SOLUTIONS

Midterm Exam

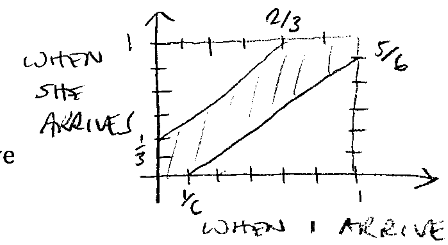
February 26 – March 1, 2013

ECEn 370: Probability Theory
Professor Neal K. Bangerter
Winter 2013

- Exam is timed with a 3-hour time limit. Time spent beyond the 3-hour time limit will be penalized at a rate of 1 point per minute
- Exam is closed book and closed course material
- A single 8.5 x 11" sheet of notes (both sides) of your own making is permitted
- A calculator is permitted (graphing or otherwise)
- Please mark your answers clearly on the provided bubble sheet, and **attach any scratch paper that you use to your exam. If there is anything that you feel is unclear in the exam, please write down your assumptions on the exam booklet and do the best you can. I will consider these cases.**
- Scriptures are permitted. ☺
- There are 27 multiple-choice problems on the exam, and a total of 45 points possible. Each problem is worth 1 or 2 points as marked on the exam.
- Please observe the honor code and refrain from discussing the test with fellow students who have not yet taken it.

1. You have set up a date for tomorrow evening at 6pm, but neither you nor your date is very punctual. Assume that you and your date will each arrive with a delay between 0 and 1 hour, with all pairs of delays being equally likely. You are willing to wait for a maximum of 20 minutes before you leave if your date doesn't show up. However, your date is only willing to wait a maximum of 10 minutes before leaving if you don't arrive. What is the probability that you and your date will meet? (2 points)

$\boxed{\text{(a) 0.4306}}$
(b) 0.4375
(c) 0.4444
(d) 0.8194
(e) None of the above



$$\frac{\text{SHADED AREA}}{\text{TOTAL AREA}} = \frac{1-\frac{1}{2}\left(\frac{2}{3}\right)^2-\frac{1}{2}\left(\frac{5}{6}\right)^2}{1} = \boxed{0.4306}$$

2. After graduating from BYU, you take your first job as an electrical engineer. Your boss knows that you took ECEn 370, and assigns every probability problem that your design team encounters to you. Unfortunately, you didn't pay much attention during ECEn 370, and have gotten into the habit of calling up two of your friends, Bob and Sue, and asking them to solve the probability problems for you.

You've done this so many times that you know the following:

- Bob succeeds in solving your problem 2/3 of the time.
- Sue succeeds in solving your problem 1/2 of the time.
- The probability that at least one of them solves your problem is 3/4.

You've just called up Bob and Sue and asked them to solve another problem. Assuming that only **one** of them succeeds in solving the problem, what is the probability that it will be Bob? (2 points)

(a) 3/12
$\boxed{\text{(b) 3/4}}$
(c) 2/3
(d) 1/4
(e) None of the above

BOB SUE

$$\Omega = \{SS, SF, FS, FF\}$$

$$P(SS) + P(SF) = \frac{2}{3}$$
$$P(FS) + P(SS) = \frac{1}{2}$$
$$P(SS) + P(SF) + P(FS) = 3/4$$
$$P(SS) + P(SF) + P(FS) + P(FF) = 1$$

4 EQNS, 4 UNKNOWNS

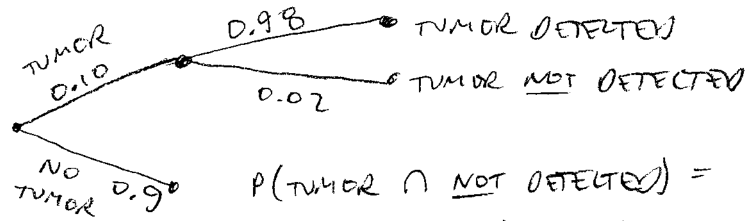
$$P(SS) = \frac{5}{12}$$
$$P(FF) = \frac{1}{4}$$
$$P(SF) = \frac{1}{4}$$
$$P(FS) = \frac{1}{12}$$

I WANT:

$$P(SF \mid \{SF, FS\}) = \frac{P(SF \cap \{SF, FS\})}{P(SF) + P(FS)} = \frac{P(SF)}{P(SF)+P(FS)} = \frac{\frac{1}{4}}{\frac{1}{4}+\frac{1}{12}} = \boxed{3/4}$$

3. You have designed a new algorithm that takes an MRI image of the brain and attempts to automatically determine if the subject has a brain tumor. If a brain tumor is actually present, your algorithm identifies it with a probability of 0.98. If a brain tumor is **not** present, your algorithm generates a false alarm (i.e., indicates that there is a brain tumor when there isn't) with probability 0.20. Assume that 10% of the brains analyzed with your new algorithm actually have a tumor. What is the probability that a brain analyzed with your new algorithm has a tumor that is not detected? (1 point)

(a) 0.2%
(b) 9.8%
(c) 1.8%
(d) 8%
(e) 7.2%
(f) None of the above



$P(\text{TUMOR} \cap \text{NOT DETECTED}) =$

$(0.10)(0.02) = 0.002 = \boxed{.2\%}$

4. Professor Bangerter buys 20 Wonka candy bars and inserts a golden ticket into 4 of them. He then randomly distributes the 20 bars to his 4 favorite students, giving each 5 bars. What is the probability that all four students receive a golden ticket? (2 points)

(a) 0.1407
(b) 0.1096
(c) 0.8000
(d) 0.2500
(e) 0.1290
(f) None of the above

X X X X ← CANDY BARS W/ GOLDEN TICKET

2nd HAS 15 PLACES WHERE NOT IN GROUP W/ 1ST...

$\frac{20}{20} \cdot \frac{15}{19} \cdot \frac{10}{18} \cdot \frac{5}{17} = \boxed{0.1290}$

1ST CAN GO ANYWHERE!

5. You are on the Monty Hall Show. There are three closed doors in front of you, and you are told that there is $1,000,000 behind one of the doors. The money is equally likely to be found behind any one of the three doors. You point to one of the doors, and then the host opens one of the **remaining** two doors after making sure the money is not behind it. At this point, you can either stick to your initial choice, or switch to the other unopened door. You win the money if it lies behind your final choice of door. What is the **difference** (rounded to the nearest dollar) in the expected value of your prize money if you (1) switch to the other unopened door after your initial guess, or (2) stick with your initial guess? (2 points)

(a) $0
(b) $250,000
(c) $333,333
(d) $666,667
(e) $1,000,000
(f) None of the above

STICK W/ INITIAL GUESS: $P(\text{WIN}) = \frac{1}{3}$

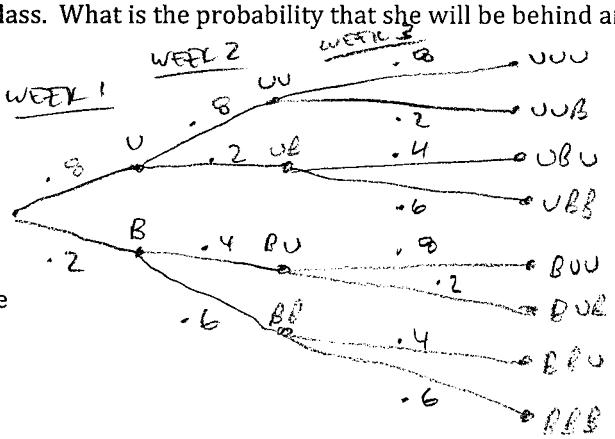
$E(\$\$) = \frac{1}{3} \cdot \$1M = \$333,333.33$

CHANGE DOOR: $P(\text{WIN}) = \frac{2}{3}$

$E(\$\$) = \frac{2}{3} \cdot \$1M = \$666,666.67$

6. Matilda is taking a probability class and at the end of each week she can be either up-to-date or she may have fallen behind. If she is up-to-date in a given week, the probability that she will be up-to-date (or behind) in the next week is 0.8 (or 0.2, respectively). If she is behind in a given week, the probability that she will be up-to-date (or behind) in the next week is 0.4 (or 0.6, respectively). Assume that Matilda is, by default, up-to-date when she starts the class. What is the probability that she will be behind after three weeks? (2 points)

(a) 0.800
(b) 0.200
(c) 0.720
(d) 0.280
(e) 0.688
(f) 0.312
(g) None of the above



$$P(\text{BEHIND AFTER 3 WEEKS}) = P(UUB) + P(UBB) + P(BUB) + P(BBB)$$

$$= (0.8)^2(0.2) + (0.8)(0.2)(0.6) + (0.2)(0.4)(0.2) + (0.2)(0.6)^2$$

$$= \boxed{0.312}$$

7. Scooby Snack allergies are exceedingly rare, but they do occur in 0.1% of students. Assume that all students enrolling in Dr. Bangerter's class are tested for a Scooby Snack allergy. If a student has a Scooby Snack allergy, the test results are positive with probability 0.95. If a student does not have a Scooby Snack allergy, the test results are negative with probability 0.90. Dr. Bangerter accidentally gives a Scooby Snack to a student who tested positive for a Scooby Snack allergy. What is the probability that the student actually has the allergy? (1 point)

(a) 0.0047
(b) 0.0094
(c) 0.0011
(d) 0.0876
(e) 0.5135
(f) None of the above

$A = \{\text{HAS ALLERGY}\}$

$B = \{\text{TESTS POSITIVE}\}$

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B|A)P(A)}{P(B)}$$

BAYES' RULE $\Rightarrow$

$$= \frac{P(B|A)P(A)}{P(B|A)P(A) + P(B|A^c)P(A^c)}$$

$$= \frac{(0.95)(0.001)}{(0.95)(0.001) + (0.10)(0.999)}$$

$$= \boxed{0.0094}$$

8. Consider an experiment involving two successive rolls of a 4-sided die in which all 16 possible outcomes are equally likely and have probability 1/16. Define the following events:

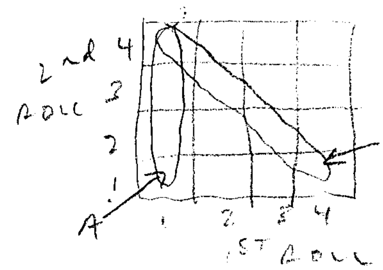
A = {1st roll is a 1}
B = {sum of the two rolls is 5}
C = {maximum of the two rolls is 2}
D = {minimum of two rolls is 2}

Which of the following is true? (2 points)

(a) Events A, B, and C are independent
(b) Events A and B are independent of each other, but events C and D are not independent of each other
(c) Events A and B are independent of each other, and events C and D are independent of each other
(d) All four events (A, B, C, and D) are independent of each other

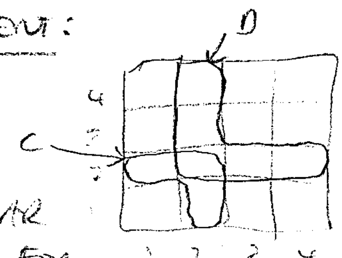
TO SEE INDEPENDENCE OF A, B:

$P(A) = \frac{4}{16} = \frac{1}{4}$
$P(B) = \frac{4}{16} = \frac{1}{4}$
$P(A \cap B) = \frac{1}{16} = P(A)P(B)$ ✓



TO SEE THAT C AND D ARE NOT INDEPENDENT:

$P(C) = \frac{3}{16}$
$P(D) = \frac{5}{16}$
$P(C \cap D) = \frac{1}{16} \neq P(C)P(D)$



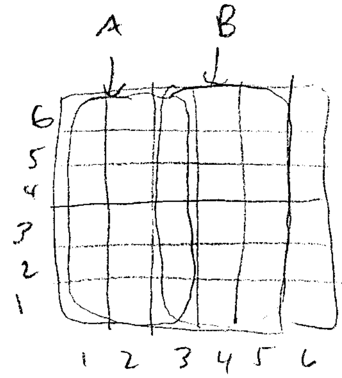
CAN USE SIMILAR ARGUMENTS FOR OTHERS..

9. Consider two independent rolls of a fair six-sided die, and the following events:

A = {1st roll is 1, 2, or 3}
B = {1st roll is 3, 4, or 5}
C = {the sum of the two rolls is 9}

Are the events A, B, and C independent? (1 point)

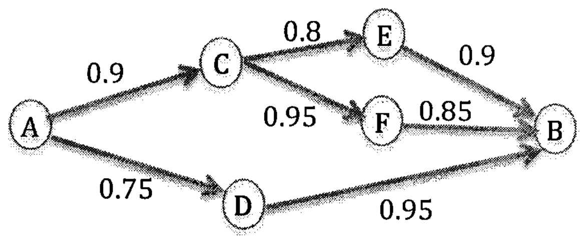
(a) Yes
(b) No



$P(A) = \frac{1}{2}$
$P(B) = \frac{1}{2}$
$P(A \cap B) = \frac{1}{6} \neq P(A)P(B)$!

SO A & B ARE NOT INDEPENDENT!

10. A computer network connects two nodes A and B through a network as shown below. The probability that each connection between intermediate nodes is "up" (i.e., that transmission is successful between those two nodes) is labeled on the diagram. We assume that link failures are independent of each other. What is the probability that there is a path connecting A and B in which all links are up? (2 points)



$C \to B$:
$(0.95)(0.85) = 0.8075$
OR $(0.8)(0.9) = 0.72$

$P(C \to B) = 1-(1-0.8075)(1-0.72)$
$= 0.9461$

$P(A \to B) = (0.9461)(0.9) \,||\, (0.75)(0.95)$
$= 0.8515 \,||\, 0.7125$
$= 1-(1-0.8515)(1-0.7125) = \boxed{0.9573}$

- $\boxed{\text{(a) } 0.957}$
- (b) 0.946
- (c) 0.851
- (d) 0.712
- (e) 0.950
- (f) None of the above

11. How many different words (letter sequences) can be obtained by rearranging the letters in the word MISSISSIPPI? (1 point)

- (a) 415,800
- $\boxed{\text{(b) } 34,650}$
- (c) 831,600
- (d) 369
- (e) None of the above

$$\binom{11}{4,4,2,1} = \frac{11!}{4!4!2!1!} = \boxed{34,650}$$

12. Let X be a discrete random variable that can assume integer values on the interval [-4, 4]. Assume that X is uniformly distributed across this interval. Let $Z = X^2 + \text{var}(X)$. What is the probability that Z will equal 7 2/3? (2 points)

- (a) 1/9
- $\boxed{\text{(b) } 2/9}$
- (c) 1/3
- (d) 4/9
- (e) 5/9
- (f) 2/3
- (g) None of the above

FIND VAR(X): $E[X] = 0$

$E[X^2] = 16\left(\frac{2}{9}\right) + 9\left(\frac{2}{9}\right) + 4\left(\frac{2}{9}\right) + 1\left(\frac{2}{9}\right) + 0\left(\frac{1}{9}\right)$
$= \frac{60}{9} = 6\,\frac{2}{3}$

SO: $\text{VAR}(X) = 6\,\frac{2}{3}$

SO:

$Z = X^2 + 6\,\frac{2}{3}$

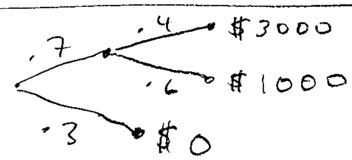
IF: $Z = 7\,\frac{2}{3}$, $X^2 + 6\,\frac{2}{3} = 7\,\frac{2}{3}$
$X^2 = 1$

$P(Z = 7\,\frac{2}{3}) = P(X^2 = 1) = \boxed{\frac{2}{9}}$

13. Consider a quiz game where a person is given two questions and must decide which one to answer first. Question 1 will be answered correctly with probability 0.7, and the person will then receive as prize $1,000, while question 2 will be answered correctly with probability 0.4, and the person will then receive as prize $2,000. If the first question attempted is answered incorrectly, the quiz terminates, and the person is not allowed to attempt the second question. If the first question is answered correctly, the person is allowed to attempt the second question. Assume that the person answering the questions is an expert in probability, and will order the questions so as to maximize his/her prize money. What is the expected value of the prize money in this case? (2 points)

(a) $960
(b) $1,080
(c) $1,260
~~(d) $1,500~~
(e) $1,680
(f) None of the above

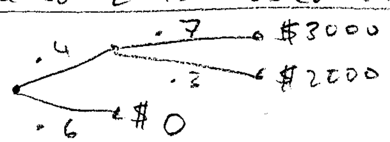
IF QUESTION 1 IS ASKED FIRST:



$E[X] = (.7)(.4)(3000) + (.7)(.6)(1000) = \boxed{\$1,260}$

↑ BIGGER!

IF QUES. 2 IS ASKED FIRST:



$E[X] = (.4)(.7)(3000) + (.4)(.3)(2000) = \$1,080$

14. You go to a party with 500 guests. **Estimate** the probability that exactly one other guest has the same birthday as you using a Poisson distribution. (Exclude birthdays on February 29th. That is, assume that there are 365 possible birthdays, and that each is equally probable for any given individual.) (1 point)

1. p < 0.2
2. 0.2 ≤ p < 0.3
3. 0.3 ≤ p < 0.4
4. 0.4 ≤ p < 0.5
5. 0.5 ≤ p < 0.6
6. p ≥ 0.6

$\lambda = np$

X = # OF GUESTS W/ SAME BIRTHDAY AS YOU = BINOMIAL W/ $n = 500$, $p = \frac{1}{365}$

$$P_X(1) \approx e^{-\lambda}\frac{\lambda^1}{1!} = \frac{e^{-\frac{500}{365}}\left(\frac{500}{365}\right)^1}{1!} = 0.3481$$

15. An internet service provider uses 25 modems to serve the needs of 2,000 customers. It is estimated that at a given time, each customer will need a connection with a probability of 0.02, independent of the other customers. In what range is the expected value of the number of modems in use at a given time? (2 points)

(a) [0, 10)
(b) [10, 15)
(c) [15, 20)
(d) [20, 30)
(e) [30, 100)
(f) [100, 200]

LET:
X = # OF CUSTOMERS WHO WANT A MODEM = BINOMIAL W/ $n = 2000$, $p = .02$

$E[X] = 40$

SO 40 CUSTOMERS ON AVERAGE WANT A MODEM AT ANY GIVEN TIME.

E[Y] WILL BE JUST UNDER 25!

LET: Y = # OF CUSTOMERS WHO GET A MODEM AT ANY GIVEN TIME

THEN:

$$P_Y(k) = \begin{cases} P_X(k), & k = 0, \dots, 24 \\ \sum_{k=25}^{2000} P_X(k), & k = 25 \\ 0 & \text{else} \end{cases}$$

SO $P_Y(25)$ WILL BE CLOSE TO 1!!

16. You just rented a large house and the realtor gave you 5 keys, one for each of the 5 doors of the house. Unfortunately, all keys look identical, so to open the front door, you try them at random. At each trial you are equally likely to choose any key, independent of whether or not you have tried it before. What is the expected number of trials you will need in order to open the front door? (1 point)

(a) 1.25
(b) 2.5
(c) 3
(d) 4
(e) 4.5
(f) 5
(g) 7.5
(h) None of the above

X = GEOMETRIC w/ $p = \frac{1}{5} \Rightarrow$ # OF TRIES UNTIL SUCCESS

$$E[X] = \frac{1}{p} = \frac{1}{\left(\frac{1}{5}\right)} = \boxed{5}$$

17. A prize is randomly placed in one of ten boxes, numbered 1 to 10. You search for the prize by asking yes-no questions. Assume that you adopt a bisection strategy, where you eliminate as close to half of the remaining boxes as possible by asking questions of the form "is it in a box numbered less than or equal to *k*?". What is the expected number of questions needed before you find the box using this strategy? (2 points)

(a) 5.5
(b) 3.4
(c) 1.7
(d) 2.7
(e) None of the above

$\Omega = \{1, 2, \ldots, 10\}$

$P(1) = P(2) = \ldots = P(10) = \frac{1}{10}$

THERE ARE 4 BOXES THAT TAKE 4 QUESTIONS TO "DISCOVER" AND 6 BOXES THAT TAKE 3 QUESTIONS!

$$\left(\frac{4}{10}\right)(4) + \left(\frac{6}{10}\right)(3) = \boxed{3.4}$$

18. Two coins are simultaneously tossed until one of them comes up a head and the other a tail. The first coin comes up a head with probability 0.7 and the second with probability 0.5. All tosses are assumed independent. What is the probability that the last toss of the first coin is a head? (1 point)

(a) 0.5
(b) 0.6
(c) 0.7
(d) 0.8
(e) 0.9
(f) None of the above

ON ANY GIVEN TOSS: $\Omega = \{HH, HT, TH, TT\}$

$P(HH) = (0.7)(0.5) = 0.35$
$P(HT) = (0.7)(0.5) = 0.35$
$P(TT) = (0.3)(0.5) = 0.15$
$P(TH) = (0.3)(0.5) = 0.15$

LAST TOSS MUST BE EITHER $\{HT\}$ OR $\{TH\}$!

SO WE WANT:

$$P(HT \mid \{HT, TH\}) = \frac{P(HT)}{P(HT) + P(TH)} = \frac{0.35}{0.35 + 0.15}$$

$$= \frac{0.35}{0.5} = \boxed{0.7}$$

19. On any given attempt at taking the ACT, your score ranges from 29 to 31, each with equal probability and independent of your score on any other attempt. If your final score is defined as the maximum of all of your attempts, how much do you improve the expected value of your final score by taking the test 3 times vs. 1 time? (2 points)

(a) Expected final score increases by 2/3 of a point
(b) Expected final score increases by 1/2 of a point
(c) Expected final score increases by 1/3 of a point
(d) Expected final score increases by 1 point
(e) None of the above

ON A SINGLE ATTEMPT

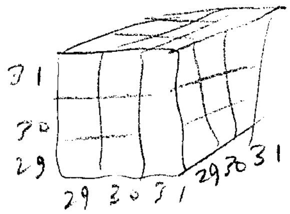
$E(X_1) = 30$

ON 3 ATTEMPTS:

$X = \max(X_1, X_2, X_3)$

$E[X] = \left(\frac{1}{27}\right)(29) + \left(\frac{7}{27}\right)(30) + \left(\frac{19}{27}\right)(31) = 30\frac{2}{3}$

INCREASE OF 2/3!



20. You toss independently a fair coin and you count the number of tosses until the first tail appears. If this number is n, you receive 2^n dollars. However, the number of tosses is limited to 20. If after the 20th toss, a tail still hasn't appeared, you receive nothing and the game ends (which, I should point out, is a serious bummer since you would have received $1,048,576 if that 20th toss had been a tail). What is the expected amount of money you receive playing this game? (1 point)

(a) $20
(b) $40
(c) $60
(d) $80
(e) None of the above

X = # OF TOSSES UNTIL 1ST TAIL ⟹ GEOMETRIC!

$P_X(k) = \left(\frac{1}{2}\right)^k$

$Z = \text{PAYOUT} = 2^{k} X$

$E[Z] = \sum_{k=1}^{20} 2^k \left(\frac{1}{2}\right)^k = \sum_{k=1}^{20} 1 = \boxed{\$20}$

21. Consider four independent rolls of a 6-sided die. Let X be the number of 1s and let Y be the number of 6s obtained. What is probability that $X = 2$ and $Y = 1$? (2 points)

(a) 0.0059
(b) 0.0040
(c) 0.1852
(d) 0.0370
(e) None of the above

6 6 6 6

TOTAL COMBINATIONS = 6^4, EACH W/ PROB. $\frac{1}{6^4}$

HOW MANY HAVE 2 1'S, 1 6, AND A 2,3,4, OR 5 ON THE OTHER?

WAYS TO PUT 2 1'S ON 4 DIES

WAYS TO PUT A 6 ON REMAINING 2 DIES

$\frac{\binom{4}{2}\binom{2}{1}4}{6^4} = \boxed{0.0370}$

LAST DIE MUST BE 2,3,4, OR 5!

22. Professor Bangerter branches out and starts offering 5 different kinds of Scooby Snack for saying the opening prayer in class. Assuming that you are equally probable to receive any one of the 5 different kinds of Scooby Snacks for saying the prayer (independent of what you receive other times), how many times do you expect to say the prayer before you have received each of the possible Scooby Snacks at least once? (2 points)

(a) 11.4
(b) ~~14.7~~
(c) 10.4
(d) 13.7
(e) None of the above

1 + E[GEOMETRIC R.V. w/ $p = \frac{4}{5}$]
+ E[GEO. R.V. w/ $p = 3/5$]
+ E[GEO. R.V. w/ $p = 2/5$]
+ E[GEO. R.V. w/ $p = \frac{1}{5}$]

$= 1 + 5/4 + 5/3 + 5/2 + 5 = \boxed{11.42}$

23. You purchased a new laptop **6 months ago**, and are now departing on a trip to the Congo to live with the great apes. You will be gone exactly **1 year**. In the user manual for your laptop, you find the following helpful information:

"Extensive statistical testing on this model of laptop has shown that the time until failure is well modeled by an exponential random variable with an expected value of 4 years (λ = 0.25)."

You know that you won't be able to replace your laptop while in the Congo. What is the probability that your laptop will fail while you are gone? (1 point)

EITHER OKAY

(a) 0.2212
(b) 0.7788
(c) 0.8048
(d) 0.1329
(e) 0.9817
(f) 0.1952
(g) None of the above

IF YOU ASSUME THE LAPTOP IS WORKING AT 6 MONTHS:

$$\int_{\frac{1}{2}}^{3/2} \lambda e^{-\lambda x}\, dx = \boxed{0.1952}$$

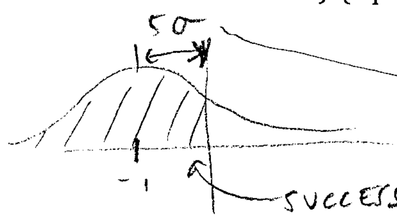
IF YOU DON'T KNOW IF THE LAPTOP BURNED OUT IN 1ST 6 MONTHS:

$$\frac{\int_{\frac{1}{2}}^{3/2} \lambda e^{-\lambda x}\, dx}{\int_{\frac{1}{2}}^{\infty} \lambda e^{-\lambda x}\, dx} = \int_0^1 \lambda e^{-\lambda x}\, dx = \boxed{0.2212}$$

24. A binary message is transmitted as a signal s, which is either -1 or +1. The communication channel corrupts the transmission with additive normal (Gaussian) noise with mean 0 and variance 0.04. The receiver concludes that the signal -1 (or +1) was transmitted if the value received is < 0 (or ≥ 0, respectively). What is the probability that a 10-bit message is transmitted correctly? (Assume that the 10-bit message is transmitted sequentially, and that any error in the transmission of a bit is independent of errors in the transmission of other bits.) (2 points)

(a) $(\Phi(0.04))^{10}$
(b) $\Phi(0.04)$
(c) $\boxed{(\Phi(5))^{10}}$
(d) $\Phi(5)$
(e) $1 - \Phi(0.04)$
(f) $1 - \Phi(25)$
(g) $(1 - \Phi(0.04))^{10}$
(h) $(1 - \Phi(25))^{10}$
(i) None of the above

$\sigma = \sqrt{0.04} = 0.2$



SUCCESSFUL TRANSMISSION w/ PROBABILITY $\Phi(5)$

So: $[\Phi(5))^{10}$

25. You have the following joint PMF of random variables X and Y:

y \ x	1	2	3	4
3	1/10	0	1/10	0
2	1/10	0	1/10	1/10
1	1/10	1/10	2/10	1/10

Evaluate the following: $E[X] + E[Y] + \text{var}(X)$ (2 points)

(a) $\boxed{5.4500}$
(b) 4.2000
(c) 3.6500
(d) 3.2500
(e) 2.7000
(f) None of the above

$$p_X(x) = \begin{cases} \frac{3}{10}, & x=1 \\ \frac{1}{10}, & x=2 \\ \frac{4}{10}, & x=3 \\ \frac{2}{10}, & x=4 \end{cases} \qquad p_Y(y) = \begin{cases} \frac{2}{10}, & y=3 \\ 3/10, & y=2 \\ \frac{5}{10}, & y=1 \end{cases}$$

$$E[X] = \frac{3}{10} + \frac{2}{10} + \frac{12}{10} + \frac{8}{10} = \frac{25}{10} = \boxed{2.5} = 5/2$$

$$E[Y] = \frac{6}{10} + \frac{6}{10} + \frac{5}{10} = \frac{17}{10} = \boxed{1.7}$$

$$E[X^2] = \frac{3}{10} + \frac{4}{10} + \frac{36}{10} + \frac{32}{10} = \frac{75}{10}$$

$$\text{var}[X] = \frac{75}{10} - \frac{25}{4} = \boxed{1.25}$$

$2.5 + 1.7 + 1.25$

$= \boxed{5.45}$

26. A random variable is called a Rayleigh random variable if its PDF is given by:

$$f_X(x) = \begin{cases} \frac{x}{\sigma^2} e^{-x^2/(2\sigma^2)} & x > 0 \\ 0 & x < 0 \end{cases}$$

The radial distance (in meters) of the landing point of a parachuting sky diver from the center of a target area is known to be a Rayleigh random variable with parameter σ^2 = 100. Find the probability that the sky diver will land within a radius of 10 m from the center of the target area. (2 points)

(a) 0.381
(b) 0.393
(c) 0.405
(d) 0.418
(e) 0.426
(f) None of the above

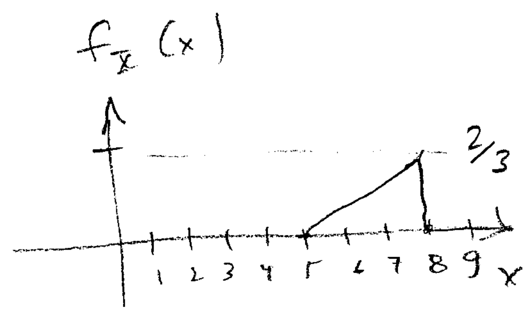
$P(0 \le X \le 10) = \int_0^{10} f_X(x)\,dx = \boxed{0.393}$

27. A random variable *X* has a probability density function given by:

$$f_X(x) = \begin{cases} \frac{2(x-5)}{9}, & \text{if } 5 \le x \le 8, \\ 0, & \text{otherwise.} \end{cases}$$

Find the cumulative distribution function $F_X(x)$ and compute the value of the following expression: $F_X(1) + F_X(7) + F_X(9)$ (2 points)

(a) 0
(b) 1
(c) 5/3
(d) 13/9
(e) 16/9
(f) 17/9
(g) None of the above



$F_X(1) = \int_{-\infty}^{1} f_X(x)\,dx = 0$

$F_X(7) = \int_{-\infty}^{7} f_X(x)\,dx = \frac{4}{9}$

$F_X(9) = \int_{-\infty}^{9} f_X(x)\,dx = 1$

$0 + \frac{4}{9} + 1 = \boxed{\frac{13}{9}}$