CSc 220

9/10/2020

Section 3.1

Induction:

Suppose we have an infinite ladder, we want to know whether we can reach every step on this ladder; We know two things:

1. We can reach the 1^{st} rung of the ladder.
2. If we can reach a particular rung of the ladder then we can reach the next rung.

Can we conclude that we can reach every rung? Yes, Mathematical induction.

. We can show that $p(n)$ is true for every positive integer n, where $P(n)$ is the "we can reach the n^{th} rung of the ladder."

show : $1+2+3+\cdots+n = \frac{n(n+1)}{2}$ $n \geq 1$, $n \in \mathbb{N}$

I: Basis: $P(1)$: $1 \stackrel{?}{=} \frac{1(1+1)}{2} = \frac{1\times 2}{2} = 1$ ✓

II: Assume $P(k)$ is true so

$$1+2+3+\cdots+k = \frac{k(k+1)}{2}$$

III: show $1+2+\cdots+k+(k+1) = \frac{(k+1)(k+1+1)}{2}$

$$1+2+\cdots+k+k+1 = \frac{k(k+1)}{2} + (k+1)$$

$$= \frac{k(k+1)+2(k+1)}{2}$$

$$= \frac{(k+1)(k+2)}{2}$$

$$1+2+2^2+\dots+2^n=2^{n+1}-1 \quad n\in\mathbb{N}$$

Basis or step I:

$p(0)$ $\quad 2^0 \overset{?}{=} 2^{0+1}-1=2^1-1=2-1=1$

step II: Assume $1+2+2^2+\dots+2^k=2^{k+1}-1$

step III:

show $1+2+2^2+\dots+2^{k+1}=2^{(k+1)+1}-1$

$$\underbrace{1+2+2^2+\dots+2^k}_{\text{I.H}}+2^{k+1}=2^{k+1}-1+2^{k+1}$$

$$=2^{k+1}+2^{k+1}-1$$

$$=2(2^{k+1})-1$$

$$=2^{1+k+1}-1$$

$$=2^{k+2}-1$$

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Remember:

$$a^m + a^n = a^m + a^n$$

$$a^m \cdot a^n = a^{m+n}$$

$$a^0 = 1$$

$$a^m \cdot b^m = (ab)^m$$

$$a^m + b^m = a^m + b^m$$

$$2+4+\cdots+2n = n(n+1) \qquad n \geq 1$$

$$P(1): \quad 2 * 1 \stackrel{?}{=} 1(1+1) = 1 * 2 = 2 \checkmark$$

$$P(k): \quad 2+4+\cdots+2k = k(k+1)$$

show

$$P(k+1) \quad 2+4+\cdots+2(k+1) = (k+1)(k+1+1)$$

$$\underbrace{2+4+\cdots+2k}_{\text{I.H}} + 2(k+1) = k(k+1) + 2(k+1)$$

$$= (k+1)(k+2)$$

$$= (k+1)(k+1+1)$$

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$$\left(\bigcup_{i=1}^{n} A_i\right)' = \bigcap_{i=1}^{n} A_i' \qquad n \geqslant 2$$

$P(2)$:

$$\left(\bigcup_{i=1}^{2} A_i\right)' = \left(A_1 \cup A_2\right)' = A_1' \cap A_2' =$$

De Morgan's Law

$$= \bigcap_{i=1}^{2} A_i'$$

$P(k)$:

$$\left(\bigcup_{i=1}^{k} A_i\right)' = \bigcap_{i=1}^{k} A_i'$$

Show:

$$\left(\bigcup_{i=1}^{k+1} A_i\right)' = \bigcap_{i=1}^{k+1} A_i'$$

Let us review some points:

$$A_1 \cup A_2 \cup A_3 \cup A_4 = \bigcup_{i=1}^{4} A_i$$

or is the same as $\bigcup_{i=1}^{3} A_i \cup A_4$

$$A_1 \cap A_2 \cdots \cap A_{100} = \bigcap_{k=1}^{100} A_k$$

$$\bigcap_{k=1}^{500} A_k = \bigcap_{k=1}^{499} A_k \cap A_{500}$$

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$$\left(\bigcup_{i=1}^{k+1} A_i\right)' = \left(\bigcup_{i=1}^{k} A_i \cup A_{k+1}\right)' \overset{\text{DeMorgan's}}{\underset{\text{Law}}{=}}$$

$$\left(\bigcup_{i=1}^{k} A_i\right)' \cap \left(A_{k+1}\right)' \overset{\text{I.H}}{=}$$

$$\bigcap_{i=1}^{k} A_i' \cap A_{k+1}' = \bigcap_{i=1}^{k+1} A'$$

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