CSC220

9/15/2020

$3n+2$ is odd $\Leftrightarrow$ $9n+5$ is even

n is an int.

I: if $3n+2$ is odd $\Rightarrow$ $9n+5$ is even

$3n+2 = 2m+1 \Rightarrow 3n = 2m-1$

$9n+5 = 3*(2m-1)+5 = 6m-3+5 =$

$6m+2 = 2(3m+1) = 2\ell$ even.

II:

if $9n+5$ is even $\Rightarrow$ $3n+2$ is odd

$3n+2$ is even $\Rightarrow$ $3n+2 = 2m \Rightarrow$

$3n = 2m-2$

$9n+5 = 3(2m-2)+5 = 6m-1$ odd

Contradiction!

If a, b, c are odd int, $ax^2+bx+c=0$

has no int. solution.

$a = 2h+1$

$b = 2i+1$

$c = 2j+1$

If $x = 2k$

$ax^2+bx+c = a(2k)^2+b(2k)+c = 4k^2(2h+1)+(2i+1)\cdot 2k+(2j+1) = 8k^2h+4k^2+4ik+2k+2j+1 = 2[\quad]+1 \neq 0$

if $x = 2k+1$

$ax^2+bx+c = a(2k+1)^2+b(2k+1)+c =$

$$(2h+1)(4k^2+4k+1)+(2i+1)(2k+1)+(2j+1)=$$

$$8k^2h+8kh+2h+4k^2+4k+1+2ik+2i+$$
$$2k+1+2j+1 \neq 0$$

$\Rightarrow$ no solution

Show by induction:

$3n^2+3n+6$ is divisible by 6. $n \in \mathbb{N}$

$p(0)$: $3*0+3*0+6=6$ $\quad 6|6$ ✓

$p(k)$ $\quad 6 \mid 3k^2+3k+6$

$p(k+1)$: $3(k+1)^2+3(k+1)+6=$

$3k^2+6k+3+3k+3+6=$

$= \underbrace{3k^2+3k+6}+6k+6$

$\stackrel{I.H}{=} 6m+6k+6$

$= 6(m+k+1)$

$= 6l$

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$$A \Rightarrow B \equiv B \vee \neg A$$

$$\neg B \Rightarrow \neg A \equiv \neg A \vee \neg(\neg B) = \neg A \vee B$$

$$\Rightarrow \quad A \Rightarrow B \equiv \neg B \Rightarrow \neg A$$

$\underbrace{\text{If } x \text{ is even}}_{A}$ then $\underbrace{x^2 \text{ is even}}_{B}$

(Session 1)

$A \Rightarrow B$

if x^2 is odd then x is odd.

x is odd iff $8 \mid x^2-1$

I: if x is odd then $8 \mid x^2-1$.

Asshme x is odd $\Rightarrow x=2k+1$

k an int

$x^2-1 = (2k+1)^2-1 = 4k^2+4k+1-1$

$= 4k^2+4k = 4k(k+1) = 4 \times 2m = 8m$

$8 \mid x^2-1$

$k \times (k+1) \doteq$ always even

$2 \times 3 = 6$
$3 \times 4 = 12$

II : if $8 \mid x^2-1$ then x is odd

$8 \mid x^2-1 \Rightarrow x^2-1=8 \cdot k$

$$x^2 = 8k+1 = \underbrace{\underbrace{1+2\underbrace{(4k)}_{\text{even}}}_{\text{even}}}_{\text{odd}}$$

$\Rightarrow x^2$ is odd

so x is odd (Page 5)

$3 \mid n^3+2n \quad n \geq 1$ induction

$p(1) \quad n^3+2n = 1+2=3 \quad 3 \mid 3$ ✓

$p(k) \quad 3 \mid k^3+2k$

or $k^3+2k=3m$

$p(k+1): \quad (k+1)^3+2(k+1)=$

$(k+1)^2 \cdot (k+1) + 2(k+1)$

$= (k^2+2k+1)(k+1) + 2(k+1)$

$= k^3+2k^2+k+k^2+2k+1+2k+2$

$= k^3+3k^2+5k+3$

$= k^3+2k+3k^3+3k+3$

$\overset{\text{I.H}}{=} 3m+3k^3+3k+3$

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$= 3m + 3(k^2 + k + 1)$

$= 3m + 3m'$

$= 3(m + m')$

$\Rightarrow 3 \mid (k+1)^3 + 2(k+1)$

- 64 is perfect square as $8 * 8 = 64$
- If s, t are odd perfect square then $s + t$ is not perfect square.

$S = n^2 \quad t = m^2$ and $S + t = p^2$

s, t are odd $\Rightarrow n^2, m^2$ are odd

$\Rightarrow n, m$ are odd

$n = 2k+1 \qquad m = 2\ell+1$

$S + t = (2k+1) + (2\ell+1) = p^2 = 2(k+\ell+1)$

hence even p must be even

$p = 2h$

$m^2 + n^2 = p^2 \Rightarrow (2k+1)^2 + (2\ell+1)^2 = (2h)^2$

$\Rightarrow 4k^2 + 4k + 1 + 4\ell^2 + 4\ell + 1 = 4h^2$

$4(k^2 + k + \ell^2 + \ell) + 2 = 4h^2$

$\underbrace{\underbrace{2(k^2 + k + \ell^2 + \ell)}_{\text{even}} + 1}_{\text{odd}} = \underbrace{2h^2}_{\text{even}}$

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Contradiction $\Rightarrow$ $t+s$ can't be perfect square.