A Simple 3D Puzzle

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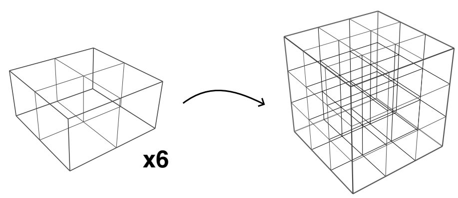
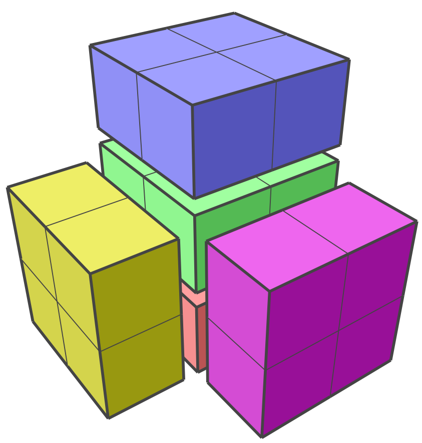
[Formats: html | pdf]

Here's a fun puzzle: Take six boxes, each $1 \times 2 \times 2$ in size, and find a way to pack them into a $3 \times 3 \times 3$ cube.

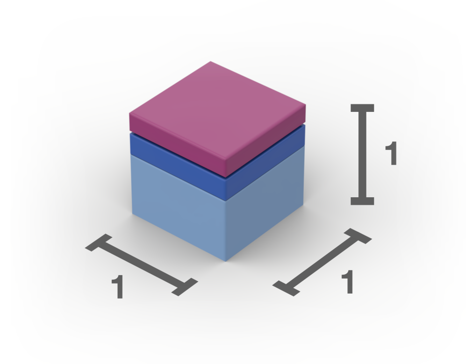


I learned about this puzzle through Donald Knuth's *The Art of Computer Programming,* §7.2.2.1. The six boxes have a total volume of 24 cubies (I'll call a $1 \times 1 \times 1$ unit a "cubie," as Knuth does). They certainly have a chance of fitting into the 27 cubie spaces of the larger $3 \times 3 \times 3$ volume. But the initial configurations I tried failed to fit more than five boxes in the space allowed:

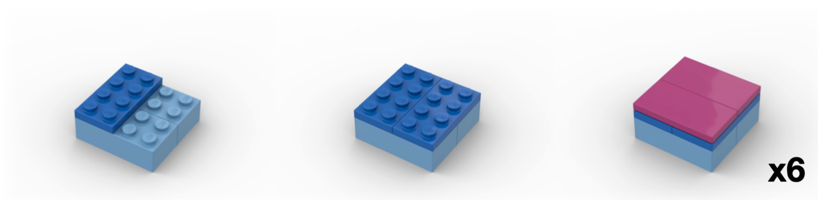


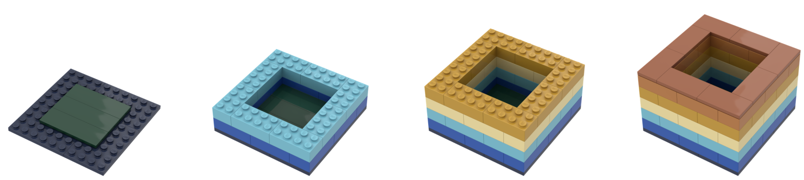
You might be able to solve this by simply thinking about it. But it's even more fun to play with a physical model.

Did you know that a 2×2 Lego brick with 2 tile-heights on top forms a perfect cube?



This allows us to construct the puzzle like so:





Here's the hodgepodge model I built with my kids' Legos: