Problem 5. [19 points] **Office Hour Optimization**

Class 0.660 (Algorithms for Introductions) is holding online office hours to help students on three problems — a, b, and c — in three corresponding breakout rooms. The TAs want to develop a 'Sort Bot' to effectively assign each student to a single room at the start of office hours. Assume there are $3n$ students, where each student i has known nonnegative integer **benefit** a_i, b_i, and c_i for being assigned to the room for problem a, b, and c, respectively.

Describe an $O(n^3)$-time algorithm to determine whether it is possible to assign the students **equally** to the three breakout rooms (i.e., n students to each room) while providing **strictly positive** help to every student, and if possible, return the maximum total benefit to students of any such assignment. Note that the assignment must not assign a student to a room for which they would get zero benefit.

設共有 3n 个 students, 第 i 个学生有 $\langle a_i, b_i, c_i \rangle$ 代表 assign 至 a. b. c 的 benefit

Example:

i	1	2	3	4	5	6
a_i	4	1	0	1	3	0
b_i	2	4	0	2	2	4
c_i	0	2	4	3	1	0

不能有 student 為 0-benefit

assign: a → 1, 5
b → 2, 6
c → 3, 4

定義 subproblem 為 $d[i,j,k]$ 為分配 $i+j+k$ 个 student 且 i 个 student 至 a 下且無人為 zero benefit 之
j 个 student 至 b
k 个 student 至 c

optimal value

其中 $i,j,k \in \{0,\dots,n\}$

設 $d[i,j,k] = -\infty$ 表示不存在該 assignment
∵ $i+j+k$ 同学生 assign 至 a, b, c 之一

recursive relation:

$$d[i,j,k] = \max\{-\infty, \max\{a_{i+j+k} + d[i-1,j,k] \text{ if } i>0 \text{ 且 } a_{i+j+k}>0,\ b_{i+j+k} + d[i,j-1,k] \text{ if } j>0 \text{ 且 } a_{i+j+k}>0,\ c_{i+j+k} + d[i,j,k-1] \text{ if } k>0 \text{ 且 } a_{i+j+k}>0\}\}$$

$d[0,0,0] = 0$

Time Complexity: $O(n^3)$