Engineering Mechanics: Dynamics

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Particle Kinematics

Rectilinear Kinematics

$v = \frac{\mathrm{d}s}{\mathrm{d}t} \quad a = \frac{\mathrm{d}v}{\mathrm{d}t} \quad a\,\mathrm{d}s = v\,\mathrm{d}v$

Projectile Motion

Horizontal Motion: $x = x_0 + (v_0)_x t$

Vertical Motion:

$v_y = (v_0)_y + at$
$y = y_0 + (v_0)_y t + \frac{1}{2}a_c t^2$
$(v_y)^2 = (v_0)_y^2 + 2a(y - y_0)$

Curvilinear Motion

Normal and Tangential Components:

$a_n = \frac{v^2}{\rho} \quad \rho = \frac{[1+(\mathrm{d}y/\mathrm{d}x)^2]^{\frac{3}{2}}}{|\frac{\mathrm{d}^2 y}{\mathrm{d}x^2}|}$
$a = \sqrt{(a_n)^2 + (a_t)^2}$

Polar Coordinates:

$\dot{\theta} = \frac{\mathrm{d}\theta}{\mathrm{d}t} \quad \ddot{\theta} = \frac{\mathrm{d}^2\theta}{\mathrm{d}t^2}$
$\dot{r} = \frac{\mathrm{d}r}{\mathrm{d}t} \quad \ddot{r} = \frac{\mathrm{d}^2 r}{\mathrm{d}t^2}$
$v_r = \dot{r} \quad v_\theta = \dot{\theta}$
$a_r = \ddot{r} - r\dot{\theta}^2 \quad a_\theta = r\ddot{\theta} + 2r\dot{\theta}^2$
$v = \sqrt{(v_r)^2 + (v_\theta)^2}$
$a = \sqrt{(a_r)^2 + (a_\theta)^2}$

Relative Motion

$v_A = v_B + v_{A/B} \quad a_A = a_B + a_{A/B}$

Particle Kinetics

Equations of Motion

Rectangular Coordinates:

$\Sigma F_x = ma_x \quad \Sigma F_y = ma_y \quad \Sigma F_z = ma_z$

Normal and Tangential Coordinates:

$\Sigma F_t = ma_t \quad \Sigma F_n = ma_n \quad \Sigma F_b = 0$

Cylindrical Coordinates:

$\Sigma F_r = ma_r \quad \Sigma F_\theta = ma_\theta \quad \Sigma F_z = ma_z$

Work and Energy

$U = \int_a^b F\,\mathrm{d}s \quad U_s = \frac{1}{2}ks^2 \quad U_W = mgy$
$T = \frac{1}{2}mv^2$

Principle of Work and Energy:

$T_1 + \Sigma U_{1-2} = T_2$

Conservation of Energy

$T_1 + V_1 = T_2 + V_2$

Linear Impulse and Momentum

$mv_1 + \Sigma\int_{t_1}^{t_2} F\,\mathrm{d}t = mv_2$

Conservation of Linear Momentum

$\Sigma\, m_i(v_i)_1 = \Sigma\, m_i(v_i)_2$

Coefficient of Restitution:

$e = \frac{(v_B)_2 - (v_A)_2}{(v_A)_1 - (v_B)_1} \quad 0 \le e \le 1$

Angular Impulse and Momentum

$(\mathbf{H}_O)_1 + \Sigma\int_{t_1}^{t_2} \mathbf{M}_O\,\mathrm{d}t = (\mathbf{H}_O)_2$
$\mathbf{H}_O = \mathbf{r} \times m\mathbf{v}$

Rigid Body Kinematics

Rotation About a Fixed Axis

$\mathbf{v} = \omega \times \mathbf{r} \quad \mathbf{a}_t = \alpha \times \mathbf{r} \quad \mathbf{a}_n = \omega \times (\omega \times \mathbf{r})$
$a = a_t + a_n$
$\omega = \frac{\mathrm{d}\theta}{\mathrm{d}t} \quad \alpha = \frac{\mathrm{d}\omega}{\mathrm{d}t} \quad \alpha\,\mathrm{d}\theta = \omega\,\mathrm{d}\omega$
$1\text{ rev} = 2\pi\text{ rad}$

Constant Angular Acceleration:

$\omega = \omega_0 + \alpha t$
$\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$
$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$

Absolute Motion Analysis

Trigonometry:

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad c^2 = a^2 + b^2 - ab\cos C$

Relative Motion Analysis: Velocity

$\mathbf{v}_B = \mathbf{v}_A + \omega \times \mathbf{r}_{B/A}$

Instantaneous Center of Zero Velocity

$\mathbf{v}_B = \omega \times \mathbf{r}_{B/IC}$

Relative Motion Analysis: Acceleration

$\mathbf{a}_B = \mathbf{a}_A + \alpha \times \mathbf{r}_{B/A} - \omega^2\mathbf{r}_{B/A}$

Rigid Body Kinetics

Equations of Motion

Translation: $\Sigma\vec{F} = m\vec{a}_G$

Rotation: $\Sigma M_G = I_G\alpha \quad \Sigma M_O = \Sigma(M_k)_O$

Work and Energy

Translation: $L = mv_G$

Rotation About a Fixed Axis:

$T = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\,\omega^2$

General Plane Motion:

$T = \frac{1}{2}mv_G^2 + \frac{1}{2}I_G\,\omega^2$
$T = \frac{1}{2}I_{IC}\,\omega^2$
$U_M = \int_{\theta_1}^{\theta_2} M\mathrm{d}\theta$

Conservation of Energy

$T_1 + V_1 = T_2 + V_2$

Impulse and Momentum

Translation: $T = \frac{1}{2}mv_G^2$

Rotation About a Fixed Axis:

$L = mv_G \quad H_G = I_G\omega$
$H_O = I_G\omega + r_G(mv_G)$

General Plane Motion:

$L = mv_G \quad H_G = I_G\omega$
$H_P = I_G\omega + d(mv_G)$

Linear Impulse: $Impulse = \int_{t_1}^{t_2} F\mathrm{d}t$

Angular Impulse: $AngularImpulse = \int_{t_1}^{t_2} M\mathrm{d}t$

Principle of Impulse and Momentum:

$m(v_{Gx})_1 + \Sigma\int_{t_1}^{t_2} F_x\,\mathrm{d}t = m(v_{Gx})_2$
$m(v_{Gy})_1 + \Sigma\int_{t_1}^{t_2} F_x\,\mathrm{d}t = m(v_{Gy})_2$
$I_G\omega_1 + \Sigma\int_{t_1}^{t_2} M_G\,\mathrm{d}t = I_G\omega_2$

Conservation of Momentum

$\Sigma(H_O)_1 = \Sigma(H_O)_2$