Formula Satisfiability

問題： 給定一 boolean formula ϕ 由下列組成：

(1). n 个 boolean variable：$x_1, x_2, \ldots, x_n$

(2). m 个 boolean connective：$\wedge, \vee, \neg, \longrightarrow, \Longleftrightarrow$

(3). 括号

求是否存在一 assignment 使得 $\phi = 1$，即 ϕ 是否為 satisfiable

$SAT = \{ \langle \phi \rangle \mid \phi \text{ is a satisfiable formula} \}$

Example：

$\phi = ((x_1 \to x_2) \vee \neg((\neg x_1 \leftrightarrow x_3) \vee x_4)) \wedge \neg x_2$

has the satisfying assignment $\langle x_1 = 0, x_2 = 0, x_3 = 1, x_4 = 1 \rangle$, since

$$\begin{aligned} \phi &= ((0 \to 0) \vee \neg((\neg 0 \leftrightarrow 1) \vee 1)) \wedge \neg 0 \\ &= (1 \vee \neg(1 \vee 1)) \wedge 1 \\ &= (1 \vee 0) \wedge 1 \\ &= 1 , \end{aligned}$$

證明： SAT 為 NP Complete

(1). $SAT \in NP$

(2). Circuit $SAT \leq_p SAT$