Axial Load

Normal Stress: $P = \frac{\sigma}{A}$

Displacement:

$\delta = \int_0^L \frac{P(x)\mathrm{d}x}{A(x)E}$ $\delta = \Sigma \frac{PL}{AE}$

$\delta_T = \alpha \Delta T L$

Torsion

Shear Stress in Circular Shaft:

$\tau = \frac{T\rho}{J}$, where

$J = \frac{\pi}{2} c^4$ Solid cross section

$J = \frac{\pi}{2}(c_o^4 - c_i^4)$ Tubular

Power: $P = T\omega = 2\pi f T$

Angle of Twist:

$\phi = \int_0^L \frac{T(x)\mathrm{d}x}{J(x)G}$ $\phi = \Sigma \frac{TL}{JG}$

Average Shear Stress in a Thin-Walled Tube:

$\tau_{avg} = \frac{T}{2tA_m}$

Shear Flow: $q = \tau_{avg} t = \frac{T}{2A_m}$

Bending

Normal Stress: $\sigma = \frac{My}{I}$

Unsymmetric Bending:

$\sigma = -\frac{M_z y}{I_z} + \frac{M_y z}{I_y}$

$\tan\alpha = \frac{I_z}{I_y}\tan\theta$

Shear

Average Direct Shear Stress:

$\tau_{avg} = \frac{V}{A}$

Transverse Shear Stress:

$\tau = \frac{VQ}{It}$

Shear Flow: $q = \tau t = \frac{VQ}{I}$

Stress in Thin-Walled Pressure Vessel

Cylinder: $\sigma_1 = \frac{pr}{t}$ $\sigma_2 = \frac{pr}{2t}$

Sphere: $\sigma_1 = \sigma_2 = \frac{pr}{2t}$

Stress Transformation Equations

$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta$
$+ \tau_{xy}\sin 2\theta$

$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta$
$+ \tau_{xy}\cos 2\theta$

Principle Stress:

$\tan 2\theta_p = \frac{\tau_{xy}}{(\sigma_x - \sigma_y)/2}$

$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm$
$\sqrt{(\frac{\sigma_x - \sigma_y}{2})^2 + \tau_{xy}^2}$

Maximum In-Plane Shear Stress :

$\tan 2\theta_s = -\frac{(\sigma_x - \sigma_y)/2}{\tau_{xy}}$

$\tau_{max} = \sqrt{(\frac{\sigma_x - \sigma_y}{2})^2 + \tau_{xy}^2}$

$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2}$

Absolute Maximum Shear Stress :

$\tau_{abs\ max} = \frac{\sigma_{max}}{2}$
for σ_{max}, σ_{min} same sign

$\tau_{abs\ max} = \frac{\sigma_{max} - \sigma_{min}}{2}$
for σ_{max}, σ_{min} opposite signs

Material Property Relations

Poisson's Ratio : $\nu = -\frac{\epsilon_{lat}}{\epsilon_{long}}$

Generalized Hooke's Law :

$\epsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)]$
$\epsilon_y = \frac{1}{E}[\sigma_y - \nu(\sigma_x + \sigma_z)]$
$\epsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)]$

$\gamma_{xy} = \frac{1}{G}\tau_{xy}$
$\gamma_{yz} = \frac{1}{G}\tau_{yz}$
$\gamma_{zx} = \frac{1}{G}\tau_{zx}$
where $G = \frac{E}{2(1+\nu)}$

Relations Between *w*, *V*, *M*

$\frac{dV}{dx} = w(x)$, $\frac{dM}{dx} = V$

Elastic Curve

$\frac{1}{\rho} = \frac{M}{EI}$

$EI\frac{d^4v}{dx^4} = w(x)$

$EI\frac{d^3v}{dx^3} = V(x)$

$EI\frac{d^2v}{dx^2} = M(x)$

Buckling

Critical Axial Load :

$P_{cr} = \frac{\pi^2 EI}{(KL)^2}$, where

$K = 1$ Pinned ends

$K = 2$ Free ends

$K = 0.5$ Fixed ends

$K = 0.7$ Pinned and fixed ends

Critical Stress :

$\sigma_{cr} = \frac{\pi^2 E}{(KL/r)^2}$, $r = \sqrt{I/A}$

$KL/r =$ Effective slenderness ratio

Secant Formula : $\sigma_{max} = \frac{P}{A}[1 + \frac{ec}{r^2}\sec(\frac{L}{2r}\sqrt{\frac{P}{EA}})]$

Energy Methods

Conservation of Energy :

$U_e = U_i$

Strain Energy :

$U_i = \frac{N^2 L}{2AE}$ Constant axial load

$U_i = \int_0^L \frac{M^2 dx}{2EI}$ Bending moment

$U_i = \int_0^L \frac{f_s v^2 dx}{2GA}$ Transverse shear

$U_i = \int_0^L \frac{T^2 dx}{2GJ}$ Torsional moment