4253 Problem Set 2 **Due F Sept 15, Noon**

1. Construct truth trees to test the following arguments for validity. If you find an argument to be invalid, determine from the tree (at least) one assignment of truth-values for which the premises are true and the conclusion is false. *[Don't forget to number the lines of your trees]*.

   (a) $A \rightarrow [(B \vee C) \rightarrow R]$, $(R \vee S) \rightarrow T \therefore A \rightarrow (C \rightarrow T)$
   (b) $\neg A \rightarrow B$, $C \rightarrow D$, $\neg B \vee D \therefore A \vee C$
   (c) $(Z \wedge M) \rightarrow (S \vee A)$, $\neg (Z \wedge S) \therefore (Z \wedge \neg A) \rightarrow \neg M$
   (d) $\neg (C \wedge B)$, $\neg C \rightarrow A$, $(A \vee E) \rightarrow (A \wedge B) \therefore A \leftrightarrow B$

2. Use truth trees to test these sentences for tautologousness. If the sentence is not a tautology determine from the tree (at least) one assignment of truth-values that shows it false. *[Don't forget to number the lines of your trees]*.

   (a) $A \rightarrow (B \rightarrow A)$
   (b) $(B \leftrightarrow A) \leftrightarrow ((B \rightarrow A) \wedge (A \vee \neg B))$

3. Use truth trees to test the following pairs of sentences for logical equivalence. If the sentences are not logically equivalent determine from the tree (at least) one assignment of truth-values that shows the sentences are inequivalent. *[Don't forget to number the lines of your trees]*.

   (a) $A \rightarrow B$ $\quad\quad$ $B \rightarrow A$
   (b) $(B \rightarrow A) \wedge (A \rightarrow B)$ $\quad\quad$ $A \leftrightarrow B$

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4. True or False? Where true, provide a proof. Where false, provide a counterexample i.e., a particular case that demonstrates that the claim is false. *All counterexamples should be in our formal language for propositional logic (i.e. not in English).*

   Notation: "|=" means *validly entails* or *semantically entails*. So, for instance, "$\Gamma \models p$" means that the argument with p as conclusion and the sentences in the set Γ as premises is valid.

   (a) A sentence of form p v q is a tautology if at least one of p and q is a tautology
   (b) For any set of sentences, Γ, if $\Gamma \models p \vee q$, then either $\Gamma \models p$ or $\Gamma \models q$.

(c) If a set of sentences is inconsistent, then the set consisting of the negations of those sentences must be consistent.

(d) If the argument from the set of premises, Γ, to the conclusion ($A \wedge -A$) is valid, then Γ is inconsistent.

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5. Prove that sentences p and q are logically equivalent *if, and only if*, $p \leftrightarrow q$ is a tautology. (*Note: proving an "if and only if" claim means that you have to prove both directions—please ask me about this in class or office hours, if you want some guidance*).

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