

CSC 220

10/6/2020

Languages



use inductive def. to define

$$L_1 = \{ a, ab, abb, \dots \}$$

$$L_1 = \{ ab^n \mid n \in \mathbb{N} \}$$

$$\text{if } n=0 \implies ab^0 = a \lambda = a$$

$$\text{if } n=1 \implies ab$$

$$n=2 \implies abb$$

\vdots

Basis: $a \in L_1$

Inductive: if $x \in L_1 \implies xb \in L_1$

$$L_1 = \{ a \}$$

$$L_1 = \{ a, ab \}$$

$\underbrace{\quad}_{x} \quad \curvearrowright$

$$L_1 = \{ a, ab, abb \}$$

\vdots
 $\underbrace{\quad}_{x} \quad \underbrace{\quad}_{x} \quad \curvearrowright$

$$L_1 = \{ a, ab, abb, \dots \}$$

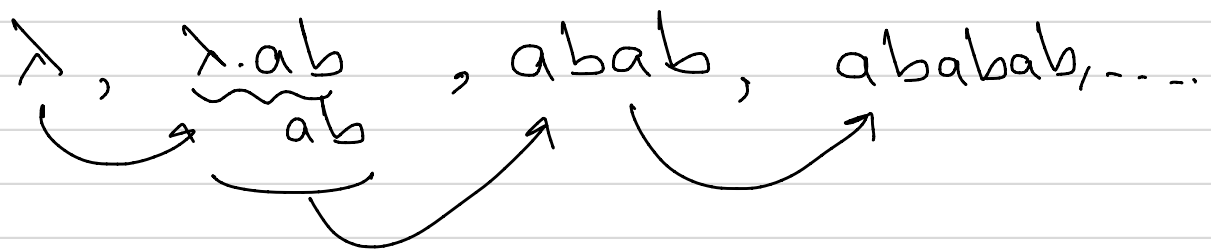
①

$$L_2 = \{ \lambda, ab, abab, \dots \}$$

$$L_2 = \{ (ab)^n \mid n \in \mathbb{N} \}$$

Basis: $\lambda \in L_2$

Inductive: if $x \in L_2 \Rightarrow xab \in L_2$



$$L_3 = \{ \lambda, ab, aabb, aaabbb, \dots \}$$

?

$$L_3 = \{ a^n b^n \mid n \in \mathbb{N} \}$$

$$n=0 \Rightarrow a^0 b^0 = \lambda \cdot \lambda = \lambda$$

$$n=1 \Rightarrow ab$$

$$n=2 \Rightarrow a^2 b^2 = aabb$$

$$n=3 \Rightarrow aaabbb$$

Basis: $\lambda \in L_3$

Inductive: if $x \in L_3 \Rightarrow axb \in L_3$

$$\begin{array}{c} \lambda \\ \downarrow \\ \underbrace{a \lambda b}_x = ab \end{array}$$

$$\begin{array}{c} a \quad ab \quad b \\ \underbrace{\quad \quad}_x = aabb \end{array}$$

$$\begin{array}{c} a \quad aaabbb \quad b \\ \underbrace{\quad \quad}_x = aaabbb \end{array}$$

⋮

③

\emptyset : a language, with no string

$\{\lambda\}$: a language, has one string

$$L_4 = \{ a^m b c^n \mid m, n \in \mathbb{N} \}$$

?

$$m=0, n=0 \quad a^0 b c^0 = \lambda b \lambda = b$$

$$m=1, n=0 \quad ab$$

$$m=0, n=1 \quad bc$$

$$m=10, n=50 \quad \dots$$

$$\begin{array}{ccc} a^m b c^n & \neq & a^n b c^n \\ \downarrow \quad \downarrow & & \downarrow \quad \downarrow \\ & & \text{both "n"} \end{array}$$

could be the same

Basis: $b \in L_4$

Inductive: if $x \in L_4 \Rightarrow ax, xc \in L_4$

Use induction to prove:

$$x \text{ string}, m, n \in \mathbb{N} \quad x^{n+m} = x^n \cdot x^m$$

induction is on n

$$P(0) \quad x^{0+m} = x^m = \cancel{x} \cdot x^m = x^0 \cdot x^m$$

$$P(k) \quad x^{k+m} = x^k \cdot x^m$$

$P(k+1)$?

$$x^{(k+1)+m} = x^{(k+m)+1} = x^{1+(k+m)}$$

$$= x^1 \cdot x^{k+m} = x \cdot x^k \cdot x^m$$

$$= x^{(1+k)} \cdot x^m$$

$$= x^{(k+1)} \cdot x^m$$

⑤