5) B-3.9) X = WAITING TIME A = FIND TAXI OR BUS AFTER 5 MIN Y = FIND Z = TAXI AFTER < 5 MIN

$P(\text{TAXI TIME} > 5 \text{ MIN}) = 1/2$

$P(A) = \frac{2}{3} + \frac{1}{3}\cdot\frac{1}{2} = \frac{5}{6}$

$P_Y(y) = \begin{cases} \frac{2}{3P(A)} & \text{if } y=0 \\ \frac{1}{6P(A)} & \text{if } y=5 \end{cases} = \begin{cases} \frac{12}{15} & y=0 \\ \frac{3}{15} & y=5 \end{cases}$

$P_Y(0) = \frac{P(Y=0 \cap A)}{P(A)} = \frac{2}{3P(A)} = \frac{12}{15}$

$P_Y(5) = 3/15$

$f_Z(z) = \begin{cases} 1/5 & 0 \le z \le 5 \\ 0 & \text{else} \end{cases}$

$F_X(x) = P(A)F_Y(x) + (1-P(A))F_Z(x)$

$$F_X(x) = \begin{cases} 0 & x<0 \\ \frac{5}{6}\cdot\frac{12}{15} + \frac{1}{6}\cdot\frac{x}{5} & 0 \le x \le 5 \\ 1 & 5 \le x \end{cases}$$

$E[X] = P(A)E[Y] + (1-P(A))E[Z] = \frac{5}{6}\cdot\frac{3}{15}\cdot 5 + \frac{1}{6}\cdot\frac{5}{2} = \frac{15}{12}$

6) B-3.10) $U \in [0,1]$ U:  $F_X(x) = u$ $S = \{x \mid 0 < F_X(x) < 1\}$

a) $X \le x$ iff $F(X) \le F(x)$

$P(X \le x) = P(F(X) \le F(x)) = P(U \le F(x)) = F(x)$ SINCE U IS UNIFORM

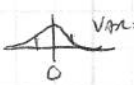
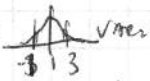
b) $F(x) = 1 - e^{-\lambda x}$ $x \ge 0$ $u \in (0,1)$ $1 - e^{-\lambda x} = u \Rightarrow x = \frac{\ln(1-u)}{-\lambda} = \boxed{-\ln(1-u)/\lambda}$

c) FOR EACH $u \in (0,1)$ THERE IS x_u $\ni$ $F(x_u - 1) < u < F(x_u)$

FOR ALL INTEGERS k:

$P(X=k) = P(F(k-1) < U < F(k)) = F(k) - F(k-1)$

MATLAB: SEE MATLAB PROBLEM #6 ON PREV PAGE

7) B-3.11) X:  Y: 

a) FIND $P(X \le 1.5)$ & $P(X<1)$

$P(X \le 1.5) = \sigma = 1 \Rightarrow \Phi(1.5) = \boxed{.9332}$

$P(X \le 1) = \Phi(1) = \boxed{.8413} = \boxed{.1587}$

$\Phi(0) - \Phi(1) = .5 - .1587$

b) PDF OF $(Y-1)/2$ = STANDARD NORMAL = 

c) $P(-1 \le Y \le 1) = \Phi(0) - [1 - \Phi(1)] = .5 - .1587$

$= \boxed{.3413}$