定義:

$$W=[w_{ij}]=\begin{cases}0 & \text{if } i=j\\ w[i,j] & \text{if } i\neq j \text{ 且 } (i,j)\in E\\ \infty & \text{otherwise}\end{cases}$$

$D=[d_{ij}]=\delta[i,j]$

$\pi=[\pi_{ij}]=$ "由 i 至 j 的 SP 中 j 的 predecesor"

characterize subproblem: 定義 $d[i,j,k]$ 為由 V_i 至 V_j 只允許經 $V_1,\dots,V_k$ 之 SP 長度

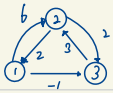
① 不包含點 k ⇒ $d[i,j,k]=d[i,j,k-1]$

② 包含點 k ⇒ $d[i,j,k]=d[i,k,k-1]+d[k,j,k-1]$

Derive Recursion function: $d[i,j,k]=\begin{cases}W[i,j] & \text{if } k=0\\ \min\{d[i,j,k-1],\ d[i,k,k-1]+d[k,j,k-1]\} & \end{cases}$

$$\pi[i,j,k]=\begin{cases}NIL & \text{if } k\neq 0 \text{ 且 } (i=j \vee W[i,j]=\infty)\\ i & \text{if } k=0 \text{ 且 } (i\neq j \wedge W[i,j]\neq\infty)\\ \pi[i,j,k-1] & \text{if } d[i,j,k-1]\leq d[i,k,k-1]+d[k,j,k-1]\\ \pi[k,j,k-1] & \text{otherwise}\end{cases}$$

Example:



$$D^{(0)}_{ij} = \begin{bmatrix} 0 & 6 & -1 \\ 2 & 0 & 2 \\ \infty & 3 & 0 \end{bmatrix} \quad D^{(1)} = \begin{bmatrix} 0 & 6 & -1 \\ 2 & 0 & 1 \\ \infty & 3 & 0 \end{bmatrix}$$

$$\pi^{(0)}_{ij} = \begin{bmatrix} Nil & 1 & 1 \\ 2 & Nil & 2 \\ Nil & 3 & Nil \end{bmatrix} \quad \pi^{(1)} = \begin{bmatrix} N & 1 & 1 \\ 2 & N & 1 \\ N & 3 & N \end{bmatrix}$$

$$D^{(2)} = \begin{bmatrix} 0 & 6 & -1 \\ 2 & 0 & 1 \\ 5 & 3 & 0 \end{bmatrix} \quad \pi^{(2)} = \begin{bmatrix} N & 1 & 1 \\ 2 & N & 1 \\ 2 & 3 & N \end{bmatrix}$$

$$D^{(3)} = \begin{bmatrix} 0 & 2 & -1 \\ 2 & 0 & 1 \\ 5 & 3 & 0 \end{bmatrix} \quad \pi^{(3)} = \begin{bmatrix} N & 3 & 1 \\ 2 & N & 1 \\ 2 & 3 & N \end{bmatrix}$$