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# MATLAB Assignment Problem 12

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## a) Plot step responses varying values of Ka

```
Ka = [0.5, 1.0, 5.0, 10.0, 20.0];
G = tf([2], conv([1,0], conv([1,1],[1,4])));
Gc1 = tf([Ka(1)], [1]);
Gc2 = tf([Ka(2)], [1]);
Gc3 = tf([Ka(3)], [1]);
Gc4 = tf([Ka(4)], [1]);
Gc5 = tf([Ka(5)], [1]);
```

## Systems for $R(s) = 1/s$ , $D(s) = 0$

```
sys1_1 = feedback(G*Gc1, [1]);
sys1_2 = feedback(G*Gc2, [1]);
sys1_3 = feedback(G*Gc3, [1]);
sys1_4 = feedback(G*Gc4, [1]);
sys1_5 = feedback(G*Gc5, [1]);

% Display performance values
stepInfo1_1 = stepinfo(sys1_1);
stepInfo1_2 = stepinfo(sys1_2);
stepInfo1_3 = stepinfo(sys1_3);
stepInfo1_4 = stepinfo(sys1_4);
stepInfo1_5 = stepinfo(sys1_5);
[y1_1,t1]=step(sys1_1);
[y1_2,t2]=step(sys1_2);
[y1_3,t3]=step(sys1_3);
[y1_4,t4]=step(sys1_4);
[y1_5,t5]=step(sys1_5);
Ess1_1 = abs(1-y1_1(end));
Ess1_2 = abs(1-y1_2(end));
```

```

Ess1_3 = abs(1-y1_3(end));
Ess1_4 = abs(1-y1_4(end));
Ess1_5 = abs(1-y1_5(end));
disp('Performance for R(s) = 1/s, D(s) = 0');
disp(sprintf('For Ka = %f. T_s: %f. PO: %f. Ess: %f', Ka(1),
    stepInfo1_1.SettlingTime, stepInfo1_1.Overshoot, Ess1_1));
disp(sprintf('For Ka = %f. T_s: %f. PO: %f. Ess: %f', Ka(2),
    stepInfo1_2.SettlingTime, stepInfo1_2.Overshoot, Ess1_2));
disp(sprintf('For Ka = %f. T_s: %f. PO: %f. Ess: %f', Ka(3),
    stepInfo1_3.SettlingTime, stepInfo1_3.Overshoot, Ess1_3));
disp(sprintf('For Ka = %f. T_s: %f. PO: %f. Ess: %f', Ka(4),
    stepInfo1_4.SettlingTime, stepInfo1_4.Overshoot, Ess1_4));
disp(sprintf('For Ka = %f. T_s: %f. PO: %f. Ess: %f', Ka(5),
    stepInfo1_5.SettlingTime, stepInfo1_5.Overshoot, Ess1_5));

% Plot stable
fig = figure;
step(sys1_1, 'r-', sys1_2, 'g-', sys1_3, 'b--');
title('Problem 12a) R(s) = 1/s. D(s) = 0. Stable Systems');
legend(sprintf('K_a = %f', Ka(1)), sprintf('K_a = %f', Ka(2)),
    sprintf('K_a = %f', Ka(3)));
uiwait(fig);

% Plot marginal stable
fig = figure;
step(sys1_4, 'k-');
title('Problem 12a) R(s) = 1/s. D(s) = 0. Marginal Stable System');
legend(sprintf('K_a = %f', Ka(4)));
uiwait(fig);

% Plot unstable
fig = figure;
step(sys1_5, 'c-');
title('Problem 12a) R(s) = 1/s. D(s) = 0. Unstable System');
legend(sprintf('K_a = %f', Ka(5)));
uiwait(fig);

Performance for R(s) = 1/s, D(s) = 0
For Ka = 0.500000. T_s: 10.416445. PO: 0.036081. Ess: 0.000294
For Ka = 1.000000. T_s: 8.843156. PO: 8.741861. Ess: 0.003795
For Ka = 5.000000. T_s: 19.705413. PO: 61.959266. Ess: 0.003374
For Ka = 10.000000. T_s: NaN. PO: NaN. Ess: 0.040034
For Ka = 20.000000. T_s: NaN. PO: NaN. Ess: 371127727772.768799

```

## Systems for $R(s) = 0$ , $D(s) = 1/s$

```

sys2_1 = feedback(G, Gc1);
sys2_2 = feedback(G, Gc2);
sys2_3 = feedback(G, Gc3);
sys2_4 = feedback(G, Gc4);
sys2_5 = feedback(G, Gc5);

% Display performance values

```

```

stepInfo2_1 = stepinfo(sys2_1);
stepInfo2_2 = stepinfo(sys2_2);
stepInfo2_3 = stepinfo(sys2_3);
stepInfo2_4 = stepinfo(sys2_4);
stepInfo2_5 = stepinfo(sys2_5);
[y2_1,t1]=step(sys2_1);
[y2_2,t2]=step(sys2_2);
[y2_3,t3]=step(sys2_3);
[y2_4,t4]=step(sys2_4);
[y2_5,t5]=step(sys2_5);
Ess2_1 = abs(0-y2_1(end));
Ess2_2 = abs(0-y2_2(end));
Ess2_3 = abs(0-y2_3(end));
Ess2_4 = abs(0-y2_4(end));
Ess2_5 = abs(0-y2_5(end));
disp('Performance for R(s) = 0, D(s) = 1/s');
disp(sprintf('For Ka = %f. T_s: %f. PO: %f. Ess: %f', Ka(1),
    stepInfo2_1.SettlingTime, stepInfo2_1.Overshoot, Ess2_1));
disp(sprintf('For Ka = %f. T_s: %f. PO: %f. Ess: %f', Ka(2),
    stepInfo2_2.SettlingTime, stepInfo2_2.Overshoot, Ess2_2));
disp(sprintf('For Ka = %f. T_s: %f. PO: %f. Ess: %f', Ka(3),
    stepInfo2_3.SettlingTime, stepInfo2_3.Overshoot, Ess2_3));
disp(sprintf('For Ka = %f. T_s: %f. PO: %f. Ess: %f', Ka(4),
    stepInfo2_4.SettlingTime, stepInfo2_4.Overshoot, Ess2_4));
disp(sprintf('For Ka = %f. T_s: %f. PO: %f. Ess: %f', Ka(5),
    stepInfo2_5.SettlingTime, stepInfo2_5.Overshoot, Ess2_5));

% Plot stable
fig = figure;
step(sys2_1, 'r-.', sys2_2, 'g-', sys2_3, 'b--');
title('Problem 12a) R(s) = 0. D(s) = 1/s. Stable Systems');
legend(sprintf('K_a = %f', Ka(1)), sprintf('K_a = %f', Ka(2)),
    sprintf('K_a = %f', Ka(3)));
uiwait(fig);

% Plot marginal stable
fig = figure;
step(sys2_4, 'k-');
title('Problem 12a) R(s) = 0. D(s) = 1/s. Marginal Stable System');
legend(sprintf('K_a = %f', Ka(4)));
uiwait(fig);

% Plot unstable
fig = figure;
step(sys2_5, 'c-');
title('Problem 12a) R(s) = 0. D(s) = 1/s. Unstable System');
legend(sprintf('K_a = %f', Ka(5)));
uiwait(fig);

Performance for R(s) = 0, D(s) = 1/s
For Ka = 0.500000. T_s: 10.416445. PO: 0.036081. Ess: 1.999413
For Ka = 1.000000. T_s: 8.843156. PO: 8.741861. Ess: 0.996205
For Ka = 5.000000. T_s: 19.705413. PO: 61.959266. Ess: 0.199325
For Ka = 10.000000. T_s: NaN. PO: NaN. Ess: 0.104003

```

*For Ka = 20.000000. T\_s: NaN. PO: NaN. Ess: 102596362907.108002*

## Systems for $R(s) = 1/s$ , $D(s) = 1/s$

```
sys3_1 = sys1_1 + sys2_1;
sys3_2 = sys1_2 + sys2_2;
sys3_3 = sys1_3 + sys2_3;
sys3_4 = sys1_4 + sys2_4;
sys3_5 = sys1_5 + sys2_5;

% Display performance values
stepInfo3_1 = stepinfo(sys3_1);
stepInfo3_2 = stepinfo(sys3_2);
stepInfo3_3 = stepinfo(sys3_3);
stepInfo3_4 = stepinfo(sys3_4);
stepInfo3_5 = stepinfo(sys3_5);
[y3_1,t1]=step(sys3_1);
[y3_2,t2]=step(sys3_2);
[y3_3,t3]=step(sys3_3);
[y3_4,t4]=step(sys3_4);
[y3_5,t5]=step(sys3_5);
Ess3_1 = abs(1-y3_1(end));
Ess3_2 = abs(1-y3_2(end));
Ess3_3 = abs(1-y3_3(end));
Ess3_4 = abs(1-y3_4(end));
Ess3_5 = abs(1-y3_5(end));
disp('Performance for R(s) = 1/s, D(s) = 1/s');
disp(sprintf('For Ka = %f. T_s: %f. PO: %f. Ess: %f', Ka(1),
    stepInfo3_1.SettlingTime, stepInfo3_1.Overshoot, Ess3_1));
disp(sprintf('For Ka = %f. T_s: %f. PO: %f. Ess: %f', Ka(2),
    stepInfo3_2.SettlingTime, stepInfo3_2.Overshoot, Ess3_2));
disp(sprintf('For Ka = %f. T_s: %f. PO: %f. Ess: %f', Ka(3),
    stepInfo3_3.SettlingTime, stepInfo3_3.Overshoot, Ess3_3));
disp(sprintf('For Ka = %f. T_s: %f. PO: %f. Ess: %f', Ka(4),
    stepInfo3_4.SettlingTime, stepInfo3_4.Overshoot, Ess3_4));
disp(sprintf('For Ka = %f. T_s: %f. PO: %f. Ess: %f', Ka(5),
    stepInfo3_5.SettlingTime, stepInfo3_5.Overshoot, Ess3_5));

% Plot stable
fig = figure;
step(sys3_1, 'r-.', sys3_2, 'g-', sys3_3, 'b--');
title('Problem 12a) R(s) = 1/s, D(s) = 1/s. Stable Systems');
legend(sprintf('K_a = %f', Ka(1)), sprintf('K_a = %f', Ka(2)),
    sprintf('K_a = %f', Ka(3)));
uiwait(fig);

% Plot marginal stable
fig = figure;
step(sys3_4, 'k-');
title('Problem 12a) R(s) = 1/s, D(s) = 1/s. Marginal Stable System');
legend(sprintf('K_a = %f', Ka(4)));
uiwait(fig);
```

```
% Plot unstable
fig = figure;
step( sys3_5, 'c-');
title('Problem 12a) R(s) = 1/s, D(s) = 1/s. Unstable System');
legend(sprintf('K_a = %f', Ka(5)));
uiwait(fig);

Performance for R(s) = 1/s, D(s) = 1/s
For Ka = 0.500000. T_s: 10.416261. PO: 0.032476. Ess: 1.997325
For Ka = 1.000000. T_s: 8.842608. PO: 8.741778. Ess: 0.984475
For Ka = 5.000000. T_s: 19.705413. PO: 61.959266. Ess: 0.195952
For Ka = 10.000000. T_s: NaN. PO: NaN. Ess:
2585492478766624341491712.000000
For Ka = 20.000000. T_s: NaN. PO: NaN. Ess: 256447780819.589355
```

## b) Plot Root Locus as Ka

```
numerator = [2];
denominator = conv([1, 0], conv([1, 1], [1, 4]));
plotRootLocus(numerator, denominator, 'Problem 12b) Root Locus Plot');
```

## Plot for selected Ka

```
Ka = 9.5;
Gc = tf([Ka], [1]);
```

## System for $R(s) = 1/s$ , $D(s) = 0$

```
sys4 = feedback(G*Gc, [1]);

% Display performance values
stepInfo4 = stepinfo(sys4);

[y4,t4]=step(sys4);
Ess4 = abs(1-y4(end));
disp('Performance for R(s) = 1/s, D(s) = 0');
disp(sprintf('For Ka = %f. T_s: %f. PO: %f. Ess: %f', Ka,
stepInfo4.SettlingTime, stepInfo4.Overshoot, Ess4));

% Plot stable
fig = figure;
step(sys4, 'r-.');
title('Problem 12b) R(s) = 1/s, D(s) = 0');
legend(sprintf('K_a = %f', Ka));
uiwait(fig);

Performance for R(s) = 1/s, D(s) = 0
For Ka = 9.500000. T_s: 218.732996. PO: 90.438203. Ess: 0.001216
```

## System for $R(s) = 0$ , $D(s) = 1/s$

```
sys5 = feedback(G, Gc);
```

```
% Display performance values
stepInfo5 = stepinfo(sys5);

[y5,t5]=step(sys5);
Ess5 = abs(0-y5(end));
disp('Performance for R(s) = 0, D(s) = 1/s');
disp(sprintf('For Ka = %f. T_s: %f. PO: %f. Ess: %f', Ka,
    stepInfo5.SettlingTime, stepInfo5.Overshoot, Ess5));

% Plot stable
fig = figure;
step(sys5, 'r-.');
title('Problem 12b) R(s) = 0, D(s) = 1/s');
legend(sprintf('K_a = %f', Ka));
uiwait(fig);

Performance for R(s) = 0, D(s) = 1/s
For Ka = 9.500000. T_s: 218.732996. PO: 90.438203. Ess: 0.105391
```

## Find Ka such that damping = 0.5

```
damping_desired = 0.5;
required_angle = pi - acos(0.5);
numerator = [2];
denominator = conv([1, 0], conv([1, 1], [1, 4]));
k = 0:0.001:10;
r = rlocus(tf(numerator, denominator), k);

disp(sprintf('Searching for Ka such that damping = %f, which means
    angle = %f', damping_desired, required_angle));
closest_angle = 0;
for i = 1:numel(r)
    this_angle = angle(r(i));
    if abs(closest_angle - required_angle) > abs(this_angle -
        required_angle)
        closest_angle = this_angle;
        bestKa = k(i);
    end
end
disp(sprintf('Found Ka = %f results in closest angle %f, only %f
    away from requirement', bestKa, closest_angle, abs(required_angle -
    closest_angle)));

Searching for Ka such that damping = 0.500000, which means angle =
    2.094395
Found Ka = 4.032000 results in closest angle 2.094395, only 0.000000
    away from requirement
```

## Show Bode Plot with the found Ka

```
Gc = tf([bestKa], [1]);
% System for R(s) = 1/s, D(s) = 0
```

```
sys6 = feedback(G*Gc, [1]);  
% System for R(s) = 0, D(s) = 1/s  
sys7 = feedback(G, Gc);  
  
bodePlot(sys6, sprintf('Problem 12d) Bode Plot R(s) = 1/s, D(s) = 0,  
    K_a = %f', bestKa));  
bodePlot(sys7, sprintf('Problem 12d) Bode Plot R(s) = 0, D(s) = 1/s,  
    K_a = %f', bestKa));  
bodePlot(G*Gc, sprintf('Problem 12d) Bode Plot R(s) = 1/s, D(s) = 0,  
    K_a = %f', bestKa));
```

## System for $R(s) = 1/s$ , $D(s) = 0$

```
Ka = 4.032;  
Gc = tf([Ka],[1]);  
sys10 = feedback(G*Gc, [1]);  
  
% Display performance values  
stepInfo4 = stepinfo(sys10);  
  
[y4,t4]=step(sys10);  
Ess4 = abs(1-y4(end));  
disp('Performance for R(s) = 1/s, D(s) = 0');  
disp(sprintf('For Ka = %f. T_s: %f. PO: %f. Ess: %f', Ka,  
    stepInfo4.SettlingTime, stepInfo4.Overshoot, Ess4));  
  
% Plot stable  
fig = figure;  
step(sys10, 'r-.');  
title('Problem 12c) R(s) = 1/s, D(s) = 0');  
legend(sprintf('K_a = %f', Ka));  
uiwait(fig);  
  
Performance for R(s) = 1/s, D(s) = 0  
For Ka = 4.032000. T_s: 15.185777. PO: 53.181740. Ess: 0.004403
```

## System for $R(s) = 0$ , $D(s) = 1/s$

```
sys5 = feedback(G, Gc);  
  
% Display performance values  
stepInfo5 = stepinfo(sys5);  
  
[y5,t5]=step(sys5);  
Ess5 = abs(0-y5(end));  
disp('Performance for R(s) = 0, D(s) = 1/s');  
disp(sprintf('For Ka = %f. T_s: %f. PO: %f. Ess: %f', Ka,  
    stepInfo5.SettlingTime, stepInfo5.Overshoot, Ess5));  
  
% Plot stable  
fig = figure;  
step(sys5, 'r-.');  
title('Problem 12b) R(s) = 0, D(s) = 1/s');
```

```
legend(sprintf('K_a = %f', Ka));  
uiwait(fig);
```

*Performance for  $R(s) = 0$ ,  $D(s) = 1/s$   
For  $K_a = 4.032000$ .  $T_s$ : 15.185777.  $PO$ : 53.181740.  $Ess$ : 0.249108*

## Systems for $R(s) = 1/s$ , $D(s) = 1/s$

```
sys3 = sys10 + sys5;  
  
% Display performance values  
stepInfo = stepinfo(sys3);  
[y,t]=step(sys3);  
Ess = abs(1-y(end));  
disp('Performance for R(s) = 1/s, D(s) = 1/s');  
disp(sprintf('For K = %f. T_s: %f. PO: %f. Ess: %f', Ka,  
    stepInfo.SettlingTime, stepInfo.Overshoot, Ess));  
  
% Plot stable  
fig = figure;  
step(sys3, 'r-.');  
title('Problem 12.2c) R(s) = 1/s, D(s) = 1/s');  
legend(sprintf('K = %f', Ka));  
uiwait(fig);  
  
function bodePlot(transfer_function, titleText)  
    plot = figure;  
    bode(transfer_function);  
    title(titleText);  
    uiwait(plot);  
end  
  
function plotRootLocus(num, den, titleText)  
    sys = tf(num, den);  
    plot = figure;  
    rlocus(sys);  
    title(titleText);  
    uiwait(plot);  
end
```

*Performance for  $R(s) = 1/s$ ,  $D(s) = 1/s$   
For  $K = 4.032000$ .  $T_s$ : 15.185777.  $PO$ : 53.181739.  $Ess$ : 0.253511*

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