

問題: 給定一數為 n , 求第 n^{th} 個 ugly number

其中: ugly number 定義為具 2, 3, 5 之質因數之數, 且含有 1

idea: 令 D_n 為第 n 個 ugly number

$$D_n = \begin{cases} 1 & \text{if } n=1 \\ \min\{2D_{\frac{n}{2}}, 3D_{\frac{n}{3}}, 5D_{\frac{n}{5}}\} & \text{otherwise} \end{cases}$$

$$D_1=1 \quad D_2=2 \quad D_3=3 \quad D_4=4$$

Algorithm: Ugly_Num(n)

1. Let $A[1..n]$ be a new array

$$A[0] = 1$$

$$i_2 = i_3 = i_5 = 0$$

$$\text{next_mul_2} = A[i_2] \times 2$$

$$\text{next_mul_3} = A[i_3] \times 3$$

$$\text{next_mul_5} = A[i_5] \times 5$$

for $i=1$ to $n-1$

$$\text{next_ugly_num} = \min(\text{next_mul_2}, \text{next_mul_3}, \text{next_mul_5})$$

$$\text{ugly}[i] = \text{next_ugly_num}$$