A random variable Y is said to have a *Poisson probability distribution* if and only if

$$p(y) = \frac{\lambda^y \cdot e^{-\lambda}}{y!}, \quad y = 0, 1, 2, ..., \lambda > 0$$

If Y is a random variable possessing a Poisson distribution with parameter λ, then

$$\mu = E(Y) = \lambda \quad \text{and} \quad \sigma^2 = V(Y) = \lambda$$

Tchebysheff's Theorem Let Y be a random variable with mean μ and finite variance σ^2. Then, for any constant $k > 0$,

$$P(|Y - \mu| < k\sigma) \geq 1 - \frac{1}{k^2} \quad \text{or} \quad P(|Y - \mu| \geq k\sigma) \leq \frac{1}{k^2}$$

Let Y denote any random variable. The *distribution function* of Y, denoted by $F(y)$, is such that $F(y) = P(Y \leq y)$ for $-\infty < y < \infty$.

A random variable Y with distribution function $F(y)$ is said to be *continuous* if $F(y)$ is continuous, for $-\infty < y < \infty$.

Let $F(y)$ be the distribution function for a continuous random variable Y. Then $f(y)$, given by

$$f(y) = \frac{dF(y)}{dy} = F'(y)$$

If the random variable Y has a density function $f(y)$ and $a < b$, then the probability that Y falls in the interval $[a, b]$ is

$$P(a \leq Y \leq B) = \int_a^b f(y)dy$$

The expected value of a continuous random variable Y is

$$E(Y) = \int_{-\infty}^{\infty} y f(y)dy,$$

provided that the integral exists.

Let $g(Y)$ be a function of Y; then the expected value of $g(Y)$ is given by

$$E[g(Y)] = \int_{-\infty}^{\infty} g(y)f(y)dy,$$

provided that the integral exists.

If $\theta_1 < \theta_2$, a random variable Y is said to have a continuous *uniform probability distribution* on the interval (θ_1, θ_2) if and only if the density function of Y is

$$f(y) = \begin{cases} \frac{1}{\theta_2 - \theta_1}, & \theta_1 \leq y \leq \theta_2, \\ 0, & elsewhere \end{cases}$$

If $\theta_1 < \theta_2$ and Y is a random variable uniformly distributed on the interval (θ_1, θ_2), then

$$\mu = E(Y) = \frac{\theta_1 + \theta_2}{2} \quad \text{and} \quad \sigma^2 = V(Y) = \frac{(\theta_2 - \theta_1)^2}{12}$$

A random variable Y is said to have a *gamma distribution with parameters* $\alpha > 0$ and $\beta > 0$ if and only if the density function of Y is

$$f(y) = \begin{cases} \frac{y^{\alpha-1}e^{-y/\beta}}{\beta^\alpha \Gamma(\alpha)}, & 0 \leq y < \infty, \\ 0, & elsewhere \end{cases}$$

where

$$\Gamma(\alpha) = \int_0^\infty y^{\alpha-1}e^{-y}dy$$

$$E(Y) = \alpha\beta \quad \text{and} \quad V(Y) = \alpha\beta^2$$

Let v be a positive integer. A random variable Y is said to have a *chi-square distribution with v degrees of freedom* if and only if Y is a gamma-distributed random variable with parameters $\alpha = v/2$ and $\beta = 2$.

$$E(Y) = \alpha\beta \quad \text{and} \quad V(Y) = \alpha\beta^2$$

If Y is a chi-square random variable with v degrees of freedom, then

$$\mu = E(Y)\ v \quad \text{and} \quad \sigma^2 = V(Y) = 2v.$$

A random variable Y is said to have an *exponential distribution with parameter* $\beta > 0$ if and only if the density function of Y is

$$f(y) = \begin{cases} \frac{1}{\beta}e^{-y/\beta}, & 0 \leq y < \infty, \\ 0, & elsewhere \end{cases}$$

If Y is an exponential random variable with parameter B, then

$$\mu = E(Y) = \beta \quad \text{and} \quad \sigma^2 = V(Y) = \beta^2$$

Tchebysheff's Theorem Let Y be a random variable with finite mean μ and finite variance σ^2. Then, for any $k > 0$,

$$P(|Y - \mu| < k\sigma) \geq 1 - \frac{1}{k^2} \quad \text{or} \quad P(|Y - \mu| \geq k\sigma) \leq \frac{1}{k^2}$$

Let X and Y be discrete random variables. The *joint* (or bivariate) *probability function* for X and Y is given by

$$p(x, y) = P(X = x, Y = y), \quad -\infty < x < \infty, -\infty < y < \infty$$

If X and Y are discrete random variables with joint probably function $p(x, y)$, then 1. $p(x, y) \geq 0$ for all x, y 2. $\Sigma_{y,y} p(x, y) = 1$, where the sum is over all values (x, y) that are assigned nonzero probabilities.

For any random variables X and Y the joint (bivariate) distribution function $F(x, y)$ is

$$F(x, y) = P(X \leq x, Y \leq y), \quad -\infty < x < \infty, -\infty < y < \infty$$

Let X and Y be continuous random variables with joint distribution function $F(x, y)$. If there exists a nonnegative function $f(x, y)$ such that

$$F(x, y) = \int_{-\infty}^{x} \int_{-\infty}^{y} f(v, w) dwd,$$

for all $-\infty < x < \infty, -\infty < y < \infty$, then X and Y are said to be *jointly continuous random variables.* The function $f(x, y)$ is called the *joint probability density function.*

If X and Y are jointly continuous random variables with a joint density function given by $f(x, y)$, then
1. $f(x, y) \geq 0$ for all x, y
2. $\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) dxdy = 1$

a. Let X and Y be jointly discrete random variables with probability function $p(x, y)$. Then the *marginal probability functions* of X and Y , respectively, are given by

$$p_1(x) = \Sigma_{all\ y} p(x, y) \quad \text{and} p_2(y) = \Sigma_{all\ x} p(x, y)$$

b. Let X and Y be jointly discrete random variables with probability function $f(x, y)$. Then the *marginal density functions* of X and Y , respectively, are given by

$$f_1(x) = \int_{-\infty}^{\infty} f(x, y) \quad \text{and} f_2(y) = \int_{-\infty}^{\infty} f(x, y)$$

If X and Y are jointly discrete random variables with joint probability function $p(x, y)$ and marginal probability functions $p_1(x)$ and $p_2(y)$, respectively, then the *conditional discrete probability function* of X given Y is

$$p(x|y) = P(X = x|Y = y) = \frac{P(X = x, Y = y)}{P(Y = y)} = \frac{p(x, y)}{p(y)}$$

If X and Y are jointly continuous random variables with joint density function $f(x, y)$, then the *conditional distribution function* of X given $Y = y$ is

$$F(x|y) = P(X \leq y|Y = y)$$

Let X and Y be jointly continuous random variables with joint density $f(x, y)$ and marginal densities $f_1(y)$ and $f_2(y)$, respectively. For any y such that $f_2(y) > 0$, the conditional density of X given $Y = y$ is given by

$$f(x|y) = \frac{f(x, y)}{f_2(y)}$$

and, for any x such that $f_1(x) > 0$, the conditional density of Y given $X = x$ is given by

$$f(y|x) = \frac{f(x, y)}{f_2(x)}$$

Let X have distribution function $F_1(x), Y$ have a distribution function $F_2(y)$, and X and Y have joint distribution function $F(x, y)$. Then X and Y are said to be *independent* if and only if

$$F(x, y) = F_1(x)F_2(y)$$

for every pair of real numbers (x, y).
If X and Y are not independent, they are said to be *dependent*.

If X and Y are discrete random variables with joint probability function $p(x, y)$ and marginal probably functions $p_1(x)$ and $p_2(y)$, respectively, then X and Y are independent if and only if

$$p(x, y) = p_1(x)p_2(y)$$

for all pairs of real numbers (x, y).

If X and Y are continuous random variables with joint density function $f(x, y)$ and marginal density functions $f_1(x)$ and $f_2(y)$, respectively, then X and Y are independent if and only if

$$f(x, y) = f_1(x)f_2(y)$$

for all pairs of real numbers (x, y).

Let X and Y have a joint density $f(x, y)$ that is positive if and only if $a \leq x \leq b$ and $c \leq y \leq d$, for constants a, b, c, and d; and $f(x, y) = 0$ otherwise. Then X and Y are independent random variables if and only if

$$f(x, y) = g(x)h(y)$$

where $g(x)$ is a nonnegative function of x alone and $h(y)$ is a nonnegative function of y alone.