Example:

```
HIRE-ASSISTANT(n)
best = 0            // candidate 0 is a least-qualified dummy candidate
for i = 1 to n
    interview candidate i
    if candidate i is better than candidate best
        best = i
        hire candidate i
```

設 interview 有 cost 為 C_i

hire 有 cost 為 C_h

可見 running time 為 $\Theta(nC_i + mC_h)$，其中 m 為 hired people 人數

但 hired 人數是隨机分布的 無法直接分析

會 depends on rank of candidates

Probabilistic Analysis：假設先依照 rank，使 input rank 為 $\langle rank(1), \dots, rank(n) \rangle$

會為原 input 之 permutation

設 applicant 進入順序為 random，∴ 構成一 uniform random permutation

⇒ $n!$ 个 rank 排列 出現 机率皆為相同

透過對 random input 用 prob analysis 可得 expected running time.

Indicator random variable

設有一 sample space 為 S 和一 event 為 A

則：indicator random variable 為：

$$I\{A\} = \begin{cases} 1 & \text{if } A \text{ occurs} \\ 0 & \text{otherwise} \end{cases}$$

Lemma 5.1
Given a sample space S and an event A in the sample space S, let $X_A = \mathrm{I}\{A\}$. Then $\mathrm{E}[X_A] = \Pr\{A\}$.

Proof By the definition of an indicator random variable from equation (5.1) and the definition of expected value, we have

$$\begin{aligned} \mathrm{E}[X_A] &= \mathrm{E}[\mathrm{I}\{A\}] \\ &= 1 \cdot \Pr\{A\} + 0 \cdot \Pr\{\overline{A}\} \\ &= \Pr\{A\}, \end{aligned}$$

where $\overline{A}$ denotes $S - A$, the complement of A. ■

example: 擲六面骰, $S = \{1, 2, 3, 4, 5, 6\}$

$\Pr\{1\} = \frac{1}{6} \dots$

設 $X_1 = I\{1\} = \begin{cases} 1 & \text{if 擲到 } 1 \\ 0 & \text{otherwise} \end{cases}$

$\therefore E[X_1] = E[I\{1\}]$

$= \Pr\{1\} = \frac{1}{6}$

設擲 n 次骰子，令 X 為擲到 1 之總次數之 random variable

$X = \sum_{i=1}^{n} X_i$ 其中 $X_i = I\{$擲第 i 次擲到 1$\}$

$\therefore E[X] = \sum_{i=1}^{n} E[X_i] = \sum_{i=1}^{n} \frac{1}{6} = \frac{n}{6}$ #

回過頭來用 indicator random variable 來 model hiring problem

設 X 為 hire 新員工次數之隨机變數

$E[X] = \sum_{x=1}^{n} x \Pr\{X=x\}$

$X_i = I\{\text{candicate } i \text{ is hired}\}$

$= \begin{cases} 1 & \text{if candidate } i \text{ is hired} \\ 0 & \text{otherwires} \end{cases}$

$\therefore X = X_1 + X_2 + \cdots + X_n$

又 $E[X_i] = \Pr\{\text{candicate } i \text{ is hired}\}$

i is hired $\Leftrightarrow$ $rank(i) > rand(j)$, $\forall j = 1, \ldots, i-1$

又為 random order, $\therefore$ $\Pr\{\text{candicate } i \text{ is hired}\} = \frac{1}{i}$

前 $1 \sim i$ 中 i 排在 i 位置之 prob: $\frac{1}{i}$

$\therefore E[X_i] = \frac{1}{i}$

$E[X] = \sum_{i=1}^{n} E[X_i] = \sum_{i=1}^{n} \frac{1}{i} = \ln n + O(1)$

$\therefore$ 可知 $m = O(\lg n)$

hiring algorithm 為 $O(nC_i + \lg n\, C_h)$