

**KINEMATICS****Particle Rectilinear Motion**Variable  $a$  :

$$v = \frac{ds}{dt} \quad a = \frac{dv}{dt} \quad a \, ds = v \, dv$$

Constant  $a = a_c$  :

$$v = v_0 + a_c t$$

$$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

$$v^2 = v_0^2 + 2a_c(s - s_0)$$

**Particle Curvilinear Motion** $x, y, z$  Coordinates :

$$v_x = \dot{x} \quad v_y = \dot{y} \quad v_z = \dot{z}$$

$$a_x = \ddot{x} \quad a_y = \ddot{y} \quad a_z = \ddot{z}$$

 $r, z, \theta$  Coordinates :

$$v_r = \dot{r} \quad v_\theta = r\dot{\theta} \quad v_z = \dot{z}$$

$$a_r = \ddot{r} - r\dot{\theta}^2 \quad a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta}$$

$$a_z = \ddot{z}$$

 $n, t, b$  Coordinates :

$$a_t = \dot{v} = v \frac{dv}{ds}$$

$$a_n = \frac{v^2}{\rho} \quad \rho = \frac{[1 + (dy/dx)^2]^{3/2}}{|\frac{d^2y}{dx^2}|}$$

$$a = \sqrt{(a_n)^2 + (a_t)^2}$$

**Relative Motion**

$$v_A = v_B + v_{A/B}$$

$$a_A = a_B + a_{A/B}$$

**Rigid Body Motion About a Fixed Axis**Variable  $\alpha$  :

$$\omega = \frac{d\theta}{dt} \quad \alpha = \frac{d\omega}{dt} \quad \alpha \, d\theta = \omega \, d\omega$$

Constant  $\alpha = \alpha_c$  :

$$\omega = \omega_0 + \alpha_c t$$

$$\theta = \theta_0 + \omega_0 t + \frac{1}{2} \alpha_c t^2$$

$$\omega^2 = \omega_0^2 + 2\alpha_c(\theta - \theta_0)$$

For Point  $P$  :

$$s = \theta r \quad v = \omega r \quad a_t = \alpha r \quad a_n = \omega^2 r$$

**Relative General Plane Motion: Translating Axes**

$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}(pin)$$

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}(pin)$$

**Relative General Plane Motion: Trans. and Rot. Axes**

$$\mathbf{v}_B = \mathbf{v}_A + \boldsymbol{\Omega} \times \mathbf{r}_{B/A} + (\mathbf{v}_{B/A})_{xyz}$$

$$\mathbf{a}_B = \mathbf{a}_A + \dot{\boldsymbol{\Omega}} \times \mathbf{r}_{B/A} + \boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{r}_{B/A}) + 2\boldsymbol{\Omega} \times (\mathbf{v}_{B/A})_{xyz} + (\mathbf{a}_{B/A})_{xyz}$$

**KINETICS****Mass Moment of Inertia**

$$I = \int r^2 \, dm$$

Parallel Axis Theorem :

$$I = I_G + md^2$$

Rad. of Gyration :  $k = \sqrt{\frac{I}{m}}$

## Equations of Motion

Particle :  $\mathbf{F} = m\mathbf{a}$

Rigid Body (Plane Motion) :

$$\Sigma F_x = m(a_G)_x \quad \Sigma F_y = m(a_G)_y$$

$$\Sigma M = I_G \alpha$$

## Principle of Work and Energy

$$T_1 + \Sigma U_{1-2} = T_2$$

## Kinetic Energy

Particle :  $T = \frac{1}{2}mv^2$

Rigid Body :

$$T = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\omega^2$$

## Work

Variable Force :

$$U_F = \int F \cos \theta \, ds$$

Constant Force :

$$U_F = (F_c \cos \theta) \Delta s$$

Weight :  $U_W = -W \Delta y$

Spring :  $U_S = -(\frac{1}{2}ks_2^2 - \frac{1}{2}ks_1^2)$

Couple Moment :  $U_M = M\Delta\theta$

## Power and Efficiency

$$P = \frac{dU}{dt} = \mathbf{F} \cdot \mathbf{v}$$

$$\epsilon = \frac{P_{out}}{P_{in}} = \frac{U_{out}}{U_{in}}$$

## Conservation of Energy

$$T_1 + V_1 = T_2 + V_2$$

Potential Energy :

$$V = V_g + V_e, \text{ where } V_g = \pm W_y,$$

$$V_e = \frac{1}{2}ks^2$$

## Principle of Linear Impulse and Momentum

Particle :

$$m\mathbf{v}_1 + \Sigma \int F \, dt = m\mathbf{v}_2$$

Rigid Body :

$$m(\mathbf{v}_G)_1 + \Sigma F \, dt = m(\mathbf{v}_G)_2$$

## Conservation of Linear Momentum

$$\Sigma(\text{sys. } m\mathbf{v})_1 = \Sigma(\text{sys. } m\mathbf{v})_2$$

Coefficient of Restitution :

$$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1}$$

## Principle of Angular Impulse and Momentum

Particle :

$$(\mathbf{H}_O)_1 + \Sigma \int \mathbf{M}_O \, dt = (\mathbf{H}_O)_2$$

where  $H_O = (d)(mv)$

Rigid Body (Plane Motion) :

$$(\mathbf{H}_G)_1 + \Sigma \int \mathbf{M}_G \, dt = (\mathbf{H}_G)_2$$

where  $H_G = I_G\omega$

$$(\mathbf{H}_O)_1 + \Sigma \int \mathbf{M}_O \, dt = (\mathbf{H}_O)_2$$

where  $H_O = I_O\omega$

## Conservation of Angular Momentum

$$\Sigma(\text{sys. } \mathbf{H})_1 = \Sigma(\text{sys. } \mathbf{H})_2$$