CSCE 4613 – Artificial Intelligence
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Assignment 2

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1 Clause Logic

1.1 What is an atom?

An *atom* is any symbol starting with a lowercase letter.

Example If we had an atom for Tyler, it could possibly be written as `tyler`. Likewise, an atom for Carson could be written as `carson`.

1.2 What is a body?

A *body* is an atom, or of the form $b_1 \wedge b_2$ where b_1 and b_2 are bodies.

Example Using the concepts in the atom definition, two bodies would be `tyler` and `carson`. Another valid body would be `tyler` $\wedge$ `carson`.

1.3 What is a definite clause?

A *definite clause* is an atom, or a rule that fits the form $h \leftarrow b$ where h is an atom and b is a body. It can be interpreted as "h if b".

Example If we have an atom `study_group`, a valid definite clause would be `study_group` $\leftarrow$ `tyler` $\wedge$ `carson`. Like the past two definitions, `tyler`, `carson`, and `study_group` also serve as definite clauses on their own.

1.4 What is a knowledge base?

A *knowledge base* is a set of definite clauses.

Example Using the past examples, a valid knowledge base would be $\{$`tyler`, `carson`, `study_group`, `study_group` $\leftarrow$ `tyler` $\wedge$ `carson`$\}$.

1.5 What is an interpretation of a knowledge base KB?

An *interpretation* of a knowledge base is an assignment of a truth value to each atom within the knowledge base.

Example Assume `tyler` and `carson` are "true" statements. Therefore, the body `study_group` $\leftarrow$ `tyler` $\wedge$ `carson`$\}$ is also true since `tyler` and `carson` are true and the logical AND of two true statements is also true.

1.6 What is a model of a knowledge base KB?

A *model* of a knowledge base is an interpretation of said knowledge base where *every* clause in that knowledge base is true.

Example For the knowledge base given in the knowledge base definition and the interpretation given in the interpretation definition, said interpretation is a model of the knowledge base since all it makes all of its constituent clauses true. However, if one of `tyler` or `carson` were false, then that interpretation would no longer be a model of that knowledge base.

2 Knowledge base

2.1 What is $KB \models g$?

$KB \models g$ means that KB logically entails g. This means that if everything in KB is true, then g must also be true. Equivalently, this can be stated as there is no interpretation in which KB is true and g is false.

2.2 What is $KB \vdash_P g$?

$KB \vdash_P g$ means that the proof procedure P can be used to derive the conjunction of atoms g from the knowledge base KB.

2.3 What is proof soundness?

A proof procedure P is *sound* if $KB \vdash_P g$ implies $KB \models g$. Equivalently, this means that if g can be derived from the proof procedure, then it must be true in all models of KB. A sound proof only derives logically entailed bodies and/or clauses. A non-sound proof would include bodies and/or clauses that are not true.

2.4 What is proof completeness?

A proof procedure P is *complete* if $KB \models g$ implies $KB \vdash_P g$. Equivalently, this means that if g is true in all models of KB, then it must be derived from the proof procedure. A complete proof derives every body and/or clause that is logically entailed by a knowledge base. An incomplete proof would fail to include some of these logically entailed bodies and/or clauses.

2.5 What is the key idea of the bottom-up proof procedure?

The key idea of the bottom-up proof procedure is that it derives a sound and complete set of proven clauses C from the bodies and clauses that make up the knowledge base KB.

Algorithm 1 Bottom-up proof procedure

$C = \{\}$
repeat
  select clause $h \leftarrow b_1 \wedge b_2 \wedge \ldots \wedge b_m \in KB$
  such that $b_i \in C\ \forall i$, and $h \notin C$
  $C = C \bigcup \{h\}$
until no more clauses can be selected

2.6 When is the bottom-up proof procedure successful?

You know that you are done in bottom-up when you see that there are no more sentences that are logically provable from the set of atoms that you have proved logically follow from the knowledge base.

2.7 When is the bottom-up proof procedure unsuccessful?

The bottom-up procedure can show that nothing logically follows from all of the sentences thus there is no successful derivation. ***(WORK ON THIS ANSWER)***

2.8 What is the key idea of the top-down proof procedure?

The key idea of the top-down proof procedure is that given a query g, the procedure searches backwards through the knowledge base KB to determine if g can be logically derived from KB.

Algorithm 2 Top-down proof procedure

$query = yes \leftarrow body$
$body = q_1 \wedge q_2 \wedge \ldots \wedge q_k$
repeat
  select $q_i \in body$
  choose clause $c = q_i \leftarrow b_c \in KB$
  replace $q_i \in body$ with b_c
until $query$ is an answer (fails if there is no clause with q_i as head)

The term **select** means any clause will work (non-deterministic). The term **choose** means a correct clause must be chosen (deterministic).

2.9 When is the top-down proof procedure successful?

A top down proof is considered successful when the first answer clause that is produced is the query, for each answer clause the clause before it proves itself, and the last answer clause is just always true.

2.10 When is the top-down proof procedure unsuccessful?

(WORK ON THIS ANSWER)

3 Predicate logic

A "Someone in Fayetteville likes basketball."

$$\exists p(LivesIn(p, Fayetteville) \wedge LikesBasketball(p))$$

Domain: People: $p \in P$

B "Everyone who likes ham also likes cheese."

$$\forall p(Likes(p, ham) \implies Likes(p, cheese))$$

Domain: People: $p \in P$

C "Mary is tall and Bill is not."

$$IsTall(Mary) \wedge \neg IsTall(Bill)$$

Domain: $\{Mary, Bill\}$

D "All of the houses near Sue's house are either large or old (but not both)."

$$\forall h(NearSue's(h) \implies IsLarge(h) \oplus IsOld(h))$$

Domain: Houses: $h \in H$

E "Between any pair of real numbers there is at least one real number."

$$\forall x \forall y \exists z(x > z > y)$$

Domain: Real numbers: $x, y, z \in \mathbb{R}$

F "There exists a dog that is cuter than all other dogs."

$$\forall d \exists c Cuter(c, d)$$

Domain: Dogs: $c, d \in D$

4 Problem formulation

A Volume of a Cylinder

Find the dimensions (radius, height) of a cylinder with given surface area A that has the largest volume V.

Optimization variable: V: The volume of the cylinder

Feasible region:

$$F = \{r, h \in \mathbb{R} : A = 2\pi r^2 + 2\pi rh\}, F \subset \mathbb{R}^2$$

Optimization objective: $f(r, h) = V$, volume of the cylinder given radius r and height h

Optimization formulation:

$$\max_V f(r, h); r, h \in F$$

Convex optimization?: ***(WORK ON THIS ANSWER)***

B Minimum average Euclidean distance

Given a set of points $S = \{x_1, x_2, ..., x_m\}$ where $x_m \in R^n$, find a point in $\mathbb{R}^n$ that has the minimum average Euclidean distance to all points in S.

Optimization variable: p: Point in n-space

Feasible region: $F = \mathbb{R}^n$

Optimization objective:

$$AvgDist(p) = \frac{1}{||S||} \sum_{x_i \in S} ||p - x_i||$$

Optimization formulation:

$$\min_{p} AvgDist(p); p \in \mathbb{R}^n$$

Convex optimization?: ***(WORK ON THIS ANSWER)***

5 Convex function

Part A

Let $f(x) : R^n \to R$, $g(x) : R^n \to R$ be convex functions. $h(x) = f(x)g(x)$. Claim: $h(x)$ is convex.

Part B

Let $f(x) : R^n \to R$, $g(x) : R \to R$ be convex functions. $h(x) = f(g(x))$. Claim: $h(x)$ is convex if $f(x)$ is non-decreasing.

Part C (Bonus)

The convex envelope of a function f (not necessarily convex), denoted conv f, is defined as the largest convex function majorized by f. Prove that conv $sin(x) = -1$.