Problem Set 3 **Due Noon, F Sept 29**

QL

1. Provide **formation trees** for the following sentences (i.e., the diagrams that show how to grammatically build up sentences), *and in each case identify the main logical operator* (i.e., main connective or quantifier). Also, specify the scope of each quantifier in every sentence.

   (a) $\exists x\,(Fx \wedge \neg Gx) \leftrightarrow (\forall x\, Fx \rightarrow \forall x\, Gx)$
   (b) $\forall x(Ax \leftrightarrow Bx) \rightarrow \exists x(Ax \wedge Bx)$
   (c) $(\forall xFx \rightarrow \forall xGx) \rightarrow \forall x(Fx \rightarrow Gx)$
   (d) $\forall x(Ax \rightarrow Bx) \vee \forall x(\neg Ax \rightarrow Bx)$

(16)

2. Translate each of the following sentences into exquisitely elegant (or at least, natural) English, using the interpretation manual F = is a fish; G = is something with gills; T = is something with a tail:
   Important: *By 'natural English' I mean: There should be no x's in your translations, and they shouldn't have artificial constructions, e.g. "There is something that is a ... and a ....", or, "Everything that is a .... is a ...."*

   (a) $\exists x\,(Fx \wedge \neg Gx)$
   (b) $\neg\exists x\,(Fx \wedge \neg Gx)$
   (c) $\forall x\, Fx \rightarrow \forall x\, Gx$
   (d) $\forall x\,(Fx \rightarrow Gx)$
   (e) $\forall x[(Fx \wedge Tx) \rightarrow Gx]$
   (f) $\forall x\,[(Fx \vee Tx) \rightarrow Gx]$
   (g) $\forall x(Fx \vee \neg Fx)$
   (h) $\forall x\, Fx \vee \forall x\, \neg Fx$

(16)

3. Translate the following sentences from English into predicate logic using the predicate symbols M (is a metal), P (is a plastic), C (conducts electricity), H (is hard), R (is a rhino), and U (is a unicorn).

   Be careful, some of these are a little tricky—the surface grammar can be deceptive about the logical structure of the sentence

   (a) Some metals conduct electricity
   (b) No metals conduct electricity
   (c) If some metals don't conduct electricity, then it's not the case that all metals do conduct electricity
   (d) All hard metals conduct electricity
   (e) All metals and plastics conduct electricity
   (f) Rhinos exist, but unicorns do not.

(12)

5. State the truth-values of the listed sentences on the following interpretation:

Domain = {1, 2, 3, 4, 5}

$F \rightarrow \{1, 2\}$,

$G \rightarrow \{1, 2, 3, 4\}$

$L \rightarrow \{<1,1>, <1,2>, <1,3>, <1, 5>, <2, 2>, <3, 1>, <3,2>,<4, 2>, <5, 2>\}$

$a \rightarrow 2$

(a) $\forall xGx$
(b) $\forall x(Fx \rightarrow Gx)$
(c) $\exists x(Gx \wedge \neg Fx)$
(d) $\forall xFx \rightarrow \forall xGx$
(e) $\forall xLxa$
(f) $\forall xLax$
(g) $\exists x \neg Lxx$
(h) $\forall x(Fa \rightarrow Lax))$

(16)

6. Use the tree test to verify that each of the following arguments is valid.
[**Note:** if you're not sure which rule to apply to some of these sentences, you should do formation trees to figure out the main logical operator]

(a) $\forall x(Fx \rightarrow Gx), \neg Ga \ \therefore\ \neg Fa$
(b) $\forall x (Fx \rightarrow Hx), \exists xHx \rightarrow Ga \ \therefore\ Fa \rightarrow Ga$
(c) $\exists x[Sx \wedge \neg (Sx \wedge Tx)] \ \therefore\ \exists xSx \wedge \exists x\neg Tx$
(d) $\neg\forall x(Fx \wedge Gx), \forall x (Hx \rightarrow Fx) \ \therefore\ \forall x Hx \rightarrow \exists x\neg Gx$

(20)

<u>**PL Metatheory**</u>

7. Prove <u>rule-soundness</u> of the tree rules for *the conditional* and *the negated conditional*.

Rule Soundness: if the premise/input is true in some case/valuation C, then all the lines in at least one of its conclusions/outputs is true in case C too.

(10)

8. Prove the <u>rule-completeness</u> of the tree rules for *the biconditional* and *the negated biconditional*.

Rule Completeness: If all the lines in one of its conclusions/outputs is true in some case/valuation C, the premise/input is true in C also.

(10)

Bonus Question

9. Consider the following, alternative tree rule for conditionals, (X):

$$\begin{array}{ccc} & p \rightarrow q & \\ / & & \backslash \\ p & & q \end{array}$$

(a) Demonstrate/show that (X) is not rule sound.
(b) Demonstrate/show that (X) is not rule complete.
(c) Demonstrate that a *tree test* that replaces the standard conditional rule with (X) (but retains the other rules) is not *sound*, by providing a closed tree for an invalid argument.
(d) Demonstrate that a *tree test* that replaces the standard conditional rule with (X) (but retains the other rules) is not *complete*, by providing an open tree for a valid argument.

(10)