Problem Set 6 **Due W Nov 29**

Remember the only permissible nodes in a register machine are ones for which:

(i) You add 1 to a given register (i.e. there is one output arrow, the "+"arrow).
(ii) You subtract one unless the register is empty (i.e. there are two output arrows, the "-" arrow and the "e" arrow)

1. Diagram register machines that compute the following functions, leaving the result in the register that takes the input value m.

   (a) $f(m) = \begin{cases} 1, & \text{if } m > 0 \\ 0, & \text{otherwise} \end{cases}$

   (b) $f(m) = \begin{cases} 0, & \text{if } m > 0 \\ 1, & \text{otherwise} \end{cases}$

(10)

2. Diagram a register machine program that computes the sum of the values of three input variables i.e., $f(l, m, n) = l + m + n$.

(10)

3. (a) Diagram a register machine that does not halt for any input

   (b) Diagram a register machine that only halts for input 0.

   (c) Consider register machines that have one halt arrow and halt for every input, yielding either output 0 or 1 in register A. Diagram a module that you can add to such machines so that the new machine (old machine + module) halts for cases where the original machine yields output 1, and otherwise does not halt.

(15)

4. A set of objects is *enumerably infinite* if its members can be placed in a one-to-one correspondence with the counting numbers (1, 2, 3, 4, …). [For instance, the even numbers can be placed in a one-to-one correspondence with the counting numbers by pairing 1 with 2, 2 with 4, 3, with 6, and in general n with 2n.]
   (a) Is the union of the set of counting numbers and the set {0, -1, -2, -3, -4, -5} enumerably infinite? If so, demonstrate that it is so. If not, explain / demonstrate why not. Your demonstration can be pretty informal.
   (b) Give an argument that the set of decimal numbers between 0 and 1 is not enumerably infinite.
   (c) We know that the set of rational numbers is enumerably infinite. Hence, the set of rational numbers between 0 and 1 is enumerably infinite. However, we can represent any such rational number as a decimal number between 0 and 1. Why does the argument you provided in part (b) not show that this set of rationals is not enumerably infinite?

(5, 10, 5)

5. Diagram a register machine that computes the function $f(n) = n + 1$. Write down an argument in the standard format (one premise that characterizes the input state, a premise or premises that characterize the operation of the machine, and conclusion that specifies that the machine halts) that is valid if and only if this machine halts for the input n = 3.

(10)

6. Diagram a register machine that computes the function:

$$f(m, n) = \begin{cases} m - n, & \text{if } n < m \\ 0, & \text{otherwise} \end{cases}$$

Write down an argument in the standard format (one premise that characterizes the input state, premises that characterize the operation of the machine, and conclusion that specifies that the machine halts) that is valid if and only if this machine halts for the input m = 3 and n = 1.

(20)

7. (a) Explain what the self-halting function, h, for functions of one variable is i.e., what does the function h(n) compute for arbitrary n? You can write down the description in a mathematically precise / formal way, but *use your words*, as they say: explain in English what the result of such a computation tells us.

(b) Given that the set of register machines that compute functions of one variable can be enumerated, prove that no register machine can compute the self-halting function for functions of one variable.

(5, 10)