Problem 4. [19 points] **Limited-Unlimited**

Given two sets of integers A and B, a **limited-unlimited sequence** of A and B is any sequence S of integers such that each integer $s \in S$ appears in either A or B, and if s appears in A then s appears at most once in S. Given a target sum m and two disjoint sets A and B, each containing exactly n **distinct positive** integers, describe an $O(nm)$-time algorithm to determine whether m is the sum of any limited-unlimited sequence S of A and B, i.e., $m = \sum_{s \in S} s$.

給定一 $A=\{a_0,\dots,a_{n-1}\}$ 和一 m

$B=\{b_0,\dots,b_{n-1}\}$

求一 Sequence S 滿足 $\sum_{s\in S} s = m$，其中 $s\in A$ or $s\in B$ 且 $s\in A$ 只能出現一次

Example: $A=\{3,5,2,6\}$ $m=12$ ⇒ $S=\langle 2,6,4\rangle$

$B=\{2,4,3,8\}$

idea: 類似於 two variable 加上 constr

定義: $d_A[i,k]$ 為 $\{a_0,\dots,a_i\}$ 中滿足一 subset $S \ni \sum_{s\in S} s = k$ 之 Boolean value

$d_B[i,k]$ 為 $\{b_0,\dots,b_i\}$ 中滿足一 sequence $S \ni \sum_{s\in S} s = k$ 之 Boolean value

則 $d_A[i,k] = d[i-1,k]$ or $d[i-1,k-a_i]$

$d_B[i,k] = d[i,k-b_i]$ or $d[i-1,k]$

Initial condition: $d_A[0,a_0]=1$ 且 $d_B[0,b_0]=1$, $d_A[i,0]=1$ 且 $d_B[i,0]=1$

Example: $A=\{3,5,2\}$ $m=5$

$B=\{2,4,3\}$

d_A:

m \ i	0	1	2
0	1	1	1
1	0	0	0
2	0	0	1
3	1	1	1
4	0	0	0
5	0	1	1

d_B:

m \ i	0	1	2
0	1	1	1
1	0	0	0
2	1	1	1
3	0	0	1
4	0	1	1
5	0	0	1

結束後，取 $d[n,m] = (d_A[n,m]$ and $d_B[n,0])$

or $(d_A[n,m-1]$ and $d_B[n,1])$

$\vdots$

or $(d_A[n,0]$ and $d_B[n,m])$

$\Big\}\ O(m)$

Time complexity: $O(mn)$ #.