# Quantitative Economics - Final Project

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## Introduction

This project aims to analyze the effects of increased tax progressivity on economy, as measured by changes to several economic indicators, including interest rates, wages, and inequality.

The majority of the classic literature studying the problem of progressive taxation focused on its function of substituting for insurance markets. According to Aiyagari (1994), the increased progressivity of the economy has beneficial insurance properties due to reducing the variation in labour income. Further analysis, including studying changes in welfare involving shifts, showed that economies with more ex-ante heterogeneity in household characteristics could benefit from progressive taxation (Holter et al. 2019). A recent numerical experiment showed that progressive taxation could lead to steady states, where the level of aggregated labor and capital inputs are substantially higher (Carroll and Young 2011). The further implications of the Carroll and Young (2011) analysis showed a decrease in income inequality, but an increase in wealth inequality. In the conclusion part, we reflect on our results in comparison to the literature, with special emphasis placed on the role of taxation in substituting for the insurance markets.

## Model specifics

The model is an extension of the Aiyagari model with balanced government budget. Labor productivity is governed by AR(1) Markov process, so we can implement Markov chain methods. We start by formulating the household problem recursively. In a stationary environment, an agent with assets a and idiosyncratic level productivity z faces the following:

$$V(a,z) = \max_{c,a'} \left\{ u(c) + \beta \sum_{z' \in Z} P(z,z') V(a',z') \right\}$$

$$y = zw$$

$$c + a' = y - \mathcal{T}(y) + (1+r)a$$

$$a' \geq -\phi \ = 0$$

Notes: Time subscripts and dependence on government policies suppressed in notation. After all, idiosyncratic tax payments result from heterogeneous labor productivity.

## Parameters calibration (besides $\beta$)

### Calibration when $\lambda = 0.0$

- Provided parameters: $\gamma = 2$, $\phi = 0$, $\rho = 0.9$ , $\sigma = 0.4$.
- Labor Share (Share of labor compensation in GDP at current prices for USA in the latest available 2019): ca. 0.6 (Groningen Growth and Development Centre 2023). Therefore, we will assume that $\alpha$ = 1 - 0.6 = 0.4

- Finding $\tilde{z}$: By recursion, we get

$$\ln z_{i,t+1} = \ln \tilde{z} + \sum_{j=0}^{\infty} \rho^j \varepsilon_{i,t+1-j},$$

Because $\epsilon_{i,t+1}$ is an i.i.d. shock with mean 0 and variance $\sigma^2$:

$$\mathbb{E} \ln z_{i,t+1} = \ln \tilde{z} + \sum_{j=0}^{\infty} \rho^j \mathbb{E}(\varepsilon_{i,t+1-j}) = \ln \tilde{z}$$

and

$$Var(\ln z_{i,t+1}) = Var(\sum_{j=0}^{\infty} \rho^j \varepsilon_{i,t+1-j}) = \sum_{j=0}^{\infty} \rho^{2j} Var(\varepsilon_{i,t+1-j}) = \frac{\sigma^2}{1-\rho^2}$$

When discretizing the productivity process, the Julia QuantEcon.tauchen function assumes i.i.d. $\epsilon_{i,t} \sim N(0, \sigma^2)$. Let's do the same. Then $z_{i,t+1}$ is distributed i.i.d. log-normally, so

$$\mathbb{E} z_{i,t} = exp(\mathbb{E} \ln z_{i,t+1} + \frac{Var(\ln z_{i,t+1})}{2}) = exp(\ln \tilde{z} + \frac{\sigma^2}{2(1-\rho^2)}) = 1$$

After logarithmic and exponential transformations and given $\rho = 0.9$ , $\sigma = 0.4$, we get

$$\tilde{z} = e^{-8/19} \approx 0.656355555$$

- Finding $\delta$ and $A$ (independent of $\lambda$) and $\tau$ for $\lambda = 0$. When $\lambda = 0$, in equilibrium w = 1, L = 1, and $\frac{wL}{AK^\alpha L^{1-\alpha}} = \frac{wL}{Y}$ = (1 - $\alpha$) = 0.6 (labor income share). Therefore:
  - Output Y = 1/0.6 = 5/3;
  - G = 1/3, because G/Y = 0.2;
  - G = $\tau \bar{y}$, but since $\bar{y}$ = 1 and G = 1/3, we get $\tau$ = 1/3 (linear tax system in $\lambda = 0$ state);
  - Formulas for the investment to output ratio I/Y, capital and labor compensation:

$$I/Y = \frac{\delta K}{AK^\alpha L^{1-\alpha}} = 0.2; \quad r = \alpha A K^{\alpha-1} L^{1-\alpha} - \delta; \quad w = (1-\alpha) A K^\alpha L^{-\alpha}$$

  - From the investment to output ratio I/Y (and knowing Y = 5/3):

$$1/5 = \frac{\delta K}{5/3} \quad => \quad \delta K = 1/3 \quad => \quad \delta = \frac{1}{3K}$$

  - From the wage equation:

$$w = (1-\alpha) A K^\alpha L^{-\alpha} \quad => w = 0.6 A K^{0.4} \quad => \quad 5/3 = AK^{0.4} \quad => \quad A = \frac{5}{3K^{0.4}}$$

  - From the capital rent equation:

$$r = \alpha A K^{\alpha-1} L^{1-\alpha} - \delta \quad => \quad 0.04 = 0.4 \frac{5}{3} \frac{1}{K^{0.4}} K^{-0.6} - \frac{1}{3K} \quad =>$$

$$=> \quad 0.1K = \frac{5}{3} K^{0.6} K^{-0.6} - \frac{5}{2} * \frac{1}{3} \quad => \quad 1/10K = 5/3 - 5/6 \quad => \quad K = 5/6 * 10 = 25/3$$

  - Therefore, we may easily calculate the following:

$$K = \frac{25}{3}; \quad \delta = \frac{1}{3K} = \frac{1}{3} * \frac{3}{25} = \frac{1}{25} = 0.04;$$

$$A = \frac{5}{3} * \frac{1}{K^{0.4}} = \frac{5}{3} * \frac{1}{\frac{25}{3}^{0.4}} \approx 0.7137$$

### Calibration when $\lambda = 0.15$

- The starting point is to calculate the tax rate, given the constant ratio of government spending to output (20%)
- Government spending:

$$G = \int_0^1 T(y_i)\, di$$

- With a tax function:

$$T(y_i) = y_i - (1-\tau)y^{(1-\tau)}\bar{y}^{-\lambda}$$

- Solve for tau, knowing that G = 0.2Y
- Starting with what is the average income in an economy with non-zero lambda

$$\bar{y} = \int_0^1 y_i d_i = w\int_0^1 z_i d_i = w$$

- In the equilibrium:

$$G = \int_0^1 \mathcal{T}(y_i) d_i = w\int_0^1 z_i d_i - (1-\tau)w\int_0^1 z_i^{1-\lambda} d_i = w - (1-\tau)w\int_0^1 z_i^{1-\lambda} d_i$$

- We further apply Tauchen's approximation to arrive at the solution

$$G \approx w - (1-\tau)w((z_{vec}^{1-\lambda})' * \lambda_z)$$

- $\lambda_z$ is a vector of z-states probabilities
- Knowing the labor share at 0.6 and government spending to output ratio at 0.2, we arrive at solution that $G = \frac{1}{3}w$

$$G \approx w - (1-\tau)w((z_{vec}^{1-\lambda})' * \lambda_z) = \frac{1}{3}w$$

- Solving for $\tau$

$$w[\frac{2}{3} - (1-\tau)((z_{vec}^{1-\lambda})' * \lambda_z)] = 0; \quad \tau = 1 - \frac{2}{3}\frac{1}{(z_{vec}^{1-\lambda})' * \lambda_z}$$

- Based on this approximation, we can find tax rate for different levels of $\lambda$. For $\lambda = 0.15$ the corresponding tax rate is approximately 0.3855 (ensuring approximate G/Y at 0.2).

# Finding $\beta$ and OPI rationale

As mentioned by Caroll and Young (2011) changes in progressivity have substantial effects, especially for households with large $\beta$. They have also found that intertemporal household's behaviour is very sensitive to $\beta$. Therefore, the final results can largely be dependent on $\beta$ calibration. We calibrated our model for $\beta$ looking for solution to Bellman equation when excess demand is (as close to) zero, given all the other parameters and knowing w = 1, r = 0.04 when $\lambda = 0$. On the other hand, solving Bellman equation-driven Aiyagari general equilibrium (we will need it for $\lambda = 0.15$ for which we do not know prices) can be computationally costly, therefore we adopted standard OPI methodology. All in all, we tried to balance numerical precision with computational possibility. We have tested various non-linear equation algorithms, but none of them arrived at an excess demand closer to zero than the guess $\beta = 0.9340109$ found with trial end error. The resulting excess demand was approx. 0.004672.

## Results discussion

The increase in tax progressiveness have led to:

- increase in r ($\approx 0.04 \Longrightarrow \approx 0.046$)
- decrease in w ($\approx 1.0 \Longrightarrow \approx 0.95$)
- increase in the tax rate ($\approx 0.33 \Longrightarrow \approx 0.38$)
- decrease in the capital to output ratio ($\approx 5.0 \Longrightarrow \approx 4.60$)
- decrease in the Gini coefficient for after-tax labor income ($\approx 0.55 \Longrightarrow \approx 0.48$)
- increase in the Gini coefficient for assets ($\approx 0.469 \Longrightarrow \approx 0.4789$)

In the same manner as showed in Carroll & Young (2011) our results showed that an increase in taxation progressiveness has led to decreased income inequality and increased asset (wealth) inequality. The change in the asset inequality is also visible in the distribution of the assets between economies - when lambda = 0.0, more households possess some assets, as compared to lambda = 0.15. This result was expected with our intuition regarding redistributive policies. The increase in the interest rate can be at play when it comes to increasing wealth inequality.

## Bibliography


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