A Simple 3D Puzzle

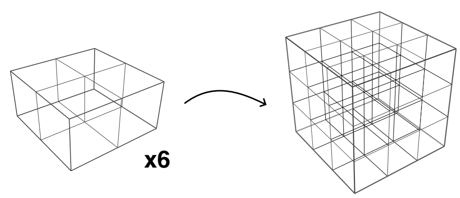
Tyler Neylon

345.2023

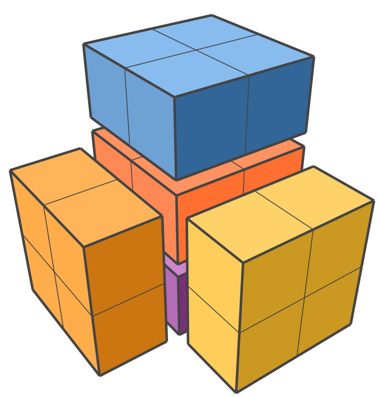
[Formats: html | pdf]

1 The Puzzle

Here's a fun puzzle: Take six boxes, each $1 \times 2 \times 2$ in size, and find a way to pack them into a $3 \times 3 \times 3$ cube.

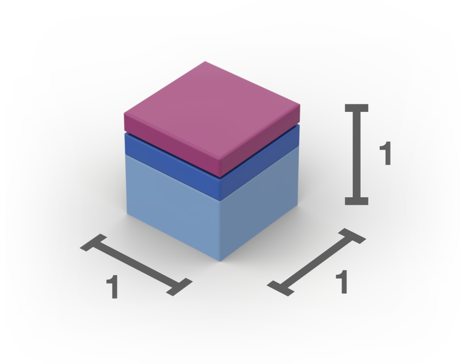


I learned about this puzzle through Donald Knuth's *The Art of Computer Programming,* §7.2.2.1. The six boxes have a total volume of 24 cubies (I'll call a $1 \times 1 \times 1$ unit a "cubie," as Knuth does). They certainly have a chance of fitting into the 27 cubie spaces of the larger $3 \times 3 \times 3$ volume. But the initial configurations I tried failed to fit more than five boxes in the space allowed:

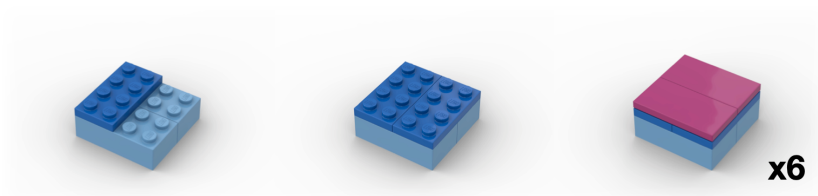


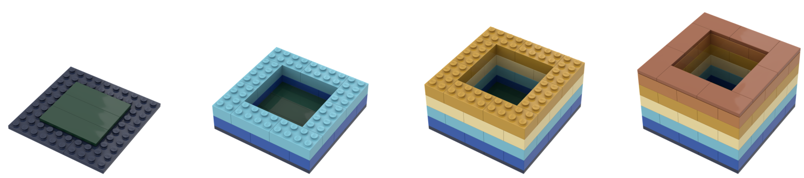
You might be able to solve this by simply thinking about it. But it's even more fun to play with a physical model.

Did you know that a 2×2 Lego brick with 2 tile-heights on top forms a perfect cube?

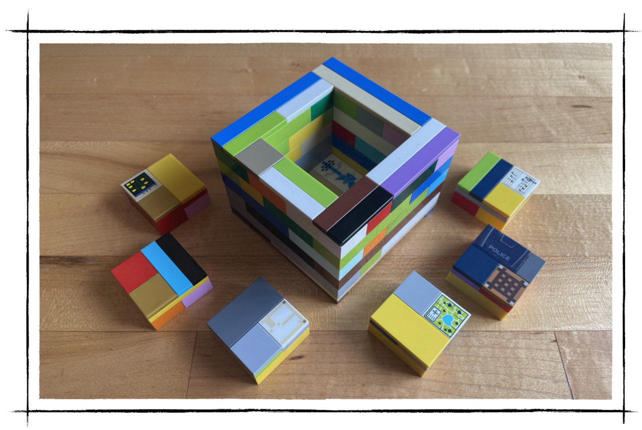


This allows us to construct the puzzle like so:





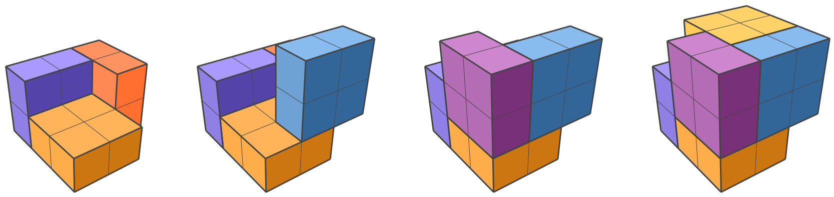
Here's the hodgepodge model I built with my kids' Legos:



I'll write a little about the math behind this puzzle below, but for now I'll give you a vertical break so you don't accidentally see the solution. Try out the puzzle first!

2 The Solution

Here's the solution:



The simplest way to characterize it is to notice that we've left the central cubie unoccupied.

As pointed out by Knuth, there's a nice observation that can help you find this solution. Define a "face cubie" as one that's adjacent to the center cubie. There are 6 of these: