9 March 2023 Buildings ● EN

Problem 5 - Buildings (Buildings)

An engineer must design a map of a city. Unfortunately, he doesn't know where the buildings are placed, but he has a visibility map from each building in all directions, along with a map showing the number of neighbouring buildings. Help him design the map of the city.

The city is represented by a square of $N \times N$ cells.

All the buildings on the same row, and on the same column, have different heights. The visibility from a building is what someone can see from the top of a building. Only higher buildings, not covered from other buildings, are visible.

As shown in the figure 1 below, if the building in front of you (\mathbf{A}) is higher than the one you're standing on, you can see it. However, if there's another lower building behind building \mathbf{A} , (building \mathbf{B}), it will not be visible, even if it's higher than the building you're standing on.

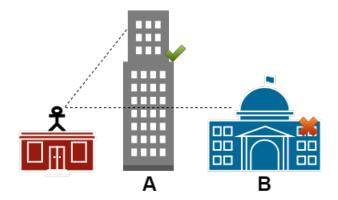


Figure 1: The building A is visible, the B is not

There are two kinds of visibility matrices. The first one is the visibility based on the direction (N, E, S, W). The second one is the visibility of neighbouring buildings that takes into account all the buildings at distance = 1 in all directions, including diagonals.

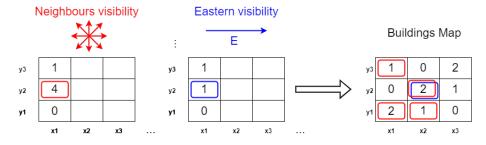


Figure:2 This image shows the neighbours and eastern visibility for column x1, for the buildings map solution. In red and blue the results for element (x1,y2)

So, given the visibility matrices in input (example on paragraph (5.1)), your goal is to create, for every test case, a matrix (the map of the city), like the buildings map in figure 2.

Figure 2 shows the visibility matrix in the Eastern direction and the neighbouring buildings matrix computed only for column $\mathbf{x1}$. Considering the building in position $(\mathbf{x1}, \mathbf{y2})$ in particular, it's possible to see the results in the Buildings Map where visible neighbouring buildings are highlighted in red, while visible buildings in the Eastern direction are highlighted in blue.

Buildings Page 1 of 4

Looking to the **east** from that building, you can see (x2, y2) because the building has a height of 2, but you cannot see the building on position (x3, y2) which has a height of 1 and is hidden by (x2,y2). Regarding the neighbouring buildings, since the height of the building is 0, you can see every neighbouring building except for the one with 0 height in position (x2,y3) which is the same height as the one you're standing on. Note that in this case, you should not consider anything on the W, N-W and S-W directions as these fall outside the matrix.

Input data

The first line of the input file contains an integer T, the number of test cases to solve.

For each test case, the first line contains an integer N: the order of the matrix.

In the following lines, the input file contains 5 visibility matrices. The first four, one for each direction (N, E, S, W) are preceded by a letter indicating the direction itself. The last one is the visibility matrix for all neighbouring buildings, which is preceded by the word "neighbours".

Output data

The output file must contain T lines.

For each test case in the input file, the output file must contain a line with the characters:

Case #t:

Where \mathbf{t} is the test case number, followed by N lines representing the corresponding building map matrix of the solution.

Note: The lines of the output file must be ordered from Case #1: to Case #T:.

Constraints

- T = 5, where T is the number of Test Cases in input.
- $2 \le N \le 14$, where N is the number of rows/columns.
- Numbers cannot repeat on each row/column in the solution.
- The solution should be valid with respect to every visibility matrix provided (direction matrices and the neighbours matrix).
- From the top of one building only higher buildings are visible.

Scoring

```
• input 1: T = 5, N = 6
```

• input 2: T = 5, N = 8

• **input 3**: T = 5, N = 10

• input 4: T = 5, N = 12

• input 5: T = 5, N = 14

Buildings Page 2 of 4

Examples

input	output
1	Case #1:
3	2 0 1
N	0 1 2
0 0 0	1 2 0
1 0 0	
1 0 1	
S	
0 2 1	
1 1 0	
0 0 0	
W	
0 1 1	
0 0 0	
0 0 1	
E	
0 1 0	
2 1 0	
1 0 0	
neighbours	
0 4 1	
4 3 0	
1 0 3	

Explanation

In the example, we have the 5 visibility matrices. The first 4 represent the 4 directions. The last matrix represents the visible neighbouring buildings. There are 5 matrices as input for each case.

The directions matrices show the number of buildings visible in a specific direction, from a specific point on the map to the edge of the map.

The neighbours matrix shows the number of buildings visible around the specific point for a distance of 1 step.

Explanation of north direction

Let's explain why the solution in the example is valid with respect to the North direction.

From the solution, you can see the first line is: $[2\ 0\ 1]$, while in the North view the corresponding line is $[0\ 0\ 0]$ because from that line it's impossible to see buildings to the North. The second line of the solution is: $[0\ 1\ 2]$, while the second line of the North view is $[1\ 0\ 0]$.

From the building **0** in the second line of the solution, if you look to the North direction you see building **2**. That's why in the corresponding North view matrix you see value **1**.

You can see an example in Figure 3.

Buildings Page 3 of 4

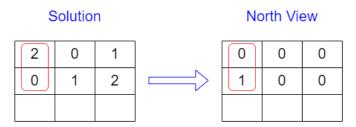


Figure 3: North visibility

Explanation of neighbours matrix

Let's explain why the solution in the example is valid with respect to the neighbours view.

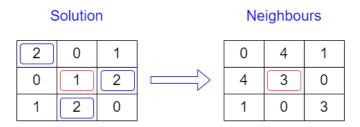


Figure 4: Neighbours visibility

From the output you can see the building in the center has an height of 1, its neighbours are: [2,0,1,0,2,1,2,0] but only the buildings of height 2 are visible. For this reason in Figure 4 the value in the center is 3.

Buildings Page 4 of 4