factors of safety in the wire rope.

Static, no bending:

$$n = \frac{F_u}{F_t} = \frac{333}{11.76} = 28.3 \quad Ans.$$

***Static, with bending*:**

Eq. (17-46): $$n_s = \frac{F_u - F_b}{F_t} = \frac{333 - 39}{11.76} = 25.0 \quad Ans.$$

Fatigue without bending:

$$n_f = \frac{F_f}{F_t} = \frac{24.2}{11.76} = 2.06 \quad Ans.$$

Fatigue, with bending: For a life of $0.1(10^6)$ cycles, from Fig. 17-21

$$(p / S_u) = 4 / 1000 = 0.004$$
$$F_f = \frac{0.004(240)(2)(72)}{2} = 69.1 \text{ kip}$$

Eq. (17-45): $$n_f = \frac{69.1 - 39}{11.76} = 2.56 \quad Ans.$$

If we were to use the endurance strength at 10^6 cycles ($F_f = 24.2$ kip) the factor of safety would be less than 1 indicating 10^6 cycle life impossible.

An Oiles SP 500 alloy brass bushing is 1 in long with a 1-in bore and operates in a clean environment at 70°F. The allowable wear without loss of function is 0.005 in. The radial load is 500 lbf. The shaft speed is 200 rev/min.

Part 1: To be demonstrated by the TA.

a. Estimate the number of revolutions for radial wear to be 0.005 in.

Part 2: To be done by students.

b. Estimate the wear expected for 1 million revolutions if the speed is reduced to 80 rev/min, the load is increased to 800 lbf, and the material is changed to Oiles SP 800.

Part 1: To be demonstrated by the TA.

a. Estimate the number of revolutions for radial wear to be 0.005 in.

The material removal rate (thickness per time) is:

$w = f_1 f_2 KFNt/(3L)$ (eq. 12-32, p. 664)

Note: eq. 12-27, p. 663 looks similar but is for flat surface wear, not curved-surface wear on a bushing.

where

P is the nominal pressure

$P = F/DL$ (eq. 12-28, p. 663)
$= (500\text{ lbf})/(1\text{in} \cdot 1\text{in})$
$= 500\text{ psi}$

V is the sliding velocity

$V = \pi DN/12$ (eq. 12-29, p. 664)
$= \pi(1\text{in})(200\text{ rev/min})/12$
$= 52.36\text{ ft/min}$

f_1 is the motion related factor

f_1 is given by Table 12-10 (p. 663)

Since the shaft is turning at 200 rev/min (constant) we are dealing with rotary motion in the table. With P = 500 psi and V = 52.36 ft/min (both found above) we get f_1 = 1.3-1.8. Of this range, the more conservative estimate is 1.8. (Alternatively, we could interpolate between the range and we would find f_1 = 1.44.) For now, use the conservative estimate:

$f_1 = 1.8$

f_2 is the environmental factor

f_2 is given by Table 12-11 (p. 663)

$f_2 = 1.0$ (<140°F in clean environment → no foreign matter)

* All problems, equations and page numbers are based on Shigley, 9th edition.

K is the wear factor

K is given in Table 12-8 (p. 662)

For Oiles 500, $K = 0.6\times10^{-10}$ $in^3\cdot min/(lbf\cdot ft\cdot hr)$

$K = 0.6\times10^{-10}$

$w = f_1f_2KFNt/(3L)$

$t = 3Lw / (f_1f_2KFN)$
$= 3(1in)(0.005\ in) / [(1.8)(1.0)(0.6\times10^{-10})(500\ lbf)(200\ rev/min)]$
$= 1389$ hrs

At 200 rev/min operating for 1389 hrs, the total number of revolutions is given by:

Revs = (200 rev/min)(1389 hrs)(60 min/hr) = $1.67\times10^7\ rev$

$\boxed{\text{Revs} = 16.7 \text{ million revolutions}}$

Part 2: To be done by students.

b. Estimate the wear expected for 1 million revolutions if the speed is reduced to 80 rev/min, the load is increased to 800 lbf, and the material is changed to Oiles SP 800.

The material removal rate (thickness per time) is:

$w = f_1f_2KFNt/(3L)$ (eq. 12-32, p. 664)

P is the nominal pressure ← increased

$P = F/DL$ (eq. 12-28, p. 663)
$= (800\ lbf)/(1in \cdot 1in)$
$= 800$ psi

V is the sliding velocity ← decreased

$V = \pi DN/12$ (eq. 12-29, p. 664)
$= \pi(1in)(80\ rev/min)/12$
$= 20.94$ ft/min

f_1 is the motion related factor

f_1 is given by Table 12-10 (p. 663)

From the table with P = 800 psi and V = 26.18 ft/min, the values of f_1 range from 1.5 to 2.0. Take 2.0 as a conservative approach in wear.

$f_1 = 2.0$

f_2 is the environmental factor

f_2 is given by Table 12-11 (p. 663)

$f_2 = 1.0$ (<140°F in clean environment → no foreign matter)

K is the wear factor

K is given in Table 12-8 (p. 662)

For Oiles 800, $K = 3\times10^{-10}\ in^3\cdot min/(lbf\cdot ft\cdot hr)$

$K = 3\times10^{-10}$

Number of revolutions = speed · time = n·t = 1 million/60 (given)

$$\begin{aligned} w &= f_1 f_2 KFNt/(3L) \\ &= (2.0)(1.0)(3\times10^{-10})(800\ lbf)(10^6/60)/(3)(1) \\ &= 2.67\times10^{-3}\ in \end{aligned}$$

$\boxed{\text{We should expect wear of } 2.67\times10^{-3} \text{ inches}}$

A certain application requires a ball bearing with the inner ring rotating. The design life is 30,000 hours at a speed of 300 rev/min. The radial load is F = 1.898 kN. The loads are unsteady, so an application factor of $a_f = 1.2$ has been recommended. The reliability goal for this design is 0.90. (See the general bearing information on the following page.)

Part 1: To be demonstrated by the TA.

a. Find the dimensionless life, x_D, and basic load rating, C_{10}
b. Choose a 02-series deep-groove ball bearing from Table 11-2
c. Find the reliability of this bearing in use

Part 2: To be done by students.

Assume the radial load from Part 1 is increased to F = 6.00kN, the reliability goal is increased to 0.95, and a cylindrical roller bearing is now to be selected.

d. Find the new dimensionless life, x_D, and basic load rating, C_{10}
e. Select an 02-series cylindrical roller bearing from the Table 11-3
f. Find the reliability of the selected bearing in use

a. Find the dimensionless life, x_D, and basic load rating, C_{10}

Dimensionless life, x_D, is the ratio of design life to L_{10} life. (see paragraph above eq. 11-4, p. 576)

$x_D = L_D / L_{10}$

$L_D = (30{,}000 \text{ hrs})(300 \text{ rev/min})(60 \text{ min/hr})$
$= 540\times10^6$ revs

$L_{10} = 10^6$ revs (given as rating lift in supplementary question information, page 2 of tutorial handout)

$x_D = 540$

The basic load rating (catalogue load rating), C_{10}, is given in eq. 11-6 (p. 578)

$$C_{10} = F_D\left[\frac{x_D}{x_0 + (\theta - x_0)\left(\ln \frac{1}{R_D}\right)^{\frac{1}{b}}}\right]^{\frac{1}{a}}$$ (eq. 11-6, p. 578)

F_D = design load
= equivalent radial load · application factor
= $F \cdot a_f$
= (1.898 kN)(1.2) (both F and a_f were given in the question.
= 2.278 kN Note that the equivalent radial load is just the radial load in this case since the load is steady and there is no axial load)

x_D = 540 (found above)

x_0 = 0.02 (given in the question supplementary info)
θ = 4.459 (given in the question supplementary info)
b = 1.483 (given in the question supplementary info)
a = 3 (ball bearing: see below eq. 11-1, p. 574)
R_D = is the design reliability
= 0.90 (given in the question)

$$C_{10} = 2.278 \text{ kN}\left[\frac{540}{0.02 + (4.459 - 0.02)\left(\ln \frac{1}{0.9}\right)^{\frac{1}{1.483}}}\right]^{\frac{1}{3}}$$
$$= 18.59 \text{ kN}$$

C_{10} = 18.59 kN

b. Choose a 02-series deep-groove ball bearing from Table 11-2

Look at the Deep groove C_{10} load rating column in Table 11-2 (column 7). Need to have a bearing with a C_{10} value at least that found in part a. Note, as indicated in table caption, all bearings are 02 series.

Choose 30mm bore bearing (C_{10} for this bearing is 19.5 kN which is greater than the 18.61 kN required). Any bearing with a larger bore would also have satisfied the design requirements.

Choose a 02-30 mm bearing

c. Find the reliability of this bearing in use

Equation 11-6 (p. 578) relates C_{10} in terms of in terms of reliability R and various other bearing parameters. We have a value of C_{10} for the bearing we selected (C_{10} = 19.5 kN) so we can solve the expression for R. This algebra is conveniently done in eq. 11-18 (p. 600).

$$R = \exp\left[-\left(\frac{x_D\left(\frac{a_f F_D}{C_{10}}\right)^a - x_0}{\theta - x_0}\right)^b\right] \qquad \text{(eq. 11-18, p. 600)}$$

$$R = \exp\left[-\left(\frac{540\left(\frac{(1.2)(1.898\text{ kN})}{19.5\text{ kN}}\right)^3 - 0.02}{4.459 - 0.02}\right)^{1.483}\right]$$

$$= 0.919$$

The reliability is 0.919

Part 2: To be done by students.

Assume the radial load from Part 1 is increased to F = 6.00kN, the reliability goal is increased to 0.95, and a cylindrical roller bearing is now to be selected.

d. Find the new dimensionless life, x_D, and basic load rating, C_{10}

Dimensionless life, x_D, is the ratio of design life to L_{10} life. (see paragraph above eq. 11-4, p. 576). It is unchanged from Part 1 above.

x_D = 540

The basic load rating (catalogue load rating), C_{10}, is given in eq. 11-6 (p. 578)

$$C_{10} = F_D \left[\frac{x_D}{x_0 + (\theta - x_0)\left(\ln \frac{1}{R_D}\right)^{\frac{1}{b}}} \right]^{\frac{1}{a}} \qquad \text{(eq. 11-6, p. 578)}$$

In this case, the radial load is increased from 1.898 kN to 3.00 kN; the constant "a" changes to 10/3 as we are working with a roller bearing (see beneath eq. 11-1 on p. 574); and the design reliability, R_D, has increased to 0.95.

F_D = $F \cdot a_f$
= (6.00 kN)(1.2)
= 7.2 kN

x_D	= 540	(found above)
x_0	= 0.02	(given in the question supplementary info)
θ	= 4.459	(given in the question supplementary info)
b	= 1.483	(given in the question supplementary info)
a	= 10/3	(roller bearing: see below eq. 11-1, p. 574)
R_D	= is the design reliability	
	= 0.95	(given in the question)

$$C_{10} = 7.20 \text{ kN} \left[\frac{540}{0.02 + (4.459 - 0.02)\left(\ln \frac{1}{0.95}\right)^{\frac{1}{1.483}}} \right]^{\frac{3}{10}}$$

$$= 54.9 \text{ kN}$$

C_{10} = 54.9 kN

e. Select an 02-series cylindrical roller bearing from the Table 11-3

From Table 11-3, column 4, the 55 mm bore bearing has a C_{10} load rating of 56.1 kN. This is greater than the required load rating of 54.9 kN above.

Choose a 02-series cylindrical roller bearing with 55 mm bore.

f. Find the reliability of the selected bearing in use

$$R = \exp\left[-\left(\frac{x_D \left(\frac{a_f F_D}{C_{10}} \right)^a - x_0}{\theta - x_0} \right)^b \right] \qquad \text{(eq. 11-18, p. 600)}$$

$$R = \exp\left[-\left(\frac{540\left(\frac{(1.2)(6.00 \text{ kN})}{56.1 \text{ kN}} \right)^{10/3} - 0.02}{4.459 - 0.02} \right)^{1.483} \right]$$

$$= 0.955$$

The reliability is 0.955