Bayesian Theorem

- Let A and B are two random variables
- P(B) is the prior probability (initial probability)
- P(A|B) is the known conditional probability
- We want to calculate the *posteriori probability* P(B|A)
  - P(A,B): in the joint probability
  - P(B|A) = P(A,B)/P(A) = P(B)P(A|B)/P(A)

Example for Bayes Rules

- The patient either has a cancer or does not.
- A prior knowledge: over the entire population, .008 have cancer
- Lab test result + or - is imperfect. It returns
  - a correct positive result in only 98% of the cases in which the cancer is actually present
  - a correct negative result in only 97% of the cases in which the cancer is not present
- What happens if a new patient for whom the lab test returns +?

Example for Bayes Rules

Pr(cancer)=0.008 Pr(not cancer)=0.992

Pr(+|cancer)=0.98 Pr(-|cancer)=0.02

Pr(+|not cancer)=0.03 Pr(-|not cancer)=0.97

Pr(+,cancer)=Pr(+|cancer)p(cancer) = 0.98* 0.008 = 0.0078

Pr(+,not cancer)=Pr(+|not cancer)Pr(not cancer) = 0.03*0.992=0.0298

Hence,

Pr(cancer|+) = Pr(cancer,+)/Pr(+) = 0.0078/(0.0078+0.0298)=0.21