Hamiltonian Cycle Problem

Define: 給定一圖為 $G=(V,E)$, G中是否具 Hamiltonian cycle?

i.e. 是否含一個 cycle 經每個點恰一次

Formal definition:

$HC = \{\langle G\rangle \mid G$ 為具HC之圖$\}$

Theorem: HC Problem is NP-Complete

① claim: $HC \in NP$

給定一 $G=(V,E)$ 和一 certificate V', 驗證V'中是否含V中所有點恰一次,且頭尾相同即可

會為 polynomial-solvable

② claim: vertex-cover $\leq_p$ Hamiltonian Cycle

給定$(G=(V,E),k)$為 vertex-cover 的 instance, 欲建構 $G'=(V',E')$為HC之instance

且 G中具size為k之clique $\Leftrightarrow$ G'具HC

Define: widget: 幫助建構G'之元件, $\forall (u,v)\in E$, 建構一个 $W_{u,v}$ 如下:

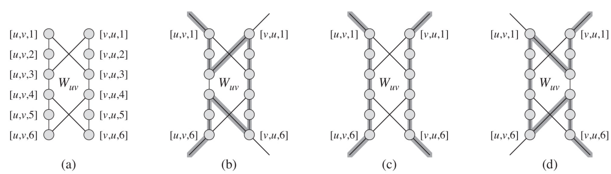

Figure 34.16 The widget used in reducing the vertex-cover problem to the hamiltonian-cycle problem. An edge (u, v) of graph G corresponds to widget W_{uv} in the graph G' created in the reduction. **(a)** The widget, with individual vertices labeled. **(b)–(d)** The shaded paths are the only possible ones through the widget that include all vertices, assuming that the only connections from the widget to the remainder of G' are through vertices $[u, v, 1]$, $[u, v, 6]$, $[v, u, 1]$, and $[v, u, 6]$.

且$W_{u,v}$具如下性質: ① 具12个 vertex 為: $[u,v,i]$, $[v,u,i]$, $\forall i=1,\dots,6$

② $W_{u,v}$ 只有 $[u,v,1]$, $[v,u,1]$, $[u,v,6]$, $[v,u,6]$ 可和外界有邊相連

③ 任何G'中HC必以上圖(b),(c),(d)三種方式走訪 $W_{u,v}$

而V'中所有除了widget之點為 $s_1,\dots,s_k$, 稱作 selector vertex