CMPUT 272 Winter 2016   Assignment 5

Due Thurs Apr 7 at 9:15AM in your seminar section's drop box across from CSC 145

**This is the mandatory assignment cover sheet;
without it, your work will not be marked.**

Submitting student: _____

Seminar section: (circle):   **Monday J1/EJ1   Tuesday J2/EJ2   Thursday J3/EJ3**

Collaborating classmates: _____

Other collaborators: _____

References other than the textbooks and handouts:

_____

_____

_____

_____

Your mark:

1	2	3	4	5	6	7	Total
/8	/8	/8	/8	/4	/8	/12	/56

_____

Even though collaboration is allowed, you must always write your solutions alone and in your own words, and properly acknowledge the sources that you used and the people with whom you worked. Your professor reserves the right to give you an oral and/or written exam to determine how well you understand what you have submitted. This may impact your mark.

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1. (Epp 7.3.25) Prove or give a counterexample: If $f : X \mapsto Y$ and $g : Y \mapsto X$ are functions such that $g \circ f = I_X$ and $f \circ g = I_Y$, then f and g are both one-to-one and onto and $g = f^{-1}$.

2. (Epp 8.3.27) Define a relation R on $\mathbb{Z}$ as follows: For all $m, n \in \mathbb{Z}$, $m \ R \ n \Leftrightarrow 4 \mid (m^2 - n^2)$. Prove that R is an equivalence relation. Describe the distinct equivalence classes of R and justify your answer.

3. (Epp 8.5.34) Suppose that R is a partial order relation on a set A and that B is a subset of A. The restriction of R to B is defined as $\{(x, y) \mid x \in B, \ y \in B, \ \text{and} \ (x, y) \in R\}$. In other words, two elements of B are related by the restriction of R to B if and only if they are related by R. Prove that the restriction of R to B is a partial order relation on B.

4. (Epp 9.2.33) Six people attend the theatre together and sit in a row with exactly six seats. There is an aisle on each end of the row.

    (a) How many ways can they be seated together in the row?
    (b) Suppose one of the six is a doctor who must sit on the aisle in case she is paged. How many ways can the people be seated together in the row with the doctor in an aisle seat?
    (c) Suppose the six people consist of three married couples and each couple wants to sit together with the husband on the left. How many ways can the six be seated together in the row?

5. In how many ways can a family of four (mother, father, and two children) be seated at a round table, with eight other people, so that the parents are seated next to each other with one child beside each parent? (Two seatings are considered the same if one can be rotated to look like the other).

6. (Epp 9.4.38) Observe that the sequence 12, 15, 8, 13, 7, 18, 19, 11, 14, 10 has three increasing subsequences of length four: 12, 15, 18, 19 and 12, 13, 18, 19 and 8, 13, 18, 19. It also has one decreasing subsequence of length four: 15, 13, 11, 10. Show that in any sequence $a_1, a_2, \ldots a_{n^2+1}$ of $n^2 + 1$ distinct real numbers, there must be a subsequence of length $n + 1$ that is either strictly increasing or strictly decreasing. Hint: Use proof by contradiction and the pigeonhole principle. For each $1 \leq k \leq n^2 + 1$, define i_k to be the maximum length of an increasing subsequence starting at a_k, and d_k to be the maximum length of a decreasing sequence starting at a_k. What happens if $(i_k, d_k) = (i_\ell, d_\ell)$ for some k and ℓ with $k \neq \ell$?

7. A computer programming team has 15 members.

    (a) How many ways can a group of eight be chosen to work on a project?
    (b) Suppose nine team members are women and six are men.
        i. How many groups of eight can be chosen that contain four women and four men?
        ii. How many groups of eight can be chosen that contain at least one man?
        iii. How many groups of eight can be chosen that contain at most three women?
    (c) Suppose two team members refuse to work together on projects. How many groups of eight can be chosen to work on a project?
    (d) Suppose two team members insist on either working together or not at all on projects. How many groups of eight can be chosen to work on a project?