ELEX 4336: Feedback Systems

LAB 4 – Block Diagram Reduction

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1 Question 1

1.1 MATLAB Code

```
% parallel configuration of two transfer functions
% Obtain the overall transfer function when the two blocks are in parallel

% G(s)
numg = [1];
deng = [500 0 0];
sysg = tf(numg, deng);

% H(s)
numh = [1 1];
denh = [1 2];
sysh = tf(numh, denh);

% H(s) + G(s)
sys = parallel(sysg, sysh);
```

1.2 MATLAB Command Window

```
sys =

  500 s^3 + 500 s^2 + s + 2
  -------------------------
     500 s^3 + 1000 s^2

Continuous-time transfer function.
```

1.3 Description

When two blocks are in parallel configuration, the resulting transfer function of the system is equivalent to the sum of the transfer function of each individual block.

Therefore,

$$T(s) = G_c(s) + G(s) = \frac{s+1}{s+2} + \frac{1}{500s^2} = \frac{(s+1)(500s^2) + (1)(s+2)}{(s+2)(500s^2)} = \frac{\mathbf{500s^3 + 500s^2 + s + 2}}{\mathbf{500s^3 + 1000s^2}}$$

2 Question 2

2.1 MATLAB Code

```
% unity feedback system
% obtain the overall transfer function of the system as shown in the figure

% G(s)
numg = [1];
deng = [500 0 0];
sysg = tf(numg, deng);

% H(s)
numh = [1 1];
denh = [1 2];
sysh = tf(numh, denh);

% overall transfer function
sys = feedback(series(sysg, sysh), 1, -1);
```

2.2 MATLAB Command Window

```
sys =

            s + 1
  --------------------------
  500 s^3 + 1000 s^2 + s + 1

Continuous-time transfer function.
```

2.3 Description

When two blocks, Gc and G, are in series configuration and the other block, H = 1, forms a unity feedback loop across the two blocks, the resulting transfer function of the system is as follows:

$$T(s) = \frac{G_c(s)G(s)}{1 + G_c(s)G(s)H(s)} = \frac{\left(\frac{s+1}{s+2}\right)\left(\frac{1}{500s^2}\right)}{1 + \left(\frac{s+1}{s+2}\right)\left(\frac{1}{500s^2}\right)(1)} = \frac{\frac{(s+1)}{(s+2)(500s^2)}}{\frac{(s+2)(500s^2) + (s+1)}{(s+2)(500s^2)}}$$

$$= \frac{(s+1)}{(s+2)(500s^2) + (s+1)} = \boldsymbol{\frac{s+1}{500s^3 + 1000s^2 + s + 1}}$$

3 Question 3

3.1 MATLAB Code

```
% non-unity feedback system
% obtain the overall transfer function of the system as shown in the figure

% G(s)
numg = [1];
deng = [500 0 0];
sysg = tf(numg, deng);

% H(s)
numh = [1 1];
denh = [1 2];
sysh = tf(numh, denh);

% overall transfer function
sys = feedback(sysg, sysh, -1);
```

3.2 MATLAB Command Window

```
sys =

            s + 2
  --------------------------
  500 s^3 + 1000 s^2 + s + 1

Continuous-time transfer function.
```

3.3 Description

When a block, H, forms a non-unity feedback loop across the other block, G, the resulting transfer function of the system is as follows:

$$T(s) = \frac{G(s)}{1+G(s)H(s)} = \frac{\left(\frac{1}{500s^2}\right)}{1+\left(\frac{1}{500s^2}\right)\left(\frac{s+1}{s+2}\right)} = \frac{\frac{1}{(500s^2)}}{\frac{(500s^2)(s+2)+(s+1)}{(s+2)(500s^2)}}$$

$$= \frac{(s+2)}{(500s^2)(s+2)+(s+1)} = \boldsymbol{\frac{s+2}{500s^3+1000s^2+s+1}}$$

4 Question 4

4.1 MATLAB Code

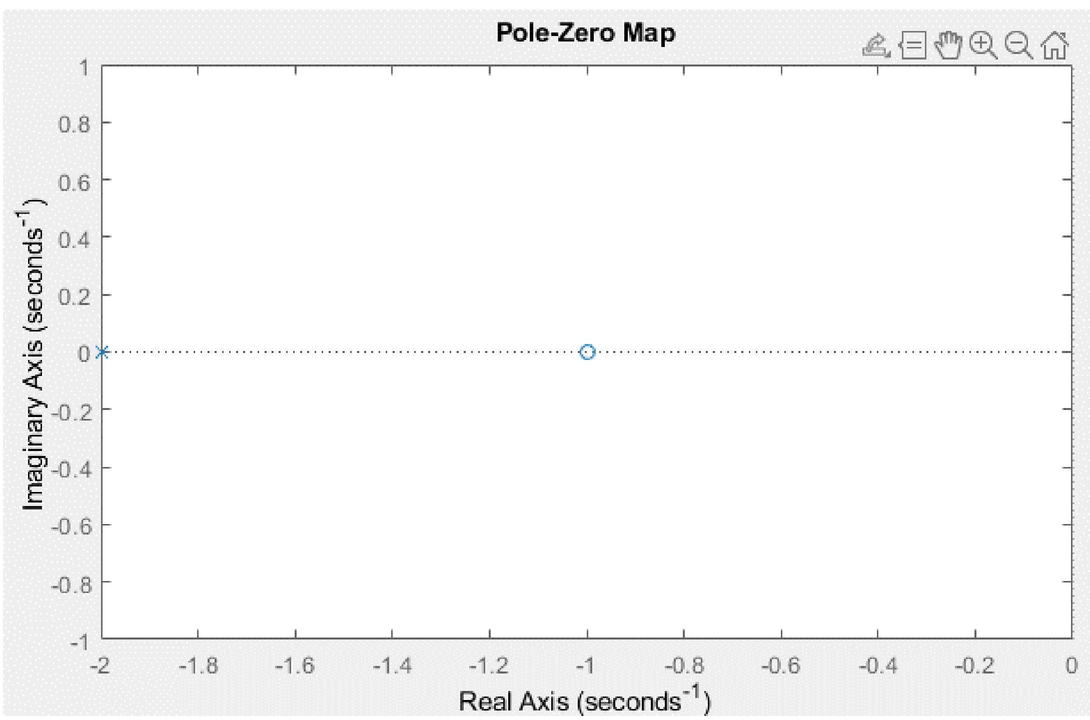
```
% plot the location of the system zeros and poles

% H(s)
numh = [1 1];
denh = [1 2];
sysh = tf(numh, denh);

% find poles and zero
p = pole(sysh);
z = zero(sysh);

% plot pole-zero map
pzmap(sysh);
```

4.2 MATLAB Plot



4.3 Description

Poles are roots of the denominator of the transfer function and zeroes are that of the numerator.

$$H(s) = \frac{s+1}{s+2}$$

In this transfer function pole is -2 and zero is -1. In the pole-zero map plotted as above, x indicates the pole and o indicates the zero.

5 Question 5

5.1 MATLAB Code

```
% multi-loop feedback system
% find closed loop transfer function and plot pole-zero map of the system

% define blocks
sysg1 = tf([1], [1 10]);
sysg2 = tf([1], [1 1]);
sysg3 = tf([1 0 1], [1 4 4]);
sysg4 = tf([1 1], [1 6]);
sysh1 = tf([1 1], [1 2]);
sysh2 = tf([2], [1]);
sysh3 = tf([1], [1]);

% construct closed loop transfer function
L1 = feedback(series(sysg3, sysg4), sysh1, +1);
L2 = feedback(series(sysg2, L1), sysh2/sysg4, -1);
sys = feedback(series(sysg1, L2), sysh3, -1);

% plot pole-zero map
pzmap(sys);
```

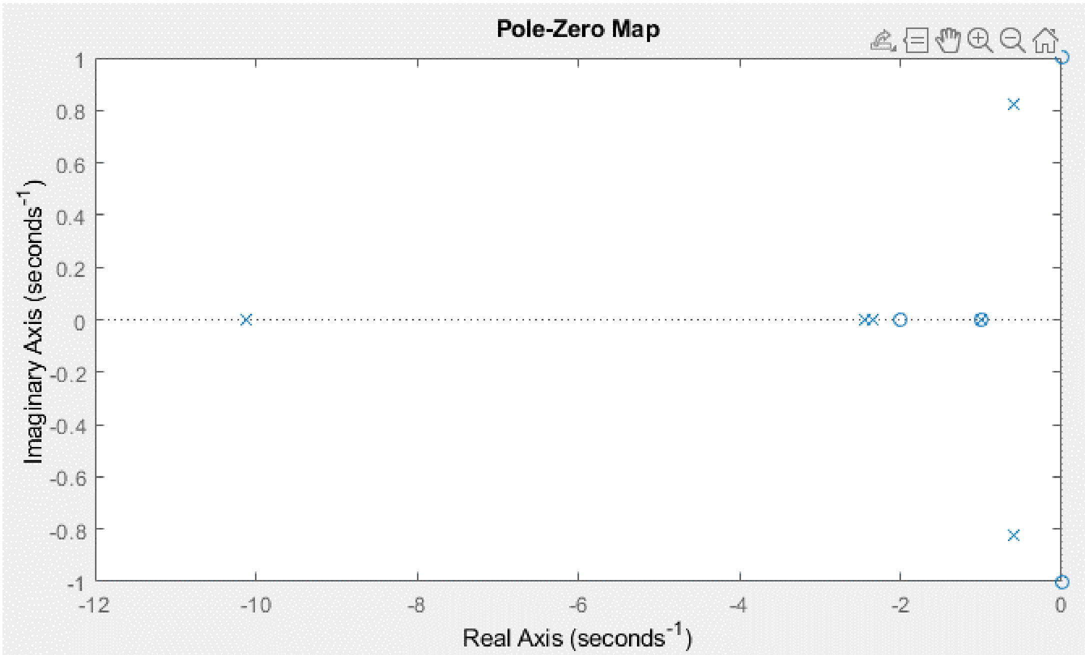
5.2 MATLAB Command Window

```
sys =

              s^5 + 4 s^4 + 6 s^3 + 6 s^2 + 5 s + 2
  ----------------------------------------------------------------
  12 s^6 + 205 s^5 + 1066 s^4 + 2517 s^3 + 3128 s^2 + 2196 s + 712

Continuous-time transfer function.
```

5.3 MATLAB Plot



5.4 Description

The feedback loop formed by H2 had to be moved next to G4 to construct the system using only series, parallel and feedback functions.

Whether this answer is correct or not will be explained in Question 6.

6 Question 6

6.1 MATLAB Code

```
% multi-loop feedback system
% simplify and calculate the transfer function of the system

% define symbols
syms s;

% define blocks
g1 = 1/(s + 10);
g2 = 1/(s + 1);
g3 = (s^2 + 1)/(s^2 + 4*s + 4);
g4 = (s + 1)/(s + 6);
h1 = (s + 1)/(s + 2);
h2 = 2;
h3 = 1;

% calculate overall transfer function
sys = (g1*g2*g3*g4)/(1 - (g3*g4*h1) + (g2*g3*h2) + (g1*g2*g3*g4*h3));

% simplify and display transfer equation
sys = simplifyFraction(sys, 'Expand', true);
pretty(sys);

% simplify the transfer function from Question 5
sys2 = (s^5 + 4*s^4 + 6*s^3 + 6*s^2 + 5*s + 2) / (12*s^6 + 205*s^5 + 1066*s^4 +
2517*s^3 + 3128*s^2 + 2196*s + 712);
sys2 = simplifyFraction(sys2, 'Expand', true);
pretty(sys2);
```

6.2 MATLAB Command Window

```
             4       3       2
            s  + 3 s  + 3 s  + 3 s + 2
------------------------------------------------
    5         4         3          2
12 s  + 193 s  + 873 s  + 1644 s  + 1484 s + 712

             4       3       2
            s  + 3 s  + 3 s  + 3 s + 2
------------------------------------------------
    5         4         3          2
12 s  + 193 s  + 873 s  + 1644 s  + 1484 s + 712
```

6.3 Description

The multi-loop feedback system was simplified using lecture method and as the lecture note. The feedback loop containing H2 was first moved next to G4 and then the feedback loop containing H1 was simplified (L1). Since H2/G4 is forming a feedback loop across the series of G2 and L1, they can be further simplified (L2). Finally, H3 is forming the feedback loop across the series of G1 and L2.

After simplifying the equations from Question 5 and 6, they turned out to be identical.

7 Question 7

7.1 MATLAB Code

```
% unity feedback system
% compute and plot unit step response of closed-loop transfer function

% define blocks
sysg = tf([1], [1 1]);  % controller
sysh = tf([1 2], [1 3]);    % plant

% construct closed loop transfer function
sys = feedback(series(sysg, sysh), 1, -1);

% plot unit step response
step(sys);
```

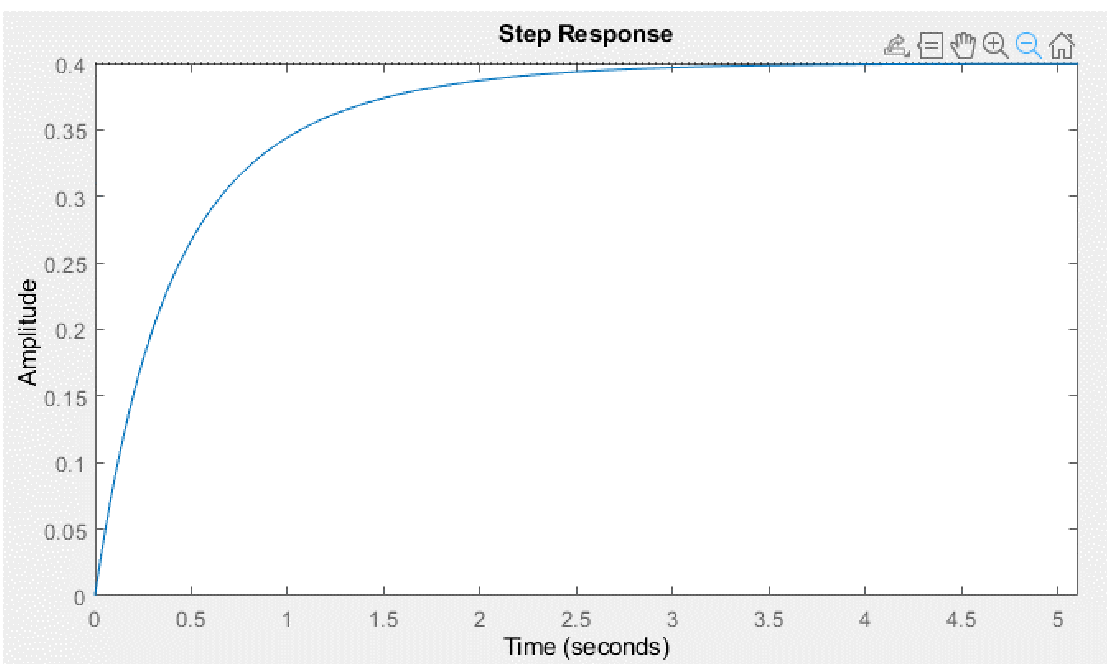
7.2 MATLAB Command Window

```
sys =

      s + 2
  -------------
  s^2 + 5 s + 5

Continuous-time transfer function.
```

7.3 MATLAB Plot



7.4 Description

When series of two blocks, G (Controller) and H (Plant), is forming a unity feedback loop, the resulting transfer function of the system is as follows:

$$T(s) = \frac{G(s)H(s)}{1 + G(s)H(s)(1)} = \frac{\left(\frac{1}{s+1}\right)\left(\frac{s+2}{s+3}\right)}{1 + \left(\frac{1}{s+1}\right)\left(\frac{s+2}{s+3}\right)(1)} = \frac{\frac{(s+2)}{(s+1)(s+3)}}{\frac{(s+1)(s+3)+(s+2)}{(s+1)(s+3)}}$$

$$= \frac{(s+2)}{(s+1)(s+3)+(s+2)} = \frac{\boldsymbol{s+2}}{\boldsymbol{s^2+5s+5}}$$

The final value of the output of the unit step response can be calculated as follows:

$$\lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s) = \lim_{s\to 0}\left(s \cdot \frac{s+2}{s^2+5s+5} \cdot \frac{1}{s}\right) = \frac{\mathbf{2}}{\mathbf{5}}$$

8 Question 8

8.1 MATLAB Code

```
% compute the closed-loop transfer function then compute and plot the step
% response to a 10 degree step input. Finally compare the step response
% when J is reduced by 20% and 50%.

% spacecraft parameters
J = 10.8E8; % moment of inertia

% controller parameters
k = 10.8E8;
a = 1;
b = 8;

% define blocks
sysg = tf([k k*a], [1 b]);  % controller
sysh1 = tf([1], [J 0 0]);   % spacecraft
sysh2 = tf([1], [(1 - 0.2)*J 0 0]); % spacecraft (J reduced by 20%)
sysh3 = tf([1], [(1 - 0.5)*J 0 0]); % spacecraft (J reduced by 50%)

% construct closed loop transfer function
sys1 = feedback(series(sysg, sysh1), 1, -1);
sys2 = feedback(series(sysg, sysh2), 1, -1);
sys3 = feedback(series(sysg, sysh3), 1, -1);

% plot step response of 10 degree step input
opt = stepDataOptions('StepAmplitude', 10); % 10 degree step input
step(sys1, sys2, sys3, opt);
```

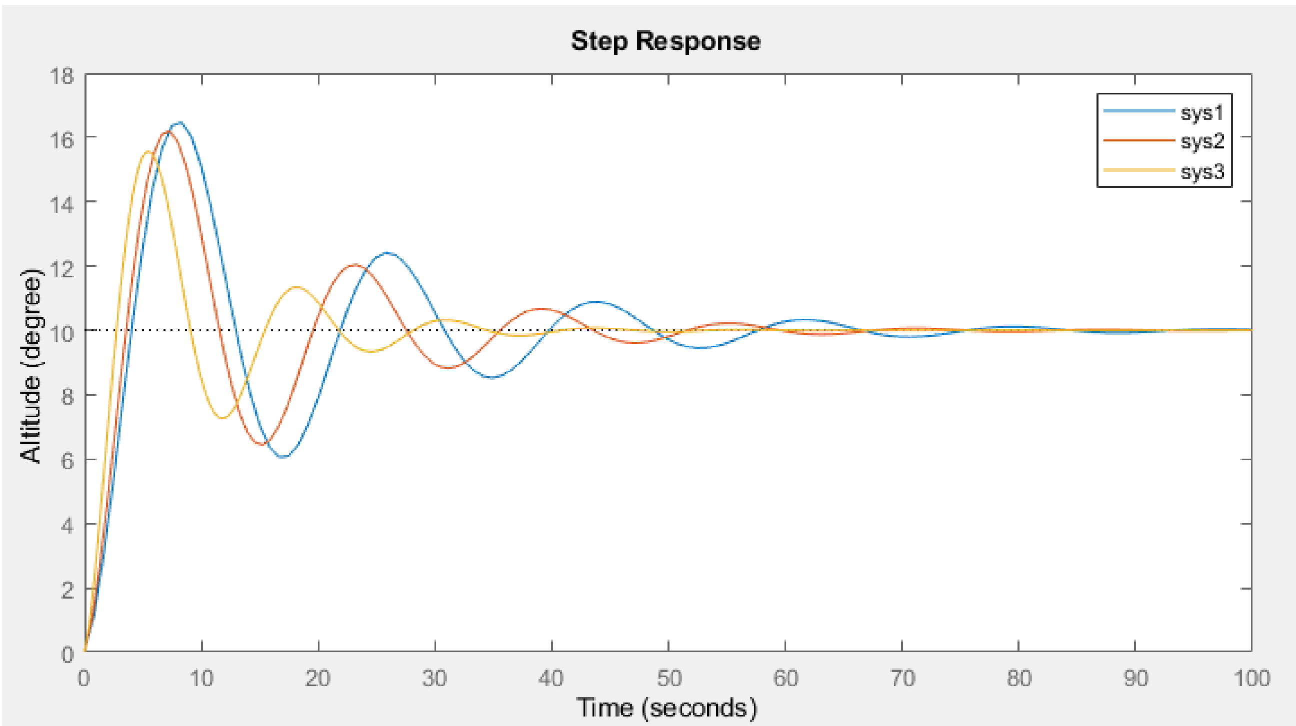
8.2 MATLAB Command Window

```
sys1 =

                  1.08e09 s + 1.08e09
  -----------------------------------------------
  1.08e09 s^3 + 8.64e09 s^2 + 1.08e09 s + 1.08e09

Continuous-time transfer function.
```

8.3 MATLAB Plot



8.4 Description

When series of two blocks, G (Controller) and H (Spacecraft), is forming a unity feedback loop, the resulting transfer function of the system is as follows:

$$T(s) = \frac{G(s)H(s)}{1+G(s)H(s)(1)} = \frac{\left(\frac{k(s+a)}{s+b}\right)\left(\frac{1}{Js^2}\right)}{1+\left(\frac{k(s+a)}{s+b}\right)\left(\frac{1}{Js^2}\right)(1)} = \frac{\frac{k(s+a)}{(s+b)(Js^2)}}{\frac{(s+b)(Js^2)+k(s+a)}{(s+b)(Js^2)}}$$

$$= \frac{k(s+a)}{(s+b)(Js^2)+k(s+a)} = \frac{\boldsymbol{k(s+a)}}{\boldsymbol{Js^3+Jbs^2+ks+ka}}$$

As spacecraft moment of inertia decreases, the system responds faster to the change in altitude.