

Problem Set 7

Due M Dec 11 (i.e., the day of the final exam), *but I will grade and comment promptly if you submit earlier.*

1. Use trees to prove the following (“ \models_K ” means is a logical truth in K. Also, P and Q are variables i.e., they can stand for arbitrary non-modal sentences)

$$(a) \models_K (\Box P \wedge \Box Q) \rightarrow \Box(P \wedge Q)$$

$$(b) \models_K \Diamond(P \wedge Q) \rightarrow (\Diamond P \wedge \Diamond Q)$$

$$(c) \models_K \Box(P \rightarrow Q) \rightarrow (\Diamond P \rightarrow \Diamond Q)$$

(24)

2. Use trees to show that the following entailments do not hold (in K). In each case, construct a counter-model from an open branch, and diagram it.

$$(a) \Diamond P, \Diamond Q \not\models_K \Diamond(P \wedge Q)$$

$$(b) \Box P \rightarrow \Box Q \not\models_K \Box(P \rightarrow Q)$$

(16)

3. Use trees to prove the following (where “ \models_α ” means is a logical truth in system α):

$$(a) \models_p P \rightarrow \Diamond P$$

$$(b) \models_\sigma \Diamond \Box P \rightarrow P$$

$$(c) \models_\tau \Diamond \Diamond P \rightarrow \Diamond P$$

$$(d) \models_{ss} \Diamond \Box P \rightarrow \Box P$$

(32)

4. Use trees to show the following expressions are not logical truths in the relevant systems, and read an interpretation off an open branch of the tree that demonstrates this. Be careful to ensure that the accessibility relations in your interpretations respect the demands of the relevant system i.e. all worlds are accessible from themselves in p-interpretations, all accessibility relations are symmetric in σ -interpretations.

$$(a) \not\models_p \Diamond \Box P \rightarrow P$$

(b) $\not\models_{\sigma} \Box P \rightarrow \Box \Box P$

(20)

5. True or false? In each case, explain why.

(a) Every logical truth in K is also a logical truth in ρ .

(b) Every sentence that is not a logical truth in S_5 is also not a logical truth in σ .

(8)