1) B-3.34

MACHINE MINTS COINS W/ $P(H) = P$

PDF $f_P(p) = \begin{cases} pe^p, & p \in [0,1] \\ 0, & \text{else} \end{cases}$

$u = p^2 \quad du = 2p\,dp$

$dv = e^p \quad v = e^p$

$uv - \int v\,du$

$p^2 \; e^p$
$2p \; e^p$
$2 \; e^p$
$0 \; e^p$

a) FIND $P(A) = \int_0^1 P(A|P=p) f_P(p)\,dp = \int_0^1 p^2 e^p\,dp \Rightarrow p^2e^p - 2pe^p + 2e^p \Big|_0^1$

$e - 2e + 2e - (2)$

A = FIRST TOSS IS H

$P(A) = \boxed{e-2}$

b) $f_{P|A}(1) = \dfrac{P(A|P=p) f_P(p)}{P(A)} = \dfrac{(e^p - 2)(pe^p)}{e-2}$

$f_{P|A}(p|A) = \begin{cases} \dfrac{p^2 e^p}{e-2} & 0 \le p \le 1 \\ 0 & \text{else} \end{cases}$

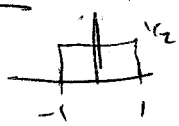
c) B = 2ND TOSS = H

$P(B|A) = \int_0^1 P(B|P=p,A) f_{P|A}(p)\,dp = \int_0^1 P(B|P=p) f_{P|A}(p)\,dp = \dfrac{1}{e-2}\int_0^1 p^3 e^p\,dp$

$P(B|A) = \dfrac{1}{e-2}\left(p^3e^p - 3p^2e^p + 6pe^p - 6e^p \Big|_0^1\right) = \dfrac{1}{e-2}\left(e^1 - 3e + 6e - 6e + (6)\right)$

$= \dfrac{1}{e-2}(-2e+6) = \boxed{.786}$

2) B-4.1



FIND PDF $\sqrt{|X|}$ & PDF $-\ln|X|$

$Y = \sqrt{|X|} \rightarrow F_Y(y) = P(Y \le y) =$

$= P(\sqrt{|X|} \le y) = P(-y^2 \le X \le y^2) = y^2$

PDF $\sqrt{|X|}$ $\boxed{f_Y(y) = 2y \quad 0 \le y \le 1}$

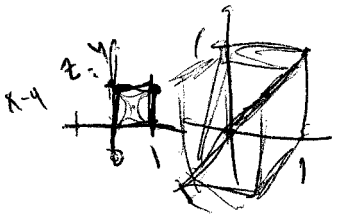
PDF $Z = -\ln|X| \rightarrow F_Z(z) = P(Z \le z) = P(-\ln|X| \le z) = P(\ln|X| \ge -z)$

$= P(X \ge e^{-z}) + P(X \le -e^{-z}) = 1 - e^{-z}$

$\boxed{f_Z(z) = e^{-z} \quad z \ge 0}$

3) B-4.5

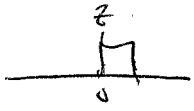
X, Y ARE IND RV $[0,1]$ FIND CDF & PDF of $|X-Y|$ $F_X = \frac{1}{2}$ $F_Y = \frac{1}{2}$



$Z = |X-Y| \rightarrow F_Z(z) = P(Z \le |X-Y|) = P(-z \le X-Y \le z)$

$P(z \le |x-y|) = 1 - (1-z)^2$

$\boxed{f_Z(z) = \begin{cases} 2(1-z), & 0 \le z \le 1 \\ 0 & \text{else} \end{cases}}$



RANGE$_Z$ = $[0,1]$

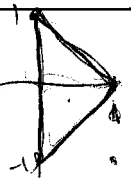
$P_Z(0) = 1 P_X(1)$

$X = Z+Y$

$Z+Y \quad Z^2 + ZY$

$Z^2 = -ZY$

4) B-4.6



$$F_Z(z) = \begin{cases} 0 & z \le 1 \\ z & 0 < z < 1 \\ 1 & z \ge 1 \end{cases}$$

CDF $|X-Y| \Rightarrow Z = |X-Y|$

$F_Z(z) = P(X-Y \le z,\ X \ge Y) + P(Y-X \le z,\ X < Y)$

$F_Z(z) = \left[\left(\frac{z}{2} + \frac{z^2}{4}\right) + \left(\frac{1}{4} - (1-z)^2/4\right)\right] = z$

$F_Z(z) = \left(\frac{z}{2} + \frac{z^2}{4}\right) +$

PDF $|X-Y|$: $\int f_Z(z)\frac{d}{dz}dz \Rightarrow$ $f_Z(z) \begin{cases} 1 & 0 \le z \le 1 \\ 0 & \text{else} \end{cases}$

5) B-4.8

FIND PDF $Z = X+Y$ X, Y = IND RV EXP W/ PARAM λ

$f_X(x)$, $f_Y(z-x)$ ARE NONZERO WHEN $x \ge 0$ $x \le z$

$$f_Z(z) = \int_{-\infty}^{\infty} f_X(x) f_Y(z-x)\,dx = \int_0^z \lambda e^{-\lambda x} \lambda e^{-\lambda(z-x)}\,dx = \lambda^2 e^{-z} \int_0^z dx = \boxed{\lambda^2 e^{-\lambda z}}$$

6) B-4.10

X, Y RV

$$p_X(x) = \begin{cases} 1/3, & x = 1,2,3 \\ 0, & \text{else} \end{cases} \qquad p_Y(y) = \begin{cases} 1/2, & y = 0 \\ 1/3, & y = 1 \\ 1/6, & y = 2 \\ 0, & \text{else} \end{cases}$$

$Z = X+Y$

RANGE $Z = \{1,2,3,4,5\}$

$p_Z(z) = 0$ if $z \ne 1,5$

$p_Z(1) = \sum_x p_X(x)\, p_Y(1-x) = p_X(1)\, p_Y(0) = 1/3 \cdot 1/2 = 1/6$

$p_Z(2) = p_X(1)\, p_Y(1) + p_X(2)\, p_Y(0) = 1/3\ 1/3 + 1/3\ 1/2 = \frac{1}{9} + \frac{1}{6} = 5/18$

$p_Z(3) = p_X(1)\, p_Y(2) + p_X(2)\, p_Y(1) + p_X(3)\, p_Y(0) = 1/3\ 1/6 + 1/3\ 1/2 + 1/3\ 1/2 = 1/2$

$p_Z(4) = p_X(2)\, p_Y(2) + p_X(3)\, p_Y(1) = 1/3\ 1/6 + 1/3\ 1/3 = 1/6$

$p_Z(5) = p_X(3)\, p_Y(2) = 1/3\ 1/6 = \frac{1}{18}$

$$\Rightarrow p_Z(z) = \begin{cases} 1/6 & z = 1 \\ 5/18 & z = 2 \\ 1/2 & z = 3 \\ 1/6 & z = 4 \\ 1/18 & z = 5 \\ 0 & \text{else} \end{cases}$$

7) B-4.14 X, Y : LIFETIME OF LIGHTBULBS λ, μ
EXP RVs w/ PARAMS λ, μ

$Z = \min\{X, Y\}$

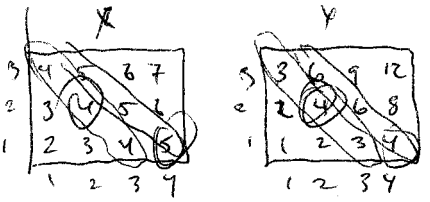
FOR ALL $z \geq 0 \Rightarrow F_Z(z) = P(\min\{X,Y\} \leq z) = 1 - P(\min(X,Y) > z) = 1 - P(X > z, Y > z)$

$= 1 - P(X>z)P(Y>z) = 1 - e^{-\lambda z} e^{-\mu z} = \boxed{1 - e^{-(\lambda+\mu)z}} = $ EXP CDF w/ PARAM $\lambda+\mu$

8) EXP: ROLL 4 SIDED (A) DIE & FAIR 3-SIDED (B) DIE

X = SUM (A, B)

~~Y = PRODUCT (A, B)~~



a) RANGE $X = \{2, 3, 4, 5, 6, 7\}$

b) PMF $X = \begin{cases} 1/12 & x=2 \\ 1/6 & x=3 \\ 1/4 & x=4 \\ 1/4 & x=5 \\ 1/6 & x=6 \\ 1/12 & x=7 \end{cases}$

c) $E(X) = \sum x\,p(x) = 2(1/12) + 3(1/6) + 4(1/4) + 5(1/4) + 6(1/6) + 7(1/12) = \boxed{4.5}$

d) $\text{Var}(X) = E\left[(X - E[X])^2\right] = \sum (x - E[X])^2 p(x) =$

$E[X^2] - (E[X])^2 = \sum x^2 p(x) - \sum x\,p(x)$

$= (2 - E[X])^2(1/12) + \text{etc}$

$\text{VAR}(X) = \left(-\frac{5}{2}\right)^2\left(\frac{1}{12}\right) + \left(-\frac{3}{2}\right)^2\left(\frac{1}{6}\right) + \left(-\frac{1}{2}\right)^2\left(\frac{1}{4}\right) + \left(\frac{1}{2}\right)^2\left(\frac{1}{4}\right) + \left(\frac{3}{2}\right)^2\left(\frac{1}{6}\right) + \left(\frac{5}{2}\right)^2\left(\frac{1}{12}\right)$

9) ~~Y~~ = PRODUCT ↑

a) RANGE $Y = \{1, 2, 3, 4, 6, 8, 9, 12\}$

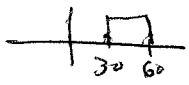
b) PMF $Y = p_Y(y) = \begin{cases} 1/12 & y=1 \\ 1/6 & y=2 \\ 1/6 & y=3 \\ 1/4 & y=4 \\ 1/6 & y=6 \end{cases} = \begin{cases} 1/12 & y = 1, 8, 9, 12 \\ 1/6 & y = 2, 3, 4, 6 \end{cases}$

c) $p_{XY}(X=4, Y=4) = \boxed{1/12}$

d) $P(X=5 \mid Y=4) = p_{X|Y}(x=5 \mid y=4) = \boxed{1/2}$

10) DRIVE 180 MI, FOR DAZE @ CONS SPEED

$X: [30, 60]$ mi/hr



$$F_X(x) = \begin{cases} 0 & x < 30 \\ \frac{x-30}{30} & 30 \le x \le 60 \\ 1 & x > 60 \end{cases}$$

Y = DURATION = $\frac{180}{X}$

a) PDF of Y?

$F_Y(y) = P(Y \le y) = P\left(\frac{180}{X} \le y\right)$

$= P\left(X \ge \frac{180}{y}\right) = 1 - P\left(X < \frac{180}{y}\right)$

$f_Y(y) =$

$$F_Y(y) = 1 - F_X\left(\frac{180}{y}\right) = \begin{cases} 0 & y < 3 \\ 1 - \frac{\frac{180}{y} - 30}{30} & 3 \le y \le 6 \\ 1 & y > 6 \end{cases} = \begin{cases} 0 & y < 3 \\ 2 - \frac{6}{y} & 3 \le y \le 6 \\ 1 & y > 6 \end{cases}$$

$6y^{-1}$ $6y^{-2}$

$$\rightarrow \boxed{f_Y(y) = \begin{cases} 0 & y < 3,\ y > 6 \\ \frac{6}{y^2} & 3 \le y \le 6 \end{cases}}$$

b) P(SPEND MORE THAN 5 HR)

$$P(Y > 5) = \int_5^\infty f_Y(y)\,dy = \boxed{\int_5^6 f_Y(y)\,dy} = 2 - \frac{6}{y}\Big|_5^6 \quad (2-1) - (2 - 6/5) = -(-1/5) = \boxed{1/5}$$

c) $E[Y] = \int_{-\infty}^{\infty} y f_Y\,dy = \boxed{\int_3^6 y f_Y(y)\,dy} = 2y - \frac{6}{y}\Big|_3^6 \quad 12 - 6 - (6 - 6) = 6$ HR