CSC 220

11/3/2020

Regular Expressions

- A language over a finite alphabet is a set of strings of letters from A.

- A language over A is a subset of A^*.

- The language that can be constructed from letters of an alphabet by operations such as union, concatenation and closure is a regular language.

- $\emptyset$, $\{\lambda\}$, $\{a\}$ for all $a \in A$

· If L and M are regular then $L \cup M, LM, L^*$ are also regular.

$A = \{a, b\} \Rightarrow \phi, \{a\}, \{b\}, \{\lambda\}$ are Reg.

$\{\lambda, b\}$ is regular $\quad \{\lambda\} \cup \{b\}$

$\{a, ab\} = \{a\} \cdot \{\lambda, b\}$ so regular

$\{\lambda, b, bb, bbb, \ldots\} = \{b^n \mid n \in \mathbb{N}\} = \{b\}^*$

is regular

- A regular language is often described by means of an algebraic expression called regular expression.

- ϕ, λ, a are regular for all $a \in A$.

- If R, S are regular expressions then

  (R), $R+S$, $R.S$, R^* are

  also regular.

- $(\neq \{$

  $\{ \longrightarrow$ Set

  $(\downarrow$ R.E

RE over $A = \{a, b\}$

λ, a, $a.a$, a^*, $a^* + b$, $a^*.b$, ...

. Hierarchy $\uparrow$ $*$ $\cdot$ $+$ $\qquad b + b.b^*$

. For each R.E., we will associate a regular language $L(E)$, where A is an alphabet and R, S are regular expressions

$L(\emptyset) = \emptyset$ $\qquad L(\lambda) = \{\lambda\}$

$L(a) = \{a\}$

- $L(R+S) = L(R) \cup L(S)$
- $L(R.S) = L(R).L(S)$

- Find the language of the regular expression $a+bc^*$.

$L(a+bc^*) = L(a) \cup L(bc^*)$

$= L(a) \cup (L(b).L(c^*))$

$= \{a\} \cup (\{b\}.\{\lambda, c, cc, ccc, \ldots\})$

$= \{a\} \cup \{b, bc, bcc, \ldots\} = \{a, b, bc, bcc, \ldots\}$

Single letter a, or strings of form b followed by zero or more c's.

$L = \{ \lambda, a, b, ab, abb, abbb, \ldots, ab^n, \ldots \}$

is regular

$\lambda + b + ab^*$

$L(a+b) = L(a) \cup L(b)$

$= \{a\} \cup \{b\} = \{a, b\} = \{b, a\}$

$= L(b+a)$

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$$L(ab) = L(a) \cdot L(b) = \{a\} \cdot \{b\} = \{ab\}$$

$$L(ba) = L(b) \cdot L(a) = \{b\} \cdot \{a\} = \{ba\}$$

$$L(ab) \neq L(ba)$$

$$ab \neq ba$$

$$L = \{a, aa, aaa, \ldots\} = \{a^n \mid n \in \mathbb{N}, n \geq 1\}$$

$$= \{a\} \cdot \{a\}^* \quad \text{or} \quad a \cdot a^*$$

$$= \{a\}\{\lambda, a, aa, aaa, \ldots\}$$

Regular Expressions R and S are equal if $L(R) = L(S)$