Problem 5. Dynamic Programming [25 points] (1 parts)

You are given a sequence of n numbers (positive or negative): 感覺跟一些有點像 longest increasing seqeuence

$$x_1, x_2, \ldots, x_n$$

Your job is to select a subset of these numbers of maximum total sum, subject to the constraint that you can't select two elements that are adjacent (that is, if you pick x_i then you cannot pick either x_{i-1} or x_{i+1}).

Explain how you can find, in time polynomial in n, the subset of maximum total sum.

給定 - A[1,...,n] 求 A 下 最大不相鄰數構成之和

ex. <1, 2, 4, 5, 9, 7>

optimal: <1, 4, 9>

DP 解: 令 d[i] 為在 A[1,...,i] 下包含 A[i] 之最大不相鄰數構成之和

則: $d[i] = \begin{cases} 0 & \text{if } i = 0 \\ \max\{x_i, 0\} & \text{if } i = 1 \\ \max\{d[i-1], d[i-2] + x_i\} & \end{cases}$

Example:

i	1	2	3	4	5	6
di	1	2	5	7	14	14

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Note: 7 問題定義和 LIS 相似