KINEMATICS

Rectilinear Motion

Variable a :

$v = \frac{\mathrm{d}s}{\mathrm{d}t} \quad a = \frac{\mathrm{d}v}{\mathrm{d}t} \quad a\,\mathrm{d}s = v\,\mathrm{d}v$

Constant $a = a_c$:

$v = v_0 + a_c t$

$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$

$v^2 = v_0^2 + 2a_c(s - s_0)$

Curvilinear Motion

x, y, z Coordinates :

$v_x = \dot{x} \quad v_y = \dot{y} \quad v_z = \dot{z}$

$a_x = \ddot{x} \quad a_y = \ddot{y} \quad a_z = \ddot{z}$

r, z, θ Coordinates :

$v_r = \dot{r} \quad v_\theta = r\dot{\theta} \quad v_z = \dot{z}$

$a_r = \ddot{r} - r\dot{\theta}^2 \qquad a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$

$a_z = \ddot{z}$

n, t, b Coordinates :

$a_t = \dot{v} = v\frac{\mathrm{d}v}{\mathrm{d}s}$

$a_n = \frac{v^2}{\rho} \qquad \rho = \frac{[1+(\mathrm{d}y/\mathrm{d}x)^2]^{3/2}}{|\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}|}$

$a = \sqrt{(a_n)^2 + (a_t)^2}$

Relative Motion

$v_A = v_B + v_{A/B}$

$a_A = a_B + a_{A/B}$

Rigid Body Motion About a Fixed Axis

Variable α :

$\omega = \frac{\mathrm{d}\theta}{\mathrm{d}t} \quad \alpha = \frac{\mathrm{d}\omega}{\mathrm{d}t} \quad \alpha\,\mathrm{d}\theta = \omega\,\mathrm{d}\omega$

Constant $\alpha = \alpha_c$:

$\omega = \omega_0 + \alpha_c t$

$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha_c t^2$

$\omega^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0)$

For Point P :

$s = \theta r \;\; v = \omega r \;\; a_t = \alpha r \;\; a_n = \omega^2 r$

Relative General Plane Motion: Translating Axes

$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A(pin)}$

$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A(pin)}$

Relative General Plane Motion: Trans. and Rot. Axes

$\mathbf{v}_B = \mathbf{v}_A + \Omega \times \mathbf{r}_{B/A} + (\mathbf{v}_{B/A})_{xyz}$

$\mathbf{a}_B = \mathbf{a}_A + \dot{\Omega} \times \mathbf{r}_{B/A} + \Omega \times (\Omega \times \mathbf{r}_B/A) + 2\Omega \times (\mathbf{v}_{B/A})_{xyz} + (\mathbf{a}_{B/A})_{xyz}$

KINETICS

Mass Moment of Inertia

$I = \int r^2\,\mathrm{d}m$

Parallel Axis Theorem :

$I = I_G + md^2$

Rad. of Gyration : $k = \sqrt{\frac{I}{m}}$

Equations of Motion

$\underline{Particle:}\ \mathbf{F} = m\mathbf{a}$

$\underline{Rigid\ Body\ (Plane\ Motion):}$

$\Sigma F_x = m(a_G)_x \quad \Sigma F_y = m(a_G)_y$

$\Sigma M = I_G \alpha$

Principle of Work and Energy

$T_1 + \Sigma U_{1-2} = T_2$

Kinetic Energy

$\underline{Particle:}\ T = \frac{1}{2}mv^2$

$\underline{Rigid\ Body:}$

$T = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2$

Work

$\underline{Variable\ Force:}$

$U_F = \int F cos\,\theta\,\mathrm{d}s$

$\underline{Constant\ Force:}$

$U_F = (F_c cos\,\theta)\Delta s$

$\underline{Weight:}\ U_W = -W\Delta y$

$\underline{Spring:}\ U_S = -(\frac{1}{2}ks_2^2 - \frac{1}{2}ks_1^2)$

$\underline{Couple\ Moment:}\ U_M = M\Delta\theta$

Power and Efficiency

$P = \frac{\mathrm{d}U}{\mathrm{d}t} = \mathbf{F}\cdot\mathbf{v}$

$\epsilon = \frac{P_{out}}{P_{in}} = \frac{U_{out}}{U_{in}}$

Conservation of Energy

$T_1 + V_1 = T_2 + V_2$

$\underline{Potential\ Energy:}$

$V = V_g + V_e,\ where\ V_g = \pm Wy,$

$V_e = \frac{1}{2}ks^2$

Principle of Linear Impulse and Momentum

$\underline{Particle:}$

$m\mathbf{v}_1 + \Sigma\int F\,\mathrm{d}t = m\mathbf{v}_2$

$\underline{Rigid\ Body:}$

$m(\mathbf{v}_G)_1 + \Sigma F\,\mathrm{d}t = m(\mathbf{v}_G)_2$

Conservation of Linear Momentum

$\Sigma(syst.\ m\mathbf{v})_1 = \Sigma(syst.\ m\mathbf{v})_2$

$\underline{Coefficient\ of\ Restitution:}$

$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$

Principle of Angular Impulse and Momentum

$\underline{Particle:}$

$(\mathbf{H}_O)_1 + \Sigma\int \mathbf{M}_O\,\mathrm{d}t = (\mathbf{H}_O)_2$

$where\ H_O = (d)(mv)$

$\underline{Rigid\ Body\ (Plane\ Motion):}$

$(\mathbf{H}_G)_1 + \Sigma\int \mathbf{M}_G\,\mathrm{d}t = (\mathbf{H}_G)_2$

$where\ H_G = I_G\omega$

$(\mathbf{H}_O)_1 + \Sigma\int \mathbf{M}_O\,\mathrm{d}t = (\mathbf{H}_O)_2$

$where\ H_O = I_O\omega$

Conservation of Angular Momentum

$\Sigma(syst.\ \mathbf{H})_1 = \Sigma(syst.\ \mathbf{H})_2$