CSC220

10/8/2020

$L_1 = \{ a^m b^n \mid m, n \in \mathbb{N} \quad m > 0 \text{ and } n > 0 \}$

Basis : $ab \in L_1$

Inductive: if $x \in L_1 \Rightarrow ax, xb \in L_1$

$L_2 = \{ a^{2n} \mid n \in \mathbb{N} \}$

$n = 0 \qquad a^0 = \lambda$

$n = 1 \qquad aa$

$n = 2 \qquad aaaa$

Basis: $\lambda \in L_2$

Inductive: if $x \in L_2 \Rightarrow xaa \in L_2$ or

$axa \in L_2$ or

$aax \in L_2$

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• $A = \{a, b\}$, define L, where all strings begin with aa.

$aa, aaa, aab, \cancel{baa}, \ldots$

Basis: $aa \in L$

Inductive: if $x \in L \Rightarrow xa, xb \in L$

$$L_1 = \{ a^n \mid n \in \mathbb{N}, \quad n = 2k+1 \}$$

$n=0$ ✗

$n=1$ ✓ a

$n=2$ ✗

$n=3$ ✓ aaa

$\vdots$

Basis: $a \in L_1$

Inductive: if $x \in L_1 \Rightarrow aax \in L_1$

or

$axa \in L_1$

or

$x.aa \in L_1$

Define L over $\{a, b\}$, where each string begins with "a" and has even Length.

λ, aa, aaaa, ab, aabbbb, aaa, ...

✗ ✓ ✓ ✓ ✓ ✗

Basis: $aa, ab \in L$

Inductive:

if $x \in L \Rightarrow xaa, xbb, xab, xba \in L$

Reversal:

$\lambda^R = \lambda$ $\quad a^R = a \quad (ab)^R = ba$

prove if $u, v \in L^*$ then $(uv)^R = v^R . u^R$

proof by induction on length of v.

$P(0)$ if $|v| = 0 \Rightarrow v = \lambda$

$$(uv)^R = (u\lambda)^R = u^R = \lambda . u^R = v^R . u^R$$

$P(k)$ $(uv)^R = v^R . u^R \; ; |v| = k$

$P(k+1)$: $\quad |v| = k+1 \quad v = w.a$

$|a| = 1 \quad |w| = k$

$$(uv)^R = (u.(wa))^R = ((uw).a)^R =$$

$$a^R . (uw)^R = a.(uw)^R \overset{I.H}{=\!=} a.(w^R u^R)$$

$$= (a w^R) . u^R = (wa)^R . u^R = v^R . u^R$$