ELEX 4336: Feedback Systems

LAB 5 – Model Identification For First Order LTI Systems

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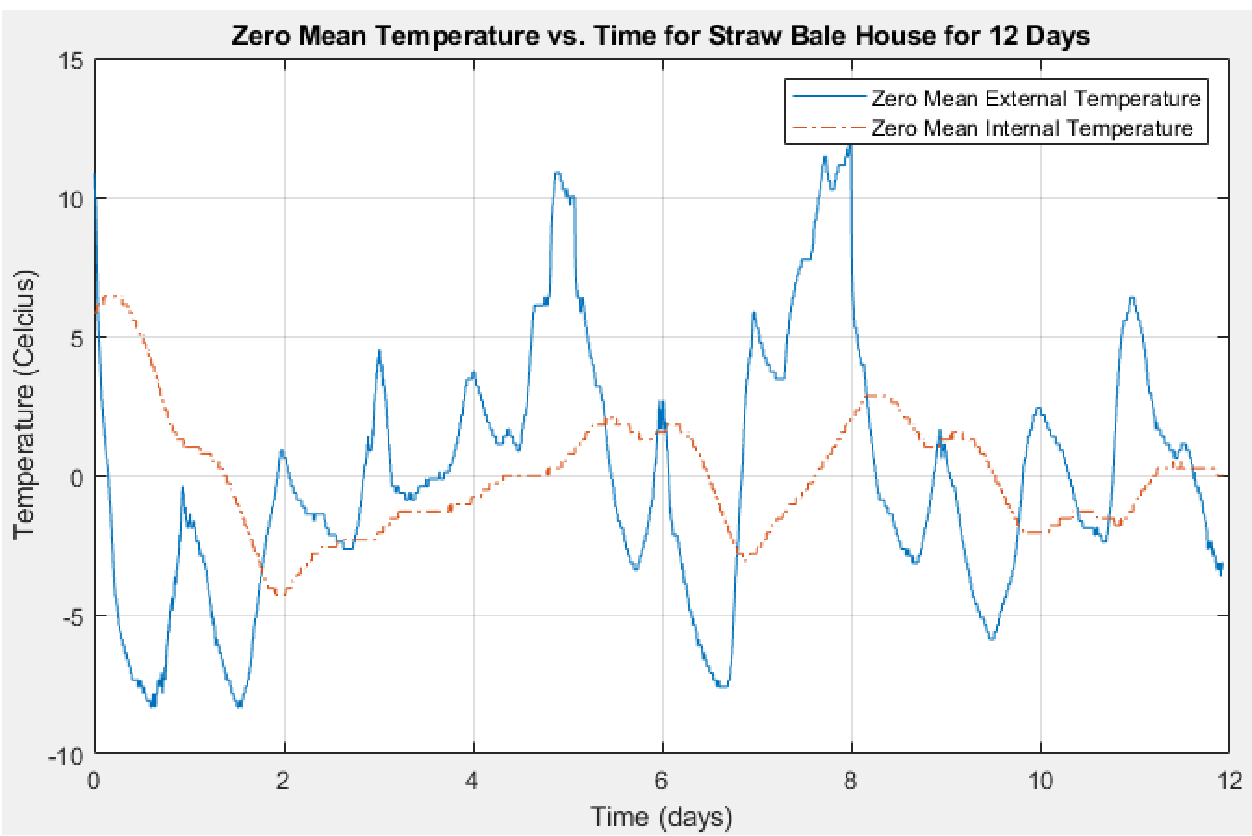
1 Step 1

1.1 MATLAB Code

```
% Step 1
% Plot the zero mean adjusted data for 12 days as shown as in Figure 1

load temps;

% Array indices
idxStart = 1;
idxEnd = 5372;  % approx. 12 days

% Calculate zero means
DeltaExtTemp = ExtTemp(idxStart:idxEnd) - mean(ExtTemp(idxStart:idxEnd));
DeltaIntTemp = IntTemp(idxStart:idxEnd) - mean(IntTemp(idxStart:idxEnd));

% Plot the data
figure(2);
t = t(idxStart:idxEnd);
plot(t/60/60/24, DeltaExtTemp, '-', t/60/60/24, DeltaIntTemp, '-.');
ylabel('Temperature (Celcius)');
xlabel('Time (days)');
title('Zero Mean Temperature vs. Time for Straw Bale House for 12 Days');
legend('Zero Mean External Temperature', 'Zero Mean Internal Temperature');
grid on;
```

1.2 MATLAB Plot



1.3 Description

The zero mean external and internal temperature was calculated using below expressions:

$$\theta_{e\ zero\ mean}(t) = \theta_e(t) - \theta_{e\ ave}(t)$$

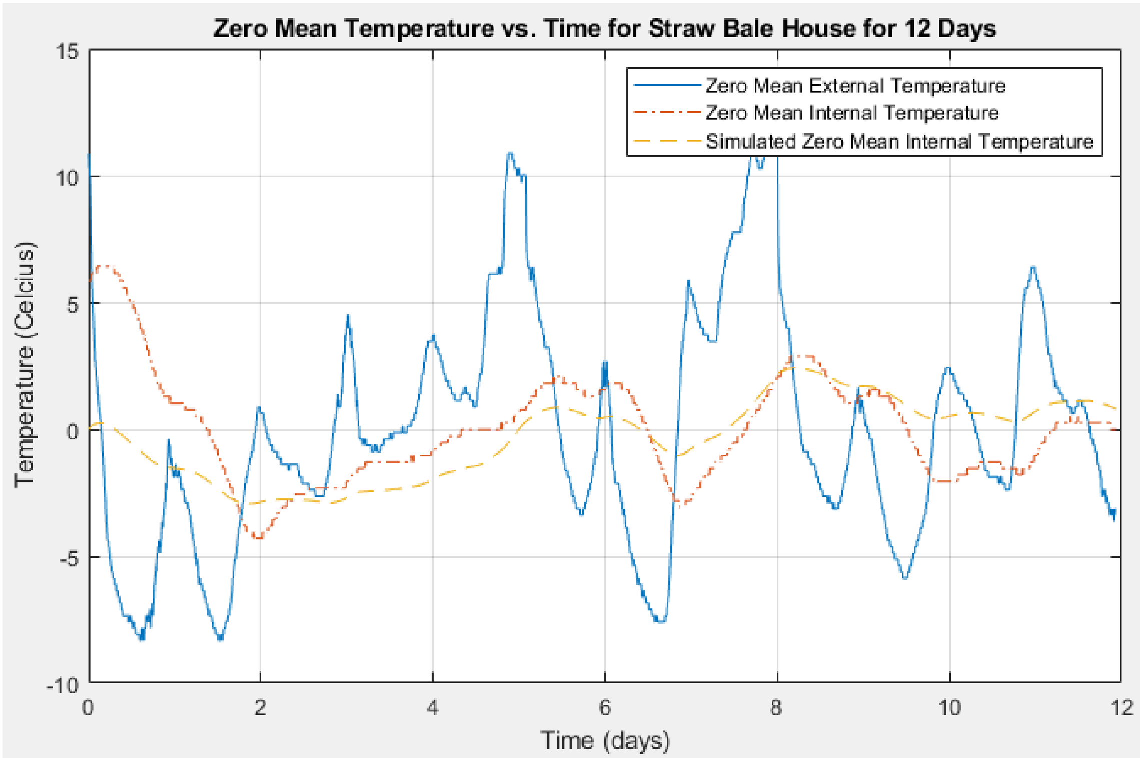
$$\theta_{i\ zero\ mean}(t) = \theta_i(t) - \theta_{i\ ave}(t)$$

2 Step 2, 3

2.1 MATLAB Code

```
% Model Equation: T(s) = K / (?*s + 1)
% Approximate the gain relationship between mean internal temperature and
% mean external temperature by assuming that K = 1 in your input output model

load temps;

% System Parameters
K = 1;
tau = 5*60*60*24; % 5 days

% System transfer function
sys = tf([K], [tau 1]);

% Array indices
idxStart = 1;
idxEnd = 5372;  % approx. 12 days

% Calculate zero means
DeltaExtTemp = ExtTemp(idxStart:idxEnd) - mean(ExtTemp(idxStart:idxEnd));
DeltaIntTemp = IntTemp(idxStart:idxEnd) - mean(IntTemp(idxStart:idxEnd));
DeltaIntTempSim = lsim(sys, DeltaExtTemp, t(idxStart:idxEnd));

% Plot the data
figure(3);
t = t(idxStart:idxEnd);
plot(t/60/60/24, DeltaExtTemp, '-', t/60/60/24, DeltaIntTemp, '-.', t/60/60/24,
DeltaIntTempSim, '--');
ylabel('Temperature (Celcius)');
xlabel('Time (days)');
title('Zero Mean Temperature vs. Time for Straw Bale House for 12 Days');
legend('Zero Mean External Temperature', 'Zero Mean Internal Temperature',
'Simulated Zero Mean Internal Temperature');
grid on;
```

2.2 MATLAB Plot



2.3 Description

The gain relationship between mean external and internal temperature was approximated by below equation (assuming $K = 1,\ \tau = 5\ days \cdot \frac{24\ hours}{1\ day} \cdot \frac{60\ minutes}{1\ hour} \cdot \frac{60\ seconds}{1\ minute} = 432000\ seconds$)

$$\tau\Delta\dot{\theta}_i(t) + \Delta\theta_i(t) = K\Delta\theta_e(t)$$

$$\frac{\Delta\theta_i(s)}{\Delta\theta_e(s)} = \frac{K}{\tau s + 1}$$

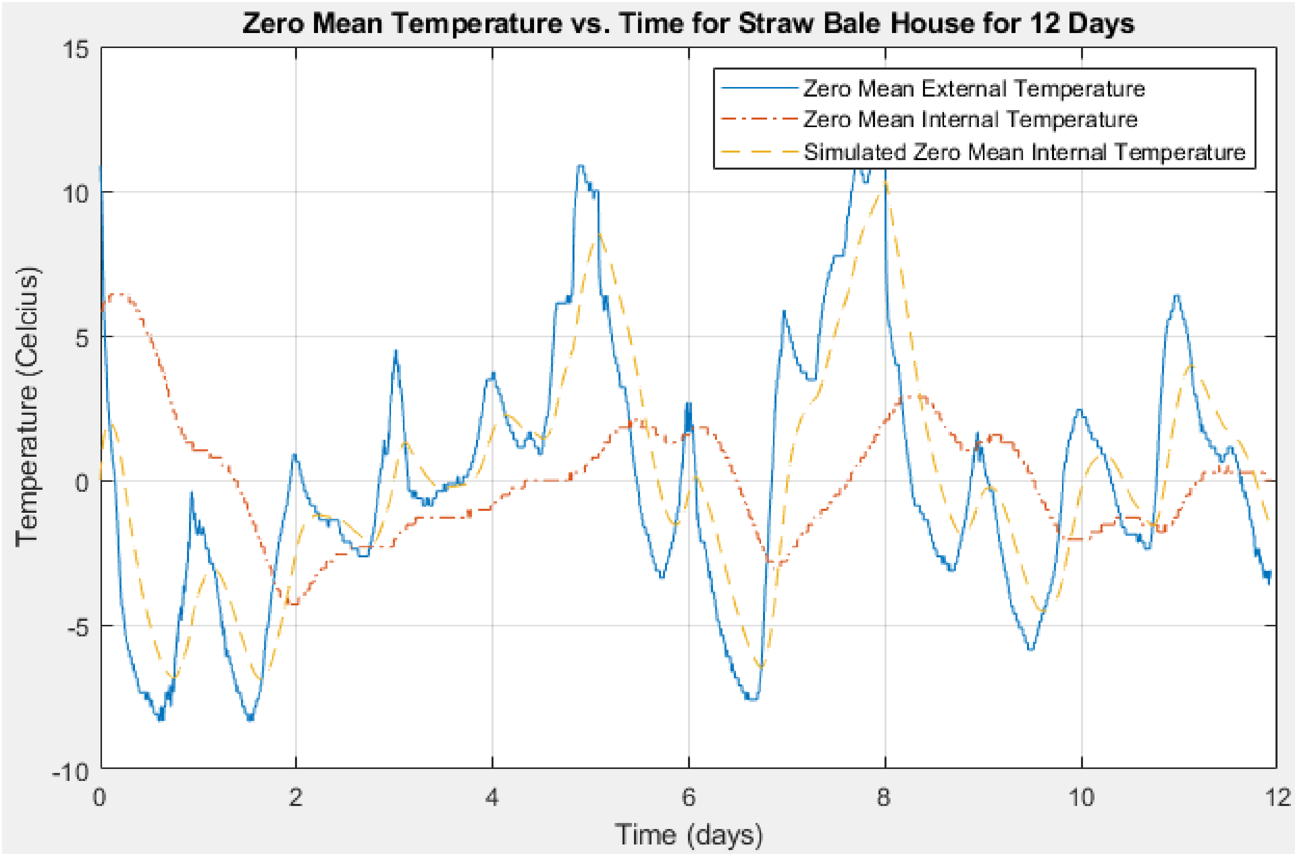
While the simulated zero mean internal temperature follows the trend of the actual zero mean temperature, $\tau = 432000\ seconds$ is too long because it has big SSE of 1.8831E04 and it does not follow the steep curves of the actual plot (it is close to y = 0)

3 Step 4

3.1 MATLAB Code

```
% Model Equation: T(s) = K / (tau*s + 1)
% Approximate the gain relationship between mean internal temperature and
% mean external temperature by assuming that K = 1 in your input output model

load temps;

% System Parameters
K = 1;
tau = 5*60*60*24; % 5 days

% System transfer function
sys = tf([K], [tau 1]);

% Array indices
idxStart = 1;
idxEnd = 5372;  % approx. 12 days

% Calculate zero means
DeltaExtTemp = ExtTemp(idxStart:idxEnd) - mean(ExtTemp(idxStart:idxEnd));
DeltaIntTemp = IntTemp(idxStart:idxEnd) - mean(IntTemp(idxStart:idxEnd));
DeltaIntTempSim = lsim(sys, DeltaExtTemp, t(idxStart:idxEnd));

% Plot the data
figure(3);
t = t(idxStart:idxEnd);
plot(t/60/60/24, DeltaExtTemp, '-', t/60/60/24, DeltaIntTemp, '-.', t/60/60/24,
DeltaIntTempSim, '--');
ylabel('Temperature (Celcius)');
xlabel('Time (days)');
title('Zero Mean Temperature vs. Time for Straw Bale House for 12 Days');
legend('Zero Mean External Temperature', 'Zero Mean Internal Temperature',
'Simulated Zero Mean Internal Temperature');
grid on;
```

3.2 MATLAB Plot



3.3 Description

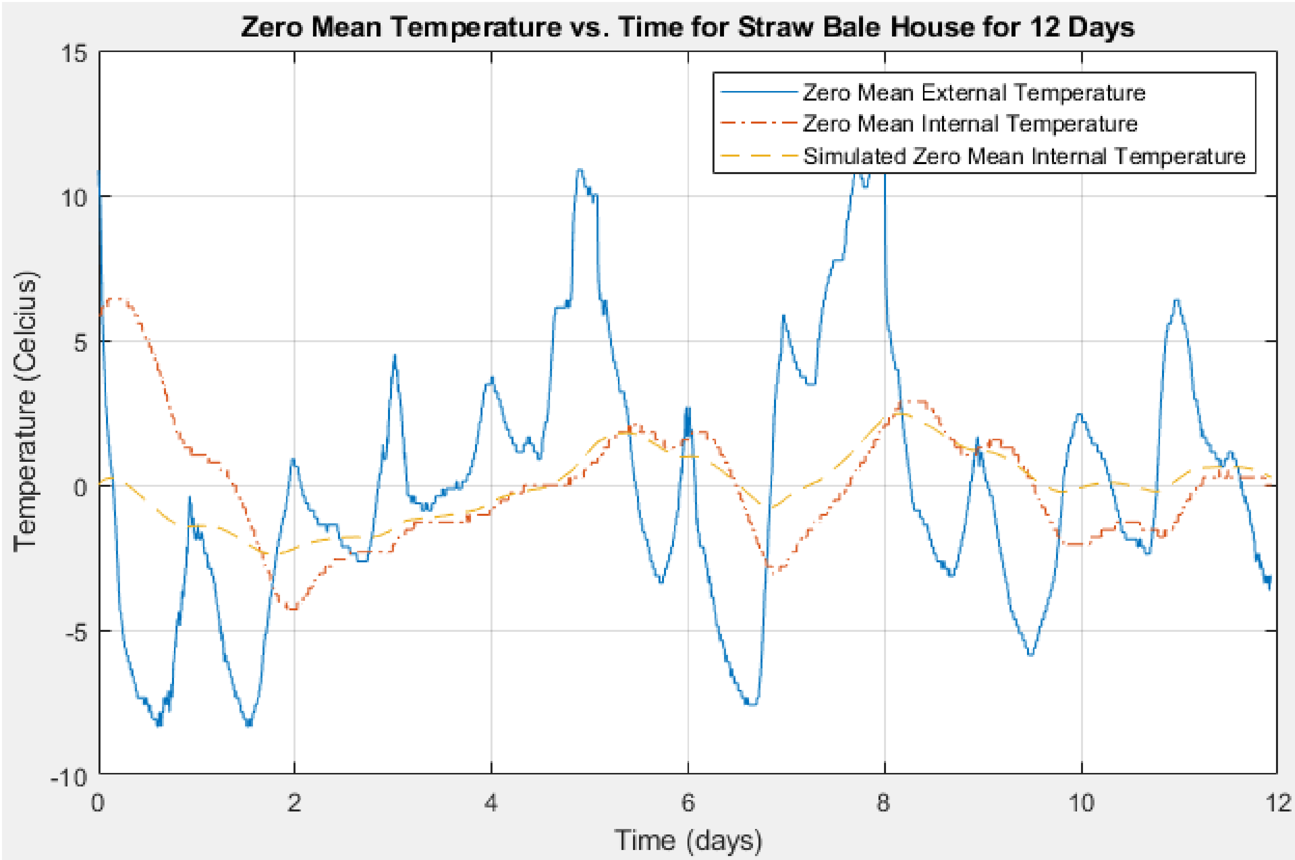
This time, I assumed that $\tau = 5\ hours \cdot \frac{60\ minutes}{1\ hour} \cdot \frac{60\ seconds}{1\ minute} = 18000$ and the simulated zero mean internal temperature did not follow the trend of the actual zero mean temperature at all. Instead, it looks more like its original input, $\Delta\theta_e(t)$. Therefore the choice of τ is too short.

4 Step 5, 6

4.1 MATLAB Code

```
% Model Equation: T(s) = K / (tau*s + 1)
% Approximate the gain relationship between mean internal temperature and
% mean external temperature by assuming that K = 1 in your input output model

load temps;

% Array indices
idxStart = 1;
idxEnd = 5372;  % approx. 12 days

% Calculate zero means
DeltaExtTemp = ExtTemp(idxStart:idxEnd) - mean(ExtTemp(idxStart:idxEnd));
DeltaIntTemp = IntTemp(idxStart:idxEnd) - mean(IntTemp(idxStart:idxEnd));

% Find best fit K and tau
K = 0;
tau = 0;
minSSE = -1;

for kTest = 0.5:0.01:2
    for tauTest = 5*60*60:100:5*60*60*24
        sys = tf([kTest], [tauTest 1]);
        DeltaIntTempSim = lsim(sys, DeltaExtTemp, t(idxStart:idxEnd));
        SSE = sum((DeltaIntTemp - DeltaIntTempSim) .^ 2);

        if minSSE == -1
            minSSE = SSE;
        elseif SSE < minSSE
            minSSE = SSE;
            K = kTest;
            tau = tauTest;
        end
    end
end

% Calculated internal temperature
sys = tf([K], [tau 1]);
DeltaIntTempSim = lsim(sys, DeltaExtTemp, t(idxStart:idxEnd));

% Plot the data
figure(4);
t = t(idxStart:idxEnd);
plot(t/60/60/24, DeltaExtTemp, '-', t/60/60/24, DeltaIntTemp, '-.', t/60/60/24,
DeltaIntTempSim, '--');
ylabel('Temperature (Celcius)');
xlabel('Time (days)');
title('Zero Mean Temperature vs. Time for Straw Bale House for 12 Days');
legend('Zero Mean External Temperature', 'Zero Mean Internal Temperature',
'Simulated Zero Mean Internal Temperature');
grid on;
```

4.2 MATLAB Plot



4.3 Description

The optimal values for K and τ (values that gives least SSE) was found, using brute force, to be $K = 0.72,\ \tau = 145300$. It took 42 minutes 37 seconds in total with SSE of 1.7045E4. A way to optimize this program would be, assuming there is only one optimal values for each K and τ, reducing the increment and range gradually. ($\tau\ within\ 145200{:}\,10{:}\,145400, then\ 145350{:}\,1{:}\,145370$, …)

Even if the first order model was a perfect match for this input/output data, my optimal τ will not necessarily turn out to be the same if any value of K was used.

Let $f(s) = \frac{1}{s+1}$. This function can be expressed as $\frac{K}{\tau} f\left(s - \left(1 - \frac{1}{\tau}\right)\right) = \frac{K}{\tau s+1}$. Which means that adjusting τ will result in phase shift and vertical expansion/compression while adjusting K will only result in horizontal expansion/compression. However, I will ignore $\tau's$ effect on phase shift as it is negligible compared to error caused by a large vertical expansion/compression.

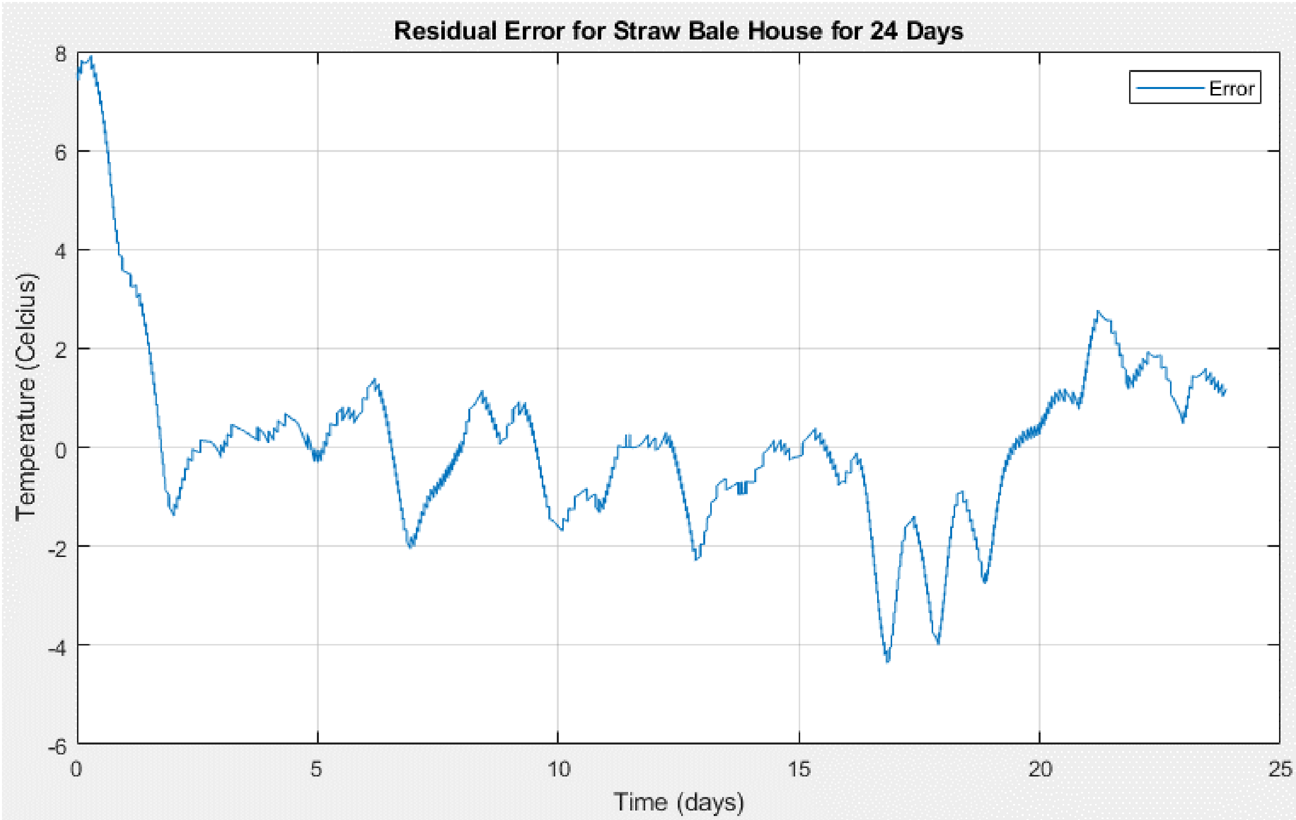
If chosen K is greater than that of the original transfer function, τ should also be greater to compensate the vertical expansion and vice versa. Therefore, the optimal value of τ will be different depending on the chosen value of K and the input data.

5 Step 7

5.1 MATLAB Code

```
% Model Equation: E(t) = DeltaIntTemp(t) - DeltaIntTempSim(t)
% Calculate and plot the residual errors

load temps;

% System Parameters
K = 0.72;
tau = 145300;
sys = tf([K], [tau 1]);

% Array indices
idxStart = 1;
idxEnd = 10744;  % approx. 24 days (full data)

% Calculate zero means
DeltaExtTemp = ExtTemp(idxStart:idxEnd) - mean(ExtTemp(idxStart:idxEnd));
DeltaIntTemp = IntTemp(idxStart:idxEnd) - mean(IntTemp(idxStart:idxEnd));

% Calculated internal temperature
DeltaIntTempSim = lsim(sys, DeltaExtTemp, t(idxStart:idxEnd));

% Calculate Residual Error
E = DeltaIntTemp - DeltaIntTempSim;

% Plot the data
figure(5);
t = t(idxStart:idxEnd);
plot(t/60/60/24, E, '-');
ylabel('Temperature (Celcius)');
xlabel('Time (days)');
title('Residual Error for Straw Bale House for 24 Days');
legend('Error');
grid on;
```

5.2 MATLAB Plot



5.3 Description

The residual errors for approximated zero mean internal temperature was calculated using below expressions:

$$E(t) = \Delta\theta_i(t) - \Delta\theta_{i\ approx.}(t)$$