CSC 220

10/6/2020

Languages

Use inductive def. to define

$L_1 = \{ a, ab, abb, \ldots \}$

$L_1 = \{ ab^n \mid n \in \mathbb{N} \}$

if $n=0 \Longrightarrow ab^0 = a\lambda = a$

if $n=1 \Longrightarrow ab$

$n=2 \Longrightarrow abb$

$\vdots$

Basis: $a \in L_1$

Inductive: if $x \in L_1 \Longrightarrow xb \in L_1$

$L_1 = \{ a \}$

$L_1 = \{ \underbrace{a}_{x}, ab \}$

$L_1 = \{ a, \underbrace{ab}_{x}, abb \}$

$\vdots$

$L_1 = \{ a, ab, abb, \ldots \}$

1

$L_2 = \{ \lambda, ab, abab, \ldots \}$

$L_2 = \{ (ab)^n \mid n \in \mathbb{N} \}$

Basis: $\lambda \in L_2$

Inductive: if $x \in L_2 \Rightarrow xab \in L_2$

$\lambda \;,\; \underbrace{\lambda . ab}_{ab} \;,\; abab \;,\; ababab, \ldots$

$L_3 = \{ \lambda, ab, aabb, aaabbb, \ldots \}$

?

$L_3 = \{ a^n b^n \mid n \in \mathbb{N} \}$

$n = 0 \Longrightarrow a^0 b^0 = \lambda . \lambda = \lambda$

$n = 1 \Longrightarrow ab$

$n = 2 \Longrightarrow a^2 b^2 = aabb$

$n = 3 \Longrightarrow aaabbb$

Basis: $\lambda \in L_3$

Inductive: if $x \in L_3 \Longrightarrow axb \in L_3$

$$\overbrace{\underbrace{a}\underbrace{\lambda}_{x}\underbrace{b}}^{\lambda} = ab$$

$$\underbrace{a}\underbrace{ab}_{x}\underbrace{b} = aabb$$

$$\underbrace{a}\underbrace{aabb}_{x}\underbrace{b} = aaabbb$$

$\vdots$

ϕ : a language, with no string

$\{\lambda\}$: a language, has one string

$$L_4 = \{ a^m b c^n \mid m, n \in \mathbb{N} \}$$

?

$m = 0$, $n = 0$ $\quad a^0 b c^0 = \lambda b \lambda = b$

$m = 1$, $n = 0$ $\quad ab$

$m = 0$, $n = 1$ $\quad bc$

$m = 10$, $n = 50$ $\quad$ - - - .

$$a^m b c^n \neq a^n b c^n$$

could be the same

both "n"

4

Basis: $b \in L_4$

Inductive: if $x \in L_4 \Rightarrow ax, xc \in L_4$

Use induction to prove:

x string, $m, n \in \mathbb{N}$ $\quad x^{n+m} = x^n . x^m$

induction is on n

$P(0)$ $\quad x^{0+m} = x^m = \lambda . x^m = x^0 . x^m$

$P(k)$ $\quad x^{k+m} = x^k . x^m$

$P(k+1)$?

$$x^{(k+1)+m} = x^{(k+m)+1} = x^{1+(k+m)}$$

$$= x^1 . x^{k+m} = x . x^k . x^m$$

$$= x^{(1+k)} . x^m$$

$$= x^{(k+1)} . x^m$$

5