1. The following 1-qubit system is twice as likely to contain a 0 than a 1 once measured:

$$\frac{\sqrt{2}}{\sqrt{3}}|0\rangle + \frac{1}{\sqrt{3}}|1\rangle$$

Give examples of qubit systems in normalized form that satisfy the following:

   (a) A 1-qubit system that will contain a 0 with a 30% chance and a 1 with a 70% chance,

   (b) A 2-qubit system that has no chance of being measured as $|11\rangle$, but an equal chance of being measured in any other state,

   (c) An n-qubit system where all possible measurements are equally likely.

   There are multiple answers since making coefficients negative doesn't change the probability, so just choose your favorite.

2. Write a matrix for the following quantum operations:

   (a) A 2×2 matrix that rotates a qubit by an angle of $\pi/3$,

   (b) A 4×4 matrix that performs the XOR operation on two qubits and stores the result in the first qubit, and

   (c) A 8×8 matrix that takes in two inputs and a scratch-pad bit (assumed to be initialized to 0) and performs the OR operation on the inputs, storing the result in the scratch-pad bit.

   There may be multiple matrices that perform the operation; chose your favorite.

3. Let M be a 2×2 unitary matrix. Recall that a matrix is unitary exactly when $MM^T = I$, that is:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} a & c \\ b & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

   Prove that if you knew the value of a, you could determine the entire matrix.

4. There are 16 possible classical logic gates that have 2 input bits and 1 output bit. Suppose that you wanted to implement some of these gates as quantum gates so that the result of the computation overwrote one of the input bits from your quantum register rather than using a scratch-pad bit. How many of these gates could you implement in this way and why wouldn't they require scratch-pad bits?