MATH / PHIL 4253 **Midterm Study Guide**

Dates: M Oct 23 and W Oct 25

You should bring a blue book for each exam.

General Guidance:

For non-meta-theory:

The problem sets are a good guide to the standard. Also, many of Jeffrey's exercises are about the required standard, although he does toss in some more sophisticated problems here and there.

For meta-theory:

As detailed in the list of metatheory topics, look first to understand the core concepts; then to produce the simpler proofs regarding individual rules, then the proofs for truth trees, and ultimately, the more complex proofs demanded for quantificational logic.

What the exams will look like:

Exam 1 will focus on Propositional Logic (PL) and exam 2 will focus on Quantificational Logic (QL).

About 75-80% of the exam will be non-metatheory. The first four problem sets are a good guide to the kinds of questions you can expect to answer on that material.

We've done a limited amount of metatheory on the problem sets. Some metatheory questions will resemble questions on the problem sets; others will center on the results proven in class.

Typically, for each topic I will pose problems in ascending order of difficulty. So, for instance, for trees it will be something like: one very short and straightforward tree, one that is a bit more challenging, and one that is at the higher end of difficulty as set by the problem sets. Similarly, for translations from English into quantificational logic, something like: one or two translations at the most elementary level, a couple of more challenging ones, and then one at the high-end. And so on. So, it is in your interest to acquire basic understanding of every aspect of the course, and then build from there.

Topics

Truth-Functional Logic and Truth Trees (chapters 1 and 2)

1. Syntax: The truth functional connectives and other symbols of the language; *Rules of Formation* (section 1.13) that specify the legal sentences / well-formed formulas (wffs) of the language; *formation trees* and associated identification of the *main connective* of any sentence.
2. Semantics: *Truth-tables / Rules of Valuation* (section 1.11) for the language.
3. Translations from English into our formal language and vice-versa.
4. Basic Logical concepts: Validity / logical consequence, consistency, logical equivalence, tautology, contradiction.
5. Operations relating to those Concepts: How to test for the above using truth-tables.
6. Proofs of, and counterexamples to, claims relating those concepts e.g. an argument is valid iff the set that consists of the premises and the negated conclusion is inconsistent.
7. Truth-Trees: Use of truth-trees to test for any of the above; providing valuations from open trees that demonstrate invalidity, inconsistency, logical inequivalence, etc. Construction of a valuation that demonstrates invalidity etc., from a completed open branch.

Quantificational Logic (Chapters 3 and 4)

1. Syntax: Quantifiers and other symbols of the language; Rules of formation (sections 3.5 and 4.5); *formation trees* and associated identification of the *main logical operator* of any sentence.
2. Semantics: *Interpretations and rules of interpretation* (sections 3.9, 3.10) for the language.
3. Translations from English into our formal language and vice-versa. Section 4.16 provides a good benchmark for the kinds of translations you should be able to perform.
4. Trees for quantificational logic: Correct use of rules UI and EI (e.g. can only apply rules when the relevant quantifier is the main operator of the sentence; EI must introduce a new constant, and so, if you wish to "interact" the content of a universally quantified sentence with that of an existentially quantified sentence you must apply EI before UI); construction of a canonical interpretation from the completed open branch of a tree; infinite trees, and the construction of the canonical interpretation for an infinite open branch.

Metatheory

Roughly in order of increasing difficulty: If you find the meta-theory very challenging, then begin with the topics at the start of the list and work down as far as you can go.

1. Definitions of Soundness, Completeness and Decidability.
2. Understanding the equivalence of the original definitions of soundness and completeness and the alternate formulations that are used in the meta-theoretical proofs. For example: the tree test is sound = if the tree test says "valid", then the argument (really) is valid. This is equivalent to: if the tree is closed, then the initial list is inconsistent. By contraposition, this is equivalent to: if the initial list is consistent, then the tree is open.
3. Rule Soundness and Rule Completeness (p. 32), and proof of same for individual rules.
4. Soundness and Completeness Proofs for Truth Trees (i.e. truth-functional logic trees)
5. Understanding the failure of Decidability for the tree test for quantificational logic with more than one variable (i.e., understanding why infinite trees for some invalid arguments implies the loss of a decision procedure for validity, even though every valid argument will still yield a finite closed tree i.e. the tree test is still complete.)
6. Konig's Lemma (proof of)
7. Soundness Proof for Quantificational Logic (notably how the proof goes though given that the rule EI is not sound).
8. Completeness Proof for Quantificational Logic (notably, path completeness p. 54, and extension to cover infinite trees (using Konig's Lemma).