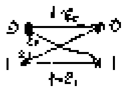
ECGN 370

HW #1

6 1.31)



a) $\boxed{p(1-\varepsilon_0)+(1-p)(1-\varepsilon_1)}$

b) $\boxed{(1-\varepsilon_1)^2(1-\varepsilon_0)}$

c) $(1-\varepsilon_0)(1-\varepsilon_0)\varepsilon_0$ or $(1-\varepsilon)^3$

$\Rightarrow \boxed{3(1-\varepsilon_0)^2\varepsilon_0+(1-\varepsilon_0)^3}$

d) $3(1-\varepsilon_0)^2\varepsilon_0+(1-\varepsilon_0)^3 > 1-\varepsilon_0$ $\quad (1-\varepsilon_0)^2(\overset{2}{\cancel{3}}\varepsilon_0+1-\cancel{\varepsilon_0}) > (1-\varepsilon_0)$

$\cancel{3\varepsilon_0 \ldots}$ $\quad 3\varepsilon_0+1 > \frac{1}{1-\varepsilon_0}$ $\quad (1-\varepsilon_0)(2\varepsilon_0+1)>1$

$(1-\varepsilon_0)2\varepsilon_0+1>\frac{1}{1-\varepsilon_0}$ $\quad 2\varepsilon_0+1-2\varepsilon_0^2-\varepsilon_0>1$

$\varepsilon_0-2\varepsilon_0^2>0 \Rightarrow \varepsilon_0(1-2\varepsilon_0)>0$

$\cancel{\varepsilon_0}-2\varepsilon_0^2 > 1$ $\quad \boxed{0<\varepsilon_0<1/2}$

$\varepsilon_0^2-\frac{1}{2}\varepsilon_0-\frac{1}{2}>-\frac{1}{2}$ $\quad \frac{1/2\pm\sqrt{1/4-0}}{2}$

$2\varepsilon=0,\ 1/2$

$\varepsilon_0(\varepsilon_0-1/2)>0$

$\frac{3}{4}\cdot\frac{1}{2}\cdot\frac{3}{8}+\frac{1}{8}$

$\varepsilon_0>1/2$

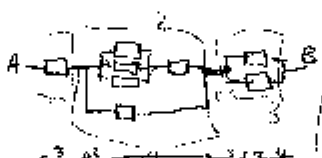
e) 101 ⇒ ~~111, 001, 100, 100~~ $(1-\varepsilon_1)^3+3(1-\varepsilon_1)\varepsilon_1$
~~000, 100, 101, 000~~ $(1-\varepsilon_0)^3+3(1-\varepsilon_0)\varepsilon_0$
111, 011, 110, 110

~~$P(101)=((1-\varepsilon_1)^3+3(1-\varepsilon_1)\varepsilon_1)^2((1-\varepsilon_0)^3+3(1-\varepsilon_0)\varepsilon_0)$~~

$P(0|101)$ $\quad P(0)=p$ $\quad P(1)=1-p$ $\quad P(101|0)=(1-\varepsilon_0)\varepsilon_0^2$ $\quad P(101|1)=\varepsilon_1(1-\varepsilon_1)^2$

$$P(0|101)=\frac{P(0)P(101|0)}{P(0)P(101|0)+P(1)P(101|1)} \Rightarrow \boxed{\frac{p(1-\varepsilon_0)\varepsilon_0^2}{p(1-\varepsilon_0)\varepsilon_0^2+(1-p)\varepsilon_1(1-\varepsilon_1)^2}}$$

9) 1.34)



$\underbrace{\left(1-(1-p)^3\right)p}_{X} \Rightarrow 1-(1-X)(1-p)$

~~$p^3+p^2+3p^4=2p^3+3p^4$~~

$P_1=p$ $\quad P_3=1-(1-p)^2$ $\quad P_2=1-\{1-[1-(1-p)^3]p\}(1-p)$

$=1-\{1-[p-p(1-p)^3]\}(1-p)$

$\Rightarrow \boxed{p\{1-(1-p)(1-p[1-(1-p)^3])\}(1-(1-p)^2)}$

b/.38) ~~$P_T = p^6$~~ ~~$P_W = (1-p)^3$~~

$P_T = \sum_{k=6}^{8} \binom{8}{k} p^k (1-p)^{8-k} = \binom{8}{6} p^6 (1-p)^2 + \binom{8}{7} p^7 (1-p)^1 + \binom{8}{8} p^8$

$P_W = \sum_{k=4}^{8} \binom{8}{k} (1-p)^k p^{8-k} = \binom{8}{4} (1-p)^4 p^4 + \binom{8}{5} (1-p)^5 p^3 + \binom{8}{6} (1-p)^6 p^2 + \binom{8}{7} (1-p)^7 p + (1-p)^8$

$P_T = \frac{8 \cdot 7}{2 \cdot 1} p^6 (1-p)^2$

$P_T = 28 p^6 (1-p)^2 + 8 p^7 (1-p) + p^8$ $\quad P_W = 70 (1-p)^4 p^4 + 56 (1-p)^5 p^3 + 28 (1-p)^6 p^2 + 8 (1-p)^7 p + (1-p)^8$

$\boxed{10 \left(\frac{P_T}{P_T + P_W} \right)}$

b/.53) $\binom{90}{2} = \frac{90!}{2(88!)}$

$\frac{\binom{89}{29}}{\binom{90}{30}} \cdot 3$

$\frac{89}{29} \quad \boxed{\frac{29}{89}}$

$\frac{89!}{29!(60!)} \frac{30!(60!)}{90!} = \frac{30}{90} \cdot 3, \frac{27}{89}$

9.38) a) $n(A) = (365)(364) \cdots (365-n+1) = \frac{365!}{(365-n)!}$ # = no same bday

$P(A) = \frac{n(A)}{n(S)} = \frac{365!}{365^n (365-n)!} = \frac{(365)(364)\cdots(365-n+1)}{365^n}$

$P(B) = 1 - P(A) = \boxed{1 - \frac{365!}{365^n (365-n)!}}$

b) $1 - \frac{365!}{365^n (365)!} \approx \boxed{.97}$

c) $1 - \frac{365!}{365^n (365-n)!} > .5 \quad .5 > \frac{365!}{365^n (365-n)!} \quad 365^n > (365)(364)\cdots(365-n+1) \cdot 2$

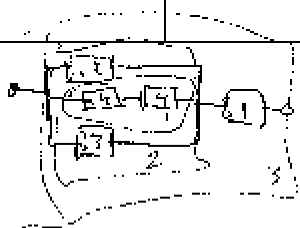
$\boxed{n = 23}$

9.45) a) $n(S) = \binom{15}{5} \quad n(A) = \binom{5}{2}\binom{10}{3} = P(A) = \frac{n(A)}{n(S)} = \frac{\binom{5}{2}\binom{10}{3}}{\binom{15}{5}}$ $\quad \frac{7 \cdot 13 \cdot 3 \cdot 11}{5}$

$\boxed{3 \cdot 4}$ $\quad \frac{1200}{3003} \approx \boxed{.4}$

b) $\frac{\binom{10}{5}}{\binom{15}{5}} = \frac{10!}{5!5!} \frac{10!}{15!} = \frac{2}{429} \approx \boxed{.0084}$

5(14)) (14.9)



$P(1\ \text{success}) = P_1 P_4 = (.6)(.6) = .36$

$P(2\ \text{success}) = 1 - P(2\ \text{fails}) = 1 - (.2)(.3)P(1\ \text{fails})$

$= 1 - (.2)(.3)(1 - .36)$

$P(3\ \text{success}) = P(2s)(.9)$

$P_1 = (.6)(.6) = \boxed{.36}$

$P_2 = 1 - (.2)(.3)(1 - .36) = 1 - (.06)(.64) = \boxed{.9616}$

$P_3 = P_2(.9) = \boxed{\cancel{.866}} = \boxed{.86544}$

MATLAB

MATLAB Code:

```
clear all, close all;
load sequence1.mat; %I ran this for each sequence 1-5
N = length(test_sequence);
```

```
count1 = sum(1==conv(1 * [1], test_sequence));
count2 = sum(1==conv(1/2 * [1 1], test_sequence));
count3 = sum(1==conv(1/3 * [1 1 1], test_sequence));
count4 = sum(1==conv(1/4 * [1 1 1 1], test_sequence));
count5 = sum(1==conv(1/5 * [1 1 1 1 1], test_sequence));

p1 = count1/N
p2 = count2/N
p3 = count3/N
p4 = count4/N
p5 = count5/N
```

Considering the actual probability (pi) of i heads in a row are:

p1 = .5 p2 = .25 p3 = .125 p4 = .0625 p4 = .03125 p5 = .015625

The results of the sequences are:

Sequence 1:
p1 = 0.5003 p2 = 0.2505 p3 = 0.1255 p4 = 0.0629 p5 = 0.0315
Sequence 2:
p1 = 0.5000 p2 = 0 p3 = 0 p4 = 0 p5 = 0
Sequence 3:
p1 = 0.5002 p2 = 0.2916 p3 = 0.1040 p4 = 0.0102 p5 = 0.0052
Sequence 4:
p1 = 0.5000 p2 = 0.4286 p3 = 0.3571 p4 = 0.2857 p5 = 0.2143
Sequence 5:
p1 = 0.5013 p2 = 0.2521 p3 = 0.1265 p4 = 0.0634 p5 = 0.0322

Sequences 2, 3, and 4 are fraudulent;