ELEX 4336: Feedback Systems

LAB 2 – Dynamical System Modelling Using MATLAB

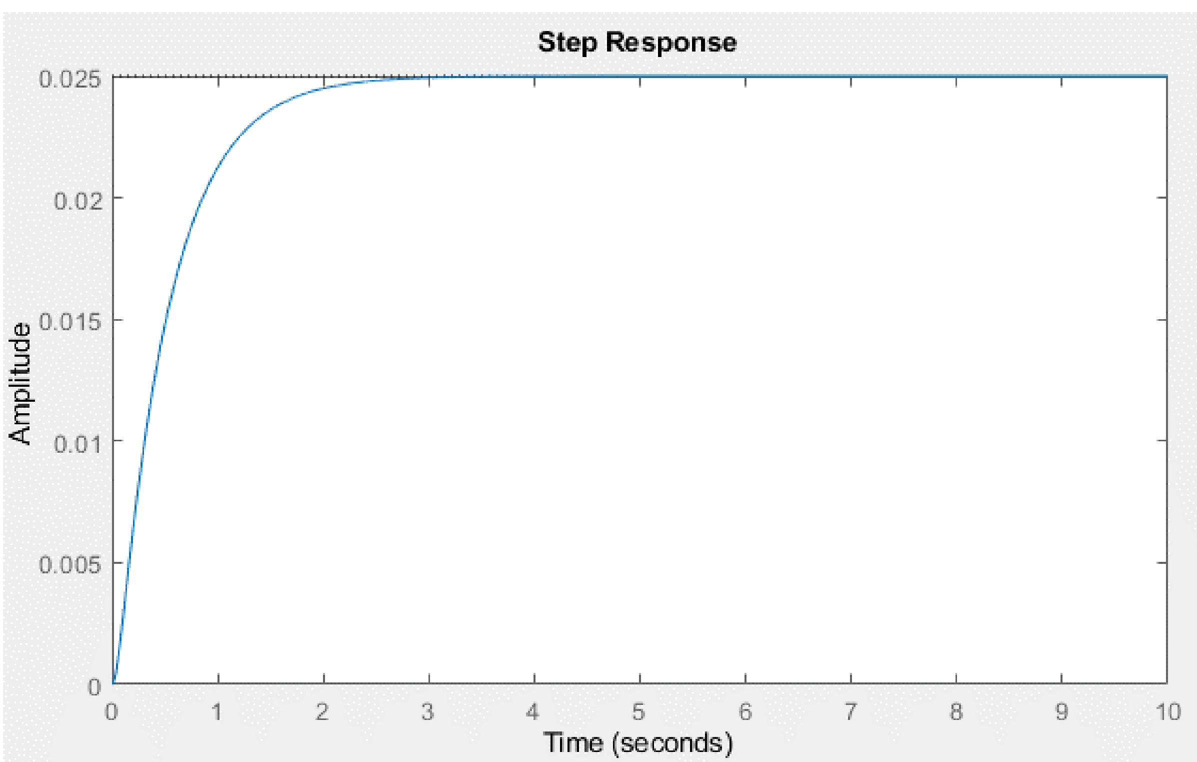
Student Name: Taeyoon Rim	Student Number: A01160214	Set: B

1 Mass Spring Damper System

1.1 MATLAB Code

```
% Model equation: y(t) = f(t) * P(D) / Q(D)
% Plot the unit step response of the mass spring damper system over [0, 10]

% system parameters
m = 1;  % unit: kg
k = 40; % unit: N/m
b = 22; % unit: N/m-sec

% transfer function
Q = [m b k];
P = [1];
sys = tf(P, Q);

% characteristic values
p = roots(Q);

% plot range
t = 0:0.001:10;

% plot the unit step response
step(sys, t);
```

1.2 MATLAB Plot



1.3 Description

This program plots the unit step response of a mass spring damper system over a range of [0, 10].

The mass spring damper system with parameters m = 1 kg, b = 22 N/m-sec, k = 40 N/m and the force balance equation can be modelled as below:

$$m\frac{d^2y(t)}{dt^2} + b\frac{dy(t)}{dt} + ky(t) = f(t)$$

$$(mD^2 + bD + k)y(t) = 1 \cdot f(t)$$

$$y(t) = \frac{1}{mD^2 + bD + k} \cdot f(t) = \frac{1}{D^2 + 22D + 40} \cdot f(t)$$

The denominator of the transfer function yields characteristic values of two real roots, -20 and -2. Therefore, the system is overdamped. The shape of the plot agrees that the system is overdamped.

When the transfer function was revised for a larger viscous damping, I expected that the response would slow down because it reduces the value of damping coefficient, ζ. For example, with b = 22 N/m-sec, the damping coefficient was found to be 1.7393. Whereas, with b = 30 N/m-sec, the damping coefficient was found to be 2.3717.

$$\omega_n{}^2 = k, \qquad 2\zeta\omega_n = b$$
$$\zeta = \frac{b}{2\omega_n} = \frac{b}{2\sqrt{k}}$$

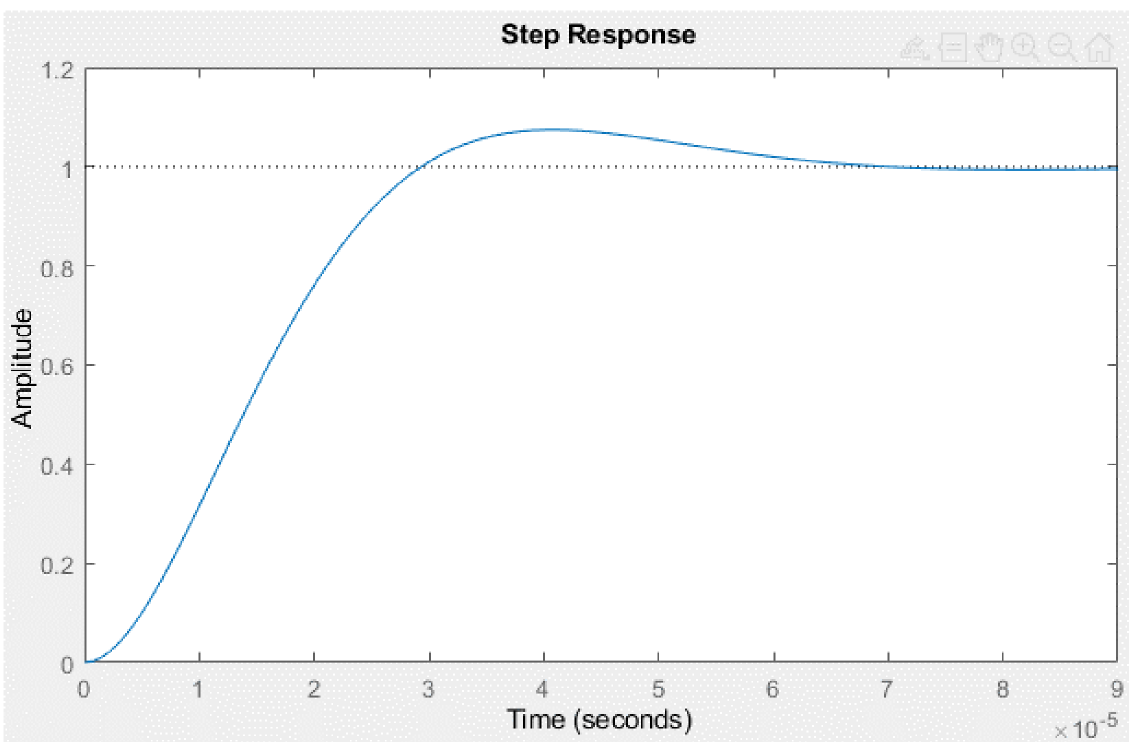
As I expected, the plot for a larger viscous damping compared to the original plot was stretched horizontally and showed a sign of slower response: Larger viscous damping does slow down the response.

2 RLC Circuit System

2.1 MATLAB Code

```
% Model equation: y(t) = V(t) * P(D) / Q(D)
% Plot the unit step response of the second order circuit system

% system parameters
L = 10 * 10^-3; % unit: H
C = 10 * 10^-9; % unit: F
R = 1.2 * 10^3; % unit: Ohms
Rcoil = 24; % unit: Ohms
Rgen = 50;  % unit: Ohms

% transfer function
Q = [L*C (R+Rcoil+Rgen)*C 1];
P = [1];
sys = tf(P, Q);

% characteristic value
p = roots(Q);

% plot the unit step response
step(sys);
```

2.2 MATLAB Plot



2.3 Description

This program plots the unit step response of a second order circuit system.

The second order circuit system with parameters R = (1.2k + 24 + 50) Ω, L = 10 mH, C = 10 nF can be modelled as below:

$$LC\frac{d^2V(t)}{dt^2} + RC\frac{dV(t)}{dt} + V(t) = V_{in}(t)$$

$$(LCD^2 + RCD + 1)V(t) = 1 \cdot V_{in}(t)$$

$$V(t) = \frac{1}{LCD^2 + RCD + 1} \cdot V_{in}(t)$$

The denominator of the transfer function yields characteristic values of two complex conjugates, -63700 ± 77086j. Therefore, the system is underdamped. The shape of the plot agrees that the system is underdamped.