MATH / PHIL 4253 **FINAL EXAM STUDY GUIDE**

Date: **M Dec 11, 10:15 a.m. – 12:15 p.m.**

You should bring a blue book

I will provide the tree rules for PL, QL, and modal logic.

General Guidance:

The problem sets are generally a good guide to the standard. However, there are topics that are fair game, but have not been covered extensively on the problem sets. This includes some meta-theory that we covered, and the proofs that the Halting problem is unsolvable and that there is no effectively computable decision procedure for first order validity.

What the exam will look like:

The exam is cumulative, and will include material from the entire course, but will be weighted towards material covered since the mid-term (i.e., from Jeffrey, chapter 5, through Priest, chapters 2 and 3).

As with the mid-term, where possible, I will pose problems in ascending order of difficulty. So, in questions on a given area, there will be an elementary problem, a more sophisticated problem or problems, and /or a considerably more challenging problem.

For the more sophisticated material, I recommend that you look first to understand the core concepts, and work from there to the more complicated demonstrations. Questions on the more advanced material will include components that give points for understanding of these core concepts.

Regarding *the more advanced material / material that we covered most recently*, you can choose **2 of 3** questions that will respectively cover *metatheory*, *register machines & halting problem type material*, and *modal logic*. The remainder of the exam will not have options and will focus on some elementary PL, perhaps with some proof/counterexample type questions, and considerably more QL (with and without identity and functions) that will involves trees, construction trees, determining truth values of sentences on a specified interpretation, translations from English to QL and from QL to English, i.e., on the core skills that we have focused on.

Topics

Truth-Functional Logic and Truth Trees (chapters 1 and 2)

1. Syntax: The truth functional connectives and other symbols of the language; *Rules of Formation* (section 1.13) that specify the legal sentences / well-formed formulas (wffs) of the language; *formation trees* and associated identification of the *main connective* of any sentence.
2. Semantics: *Truth-tables / Rules of Valuation* (section 1.11) for the language.
3. Translations from English into our formal language and vice-versa.
4. Basic Logical concepts: Validity / logical consequence, consistency, logical equivalence, tautology, contradiction.
5. Operations relating to those Concepts: How to test for the above using truth-tables.
6. Proofs of, and counterexamples to, claims relating those concepts e.g. an argument is valid iff the set that consists of the premises and the negated conclusion is inconsistent.
7. Truth-Trees: Use of truth-trees to test for any of the above; providing valuations from open trees that demonstrate invalidity, inconsistency, logical inequivalence, etc. Construction of a valuation that demonstrates invalidity etc., from a completed open branch.

Quantificational Logic (Chapters 3 and 4)

1. Syntax: Quantifiers and other symbols of the language; Rules of formation (sections 3.5 and 4.5); *formation trees* and associated identification of the *main logical operator* of any sentence.
2. Semantics: *Interpretations and rules of interpretation* (sections 3.9, 3.10) for the language.
3. Translations from English into our formal language and vice-versa. Section 4.16 provides a good benchmark for the kinds of translations you should be able to perform.
4. Trees for quantificational logic: Correct use of rules UI and EI (e.g. can only apply rules when the relevant quantifier is the main operator of the sentence; EI must introduce a new constant, and so, if you wish to "interact" the content of a universally quantified sentence with that of an existentially quantified sentence you must apply EI before UI); construction of a canonical interpretation from the completed open branch of a tree; infinite trees, and the construction of the canonical interpretation for an infinite open branch.

Identity (chapter 5)

1. Syntax and Semantics for the identity symbol.
2. Tree rules for identity
3. Definite Descriptions
4. Expressing numerical claims using identity
5. Modified Procedure for constructing counterexamples from open branches.

Functions (chapter 6, but *omit the sections we didn't cover*)

1. Syntax and Semantics for function symbols

2. Restrictions on use of UI tree rule for function symbols

Uncomputability (Chapter 7, but *omit section 7.3*)

1. Register Machines: what they are; the two basic operations in register machine programs; design of elementary register machine programs.
2. The Church-Turing Thesis
3. The Unsolvability of the Halting Problem: Enumerability; enumeration of register machine programs that compute functions of one variable; the self-halting function; demonstration that the self-halting function is not computable.
4. Describing Register Machine Programs in Logical Notation:

Undecidability (Chapter 8, but *omit sections 8.4-8.8)*

1. Describing Register Machine Programs in Logical Notation (continued): In particular, writing an argument that is valid iff a register machine P_m halts when applied to input n.
2. Proof of Church-Turing Theorem**:** The decision problem for first order validity / consistency is unsolvable by Register machine programs.

Modal Proposition Logic

Basic Modal Logic (Priest, Chapter 2)

1. Syntax and (Possible Worlds) Semantics for Modal Propositional Logic: Interpretations; accessibility relation; definitions of logical truth and of valid inference in modal logic; system K; vacuous truth of necessity claims.
2. Tree rules for necessity and possibility operators; modal logic trees, and construction of counter-models from open branches.

Normal Modal Logic (Priest, Chapter 3)

1. Extendability, Reflexivity, Symmetry, and Transitivity
2. Associated tree rules and their application.

Metatheory Summary

The initial material in the list is roughly in order of increasing difficulty: If you find the meta-theory very challenging, then begin with the topics at the start of the list and work down from 1-8 as far as you can go. 9-10 have their own challenges but can be tackled independently of 1-8.

1. Definitions of Soundness, Completeness and Decidability.
2. Understanding the equivalence of the original definitions of soundness and completeness and the alternate formulations that are used in the meta-theoretical proofs. For example: the tree test is sound = if the tree test says "valid", then the argument (really) is valid. This is equivalent to: if the tree is closed, then the initial list is inconsistent. By contraposition, this is equivalent to: if the initial list is consistent, then the tree is open.
3. Rule Soundness and Rule Completeness (p. 32), and proof of same for individual rules.
4. Soundness and Completeness Proofs for Truth Trees (i.e. truth-functional logic trees)
5. Understanding the failure of Decidability for the tree test for quantificational logic with more than one variable (i.e., understanding why infinite trees for some invalid arguments implies the loss of a decision procedure for validity, even though every valid argument will still yield a finite closed tree i.e. the tree test is still complete.)
6. Konig's Lemma (proof of)
7. Soundness Proof for Quantificational Logic (notably how the proof goes through given that the rule EI is not sound).
8. Completeness Proof for Quantificational Logic (notably, path completeness p. 54, and extension to cover infinite trees (using Konig's Lemma).
9. Proof of the unsolvability of the Halting Problem
10. (The closely related) Proof that there is no effectively computable decision procedure for validity.