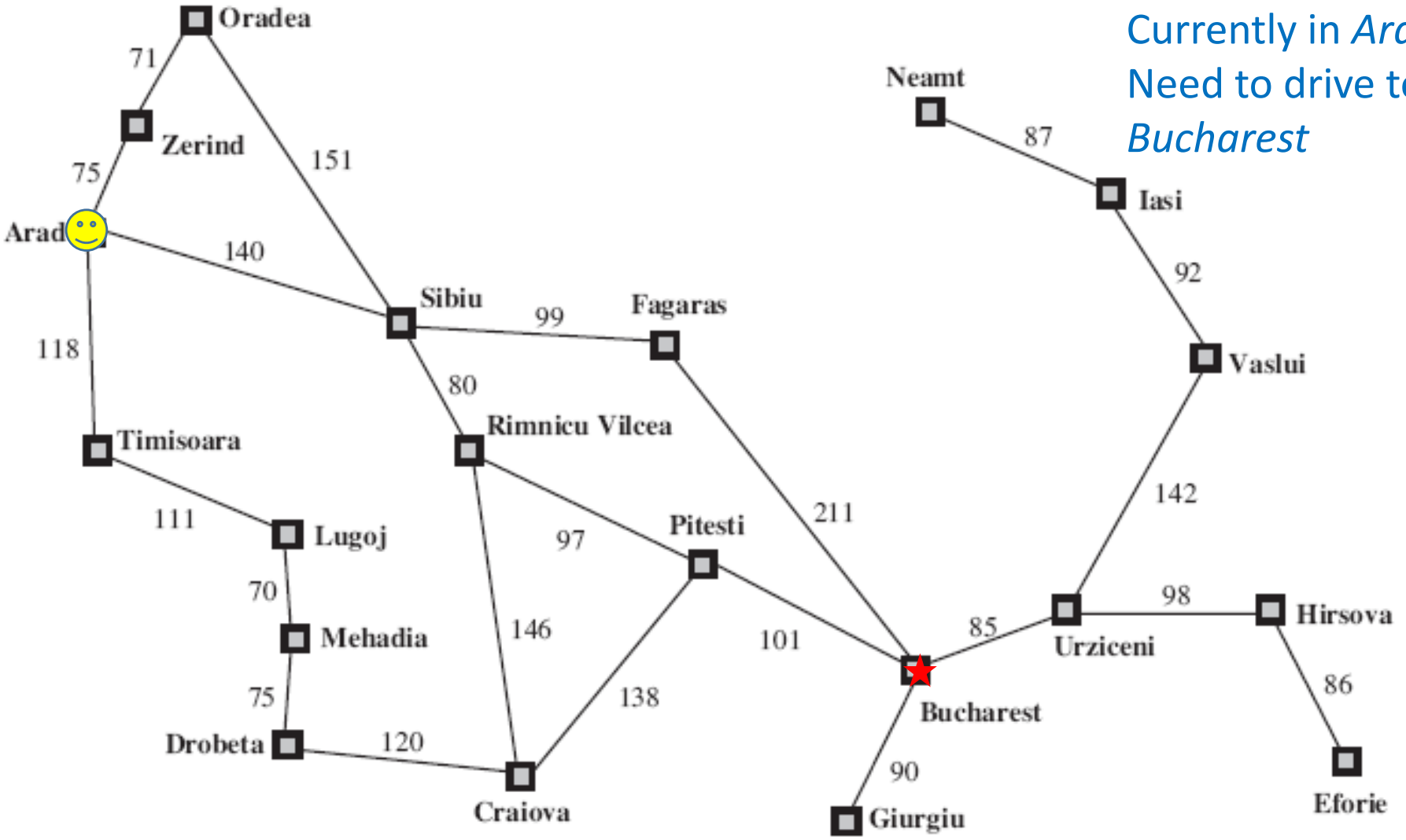
Solving Problems by Searching

COEN166

Artificial Intelligence

Navigation in Romania



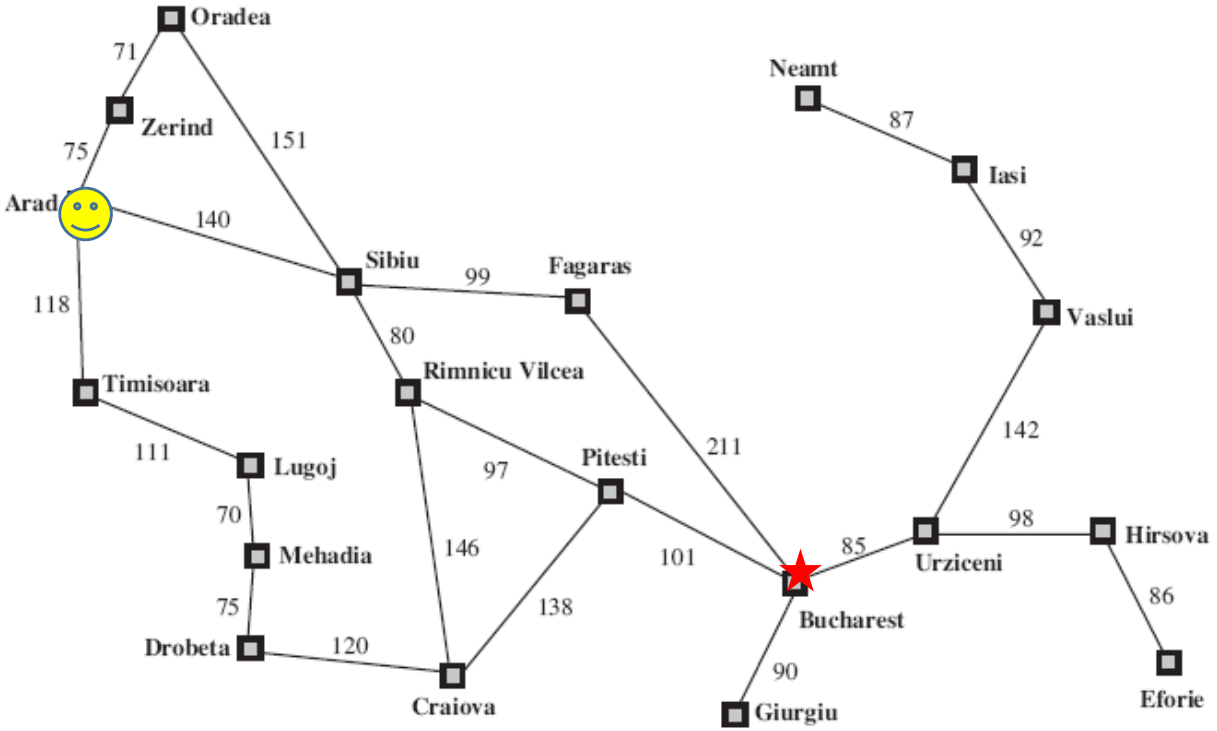
Agent:
Currently in *Arad*;
Need to drive to
Bucharest

Problem-Solving Agents

- **Goal formulation**
  - Goals help organize behavior by limiting
    - the **objectives** that the agent is trying to achieve, and hence
    - the **actions** it needs to consider
  - *Goal:* Get to Bucharest from Arad
- **Problem Formulation:** the process of deciding what actions and states to consider, given a goal
  - *Actions:* driving from one major town to another
  - *States:* being in a particular town

Problem-Solving Agents

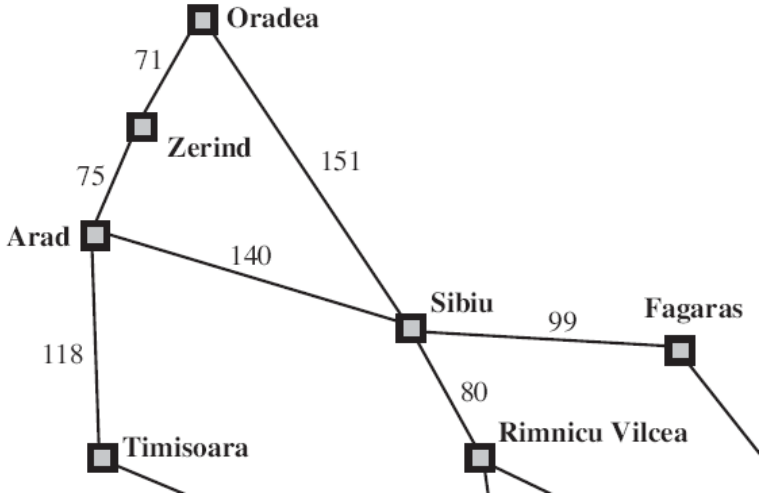
- **Search:** the process of looking for a sequence of actions that reaches the goal
  - Where to go from Arad? Sibiu, Timisoara, or Zerind?
    - Environment unknown – *trying one of the actions at random*
    - Environment observable (a map of Romania) – *decide what to do by first examining future actions*



Problem-Solving Agents

- **Search Algorithm:** takes a problem as input and returns a solution in the form of an action sequence
- **Solution:** an action sequence that leads from the initial state to a goal state
- **Execution Phase:** once a solution is found, the actions it recommends can be carried out

Well-defined Problems and Solutions



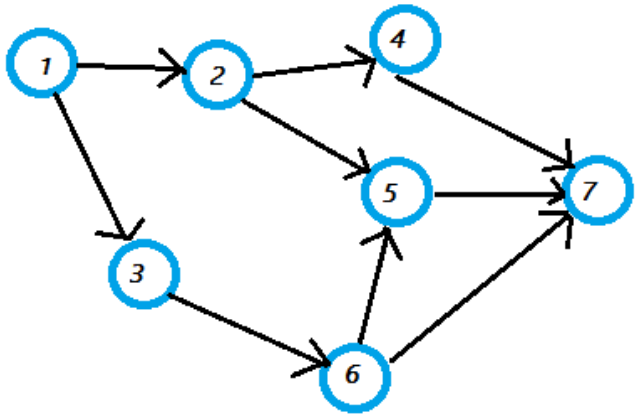
- Problem Definition - 5 components
  1. Initial State
     - ***In*(*Arad*)**
  2. Actions (available to the agent)
     - Given a state *s*, **Actions(*s*)** returns the set of actions that can be executed in *s*
     - Each of these actions is *applicable* in *s*
     - **Actions(*In*(*Arad*)): {*Go*(*Sibiu*), *Go*(*Timisoara*), *Go*(*Zerind*)}**
  3. Transition Model (a description of what each action does)
     - **Result(*s*,*a*)**: returns the state that results from doing action *a* in state *s*
     - **Result(In(Arad), Go(zerind)) = In(Zerind)**

Well-defined Problems and Solutions

- Problem Definition - 5 components

  The *State Space:* the set of all states reachable from the initial state by any sequence of actions.

  - Implicitly defined by initial state, actions, transition model
  - A directed graph (e.g. the map of Romania)
    - Nodes: the states
    - Links: actions

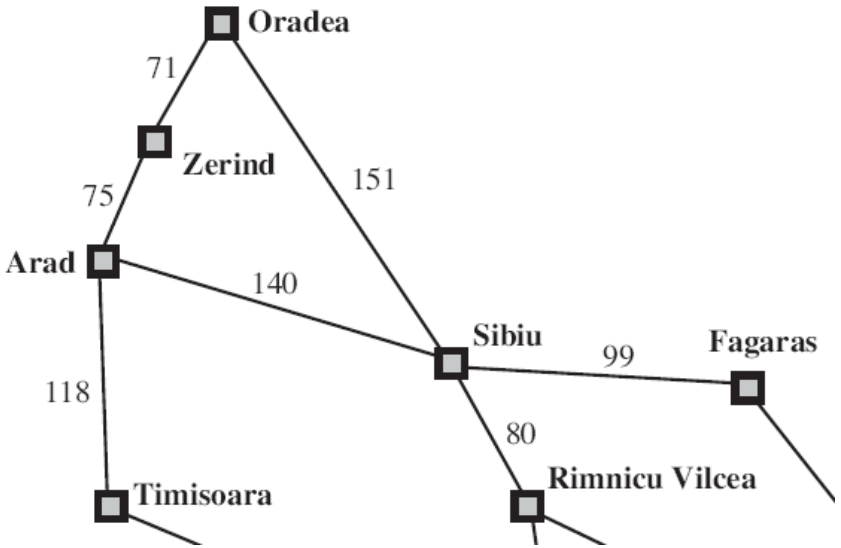


  - A *path* in the state space: a sequence of states connected by a sequence of actions

Well-defined Problems and Solutions

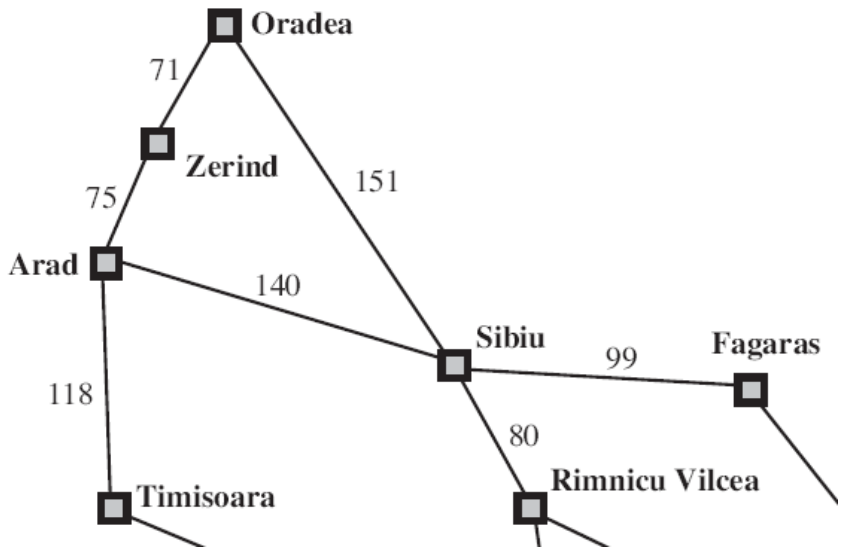
- Problem Definition - 5 components
  4. *Goal test:* determines whether a given state is a goal state
     - e.g. In the Romania Navigation problem: the goal state is the singleton set {*In*(*Bucharest*)}

  5. *Path cost function*: assigns a numeric cost to each path
     - e.g. Romania: the cost of a path can be its length in kilometers
     - step cost ***c(s,a,s')***: the cost of taking action ***a*** in state ***s*** to reach state ***s'***



Well-defined Problems and Solutions

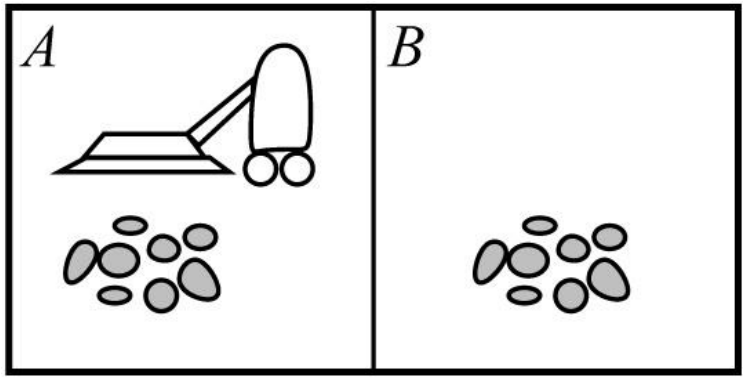
- A solution to a problem
  - An action sequence that leads from the initial state to a goal state.
  - Solution quality: measured by the path cost function
  - Optimal solution: lowest path cost



Vacuum World

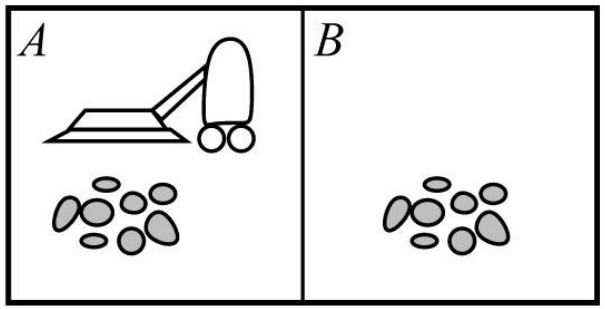
- States
  - 2 agent locations (squares), each location might or might not contain dirt
  - $2 \times 2^2 = 8$ states
  - If *n* locations, then how many states?

    $n \times 2^n$ states



Vacuum World

- Initial state
  - Any state can be designated as the initial state
- Actions
  - Left, Right, Suck
  - Larger environments: Up, Down



- Transition Model
  - Every action has an effect, except
    - Moving Left in the leftmost square,
    - Moving Right in the rightmost square,
    - Sucking in a clean square

Vacuum World

The complete state space (and transition model)

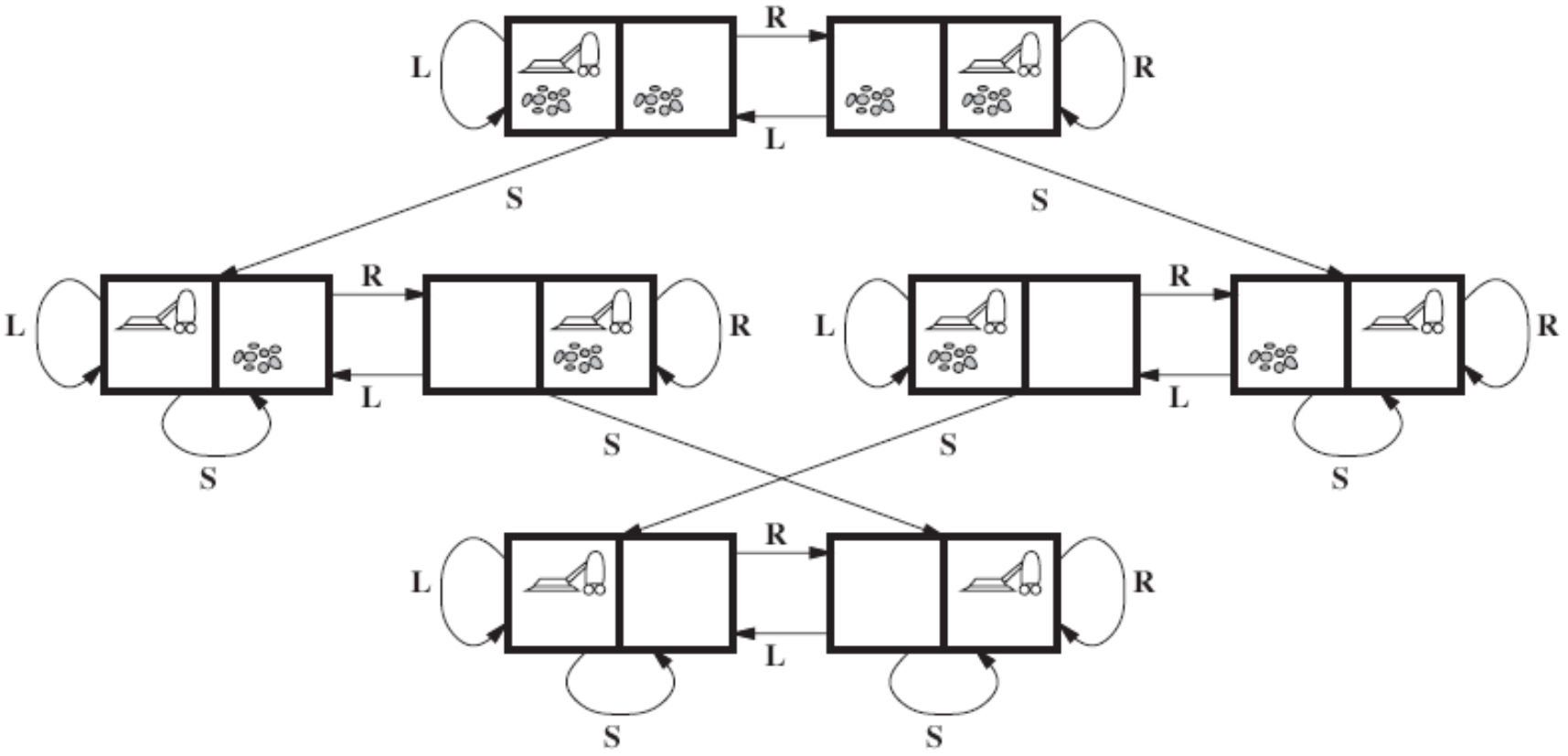


Figure 3.3 **FILES: figures/vacuum2-state-space.eps (Tue Nov 3 16:24:01 2009).** The state space for the vacuum world. Links denote actions: L = *Left*, R = *Right*, S = *Suck*.

Vacuum World

- Goal Test
  - Check whether all squares are clean
- Path cost
  - Each step costs 1 (i.e. 1 per action)
  - The path cost is the number of steps (actions) in the path

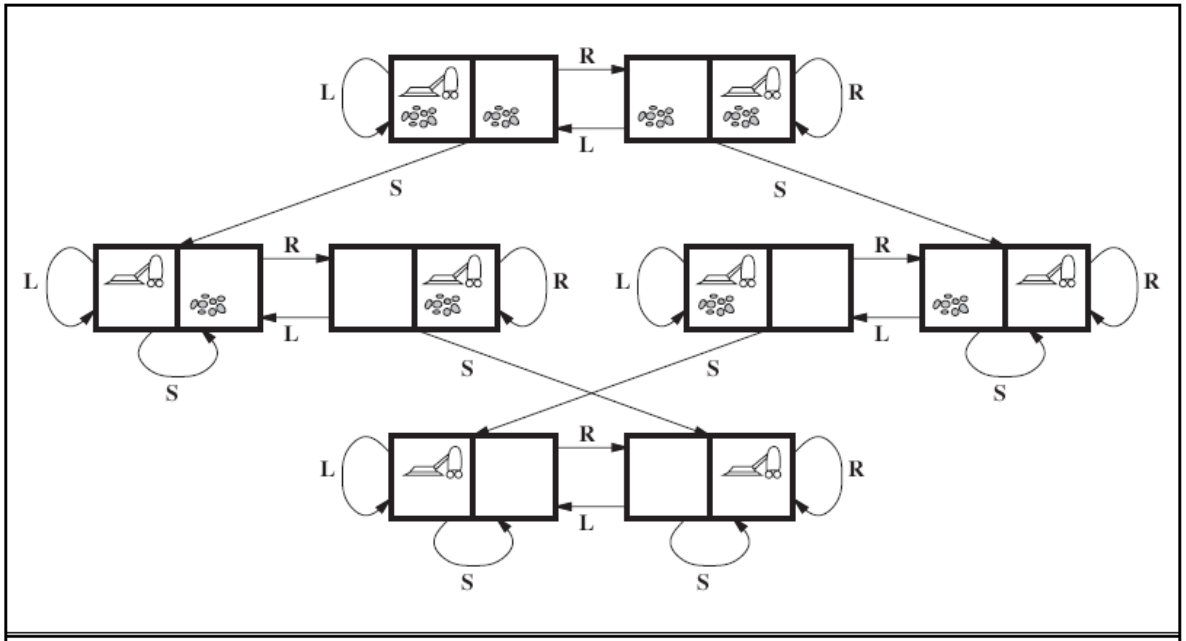
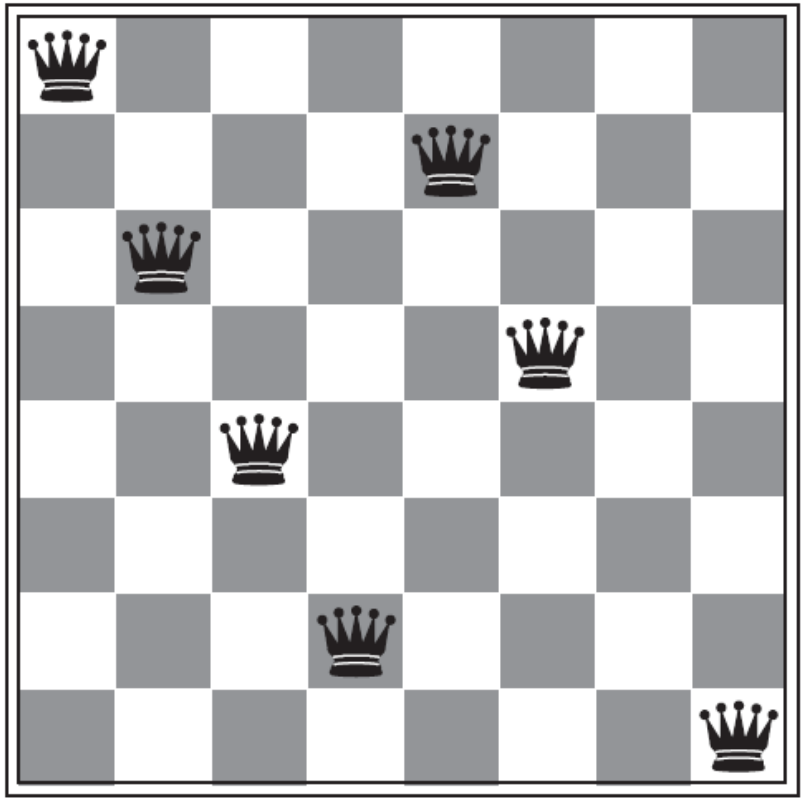


Figure 3.3 **FILES: figures/vacuum2-state-space.eps (Tue Nov 3 16:24:01 2009).** The state space for the vacuum world. Links denote actions: L = *Left*, R = *Right*, S = *Suck*.

The 8-Queens Problem

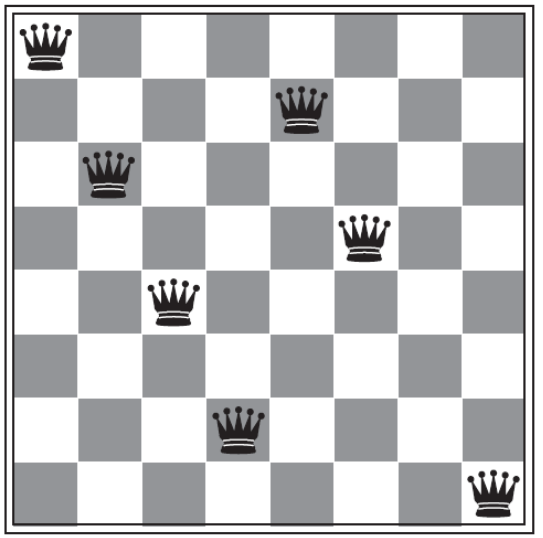
- Goal: place 8 queens on a chessboard such that no queen attacks any other
- Two queens attack each other if
  - They are in the same row, or column, or diagonal line

Almost a solution



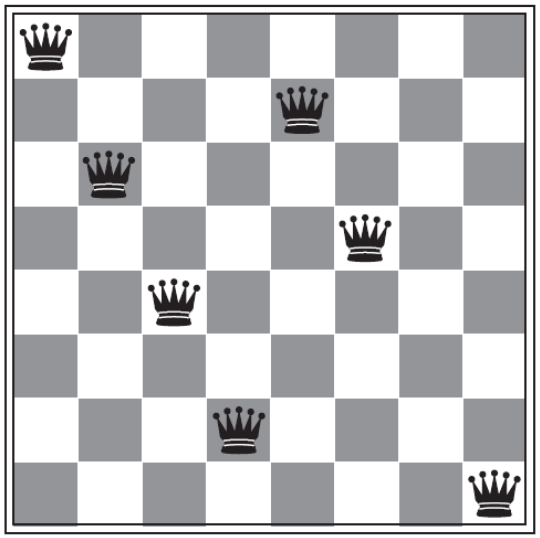
The 8-Queens Problem

- Incremental formulation: starts with an empty state, then each action adds a queen to the state
  - States?
    - any arrangement of $n \leq 8$ queens on the board
    - Or arrangements of $n \leq 8$ queens in the leftmost n columns, 1 per column, such that no queen attacks any other
  - Initial state?
    - no queens on the board
  - Actions?
    - add a queen to any empty square
    - Or add queen to leftmost empty square such that it is not attacked by other queens



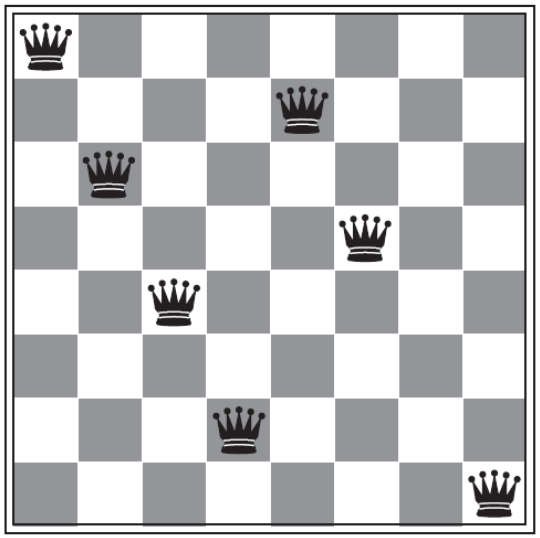
The 8-Queens Problem

- Incremental formulation: starts with an empty state, then each action adds a queen to the state
  - Transition model?
    - Returns the board with a queen added to the specified square
  - Goal test?
    - 8 queens are on the board, none attacked
  - Path cost?
    - 1 per move



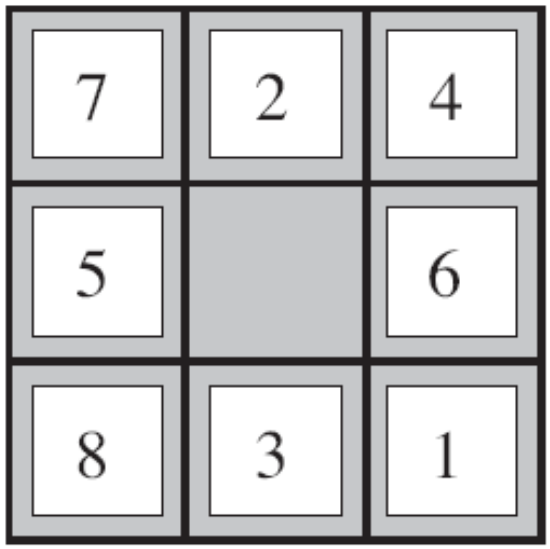
The 8-Queens Problem

- Complete-state formulation: starts with all 8 queens on the board, and moves them around

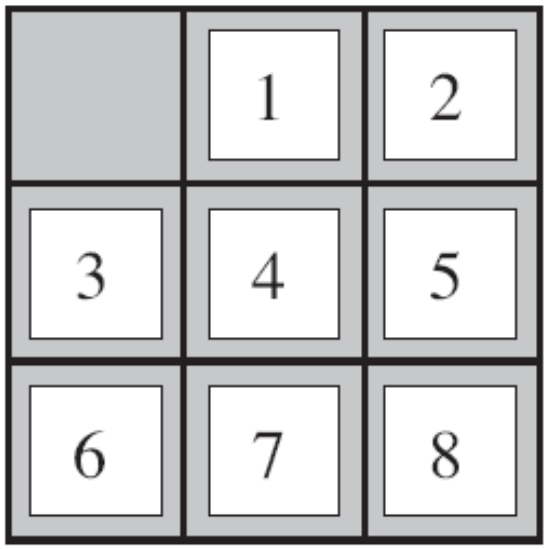


The 8-puzzle

- A tile adjacent to the blank space can slide into the space
- Objective: reach a specified goal state



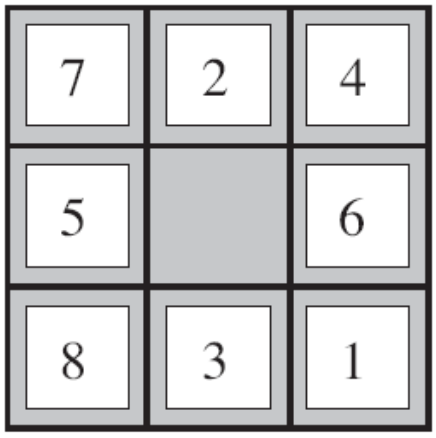
Start State



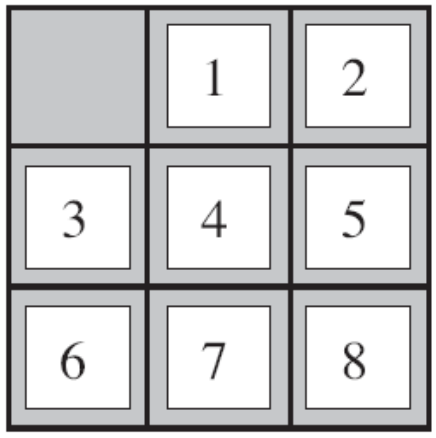
Goal State

The 8-puzzle

- States?
  - any location combination of 8 tiles and the blank
- Initial state?
  - given
- Actions?
  - Movements of the blank space: left, right, up, down, or a subset of these



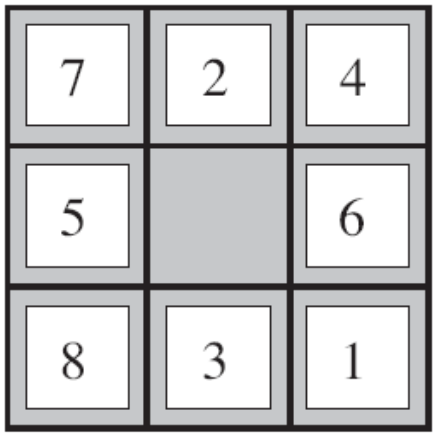
Start State



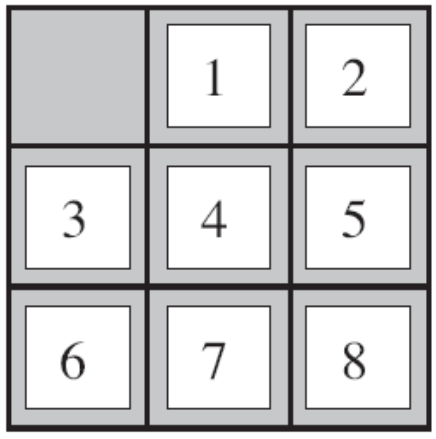
Goal State

The 8-puzzle

- Transition model?
  - given a location combination and a movement, this returns the resulting location combination
- Goal test?
  - check whether the state matches the goal configuration
- Path cost?
  - (each step costs 1) the number of steps in the path



Start State



Goal State

- Donald Knuth (1964): starting with the number 4, a sequence of factorial, square root, and floor operations will reach any desired positive integer.

$$\left\lfloor \sqrt{\sqrt{\sqrt{\sqrt{\sqrt{(4!)!}}}}} \right\rfloor = 5$$

- States?
  - Positive numbers
- Initial state?
  - 4
- Actions?
  - Apply factorial, square root, or floor operation

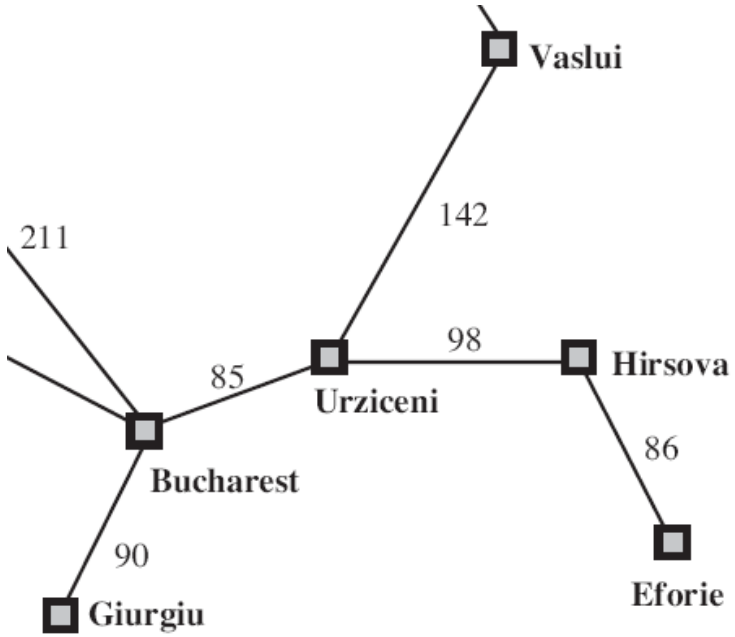
- Donald Knuth (1964): starting with the number 4, a sequence of factorial, square root, and floor operations will reach any desired positive integer.

$$\left\lfloor \sqrt{\sqrt{\sqrt{\sqrt{\sqrt{(4!)!}}}}} \right\rfloor = 5$$

- Transition model?
  - Definition of the operations
- Goal test?
  - State is the desired positive integer
- Infinite state space

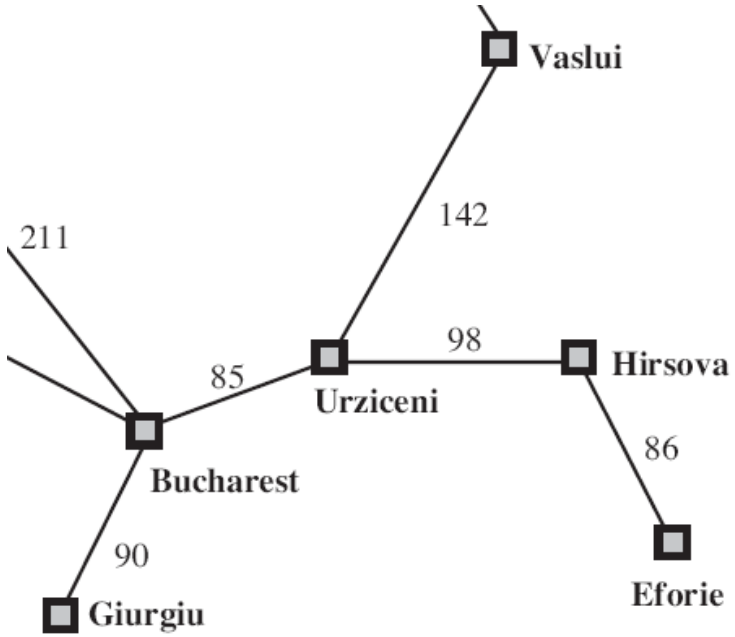
Touring problems

- "Visit every city in Romania at least once, starting and ending in Bucharest"
- Each state?
  - includes the
    current location, and
    the set of cities already visited
- Initial state?
  - In(Bucharest),
    Visited(Bucharest)



Touring problems

- "Visit every city in Romania at least once, starting and ending in Bucharest"
- Intermediate state (typical):
  - In(Vaslui), Visited({Bucharest, Urziceni, Vaslui})
- Goal test:
  - check whether the agent is in Bucharest, and all 20 cities have been visited



River Crossing Problem





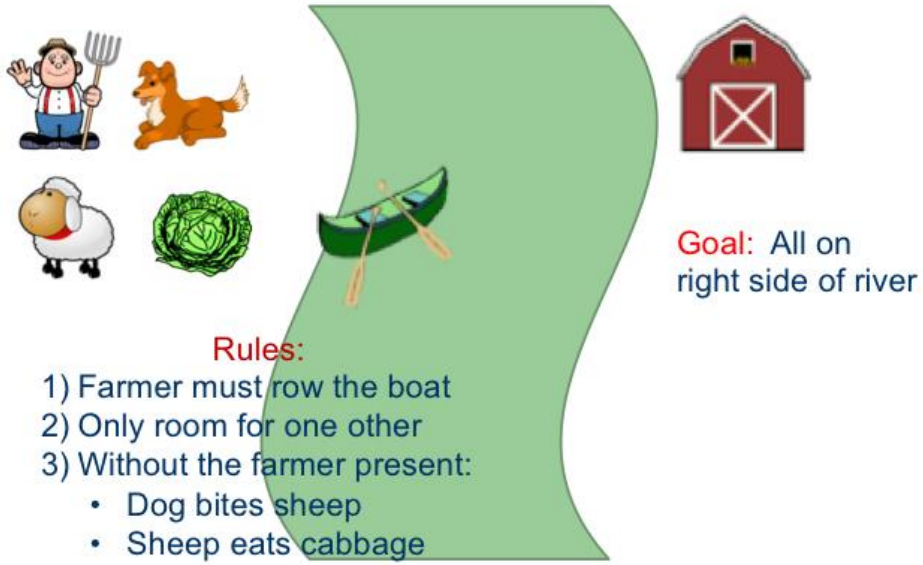

Goal: All on right side of river

Rules:

1) Farmer must row the boat
2) Only room for one other passenger
3) Without the farmer present:
   - Dog bites sheep
   - Sheep eats cabbage

Assume: a **state** describes who is (are) on the right side of the river

River Crossing Problem

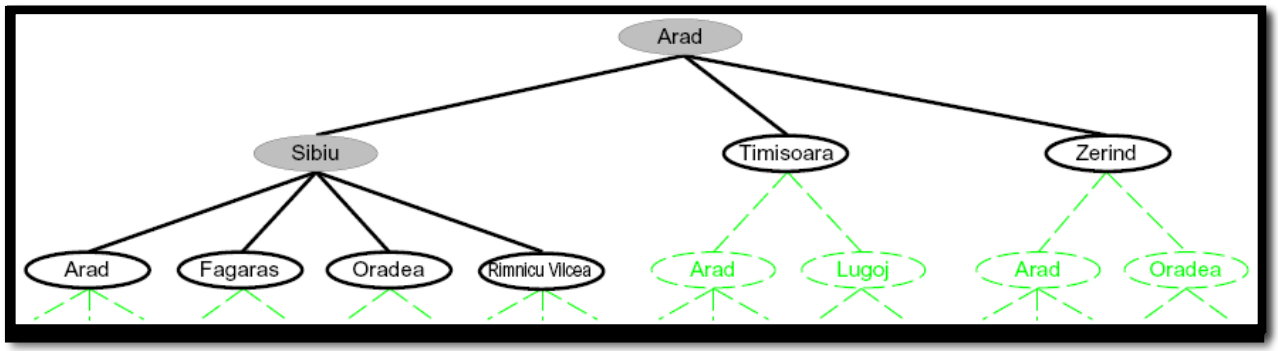


- Cost function?
  - Number of trips

- Goal Test? (Goal state)
  - Everyone is on the right side of the river
- States?
  - D, S, C, FS, DC, FDC, FDS, FCS, FDCS
- Possible Actions?
  - F>, F<, FC>, FC<, FD>, FD<, FS>, FS<

Searching for Solutions

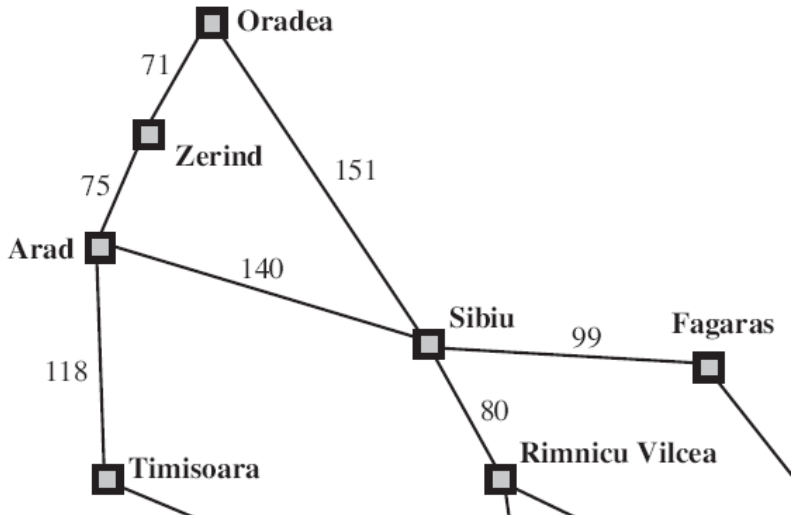
- Having formulated the problems, how do we solve them?
  - Solution: a sequence of actions leading to the goal state
  - Searching algorithms: consider various possible action sequences



- **Search Tree**
  - **Root:** initial state
    - In(Arad)
  - **Branches:** actions
  - **Nodes:** states in the state space
    - Parent node, child nodes
    - Leaf node: a node with no children

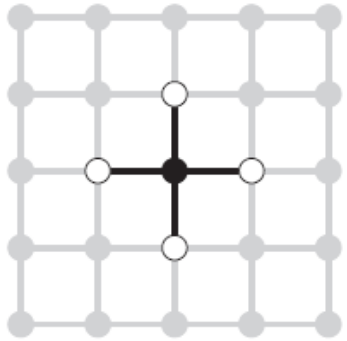
Tree-Search

- We expand the current state and generate a new set of states
- Search Strategy: how to choose which state to expand next?
- Repeated state
  - Arad-Sibiu-Arad
  - Loopy path
    - The complete search tree for Romania is *infinite*
    - Can cause certain algorithms to fail



- Redundant paths
  - More than one way to get from one state to another
    - Arad-Sibiu (140 km long) vs. Arad-Zerind-Oradea-Sibiu (297 km long)

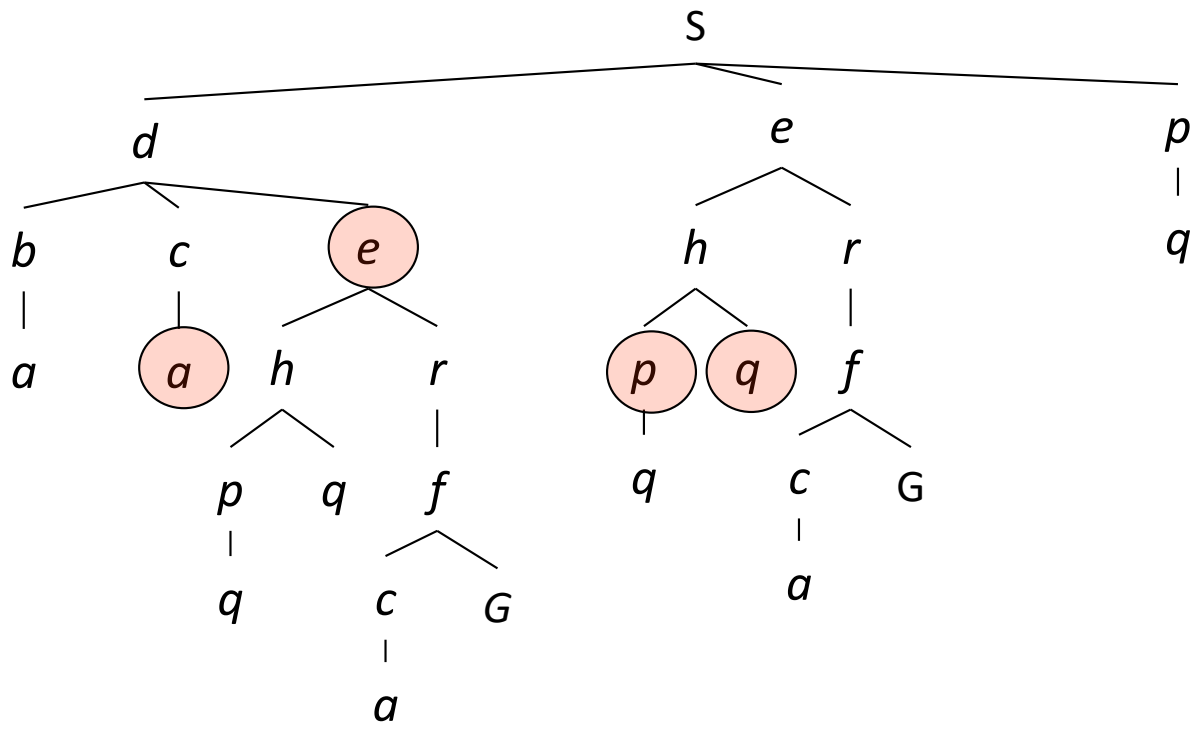
Tree-Search



- Route-finding on a rectangular grid (suppose it is infinitely large)
  - Each state: 4 successors
  - A search tree of depth d that includes repeated states: 4^d leaves
  - Only about $2d^2$ distinct states within d steps of any given state
    - $d = 20$, a trillion nodes, but only about 800 distinct states
- Algorithms that forget their history are doomed to repeat it!
  - Avoid exploring redundant paths by remembering where one has been
    - Graph-Search

Graph-Search

- Idea: never expand a state twice
- Example: in the following breadth-first graph search, we do not expand the circled nodes

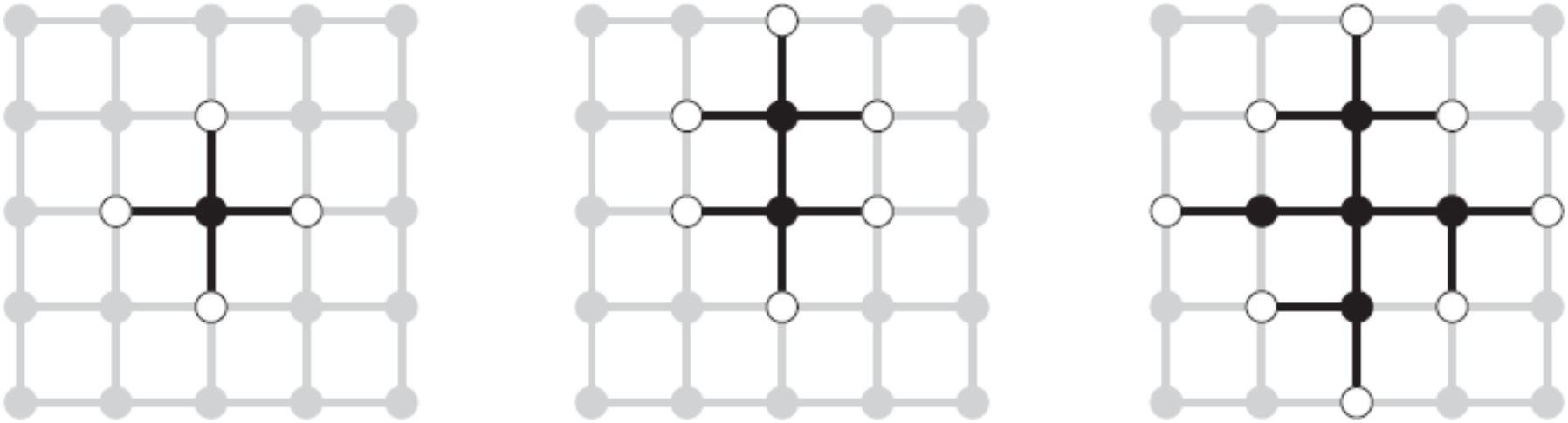


Graph-Search

- Idea: never expand a state twice
- How to implement:
  - Tree search + set of expanded states ("closed set")
  - Expand the search tree node-by-node, but…
  - Before expanding a node, check to make sure its state has never been expanded before
  - If not new, skip it, if new add to closed set
- Important: store the closed set as a set, which does not have repeated elements

Graph-Search

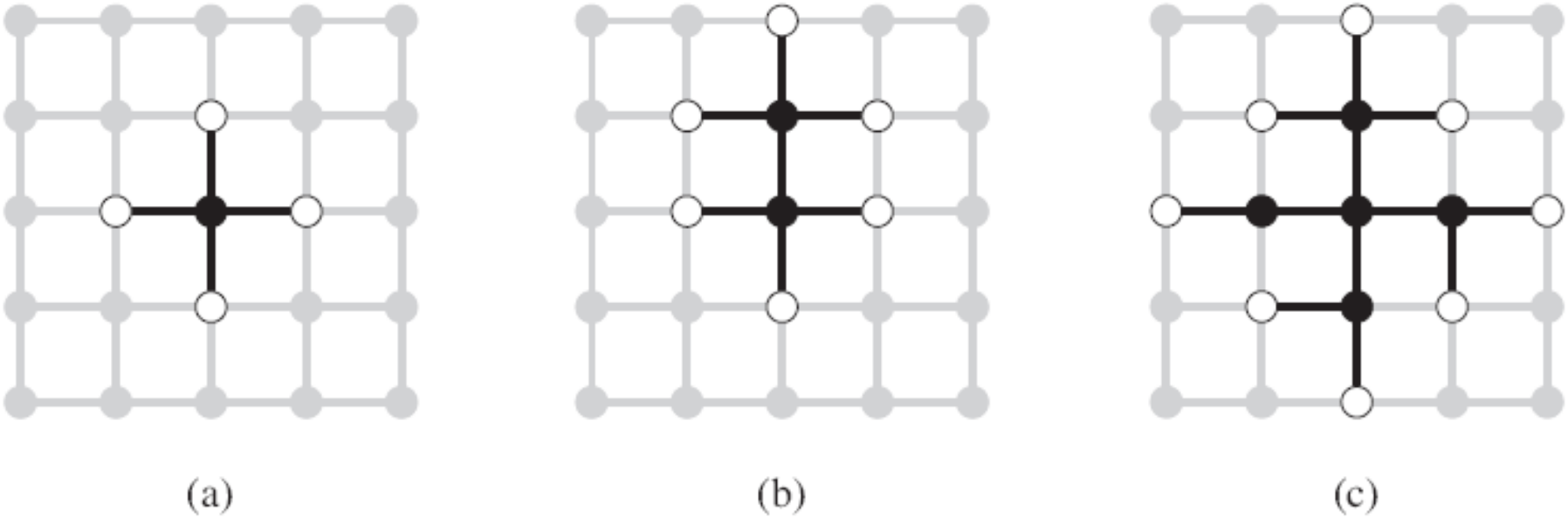
- Rectangular-grid problem
  - Black nodes: the explored region of the state space
  - White nodes: frontier (to be explored)
  - Gray nodes: unexplored region



(a) (b) (c)

Graph-Search

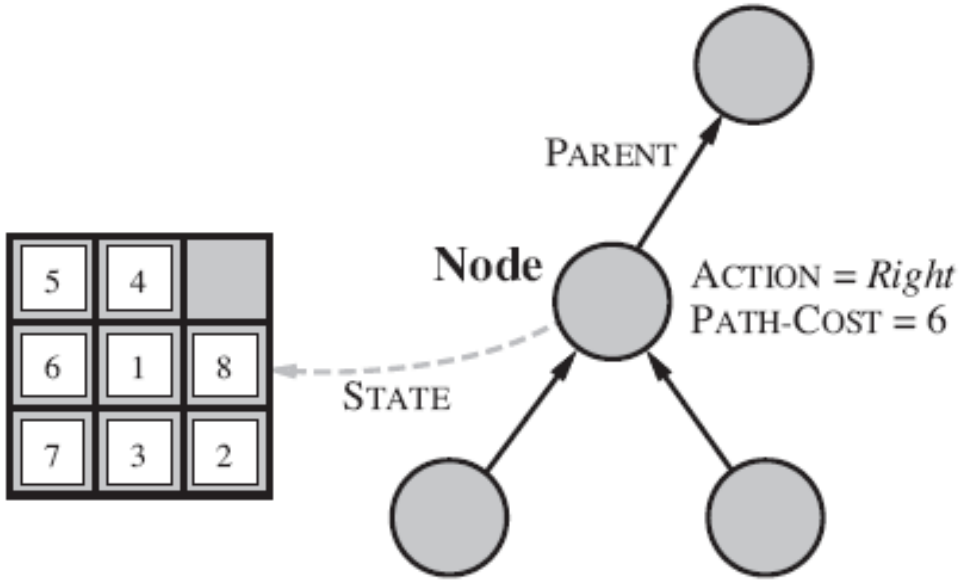
- Separation Property
  - The frontier (white) always separates the explored (black) from the unexplored (gray)
  - (a) just the root has been explored
  - (b) one leaf node has been expanded
  - (c) the remaining successors of the root have been expanded in clockwise order



(a) (b) (c)

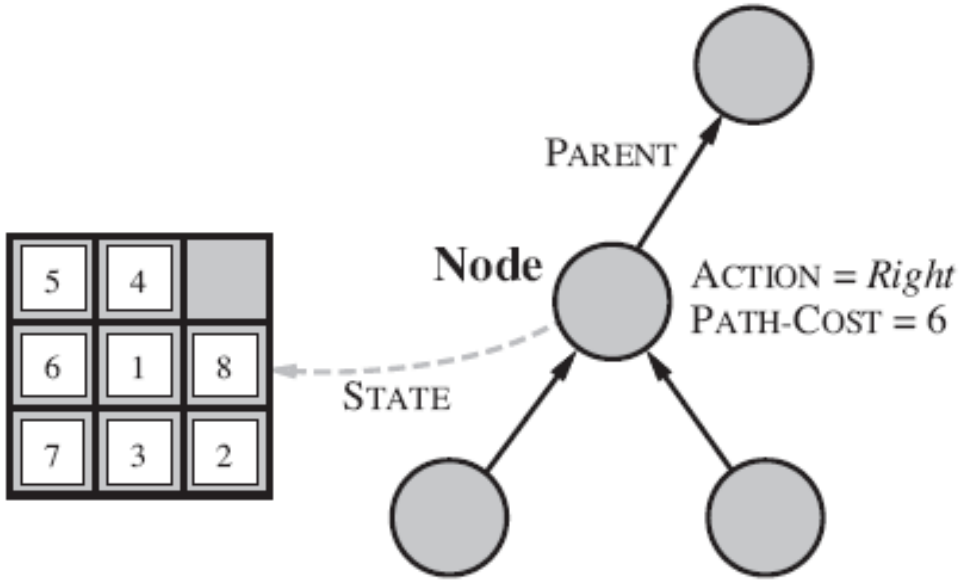
Data Structure of Search Algorithms

- Data structure: keeps track of the search tree
- Data structure for node n
  - n.State: the state in the state space to which the node corresponds.
  - n.Parent: the node in the search tree that generated this node.
  - Arrows point from child to parent



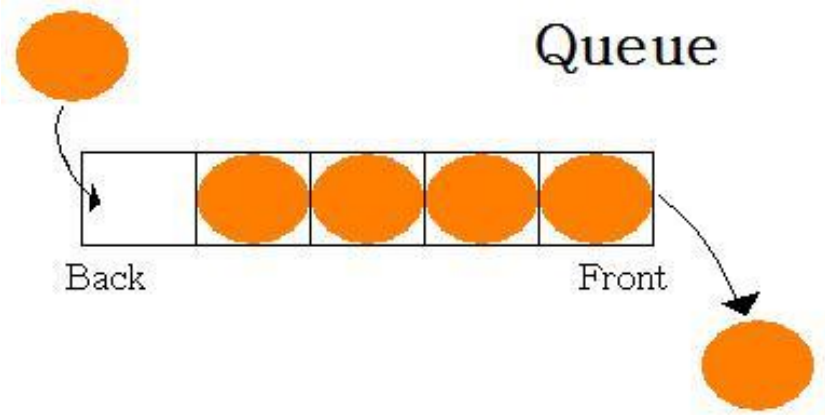
Data Structure of a Node

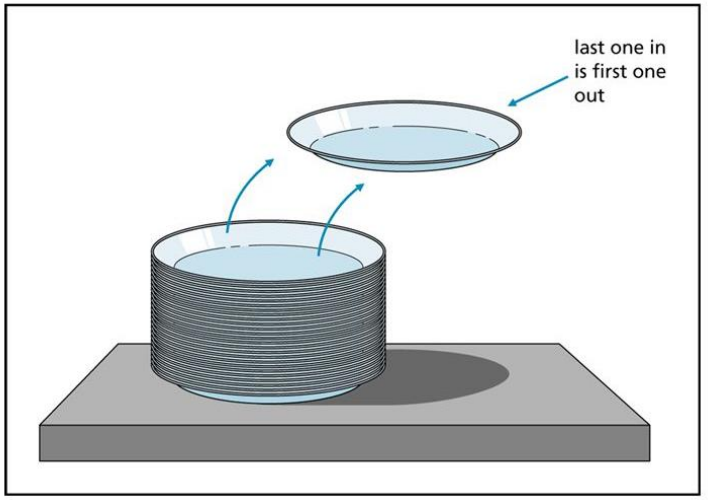
- Data structure for node n
  - n.Action: the action that was applied to the parent to generate the node.
    - n.Action: movement of the blank tile
    - Draw the state of the parent node?
  - n.Path-Cost: the cost, denoted by $g(n)$, of the path from the initial state to the node, as indicated by the parent pointers.



Frontier Set

- The set of Frontier nodes
  - a FIFO queue, or
  - a LIFO stack, or
  - a Priority queue





Performance Measurement

- Completeness: Is the algorithm guaranteed to find a solution when there is one?

- Optimality: Does the strategy find the optimal solution (w.r.t some performance measure)?
- Time complexity: How long does it take to find a solution?
  - Measured in terms of the number of nodes generated/explored/expanded/visited during the search

- Space complexity: How much memory is needed to perform the search?
  - Measured in terms of the maximum number of nodes stored in memory

Parameters

- b - Branching factor
  - Maximum number of successors of any node
- d - The *depth* of the shallowest goal node (shallowest solution)
  - i.e. the number of steps along the path from the root
- m – the *maximum length* of any path in the state space
  - For tree search
    - m can be much larger than d
    - m is infinite if the tree is unbounded

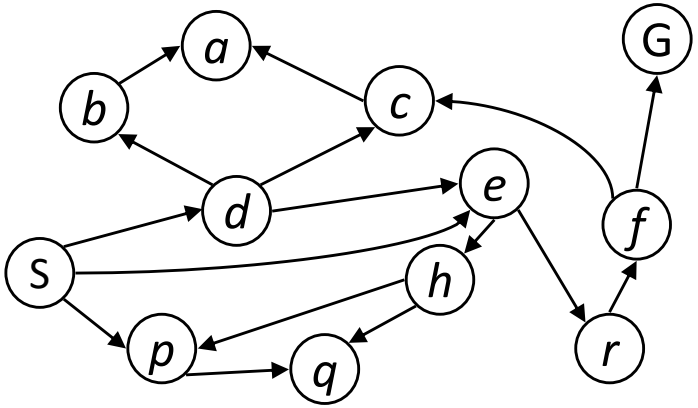
Performance Measurement

- Search Cost
  - Time complexity: the amount of time taken by the search
- Path Cost
  - The path cost of the solution found (e.g. the total length of the path in kilometers)

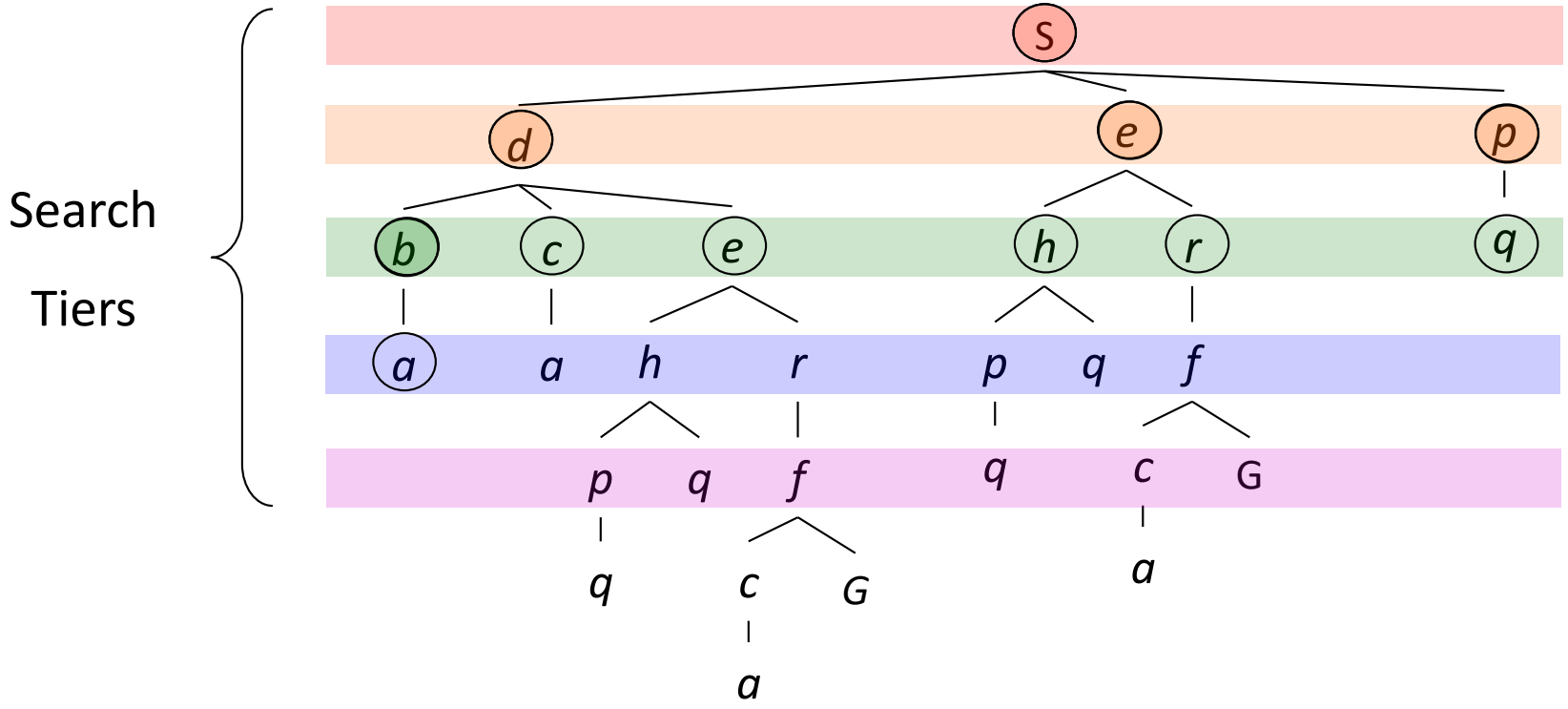
Uninformed Search

- Also called: Blind Search
- The strategies have no additional information about states
  - e.g. no information about how close the states are to the goal state
- Can only
  - Generate successors
  - Distinguish a goal state from a non-goal state
- You have no clue whether one non-goal state is better than any other
- There are different uninformed search strategies
  - They are distinguished by the **order** in which nodes are expanded

Breadth-First Search

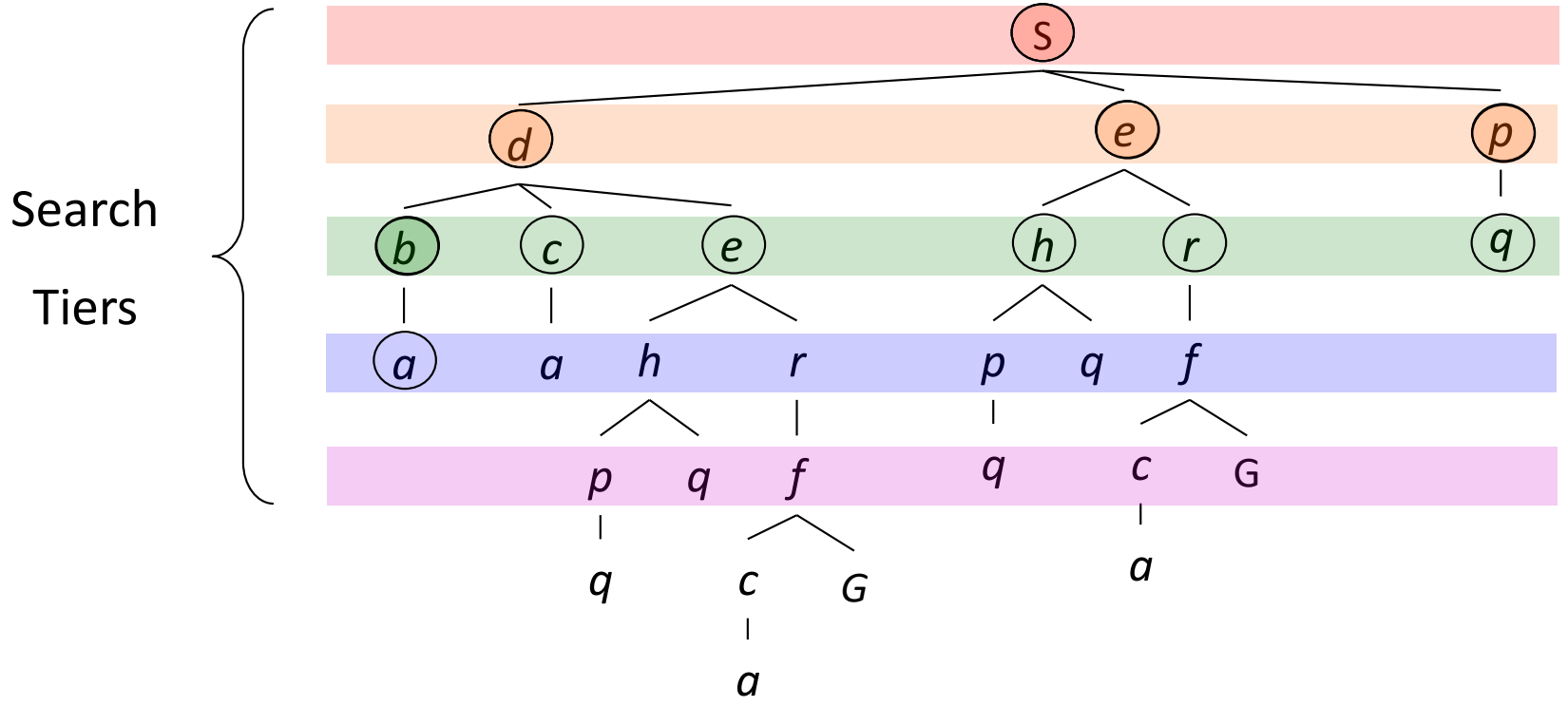


Initial State: S
Goal State: G
Explored: color circled
Frontier: white circled
Unexplored: uncircled



Breadth-First Search

- All the nodes are expanded at a given depth in the search tree before any nodes at the next level are expanded
- The shallowest unexpanded node is chosen for expansion
  - Achieved by using a FIFO queue for the frontier

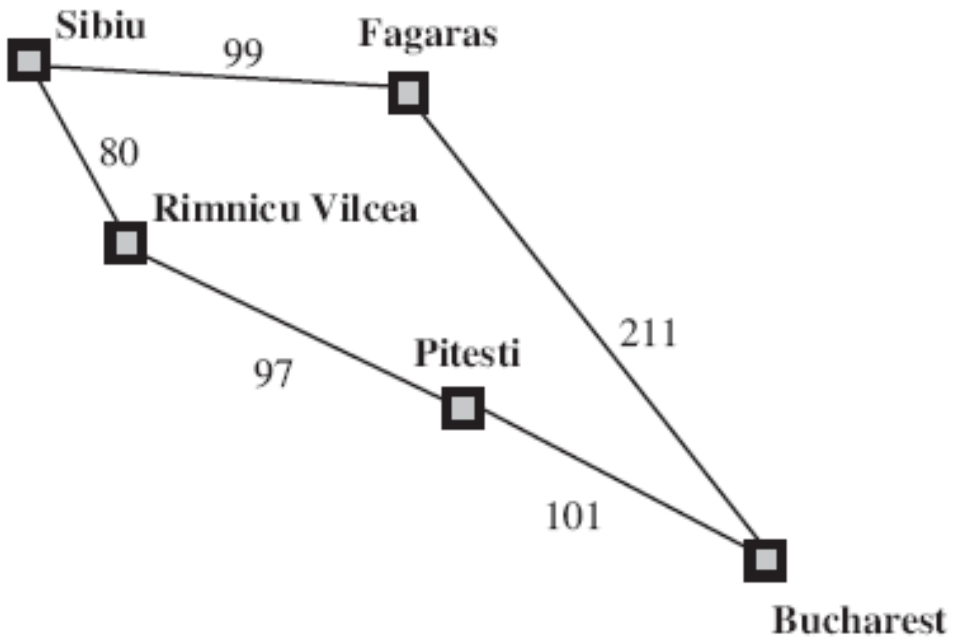


Breadth-First Search

- Optimality
  - In terms of solution path cost
  - Optimal when all step costs are equal (i.e. deeper solutions are always less optimal)
  - Always expands the shallowest unexpanded node
- Complete?
  - Yes if the branching factor b is finite

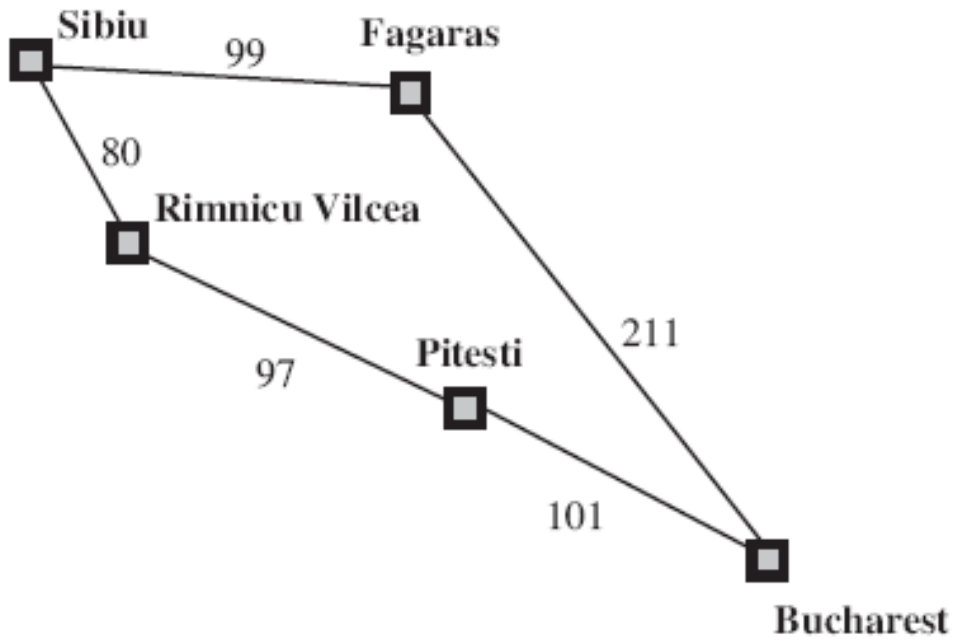
Uniform-cost Search

- Expands the node n with the *lowest path cost* $g(n)$
  - Done by storing the frontier as a priority queue ordered by g
  - *Path cost* $g(n)$: the cost of the path from the initial state to the node
- Example: get from Sibiu to Bucharest



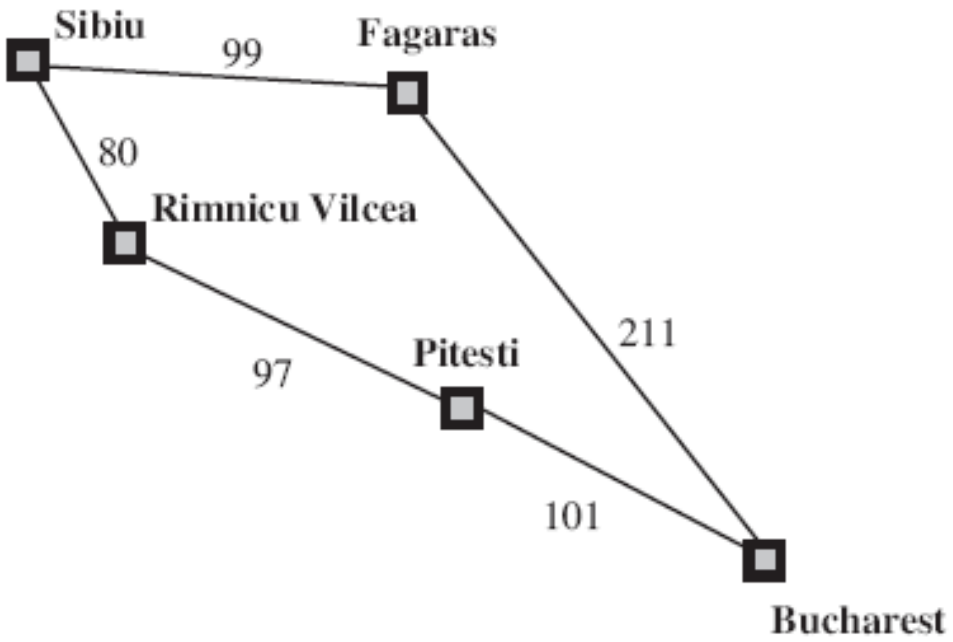
Uniform-cost Search

- Guided by path costs (rather than steps)
- Does not care about the number of steps a path has
- Only cares about the total cost of a path



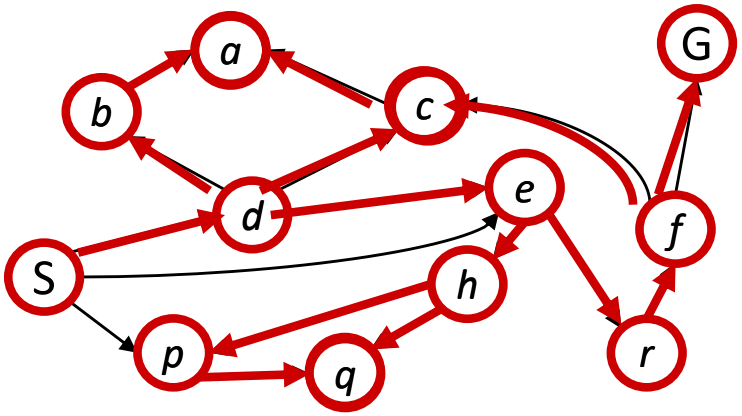
Uniform-cost Search

- Optimal or not?
  - Yes



- Completeness?
  - Guaranteed if the cost of every step is greater than a small positive value ε
  - If there's a path with an infinite sequence of zero-cost actions, then it will get stuck in an infinite loop

Depth-First Search



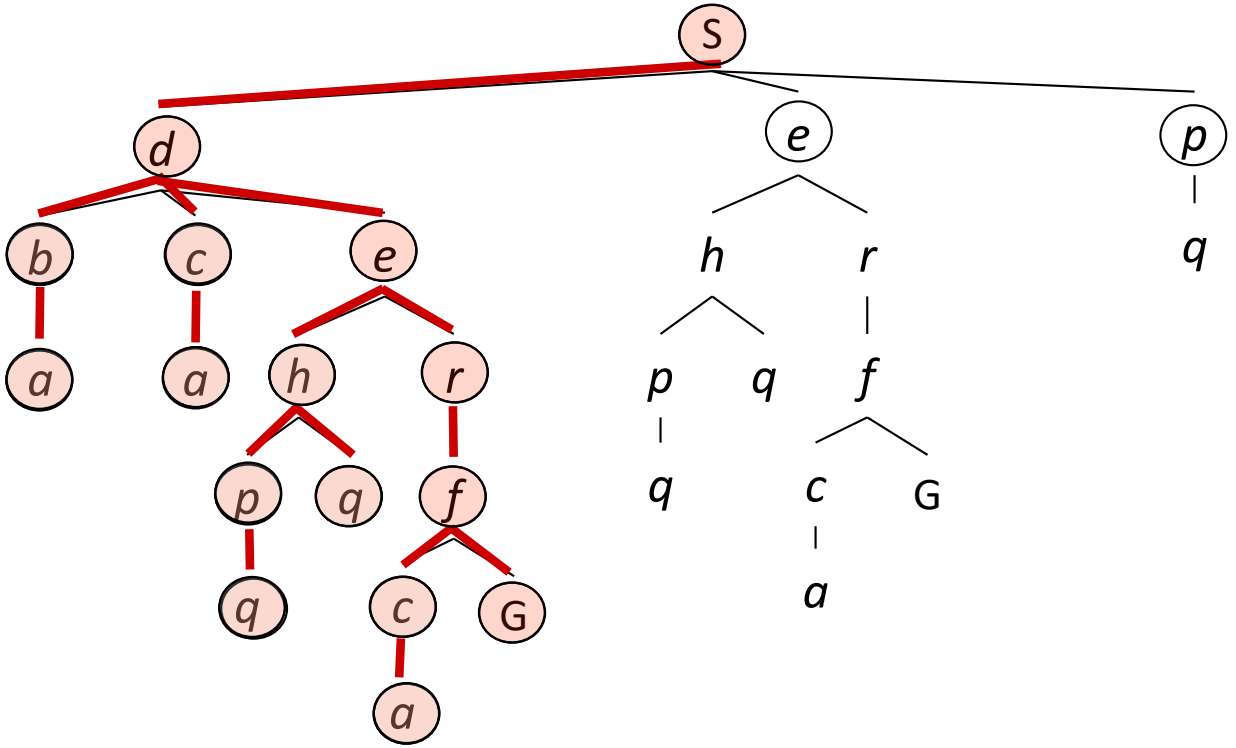
Initial State: S

Goal State: G

Explored: color circled

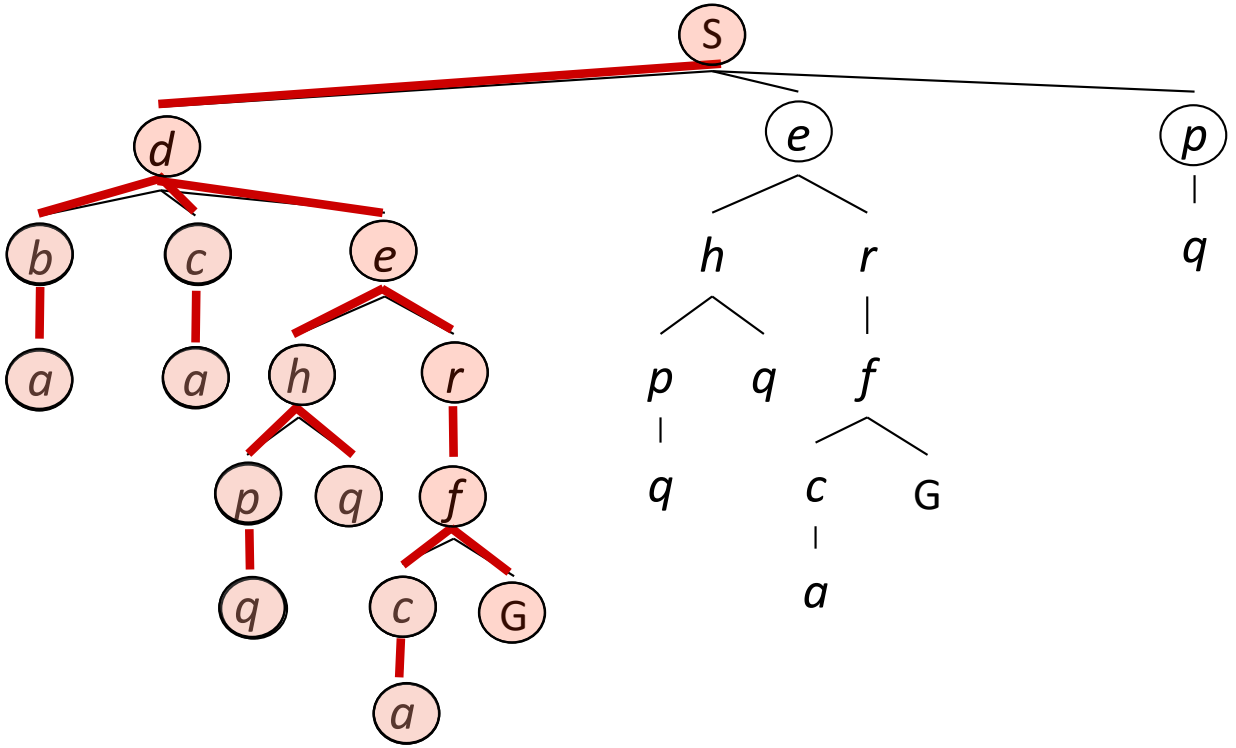
Frontier: white circled

Unexplored: uncircled



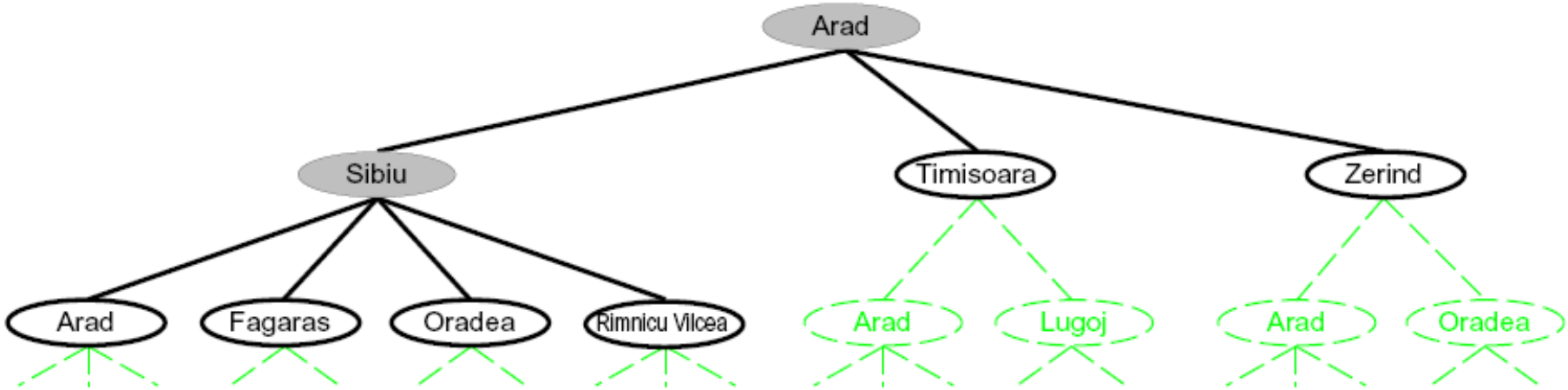
Depth-First Search

- Expands the deepest node in the current frontier of the search tree
- LIFO stack – the most recent generated node is chosen for expansion
- Explored nodes with no descendants are removed from memory



Depth-First Search

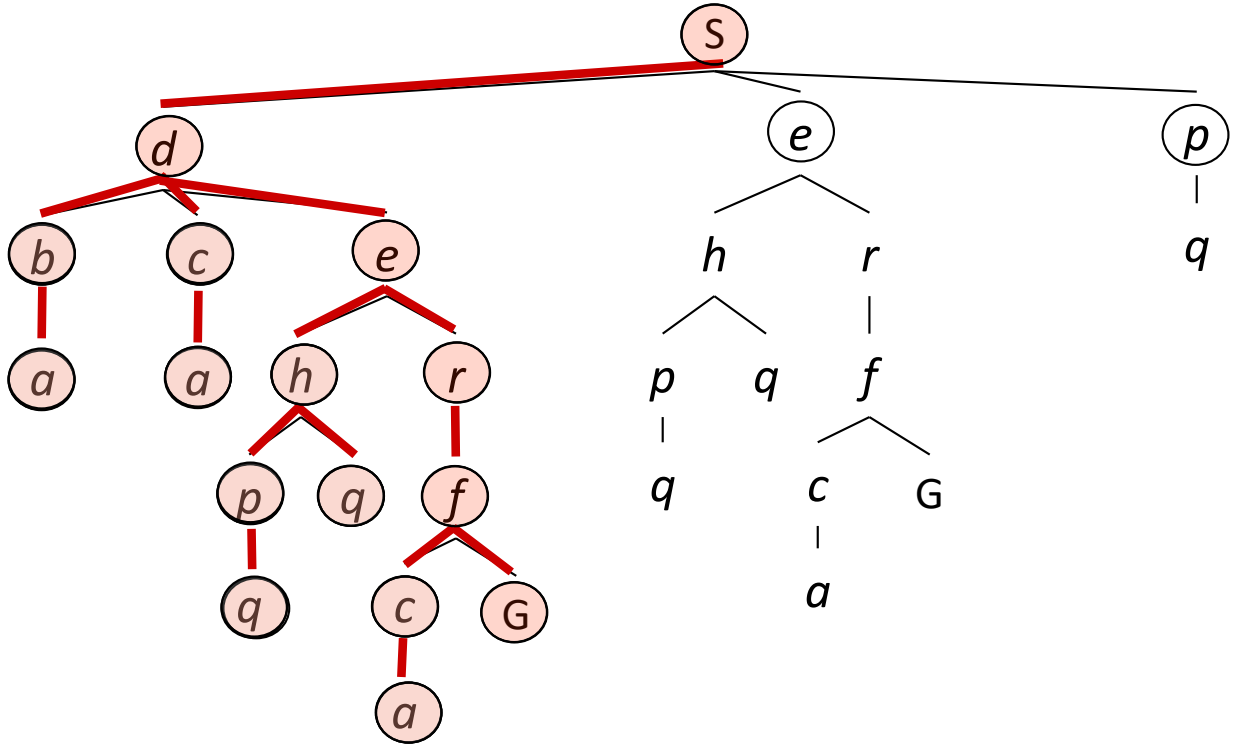
- Complete or not?
  - Depth-first Graph-search: Yes (avoids repeated states)
  - Depth-first Tree-search: No



- Arad-Sibiu-Arad-Sibiu loop forever!

Depth-First Search

- Optimal or not? (in terms of the cost of the solution path)
- (hint: If node **e** is the goal node …)



- Answer: No!
- The solution returned by the DFS will get to it in 2 steps instead of 1 step

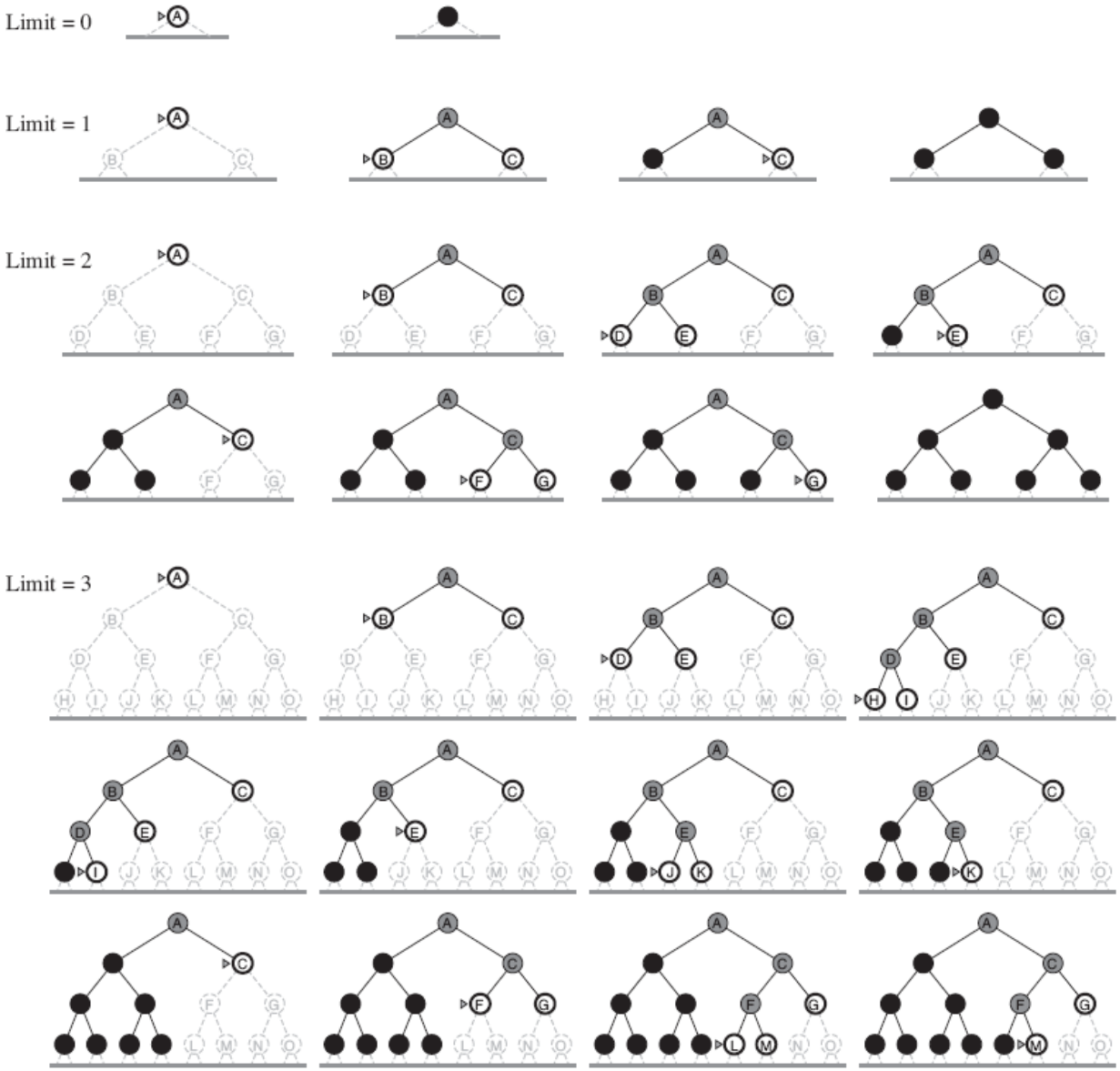
Depth-Limited Search

- Supply DFS with a predetermined depth limit l
  - Solves the infinite path problem
- Complete or not?
  - If $l < d$: incomplete! (e.g. when d is unknown)
- Optimal or not?
  - If $l > d$: Not guaranteed.
- Two kinds of failure
  - *Standard* failure: no solution
  - *Cutoff* failure: no solution within the depth limit

Iterative Deepening DFS

- Repeatedly applies depth-limited search with increasing limits l
- Terminates an iteration when a solution is found or if the depth-limited search returns *failure* (no solution for that depth limit)
- Combines the benefits of DFS and BFS

Iterative Deepening DFS

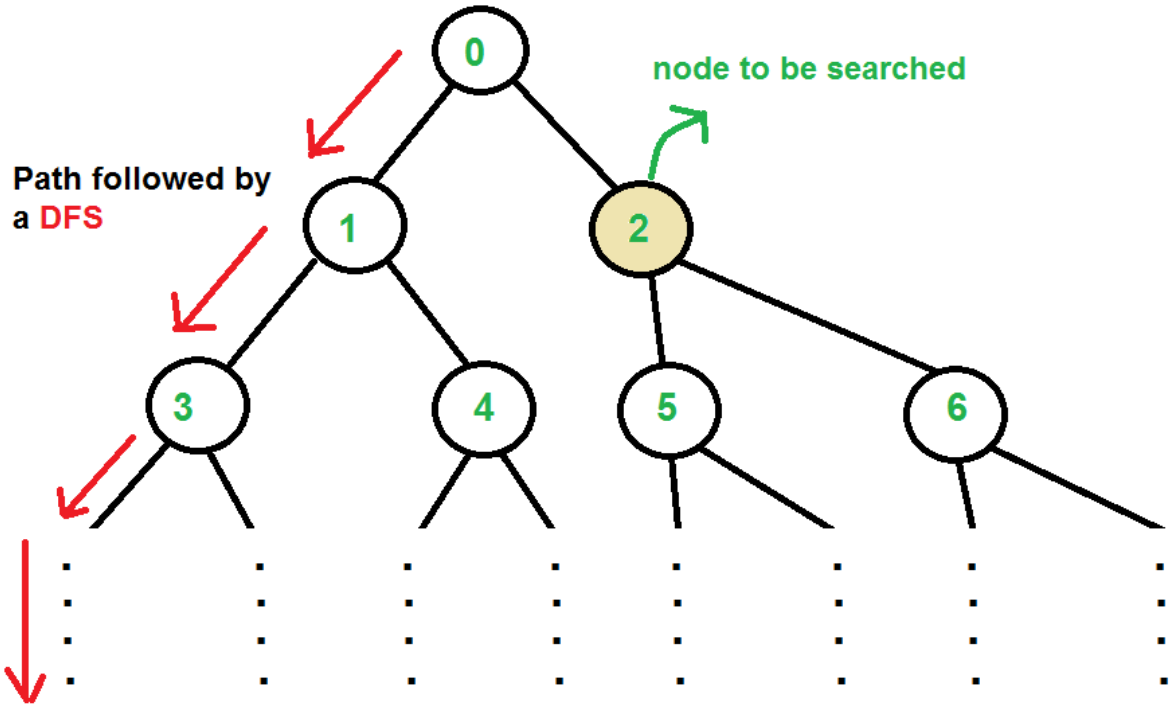


Four iterations of iterative deepening search on a binary tree

Suppose: M is the goal node

Black circles: nodes removed from memory

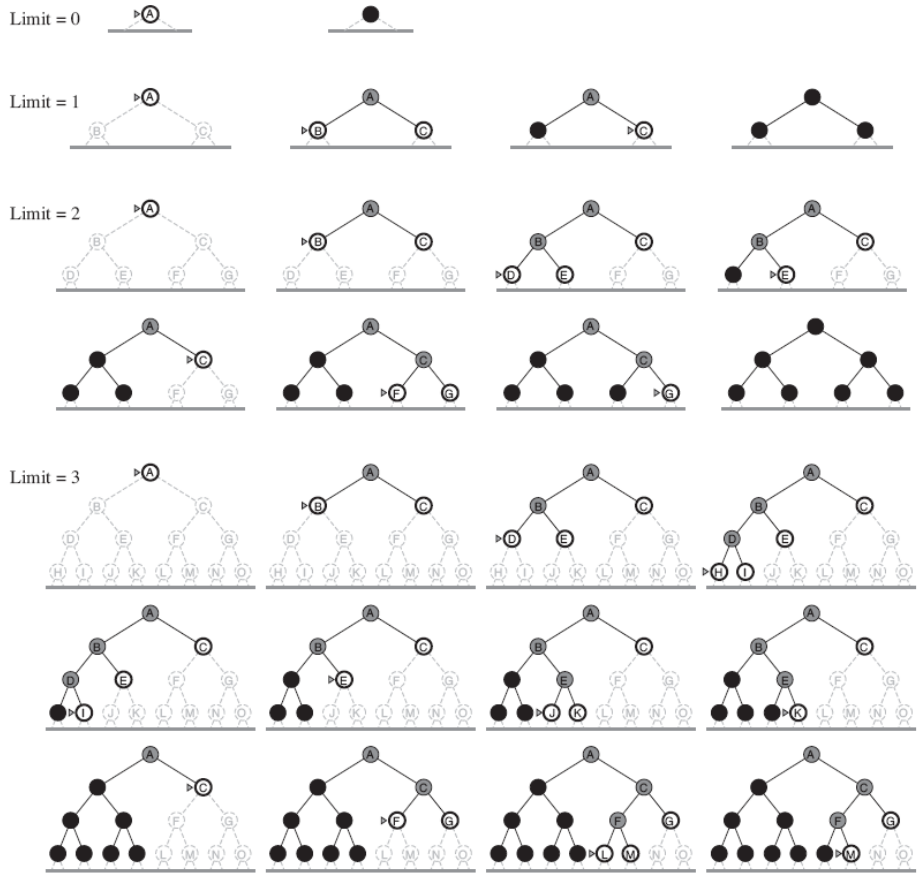
Iterative Deepening DFS



- We want to find node ‘2’ of the given “deep” tree.
- A DFS starting from node ‘0’ will dive left, towards node 1 and so on
- Hence, a DFS wastes a lot of time in coming back to node 2
- An Iterative Deepening DFS overcomes this and quickly finds the desired node.

Iterative Deepening DFS

- Complete? Yes when b is finite
- Optimal? Yes in the sense that it can always find the shallowest solution.



Informed Search Strategies

- Uses problem-specific knowledge beyond the definition of the problem itself
- Can find solutions more efficiently than can an uninformed strategy

Heuristic Function $h(n)$

- Heuristic function

  $h(n)$= estimated cost of the cheapest path from node n to a goal state

- Let $h(n)$ be
  - Arbitrary
  - Nonnegative
  - Problem-specific functions
  - Define: if n is a goal node, then $h(n) = 0$

Greedy Search

- Expands the node that seems closest to the goal
- Evaluates nodes by using a heuristic function $h(n)$
- Example: route-finding problem in Romania
  - Use straight-line distance heuristic h_{SLD}

The straight-line distance to Bucharest

Arad	366	**Mehadia**	241
Bucharest	0	**Neamt**	234
Craiova	160	**Oradea**	380
Drobeta	242	**Pitesti**	100
Eforie	161	**Rimnicu Vilcea**	193
Fagaras	176	**Sibiu**	253
Giurgiu	77	**Timisoara**	329
Hirsova	151	**Urziceni**	80
Iasi	226	**Vaslui**	199
Lugoj	244	**Zerind**	374

Greedy Search

The straight-line distance to Bucharest

Arad	366	Mehadia	241
Bucharest	0	Neamt	234
Craiova	160	Oradea	380
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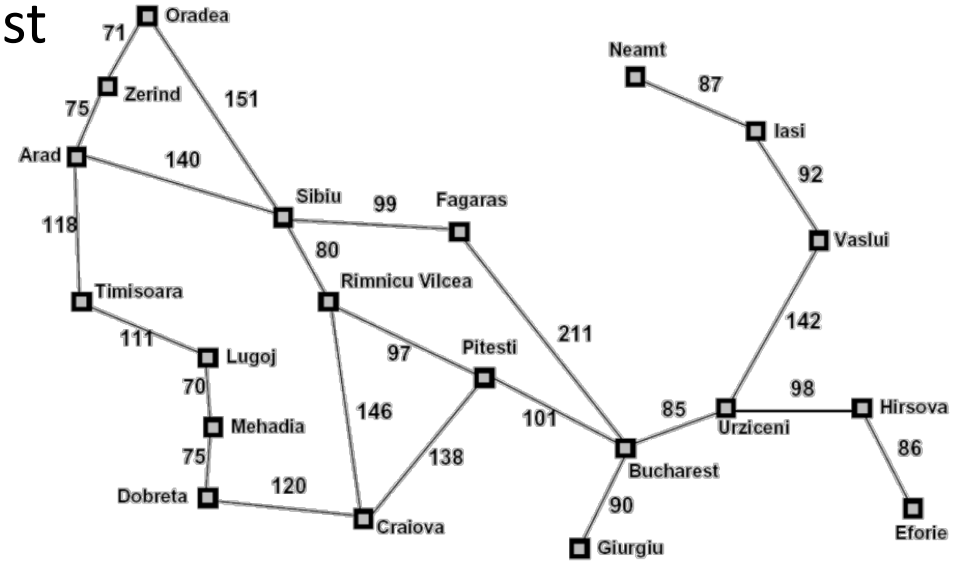
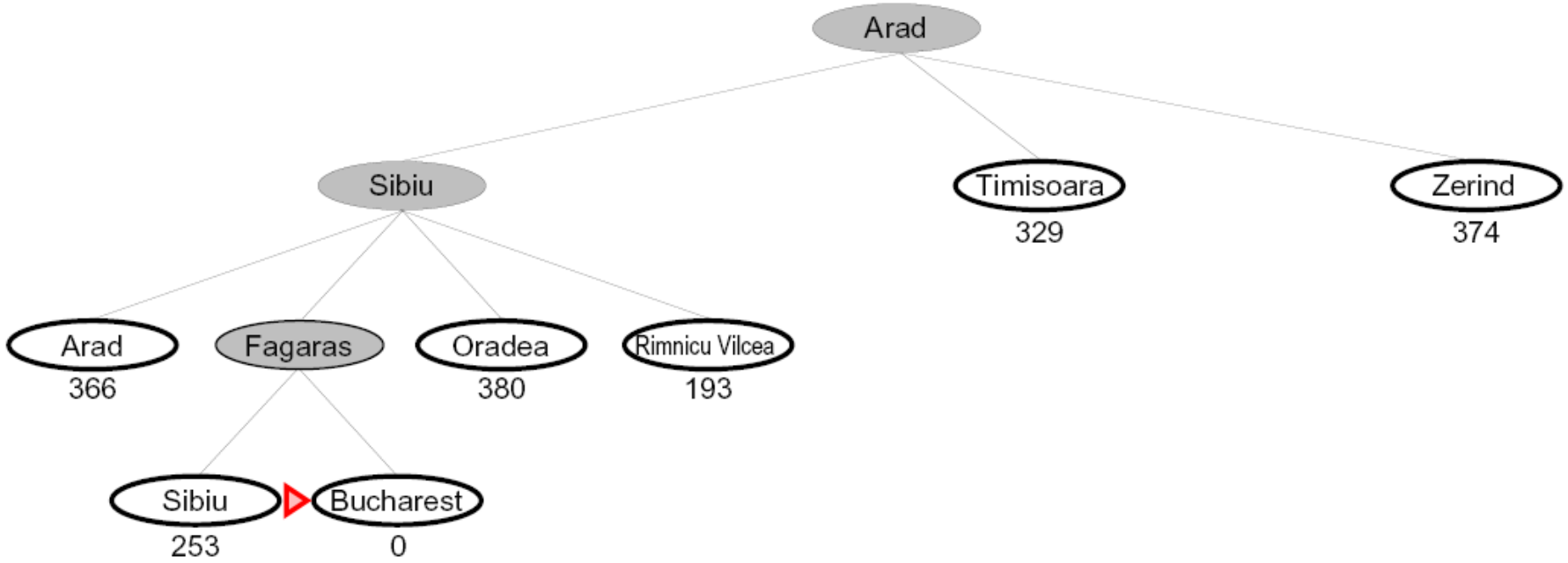




Greedy Search

- Why is it called “greedy”?
  - At each step, it tries to get as close to the goal as it can

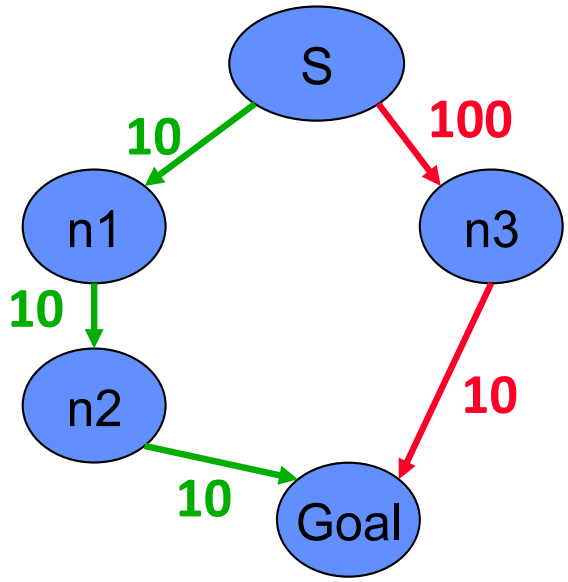
Greedy Search

- Not optimal
  - It ignores the cost of getting to n
  - Can be led astray exploring nodes that cost a lot but seem to be close to the goal

S to n1: → step cost = 10

S to n3: → step cost = 100

h(n1) = 20



h(n3) = 10

Greedy Search

- Completeness
  - Greedy tree search: Incomplete even in a finite state space (does not guarantee to find a solution)
  - The graph search version: Complete in finite state spaces, but not in infinite ones

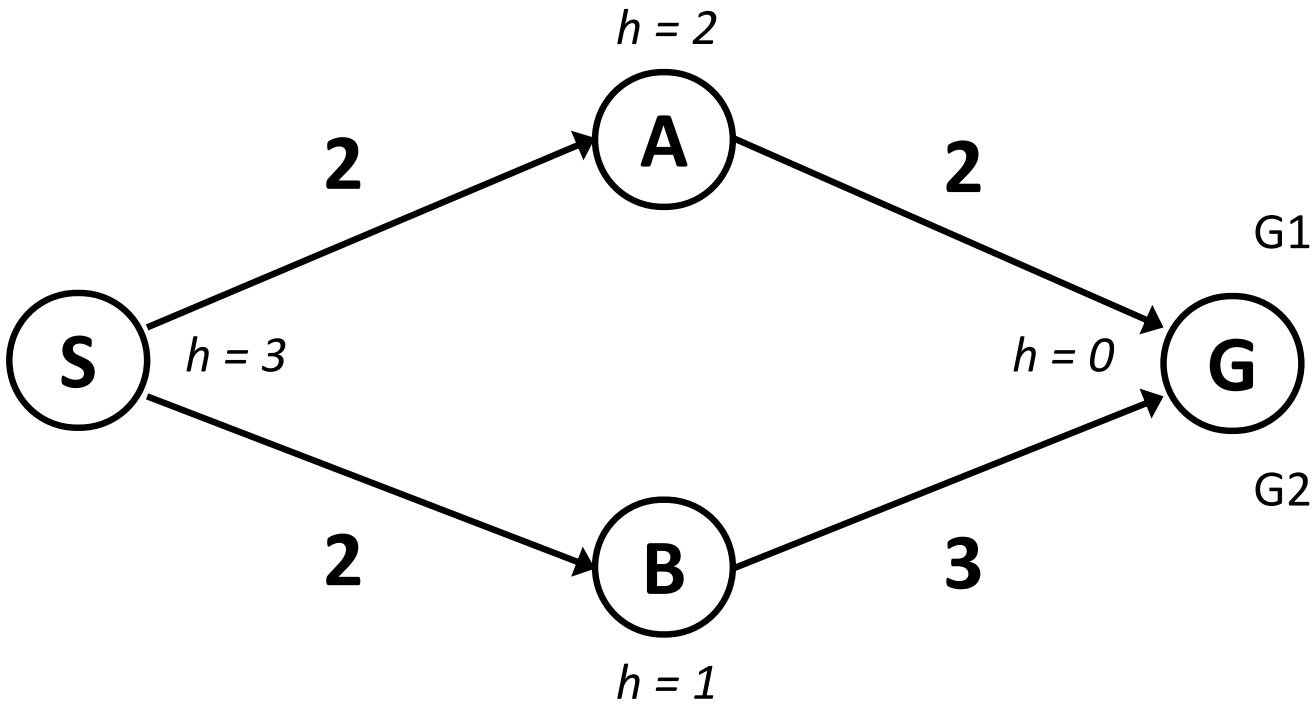
A^* Search

- Minimizes the total estimated solution cost

$$f(n) = g(n) + h(n)$$

  - $g(n)$ - the path cost from the initial node to node n
  - $h(n)$ - the estimated cost to get from node n to the goal node
  - $f(n)$ – estimated cost of the best path that continuous from node n to a goal

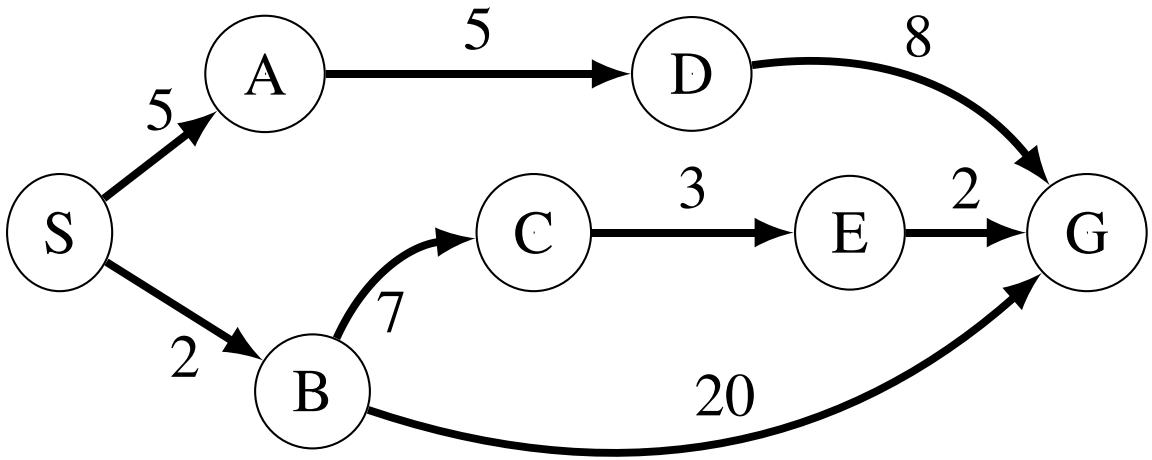
A* Search Example



- Start from S, G is the goal (G=G1=G2, achieved from different path)
- Expanded nodes in order: S, B, A, G1
- Solution path: S->A->G1
- Solution path cost: 4

BFS Example

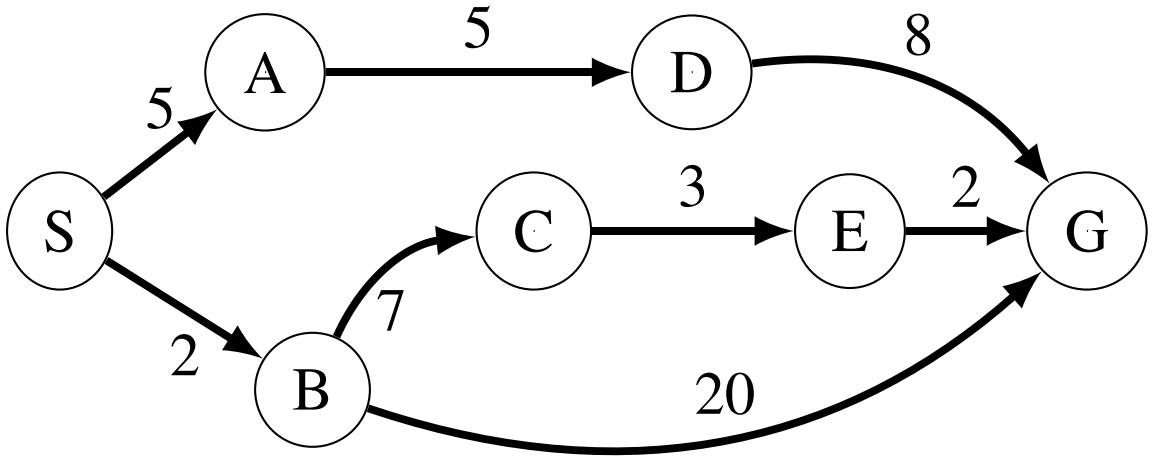
- S is the start node, G is the goal node, use BFS to find a path from S to G. List the expanded nodes in order, give the solution path, and solution path cost. Use alphabetical order to break ties.



- Answer:
- Expanded nodes: S,A,B,D,C,G
- Solution path: SBG
- Path cost: 22

DFS Example

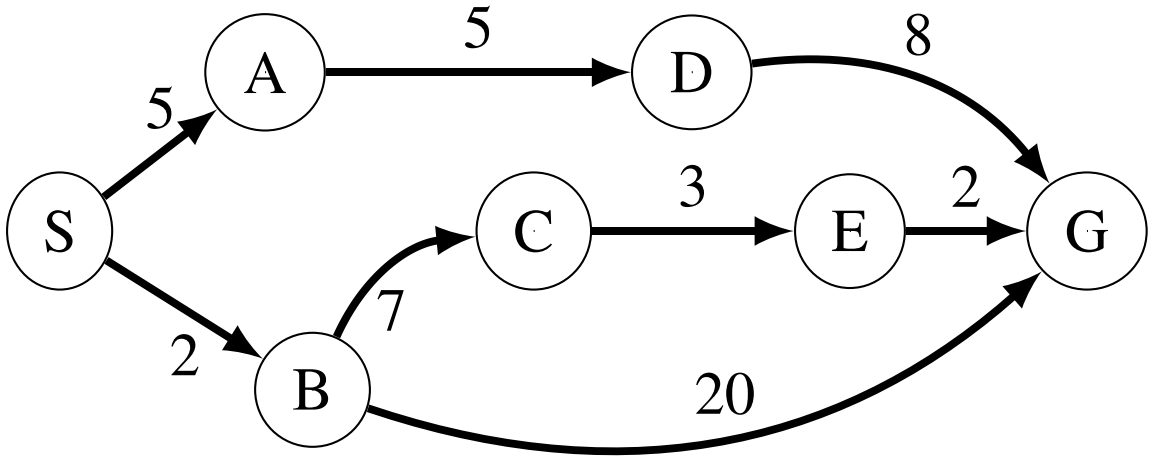
- Solve the same problem by depth-first search.



- Answer:
- Expanded nodes: S,A,D,G
- Solution path: S,A,D,G
- Path cost: 18

IDDFS Example

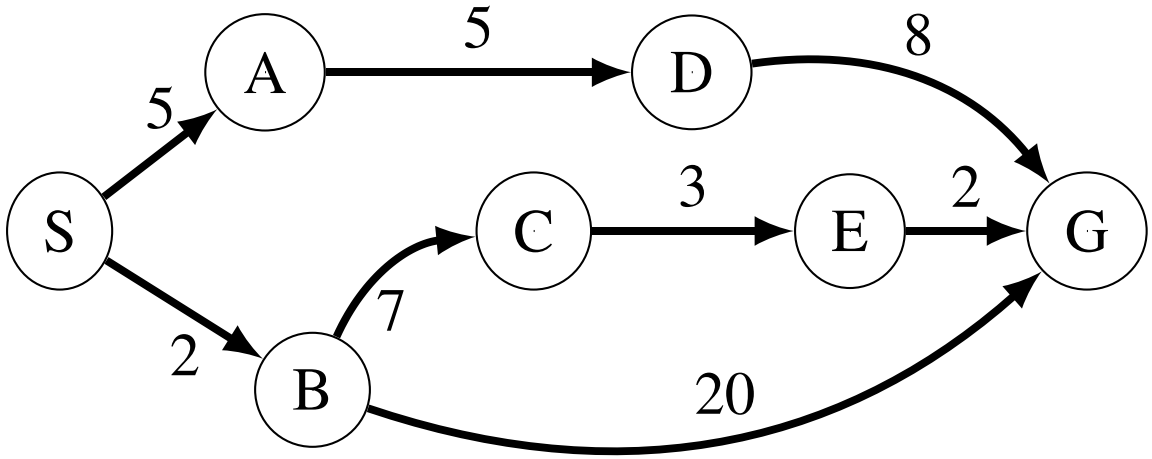
- Solve the same problem by the Iterative Deepening DFS strategy.



- Answer:
- Expanded nodes: S; SAB; SADBCG.
- Solution path: SBG
- Path cost: 22

UCS Example

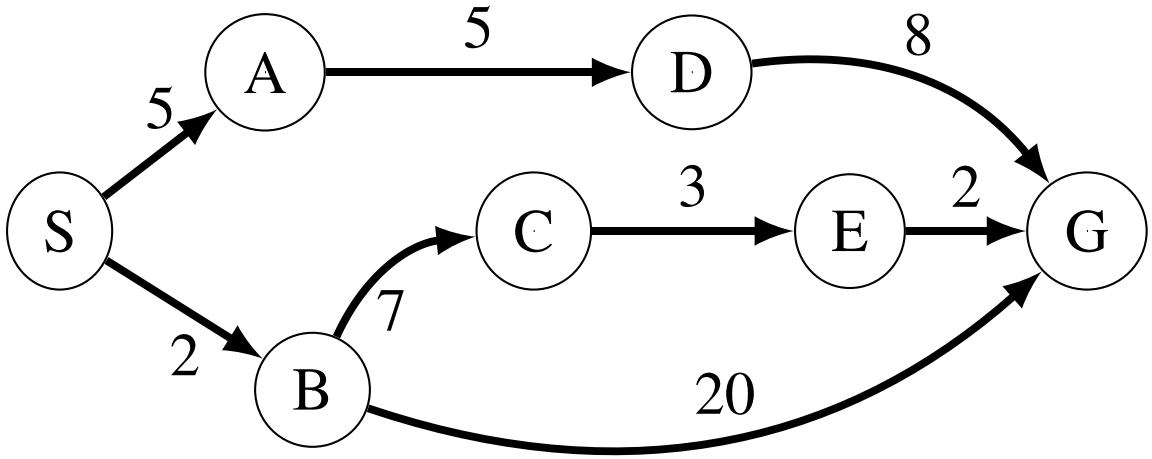
- Solve the same problem by the uniform-cost search.



- Answer:
- Expanded nodes: S,B,A,C,D,E,G
- Solution path: SBCEG
- Path cost: 14

Greedy Search Example

- Solve the same problem by greedy search. The heuristic function values are given.

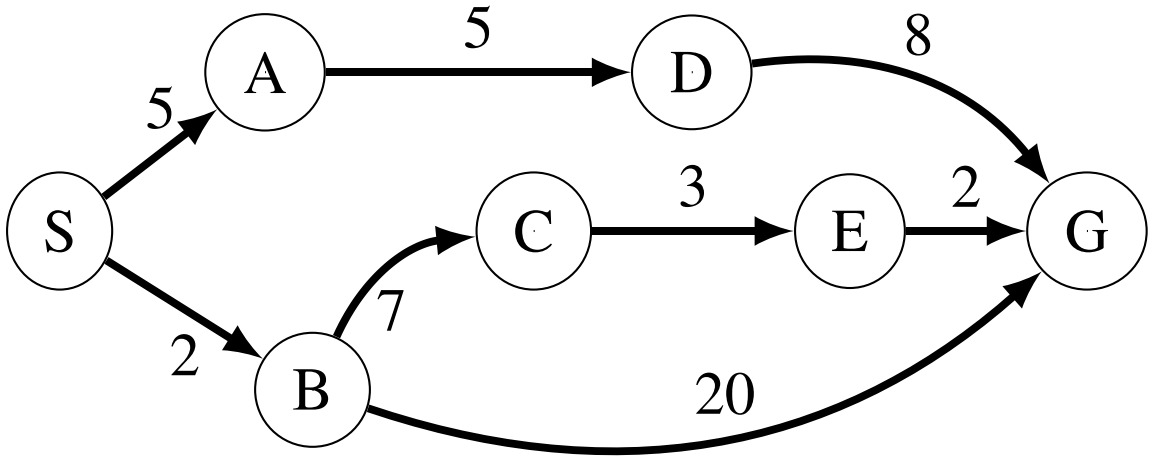


- Answer:
- Expanded nodes: S,A,D,G
- Solution path: S,A,D,G
- Path cost: 18

h(S)=12, h(A)=8,
h(B)=12, h(C)=3,
h(D)=7, h(E)=1,
h(G)=0.

A* Search Example

- Solve the same problem by the A* search strategy.



- Answer:
- Expanded nodes: S,A,B,C,E,G
- Solution path: SBCEG
- Path cost: 14

h(S)=12, h(A)=8,
h(B)=12, h(C)=3,
h(D)=7, h(E)=1,
h(G)=0.