Problem Set 4 **Due noon, F Oct 13**

1. Use the tree test to show that the following arguments are invalid. In each case construct *a canonical interpretation* from an open branch (i.e. using the rules specified in Jeffrey pp. 55-56] and verify that it demonstrates the argument invalid.

(a) $\forall x((Fx \wedge Gx) \rightarrow Hx), \exists xFx \wedge \exists x\, Gx \ \therefore\ \exists xHx$

(b) $\forall x(Fx \rightarrow Gx), \forall x(Hx \rightarrow Gx), \exists xHx \ \therefore\ \exists x(Fx \wedge Hx)$

(16)

2. Let the domain = the set of people. Symbolize the following sentences, using P = is a philosopher, W = is wise, Txy = x teaches y, Lxy = x learns from y, s = Socrates, p = Plato.

(a) Everyone learns from someone
(b) Someone learns from everyone.
(c) Someone learns from everyone who learns from them.
(d) Someone learns from no-one
(e) Everyone has someone they learn from who also learns from them.
(f) Some wise people aren't philosophers, but all philosophers are wise
(g) Anyone who was taught by Socrates was wise
(h) If everyone is a philosopher, then Socrates is a philosopher
(i) If anyone is a philosopher, then Socrates is a philosopher
(j) Anyone who is taught by Socrates learns from Socrates
(k) Whoever learns from Plato, learns from Socrates
(l) If no-one is wise, then no-one is a philosopher
(m) If everyone is taught by a philosopher, then everyone will be wise

(26)

3. Translate each of the following arguments into English, interpreting L as *loves*, a as *Alma*, and b as *the Baron*. Using trees, determine whether or not each argument is valid. If invalid, obtain a canonical interpretation from the tree and demonstrate that the premises are true and the conclusion is false on that interpretation; if valid, informally explain why the English premises entail the English conclusion.

(a) $\forall y\, Lya \rightarrow \forall y\, Lay, Lba \therefore Lab$
(b) $\forall x\forall y\, (Lxy \rightarrow Lyx], Lba \therefore Lab$
(c) $\exists y\forall x\, Lxy \therefore \exists x\exists y(Lxy \wedge Lyx)$
(d) $\exists x\forall y(Lxy \rightarrow Lyx), \forall x\, Lxb \therefore \forall x\, Lbx$

(40)

4. Construct enough of the tree for the following argument to show that it is infinite. Infer the interpretation with an infinite domain supported by an open branch of the tree, and explain why the interpretation makes the premises true and the conclusion false.

$$\forall x \exists y(Lxy \wedge Gyx),\ Laa \therefore\ Gaa$$

(8)

6. In performing the tree test in quantificational logic, a tree has not been completed until either (i) it closes, or (ii) in each open branch, you have performed UI on each universal generalization in that branch *using every name that appears in that branch*. Suppose we relaxed this requirement in the following way: You are only required to perform UI using one name in each branch to complete the tree.

(a) Would the modified tree test be sound?
(b) Would the modified tree test be complete?

In each case: if you think the tree test would still be adequate, informally explain why; If you think the tree test would be inadequate in the relevant respect, give a counterexample i.e., a tree constructed in conformity with the modified procedure that demonstrates that the tree test lacks the relevant property.

(10)

Bonus Question: *Propositional Logic Review*

7. True or False? Where true, provide a proof. Where false, provide a counterexample i.e., a particular case that demonstrates that the claim is false. All counterexamples should be in our formal language (i.e. not arguments or sentences in English).

(a) A sentence of form p v q is a tautology only if at least one of p and q is a tautology
(b) If $\{p_1, p_2, \ldots, p_n\}$ is inconsistent, then $\{p_1, p_2, \ldots, p_n\} \models q$, for arbitrary q.
(c) If $p \wedge q$ is a contradiction, then $p \wedge \neg q$ is a tautology.
(d) If $\Gamma \cup \{p\} \models q$, then $\Gamma \models p \rightarrow q$.

(20)