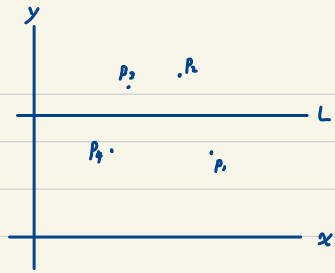
(5%) The constrained 1-center problem: Given n planar points and a straight line L, find a smallest circle, whose center is restricted to lying on L, to cover these n points. The following lists an algorithm for solving this problem. Evaluate the time complexity, T(n), of this algorithm using the recurrence relation of T(n).



Input: n points and a straight line L: $y = y'$.

Output: The constrained 1-center on L.

Step 1. If n is no more than 2, solve this problem by a brute-force method.

Step 2. Form disjoint pairs of points $(p_1, p_2), (p_3, p_4), \ldots, (p_{n-1}, p_n)$. If n is odd, let the final pair be (p_n, p_1).

Step 3. For each pair of points, (p_i, p_{i+1}), find the point $x_{i,i+1}$ on L such that $d(p_i, x_{i,i+1}) = d(p_{i+1}, x_{i,i+1})$.

Step 4. Find the median of the $\lceil n/2 \rceil$ numbers of $x_{i,i+1}$'s. Denote it as x_m.

Step 5. Calculate the distance between p_i and x_m for all i. Let p_j be the point which is the farthest from x_m. Let x_j denote the projection of p_j onto L. If x_j is to the left (right) of x_m, then the optimal solution, x^*, must be to the left (right) of x_m.

Step 6. If $x^* < x_m$, for each $x_{i,i+1} > x_m$, prune the point p_i if p_i is closer to x_m than p_{i+1}; otherwise prune the point p_{i+1}. If $x^* > x_m$, for each $x_{i,i+1} < x_m$, prune the point p_i if p_i is closer to x_m than p_{i+1}; otherwise prune the point p_{i+1}.

Step 7. Go to Step 1.

Time Complexity: step 3. 对每一个 pair (p_i, p_{i+1}) 找到一 $x_{i,i+1}$ 在 L 上使得 $d(p_i, x_{i,i+1}) = d(p_{i+1}, x_{i,i+1}) \Rightarrow O(n)$

step 4. 找 $x_{i,i+1}$ 之 median, 令為 x_m : $O(n)$

step 5. 計算 $d(x_m, p_i)$, $\forall i$

而 p_j 為 $d(x_m, p_i)$ 最大之點

令 p_j 在 L 的投影為 x_j

若 x_j 在 x_m 之左(右)側, 則 x^* 在 x_m 之左(右)側

step 6. 若 $x^* < x_m$, 捨棄 $x_{i,i+1} > x_m$ 之較靠近 x_m 之點

step 7. 遞迴直至 B.C. $\Rightarrow T(n) = T(\frac{3}{4}n)$

$\therefore T(n) = T(\frac{3}{4}n) + O(n) = O(n)$.