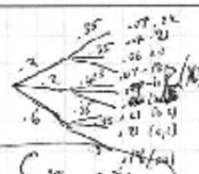
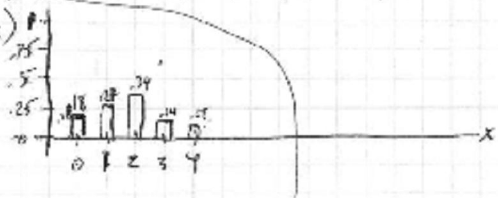
Ty Madsen
ECON 390
HW #3

1) B-2.1)



$\Omega = \{(0,0),(0,1),(0,2),(1,0),(1,1),(1,2),(2,0),(2,1),(2,2)\}$

$p(x) = P(X = x)$



$$P_X(x): \begin{cases} .18 & 0 \\ .22 & 1 \\ .34 & 2 \\ .14 & 3 \\ .07 & 4 \end{cases}$$

2) S-2.1) Throw fair die $\quad \Omega = \{1,2,3,4,5,6\}$

$X = \begin{cases} 1 & \text{if even} \\ 0 & \text{if odd} \end{cases}$

a) $R_X = \{0, 1\}$

b) $p(x=1)$ $\quad p(x=0)$

$P(X=1) = 1/2 \quad P(X=0) = 1/2$

3) S-2.2) Toss coin 3x $\quad \Omega = \{(HHH),(HHT),(HTH),(HTT),(THH),(THT),(TTH),(TTT)\}$

$X = \#$ of heads $\quad P(H) = p \quad P(T) = 1-p$

a) $R_X = \{0,1,2,3\}$

b) $P(X=0) = (1-p)^3$
$P(X=1) = 3(1-p)^2 p$
$P(X=2) = 3(1-p)p^2$
$P(X=3) = p^3$

$(1-p)^3 + 3(1-p)^2p + 3(1-p)p^2 + p^3$
$1 - 3p + 3p^2 + 3p - 3p^2 \ldots$
$(1-2p+p^2)(1-p) \ldots + p^3$
$1 - 3p + \ldots - p^3 + p^3 = 1$

4) S-2.3) $\{a, b, c, d\}$ $\quad p(a) = 1/2 \quad p(b) = 1/4 \quad p(c) = p(d) = 1/8$

a 0
b 10
c 110
d 111

X = length of code $\quad$ a) $R_X = \{1, 2, 3\}$

b) $P(X=1) = P(a) = 1/2$
$P(X=2) = P(b) = 1/4$
$P(X=3) = P(c) + P(d) = 1/4$
~~$P(X>3) = P(\emptyset) = 0$~~

7) B-2.13) 5 Nat 2 4 = $X = \#$ of girls . $2^5 = 32$

$Y = X+2$

$P(X=5) = \frac{1}{32}$

$P(Y=0) = \emptyset$

$P(Y=1) = \emptyset$

$P(Y=2) = 1/32$

$P(Y=3) = 5/32$

$P(Y=4) =$

$\cancel{P(?) = \binom{5}{?} \ldots}$

$\sum_{(x \mid x+2=y)} ?$ $\quad 0 \le x \le 5$

$$p_Y(y) = \begin{cases} \binom{5}{y-2}\left(\frac{1}{2}\right)^5 & 2 \le y \le 7 \\ 0 & \text{otherwise} \end{cases}$$

8) B-2.14) $X =$ $\quad R_x$

a) $p_Y(y) = \sum_{(x \mid x \bmod (3) = y)} p_X(x)$ $\quad R_y = \{0,1,2\}$

$P(y=0) = P(x=0)+P(x=3)+P(x=6)+P(x=9) = 4/10 = 2/5 = P(y=0)$

$P(y=1) = P(x=1)+P(x=4)+P(x=7) = 3/10$ $\quad 3/10 = P(y=1)$

$P(y=2) = P(x=2)+P(x=5)+P(x=8) = 3/10$ $\quad 3/10 = P(y=2)$

0 otherwise

b) $p_Y(y) = \sum_{(x \mid 5 \bmod (x+1))} p_X(x)$ $= R_x$ ~~{0,1,2,3,4,5}~~ $R_y = \{0,1,2,5\}$

$P(y=0) = P(x=0)+P(x=4) = 1/5$ $\quad 1/5 = P(y=0)$

$P(y=1) = P(x=1)+P(x=3) = 1/5$ $\quad 1/5 = P(y=1)$

$P(y=2) = P(x=2) = 1/10$ $\quad 1/10 = P(y=2)$

$P(y=5) = P(x=5)+P(x=6)+P(x=7)+P(x=8)+P(x=9) = 5/10 = 1/2 = P(y=5)$

0 otherwise

9) B-2.15)

$k = 1/(2n+1)$ $\quad [-n, n]$

$Y = \ln X$ $\quad X = a^{|k|}$ $\quad a > 0$ $\quad p_Y(y) = \sum_{(x \mid \ln X = y)} p_X(x)$ $\quad p_X(x) = \sum_{(k \mid a^{|k|} = x)} p_K(k)$ $\quad x > 0$

$p_K(k) =$

$|R_K| = 2n+1$ $\quad p_Y(y) = \sum_{(x \mid \ln x = y)} \sum_{(k \mid a^{|k|} = x)}$

$y = \ln(a^{|k|}) = k \ln a$

$$p_Y(y) = \begin{cases} \frac{2}{2n+1} & \text{if } y = \ln a, 2\ln a, \ldots k \ln a \\ \frac{1}{2n+1} & \text{if } y = 0 \\ 0 & \text{others} \end{cases}$$

5) B-24 X = # of Modems ~~___~~ in use

a) $\boxed{P_X(k) = \binom{1000}{k}(.01)^k(.99)^{1000-k} \quad k=0,1,2,\dots,49}$

$\boxed{P_X(50) = \sum_{k=50}^{1000}\binom{1000}{k}(.01)^k(.99)^{1000-k}}$

b) $\lambda = 1000(.01) = 10$

$$\boxed{\begin{array}{l} P_X(k) = e^{-10}\frac{10^k}{k!} \quad k=0,1,2,\dots,49 \\ P_X(50) = \sum_{k=50}^{1000} e^{-10}\frac{10^k}{k!} \end{array}}$$

c) $\boxed{P_X(51) = \sum_{k=51}^{1000}\binom{1000}{k}(.01)^k(.99)^{1000-k}}$

Poisson $\boxed{P(51) = \sum_{k=51}^{1000} e^{-10}\frac{10^k}{k!}}$

6) S-2.15)

$P_X(k) = P(X=k) = \binom{n}{k}p^k(1-p)^{n-k} \quad k=0,1,\dots,n \quad = \sum_k P_X(x_k) = 1 \quad (a+b)^n = \sum_{k=0}^{n}\binom{n}{k}a^k b^{n-k}$

$\sum_{k=0}^{n} P_X(k) = \sum_{k=0}^{n}\binom{n}{k}p^k(1-p)^{n-k} = 1$

$= \sum_{k=0}^{n}\frac{n!}{k!(n-k)!}p^k(1-p)^{n-k} = \frac{n!}{0!(n!)}p^0(1-p)^n + \frac{n!}{1!(n-1)!}p^1(1-p)^{n-1} + \frac{n!}{2!(n-2)!}p^2(1-p)^{n-2} + \sum_{k=3}^{n}\frac{n!}{k!(n-k)!}p^k(1-p)^{n-k}$

$= (1-p)^n + np(1-p)^{n-1} + \frac{n(n-1)}{2}p^2(1-p)^{n-2} + \sum$

$= p\binom{n}{k}$

$(1-p)^{n-2}[(1-p)^2 + np(1-p) + \frac{n^2-n}{2}p^2] + \sum$

$= (1-p)^{n-2}[(1-p)^2 + np - np^2 + \frac{n(n-1)}{2}p^2] + \sum$

$np^2(1+\frac{(n-1)}{2})$

$\sum_{k=0}^{n}\binom{n}{k}p^k(1-p)^{n-k} = (p+1-p)^n = 1^n = \boxed{1}$

b) $P(X>1) \quad n=6 \quad p=.1$

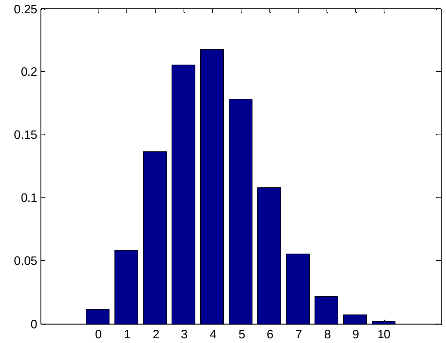
$1 - P(X=0) - P(X=1) = 1 - \left(\binom{6}{0}.1^0(.9)^6\right) - \binom{6}{1}(.1)^1(.9)^5$

$= 1 - .5314 - .3543 = 1 - .81$

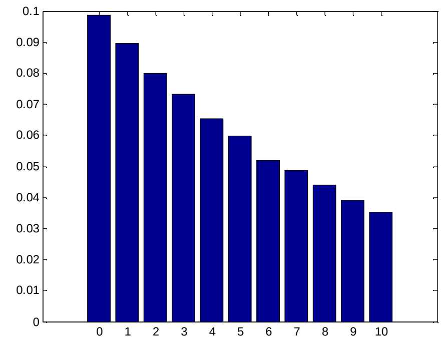
$= 1 - .8857$

$= \boxed{.1142}$

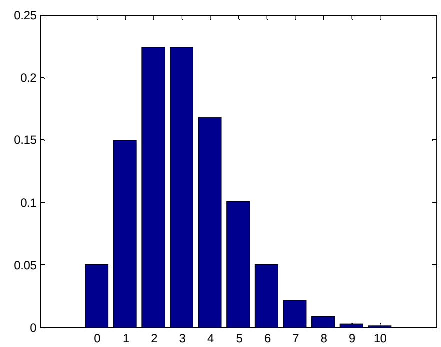
```
trials = 100000;

X_vector = random('Binomial', 20, 0.2, [1,
trials]);
for i =1:11
    count (i, 1) = i-1 ;
    count (i, 2) = sum( (i-1) == X_vector );
end
pvecX = count(:, 2)/trials;
```



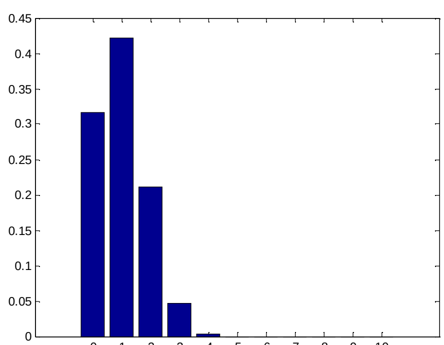

```
trials = 100000;

bar ( count(:, 1), pvecX);
Y_vector = random('geo', .1, [1, trials]);
for i =1:11
    count (i, 1) = i-1 ;
    count (i, 2) = sum( (i-1) == Y_vector );
end
pvecY = count(:, 2)/trials;
bar ( count(:, 1), pvecY);
```



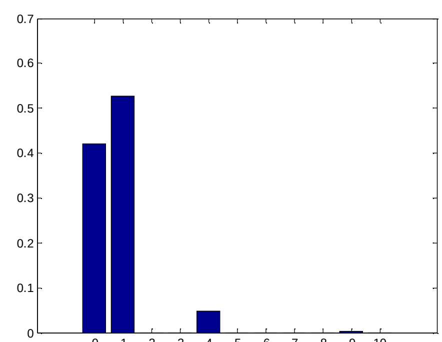

```
trials = 10000000;

Z_vector = random('poiss', .3, [1, trials]);
for i =1:11
    count (i, 1) = i-1 ;
    count (i, 2) = sum( (i-1) == Z_vector );
end
pvecZ = count(:, 2)/trials;
bar ( count(:, 1), pvecZ);
```



The simulation of the PMF of Y=(X1-1)^2 compares well with the probability mass function you computed analytically, because the only possible values are 0 for (X=1), 1 for (X=0 and 2), 4 for (X=3) and 9 for (X=4)



PMF of X1



PMF of Y