HW #1 / 1, 2, 5, 6, 14, 17, 20, 24, 27

1) ROLL 6-SIDED DIE (EXPERIMENT)
$A = \{\text{Roll is even}\} = \{2,4,6\}$
$B = \{\text{Roll is} > 3\} = \{4,5,6\}$

$(A \cup B)^c = (\{2,4,6\} \cup \{4,5,6\})^c = (\{2,4,5,6\})^c = \underset{A^c}{\{1,3,5\}} \cap \underset{B^c}{\{1,2,3\}} = \{1,3\} = A^c \cap B^c$ ✓

$(A \cap B)^c = (\{2,4,6\} \cap \{4,5,6\})^c = (\{4,6\})^c = \{1,2,3,5\} = \underset{A^c}{\{1,3,5\}} \cup \underset{B^c}{\{1,2,3\}} = A^c \cup B^c$ ✓

2) a) $A^c = (A^c \cap B) \cup (A^c \cap B^c) = A^c \cap (B \cup B^c) = A^c \cap \Omega = A^c$

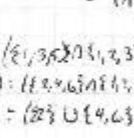
$B^c = (A \cap B^c) \cup (A^c \cap B^c) = (A \cup A^c) \cap B^c = \Omega \cap B^c = B^c$

b) $(A \cap B)^c = (A^c \cap B) \cup (A^c \cap B^c) \cup (A \cap B^c)$
$(A \cap B)^c = A^c \cup B^c$
FROM a) $A^c = (A^c \cap B) \cup (A^c \cap B^c)$ $\Rightarrow (A^c \cap B) \cup \cancel{(A^c \cap B^c)} \cup (A \cap B^c) \cup (A^c \cap B^c)$
$B^c = (A \cap B^c) \cup (A^c \cap B^c)$
$= (A^c \cap B) \cup (A^c \cap B^c) \cup (A \cap B^c)$

c) $A = \{1,3,5\}$
$B = \{1,2,3\}$ $(A \cap B)^c = (\{1,3,5\} \cap \{1,2,3\})^c = (\{1,3\})^c = \{2,4,5,6\}$
$(A^c \cap B) \cup (A^c \cap B^c) \cup (A \cap B^c) = (\{2,4,6\} \cap \{1,2,3\}) \cup (\{2,4,6\} \cap \{4,5,6\}) \cup (\{1,3,5\} \cap \{4,5,6\}) =$
$= (\{2\} \cup \{4,6\} \cup \{5\}) = \{2,4,5,6\}$

5) $P(B) = 40\%$
$P(G) = 60\%$
$P(C) = 70\%$

$B = G \cap C$



$P(C|G) = 40$ $P(C|G) = 2/3$
$P(\bar{C}|G) = 1/3$

$P(\bar{G}|\bar{C}) = \dfrac{P(\bar{G} \cap \bar{C})}{P(\bar{C})}$

$P(G|C) = \dfrac{P(G \cap C)}{P(C)} = \dfrac{40}{70\%} = \cancel{60}$ $= \cancel{P(G \cap C)}$ 28%

$P(G \cap C) = \frac{4}{7}$

$= \dfrac{P((G \cup C)^c)}{P(\bar{C})} = \dfrac{(P(G) + P(C) - P(G \cap C))^c}{\cancel{30}} = \dfrac{(90\%)^c}{} = \dfrac{10\%}{\cancel{30\%}} = \frac{1}{7} = \boxed{.1}$

$P(G^c \cap C^c) = 1 - P(G \cup C) = 1 - (.9)$

6) EXPERIMENT = 1 ROLL OF DIE
$\Omega = \{1,2,3,4,5,6\}$

$P(1) = 1/9$
$P(2) = 2/9$
$P(3) = 1/9$
$P(4) = 2/9$
$P(5) = 1/9$
$P(6) = 2/9$

$P(<4) = P(1) + P(2) + P(3) = \boxed{4/9}$

14) a) $P(1_2 \cap 1_1) = P(1_1) P(1_2 | 1_1) = \frac{1}{6}\left(\frac{1}{6}\right) = \frac{1}{36} \times 6 = \boxed{1/6}$
$P(\text{DOUBLES}) = 6/36$

b) $P(\text{sum} \le 4) = 6/36 = 1/6$
$P(\text{DOUBLES} | \text{sum} \le 4) = \dfrac{P(\text{DOUBLES} \cap \text{sum} \le 4)}{P(\text{sum} \le 4)} = \dfrac{2/36}{1/6} = \boxed{\cancel{1/3}} = 2/6 = \boxed{1/3}$

c) $P(\text{AT LEAST 1 } 6) = \boxed{11/36}$

d) $P(\text{DIFF}) = 5/6$ $P(6) = 1/36$
$P(6 | \text{DIFF}) = \dfrac{P(6 \cap \text{DIFF})}{P(\text{DIFF})} = \dfrac{10/36}{5/6} = \dfrac{2}{6} = \boxed{1/3}$

17) $P(G) = 95/100 = 19/20$

~~$P(A) + P(G) + P(G_2) + P(G_3) + P(G_4)$~~ $= \frac{4}{20} = \boxed{\frac{1}{5}}$

$G_4 = G_1 \cap G_2 \cap G_3 \cap G_4 \Rightarrow P(G_1)P(G_2|G_1)P(G_3|G_1 \cap G_2)P(G_4|G_1 \cap G_2 \cap G_3)$

$= \frac{95}{100}\left(\frac{94}{99}\right)\left(\frac{93}{98}\right)\left(\frac{92}{97}\right) = \boxed{.812}$

20) TIMID → $P_d > 0$ (DRAW), $1 - P_d$ (LOSE)
BOLD → P_w (WIN), $1 - P_w$

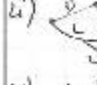
a)

i) $P_w(P_w) \Rightarrow P_w^n$ | n = # of GAMES

$P_w^2(\ldots) + P_w^2(1-P_w) + P^2(1-P_w)$

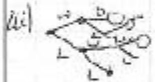
$P_w^2 + 2P_w^2(1-P_w)$

ii) $P = \boxed{P_w^2 + 2P_w^2(1-P_w)}$



$P_d(P_d)P_w = \boxed{P_d^2 P_w}$

iii)



$P_w(P_d) + P_w(1-P_d)(P_w) + (1-P_w)(P_w)(P_w)$

$P_wP_d + P_w^2 - P_w^2P_d + P_w^2 - P_w^3$

$= \boxed{P_wP_d + 2P_w^2 - P_w^2P_d - P_w^3} = P_wP_d + P_w(1-P_d)P_w + (1-P_w)P_w^2$

b) w/ $P_w < 1/2$ iii → $\frac{1}{2}P_d + \frac{1}{2} - \frac{1}{4}P_d - \frac{1}{8} = \frac{1}{4}P_d + \frac{3}{8}$

ASSUME P_w INFINITELY CLOSE TO 1/2

SO WITH $P_w = .45$ AND $P_d = .9$ iii) ⇒ P(BORIS WINS) = .45(.9) + .45(.1).45 + (.55).45² = ~~…~~ .554

WHEN BORIS CAN CHOOSE HIS PLAYING STYLE KNOWING THE RESULT OF THE FIRST GAME OR EACH GAME, BUT HIS OPPONENT CANNOT. SO HE GAINS AN ADVANTAGE BY KNOWING THE OUTCOME OF THE MATCH.

24) KNOWING WHO THE OTHER PRISONER WHO WILL BE RELEASED DOES NOT CHANGE HIS PROBABILITY OF RELEASE, THE PROBABILITY THAT HE IS RELEASED GIVEN ONE OTHER IS ALREADY RELEASED IS 1/2.

$P(R) = 2/3 = P$

HOWEVER SINCE HE IS JUST AS LIKELY TO BE RELEASED FIRST AS THE OTHERS HE HAS $1/3 + 1/6 + 1/6$ CHANCE $= 2/3$.

$P = A, B, X$

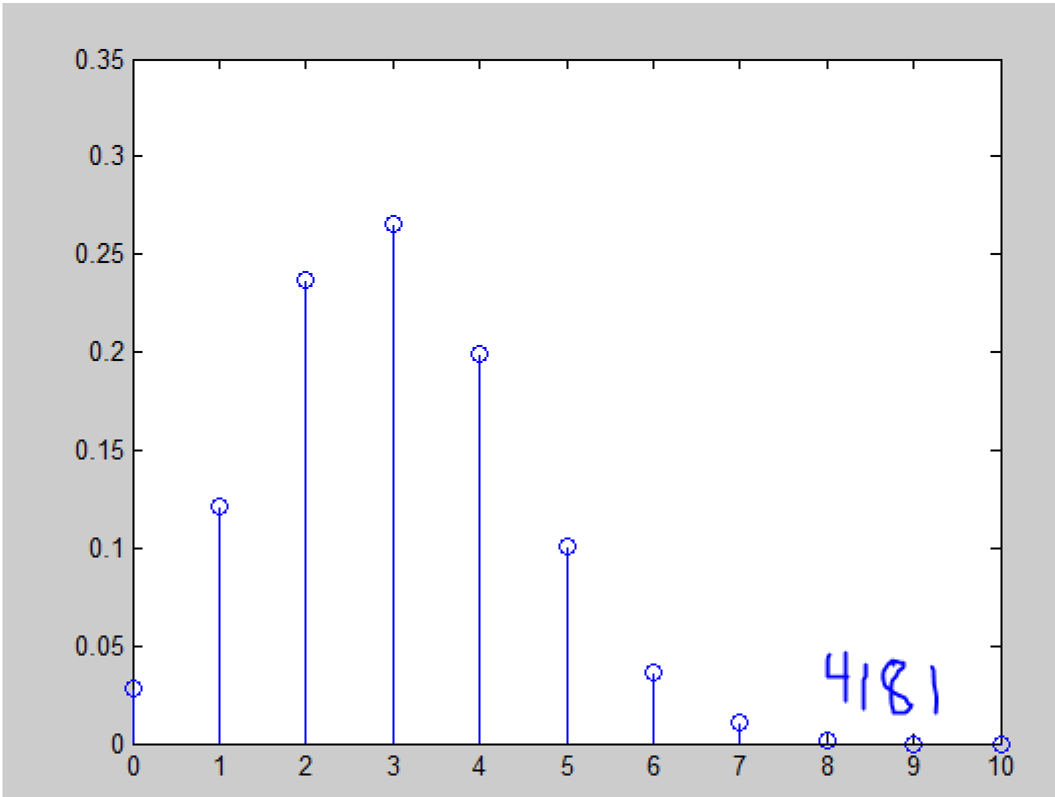
$P(X) = P(A)P(X|A) + P(B)P(X|B) + P(X) = \frac{1}{6} + \frac{1}{6} + \frac{1}{3} = \frac{2}{3}$

$\quad\; \frac{1}{3}\;\frac{1}{2} \quad + \frac{1}{3}\;\frac{1}{2} \quad + \frac{1}{3}$

27) A = n tosses
B = all tosses

$P(\text{BOB WINS}) = P(A^+)\frac{1}{2} + P(B^+)\frac{1}{2}$ ~~…~~ $+ (1 - 2P(B^+))\frac{1}{2}$

$= P(B^+) + \frac{1}{2} - P(B^+) = \boxed{1/2}$


0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0
1
2
3
4
5
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7
8
9
10
4181