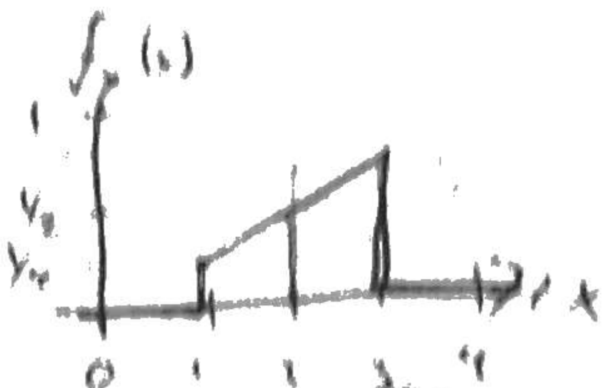
ECEN 370

Ty Madsen — HW #7

1) ~~B~~- 3.15

See HW #6

2) B- 3.18



$f_X(x) = \begin{cases} x/4, & \text{if } 1 < x \le 3 \\ 0, & \text{else} \end{cases}$ $\quad A = \{X \ge 2\}$

a) $E[X] = \int_{-\infty}^{\infty} x f_X(x)\,dx = \int_1^3 x^2/4 = \frac{x^3}{12}\Big|_1^3 = \frac{26}{12} = \boxed{\frac{13}{6}}$ $E[X]$

$\boxed{P(A) = \boxed{5/8}} = \int_2^3 x/4 = \frac{x^2}{8}\Big|_2^3 = \cancel{9/8} - 4/8 = 5/8$

$f_{X|A}(x) = \frac{\int_2^3 x/4}{\int_{2}^{3}} = \frac{f_X(x)}{P(A)} = \frac{x/4}{5/8} = \boxed{2x/5} \Rightarrow \boxed{\begin{cases} 2x/5, & 2 \le x \le 3 \\ 0, & \text{else} \end{cases}}$ $f_{X|A}(x)$

$E[X|A] = \int_{-\infty}^{\infty} x f_{X|A}(x) = \int_2^3 x \cdot 2x/5 = \int \frac{2x^2}{5} = \frac{2x^3}{15}\Big|_2^3$

$= \frac{54}{15} - \frac{16}{15} = \frac{38}{15} = \cancel{\boxed{\ }} = \boxed{38/15}$ $E[X|A]$

b) $Y = X^2$

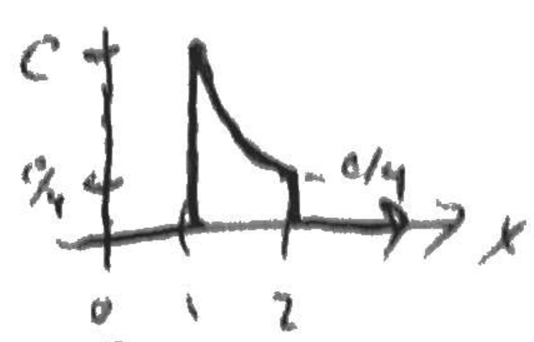
$E[Y] = \int_1^3 x^2 f_X(x)\,dx = \int_1^3 x^3/4 = \frac{x^4}{16}\Big|_1^3 \quad \frac{81}{16} - \frac{1}{16} = \frac{80}{16} = \boxed{5}$

$Var[Y] = \int_1^3 x^4 f_X(x)\,dx = \int x^5/4 = \frac{x^6}{24}\Big|_1^3 \quad \frac{729}{24} - \frac{1}{24} = \frac{728}{24} = \frac{91}{3}$

$= \frac{91}{3} - 25 = \boxed{\frac{16}{3}}$ $VAR[Y]$

3) B- 3.19

$f_X(x) = \begin{cases} cx^{-2}, & 1 \le x \le 2 \\ 0, & \text{else} \end{cases}$



a) $\int_1^2 f_X(x) = 1 \Rightarrow \int_1^2 cx^{-2}\,dx = -cx^{-1}\Big|_1^2 = 1 \Rightarrow -c/2 - (-c) = 1 \Rightarrow c(1 - 1/2) \Rightarrow c = 2$

$c = 1 + c/2$

$\boxed{c = 2}$

b) $A = \{X > 1.5\}$

$P(A) = \int_{1.5}^2 2x^{-2} = -2/x\Big|_{1.5}^2 = -1 + 4/3 = \boxed{1/3}$ $P(A)$

$f_{X|A} = \frac{f_X(x)}{P(A)} = \frac{2/x^2}{1/3} = \boxed{\begin{cases} 6/x^2, & 1.5 \le x \le 2 \\ 0, & \text{else} \end{cases}}$

c) $Y = X^2$

$E[X^2|A] = \int_{1.5}^2 x^2 f_{X|A}(x|A) = \int_{1.5}^2 6 = 6x\Big|_{1.5}^2 \quad 12 - 9 = \boxed{3}$

$VAR[Y|A] = VAR[X^2|A] : \quad x^4 \frac{6}{x^2} = \int_{1.5}^2 6x^2 \quad \frac{6x^3}{3}\Big|_{1.5}^2 \quad ~~\frac{16}{3} - \frac{9}{4} = \frac{64}{12} - \frac{27}{12} = \frac{37}{12}~~$ $E[Y^2|A]$

$= 2x^3\Big|_{1.5}^2 = 16 - \frac{27}{4} = \frac{64}{4} - \frac{27}{4} = \frac{37}{4}$

$= \frac{37}{4} - \cancel{\frac{81}{12}}\ \frac{36}{4} = \boxed{\frac{1}{4}}$

4) S-3.29

$f_{XY}(x,y) = \begin{cases} 4xy & 0 < y < 1,\ 0 < x < 1 \\ 0 & \text{else} \end{cases}$

$f_X(x) = 2x \quad 0 < x < 1$

$f_Y(y) = 2y \quad 0 < y < 1$

$\boxed{f_{Y|X}(y|x) = \frac{f_{XY}(x,y)}{f_X(x)} = \frac{4xy}{2x} = 2y \quad \begin{matrix} 0 < y < 1 \\ 0 < x < 1 \end{matrix} \qquad f_{X|Y}(x|y) = \frac{f_{XY}(x,y)}{f_Y(y)} = \frac{4xy}{2y} = 2x \quad \begin{matrix} 0 < y < 1 \\ 0 < x < 1 \end{matrix}}$

$f_{Y|X}(y|x) = \frac{f_{XY}(x,y)}{f_X(x)} = ~~\frac{4xy}{2x} = \frac{4xy}{4xy} = \frac{2y}{2x} = \frac{4}{x}~~$

$f_{X|Y}(x|y) = \frac{f_{XY}(x,y)}{f_Y(y)} = \frac{4xy}{2x}\ \frac{2y}{2y} = 1$

5) S 3.30

$f_{XY}(x,y) = \begin{cases} 2 & 0<y\le x\le 1 \\ 0 & \text{else} \end{cases}$

$f_X(x) = 2x \quad 0<x<1$

$f_Y(y) = 2(1-y) \quad 0<y<1$

$f_{Y|X}(y|x) = \frac{f_{XY}(x,y)}{f_X(x)} = \frac{2}{2x} = \boxed{\frac{1}{x} \quad 0<y\le x<1}$

$f_{X|Y}(x|y) = \frac{f_{XY}(x,y)}{f_Y(y)} = \frac{2}{2(1-y)} = \boxed{\frac{1}{1-y} \quad 0<y\le x<1}$

6) S-3.40

$E(Y|X) = \int_0^x y f_{Y|X}(y|x) = \int_0^x \frac{y}{x}\,dy = \frac{y^2}{2x}\Big|_0^x = \frac{x^2}{2x} = \quad 0<x<1 \quad \boxed{\frac{x}{2} \quad 0<x<1}$

$E[X|Y] = \int_y^1 x f_{X|Y}(x|y) = \int_y^1 \frac{x}{1-y}\,dx = \frac{x^2}{2-2y}\Big|_y^1 = \frac{1}{2(1-y)} - \frac{y^2}{2(1-y)} \quad \frac{1-y^2}{2(1-y)} = \frac{(1-y)(1+y)}{2(1-y)} = \boxed{\frac{1+y}{2} \quad 0<y<1}$

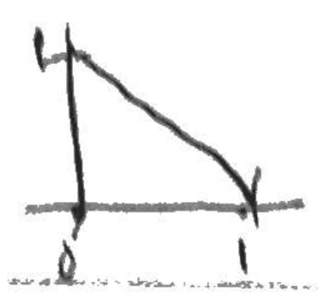
7) S-3.41

$Var(Y|X) = \int y^2 f_{Y|X}(y|x)\,dy = \int_0^x \frac{y^2}{x}\,dy = \frac{y^3}{3x}\Big|_0^x = \frac{x^3}{3x} = \frac{x^2}{3}(E(Y^2|X)) \Rightarrow \frac{x^2}{3} - \frac{x^2}{4} = \boxed{\frac{x^2}{12}}$

$Var(X|Y) = \int x^2 f_{X|Y}(x|y)\,dy = \int_y^1 \frac{x^2}{1-y}\,dx = \frac{x^3}{3(1-y)}\Big|_y^1 \quad \frac{1-y^3}{3(1-y)} = \frac{(1+y+y^2)}{3} \Rightarrow \frac{1+y+y^2}{3} - \frac{1+2y+y^2}{4}$

$\Rightarrow \frac{4+4y+4y^2-3-6y-3y^2}{12} = \frac{1-2y+y^2}{12} = \boxed{\frac{(1-y)^2}{12}}$

8) B-3.23



a) $\boxed{f_{XY}(x,y) = \begin{cases} 2, & 0<y<1,\ 0\le x<1-y \\ 0, & \text{else} \end{cases}}$

b) $f_Y(y) = \int_{-\infty}^{\infty} f_{XY}(x,y)\,dx = \int_0^{1-y} 2\,dx = 2x\Big|_0^{1-y} = \boxed{2(1-y) \quad 0\le y\le 1}$

c) $f_{X|Y}(x|y) = \frac{f_{XY}(x,y)}{f_Y(y)} = \frac{2}{2(1-y)} = \boxed{\frac{1}{1-y} \quad 0\le x\le 1-y}$

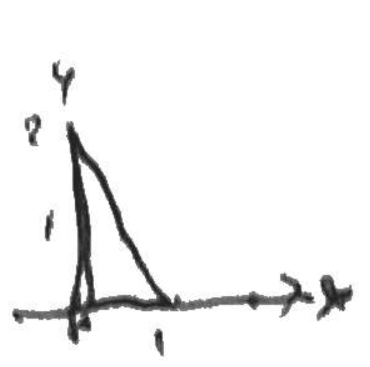
d) $E[X|Y=y] = \int_{-\infty}^{\infty} x f_{X|Y}(x|y)\,dx = \int_0^{1-y} \frac{x}{1-y} = \frac{x^2}{2(1-y)}\Big|_0^{1-y} = \boxed{\frac{1-y}{2} \quad 0<y<1}$

$E[X] = E[X|Y]\, f_Y(y) = \int_0^1 \frac{1-y}{2} f_Y(y)\,dy = \int \frac{1-y}{2}\ \cancel{(2(1-y))} = \int_0 (1-y)^2 =$

$= \int_0^1 \frac{1}{2} - \frac{y}{2}(y f_Y(y)\,dy) = \frac{1}{2} - \frac{1}{2}\int y f_Y(y) = \frac{1}{2} - \frac{E[Y]}{2} = \boxed{\frac{1-E[Y]}{2}}$

e) $E[X] = E[Y] = \frac{1-E[X]}{2} = 3E[X] = 1 \Rightarrow \boxed{E[X] = 1/3}$

9) B-3.24



$f_{XY}(x,y) = \begin{cases} 1 & 0\le y\le 2,\ 0\le x\le 1-\frac{y}{2} \\ 0, & \text{else} \end{cases} \quad f_Y(y) = \int_0^{1-y/2} f_{XY}(x,y)\,dx = x\Big|_0^{1-y/2} = \boxed{1-y/2 \quad 0\le y\le 2}$

$f_{X|Y} = \frac{1}{1-y/2} = \frac{2}{2-y} \quad 0\le x\le 1-y/2$

$E[X|Y] = x f_{X|Y}\,dx = \int_1^{1-y/2} \frac{2x}{2-y} = \frac{x^2}{2-y}\Big|_0^{1-y/2} \quad \frac{\left(\frac{2-y}{2}\right)^2}{2-y} = \boxed{\frac{2-y}{4} \quad 0\le y\le 2}$

$E[X] = \int_0^2 \frac{2-y}{4} f_Y(y)\,dy = \frac{1}{2} - \frac{1}{4}\int_0^2 y f_Y(y) = \frac{1}{2} - \frac{1}{4}E[Y] = \frac{2-E[Y]}{4}$

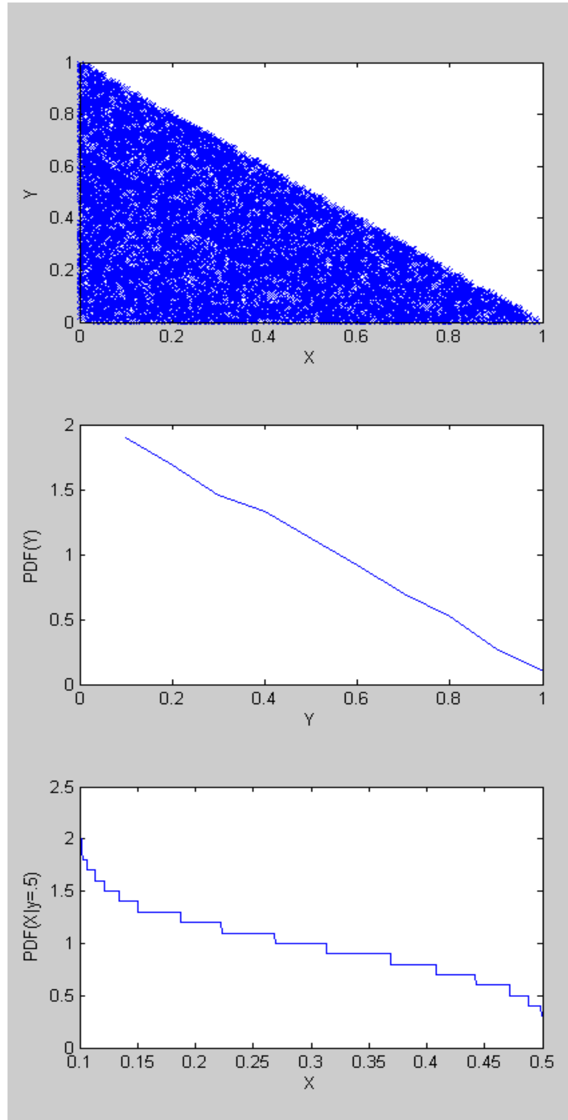
$E[Y|X] = y f_{X|Y}$

$E[Y] = \int_0^1 (1-x) f_X(x)\,dx = 1 - E[X] \Rightarrow E[X] = \frac{2-1+E[X]}{4} \quad 3E[X] = 1 \quad \boxed{E[X] = 1/3 \\ E[Y] = 2/3}$

3-0235 — 50 SHEETS — 5 SQUARES
3-0236 — 100 SHEETS — 5 SQUARES
3-0237 — 200 SHEETS — 5 SQUARES
3-0137 — 200 SHEETS — FILLER
COMET

Problem 8)

E[X] = .3331



Problem 9)

E[X] = .328

E[Y] = .664