

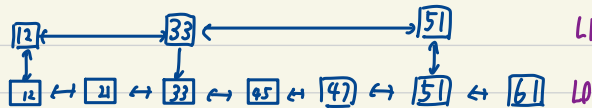
skip list pros:

1. easy implement
2. maintain a dynamic set of n elements
in $O(\lg n)$ time per operation in expected running time

Review: Linked List: $\rightarrow \square \leftrightarrow \square \leftrightarrow \square \leftrightarrow \square \rightarrow Nil$

Search: $O(n)$

Two Sorted Linked List:



- minimize the distance travel.

Search for 47: $12 \rightarrow 33 \rightarrow 33 \rightarrow 45 \rightarrow 47$
L1 L1 L2 L2 L2

- How to structure double sorted linked list?

Search (x) [2 linked list]

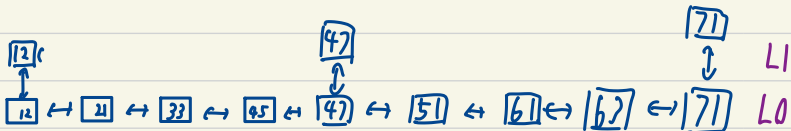
在 L1 走訪至不超過 x 之最大 node

走訪至 L2

最終走訪至 x , 或找不到 x

Analysis: Search cost $\approx |L1| + \frac{|L2|}{|L1|} = \theta(2\sqrt{n})$

minimize this: when $|L1|^2 = |L2| = n \Rightarrow |L1| = \sqrt{n}$

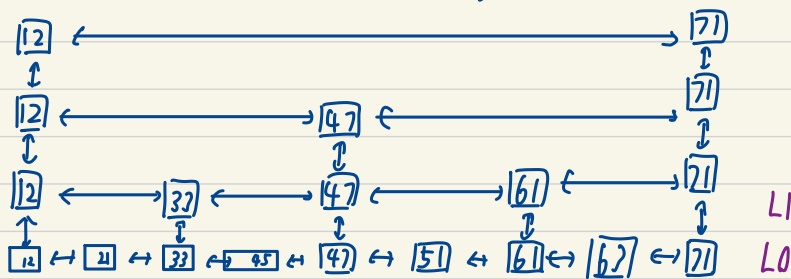


not enough! $\Rightarrow O(\lg n)$

- two Sorted Linked List $\Rightarrow O(2\sqrt{n})$

- three Sorted Linked List $\Rightarrow O(3\sqrt[3]{n})$

- k Sorted Linked List $\Rightarrow O(k \cdot \sqrt{n})$
- $\lg n$ Sorted Linked List $\Rightarrow O(\lg n \cdot \sqrt{\lg n}) = O(\sqrt{2} \lg n) \Rightarrow$ 只适用于 static set



Insert(x)

先用 $\text{Search}(x)$ 找到 x 在 bottom list 的合适位置

并插入至其它的 list (用 flip fair coin 决定)

Flip fair coin

可能已经 created

if Heads, promote x to next level up and repeat

else stop

Note: promote level up 可能会 create new linked list \Rightarrow 增加 level of linked list

\Rightarrow merry \Rightarrow randomized search cost

worst case: - 一直是 Head $\Rightarrow \infty$?

Delete(x):

先用 $\text{Search}(x)$ 找到 x , 把 x 的所有 level 删掉

Warmup Lemma: # of levels in n -element skip list is $O(\lg n)$ with high probability \Rightarrow prob: $1 - \frac{1}{n^\alpha}$, where $\alpha > 1$