Problem Set 5   **Due F Nov 10**

1. Evaluate the validity of the following argument using a tree.   If the argument is invalid, specify a canonical interpretation from an open branch, and explicitly verify that the interpretation shows the argument invalid. Remember: *the procedure for constructing a canonical interpretation differs for trees that involve identity statements*.

$$\exists y(Lay \wedge \neg a = y), Lab \therefore \neg a = b$$

(10)

2. Assess whether the following statement is a logical truth using a tree. If it's not a logical truth, specify a canonical interpretation from an open branch and explicitly verify that the interpretation makes the sentence false.

$$\forall x \forall y \forall z[(\neg x = y \wedge \neg y = z) \rightarrow \neg x = z]$$

(10)

3. Translate the following arguments into QL and assess their validity using a tree. If the argument is invalid, specify a canonical interpretation from an open branch, and explicitly verify that the interpretation shows the argument invalid. *Note: the arguments involve definite descriptions.*

(a) The black cat is on the mat. So, some cat on the mat is black.

(b) The grey cat is on the mat. So, every cat is on the mat.

(20)

4. Take as your domain the set of people. Let Lxy stand for x likes y, Tx stand for x is tall, a = Art, and b = Betty. Symbolize the following sentences.

   (a) Everyone likes at least one person.
   (b) Everyone likes at most one person.
   (c) Everyone likes exactly one person.
   (d) Everyone likes at least two people.
   (e) Everyone likes at most two people.
   (f) Everyone likes exactly two people.
   (g) Everyone like only themselves [i.e. we like ourselves and no-one else]
   (h) Everyone likes some other person [i.e. someone other than themself]
   (i) Some people like only other people [i.e. people other than themself]
   (j) Only Art likes Betty.
   (k) The tall person likes Art
   (l) The person who likes Art likes Betty
   (m) The person whom Art likes likes Betty.
   (n) Betty likes the person whom Art likes
   (o) Art likes tall people

(30)

1

5. Take as your domain the set of people. Translate (a) – (d) into reasonably natural English (no variables etc.), and translate (e) – (j) into QL using:

    Predicates: Gxy = x is a grandparent of y, Pxy = x is a parent of y
    Functions: fx = the father of x; mx = the mother of x, sx = the son of x
    Names: j = Jack, l = Laura

Please note that in translating (e)-(j) you are restricted to using the predicates and functions listed above.

(a) ∃x (x = fj v x = mj) → ∃xPxj

(b) ¬∃xPjx → ¬∃x Gjx

(c) ∃xGffjx

(d) ∃x([(Pxj ∧¬x = mj) ∧∀y((Pyj ∧ ¬y = mj) → x = y)] ∧ x = fj)

(e) Laura is Jack's mother

(f) Laura is a parent of Jack.

(g) The father of Jack's mother is a grandparent of Jack.

(h) If Laura is Jack's mother, then Jack is Laura's son

(i) Jack's mother's mother is not a parent of Jack.

(j) If Jack has a father, then Jack has a parent

(30)