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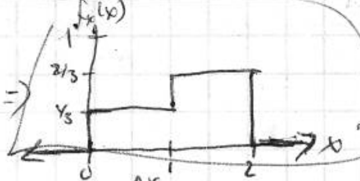
ECEN 370 | HW # 6 | 2/23/15

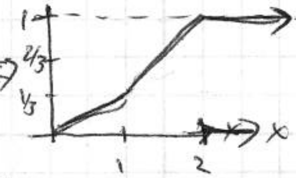
1) $\underline{S-2.21)}$

$$f_X(x)=\begin{cases} 1/3 & 0<x<1 \\ 2/3 & 1<x<2 \\ 0 & \text{OTHERWISE}\end{cases} \Rightarrow$$



$$F_X(x)= P(X\leq x)= \int_{-\infty}^{x} f_X(x)\,dx = \int_0^x f_X(x)\,dx$$

$$F_X(x) \Rightarrow \begin{cases} 0 & 0>x \\ 1/3\,x & 0<x<1 \\ 2/3\,x & 1<x<2 \\ 1 & 2>x \end{cases} \Rightarrow$$



2) $\underline{B-3.5)}$

$A = bh/2$

$b \quad h_x = h-x \quad A_x = b(h-x)^2/2h$

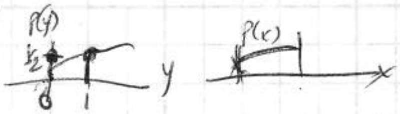
$b_x = (h-x)b/h$

$$F_X(x) = 1-P(X>x) = 1-\frac{A_x}{A} = 1-\frac{b(h-x)^2/(2h)}{bh/2} = 1-\left(\frac{h-x}{h}\right)^2$$

CDF: $F_X(x) = \begin{cases} 0 & \text{for } x<0 \\ 1 & x>h \end{cases}$ F CDF

PDF = $f_X(x)\,\dfrac{dF_X(x)}{dx} = \begin{cases}\dfrac{2(h-x)}{h^2} & 0\leq x\leq h \\ 0 & \text{OTHERWISE}\end{cases}$

3) $\underline{B-3.6)}$ $p=.5$ X: Time Y: # of PEOPLE



$$F(X\leq x) = \begin{cases} 0 & x<0 \\ \tfrac{1}{2}(2-e^{-\lambda x}) & x\geq 0\end{cases}$$

$$F_X(x) = P(X\leq x) = \tfrac{1}{2}\underbrace{P(X\leq x \mid Y=0)}_{1} + \tfrac{1}{2}\underbrace{P(X\leq x\mid Y=1)}_{1-e^{-\lambda x}}$$

$\tfrac{1}{2}(2-e^{-\lambda x})$

4) $\underline{B-3.7)}$

X: DIST TO CENTER

$$F_X(x) = P(X\leq x) = \frac{\pi x^2}{\pi r^2} = \frac{x^2}{r^2} \quad 0\leq x\leq r$$

$$F_X(x) = \begin{cases}\left(\frac{x}{r}\right)^2 & x\in[0,r] \\ 0 & x<0 \\ 1 & x>r\end{cases}$$

a)

$$\frac{dF(x)}{dx} = \frac{2x}{r^2}$$

$$f_X(x) = \begin{cases}\dfrac{2x}{r^2} & 0\leq x\leq r \\ 0 & \text{OTHERWISE}\end{cases}$$

$$E[X] = \int_0^r \frac{2x^2}{r^2}\,dx = \frac{2x^3}{3r^2}\Big|_0^r = \boxed{\frac{2r}{3}}$$

$$E(x^2) = \int_0^r \frac{2x^3}{r^2}\,dx = \frac{2x^4}{4r^2}\Big|_0^r\ \boxed{\frac{r^2}{2}}$$

$$VAR(X) = \frac{r^2}{2} - \frac{4r^2}{9} = \frac{r^2}{18} = \boxed{\frac{r^2}{18}} \quad VAR(X)$$

$1/t < s \quad F_S(s) = P(S\leq s) = P(X\leq t)P(S\leq s\mid X\leq t) + P(X>t)P(S\leq s\mid X>t)$

$P(X\leq t) = \frac{t^2}{r^2} \quad P(X>t) = 1-\frac{t^2}{r^2}$

$S=0 \quad X>t$

$P(S\leq s\mid X>t) = 1$

$$P(S\leq s\mid X\leq t) = P(1/X\leq s\mid X\leq t) = \frac{P(1/s\leq X\leq t)}{P(X\leq t)}$$

$$= \frac{\dfrac{\pi t^2 - \pi(1/s)^2}{\pi r^2}}{\dfrac{\pi t^2}{\pi r^2}} = 1-\frac{1}{t^2s^2}$$

$$\Rightarrow 1-\frac{1}{s^2r^2}$$

b) $S = 1/X$ ~~$X\leq t$~~

$S=0$ OTHERWISE

$$F_S(s) = \begin{cases} 1-P(X\leq t) = 1-\dfrac{t^2}{r^2} & 0<s<1/t \\ 1-\dfrac{1}{s^2r^2} & 1/t\leq s \\ 0 & s<0\end{cases}$$