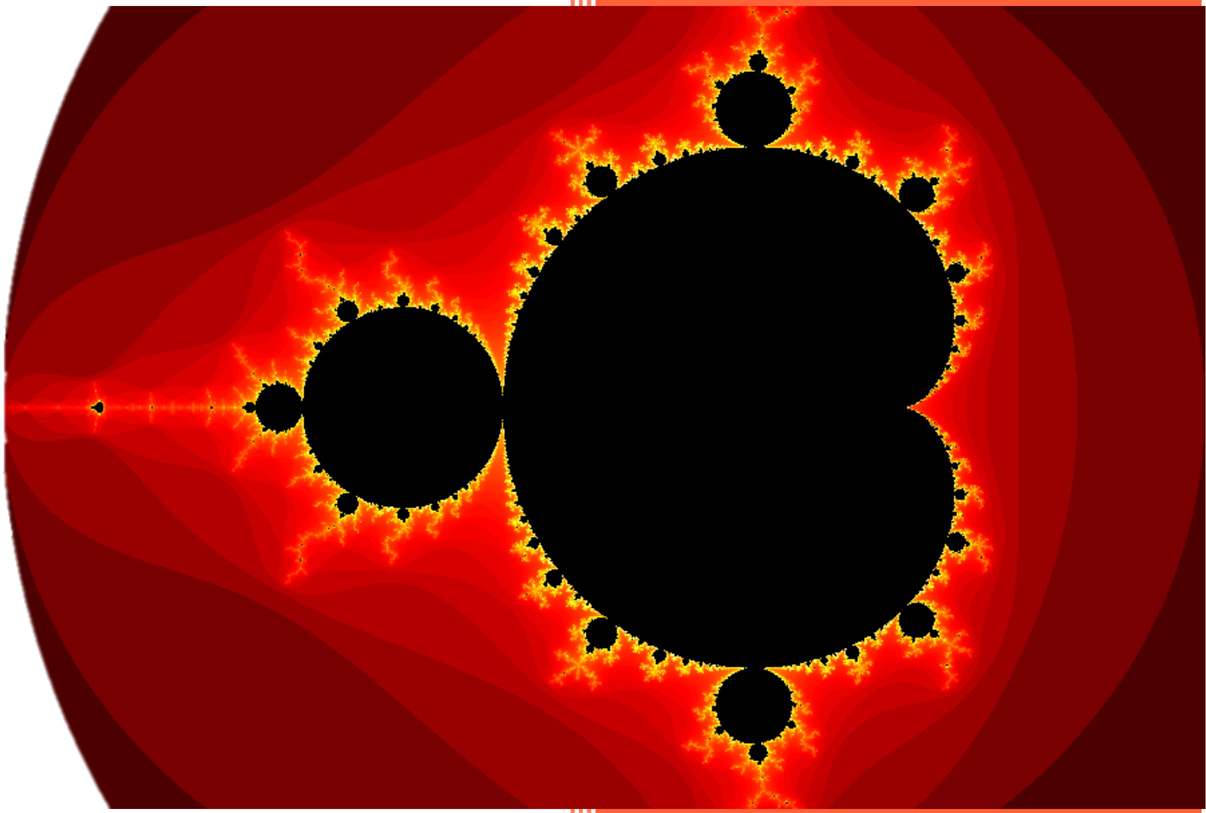
2017

Mandelbrot set



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Contents

The Mandelbrot Set

The term Mandelbrot set is used to refer both to a general class of fractal sets and to a particular instance of such a set. In general, a Mandelbrot set marks the set of points in the complex plane such that the corresponding Julia set is connected and not computable.

The Mandelbrot set is the set obtained from the quadratic recurrence equation

$$z_{n+1} = z_n^2 + \mathrm{C}$$

With $z_0 = \mathrm{C}$, where points C in the complex plane for which the orbit of z_n does not tend to infinity are in the set. Setting z_0 equal to any point **in the set** that is not a periodic point gives the same result (Weisstein, n.d.).

The Mandelbrot set is the dark glob in the centre of the picture. The colour of the pixels outside indicate how many iterations it took for each of those pixels until the condition for being outside the Mandelbrot set was satisfied.

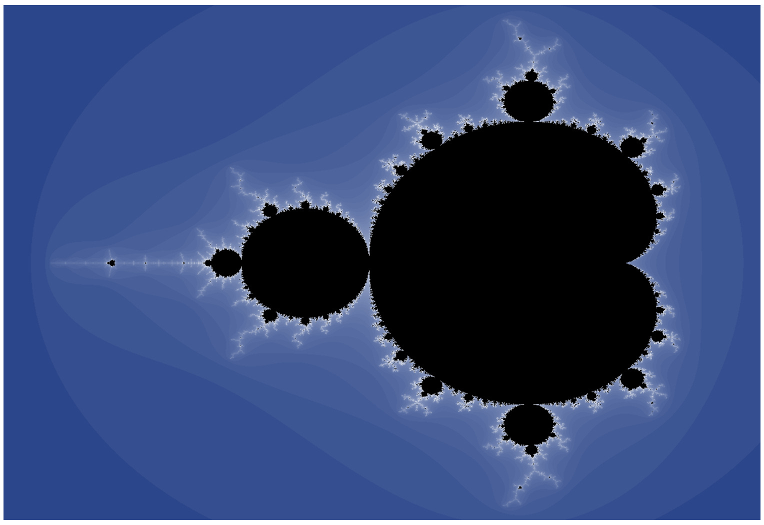


Figure 1. Mandelbrot Set (Mandelbrot Set, n.d.)

The Algorithm

In pseudocode, the algorithm looks as the following:

```
For each pixel (Px, Py) on the screen, do:
{
  x0 = scaled x coordinate of pixel (between (-2.5, 1))
  y0 = scaled y coordinate of pixel (between (-1, 1))
  x = 0.0
  y = 0.0
  iteration = 0
  max_iteration = 1000
  while (x*x + y*y < 2*2  AND  iteration < max_iteration) {
    xtemp = x*x - y*y + x0
    y = 2*x*y + y0
    x = xtemp
    iteration = iteration + 1
  }
  color = palette[iteration]
  plot(Px, Py, color)
}
```

Figure 2. Pseudocode (En.wikipedia.org, 2017)

The Problem

Given a sample code that can generate a section of the Mandelbrot set using a Central Processing Unit, there is a requirement to optimise and port the execution of the application to a General Purpose Graphic Processing Unit using Nvidia's CUDA API.

The Sample Code

Compiling the code generates an executable, which, when executed, saves a section of an instance of the Mandelbrot set to a ppm image file with a size of 4096x4096 pixels.

The main program is composed of three functions with the following execution times:

- *alloc_2d* – **13 ms:** reserves memory for the image to be generated;
- *calc_mandel* – **3420 ms:** fills the pixels with the right colours reflecting the Mandelbrot set;
- *screen_dump* – **236 ms:** saves the generated data to an image file, for a better visualisation;

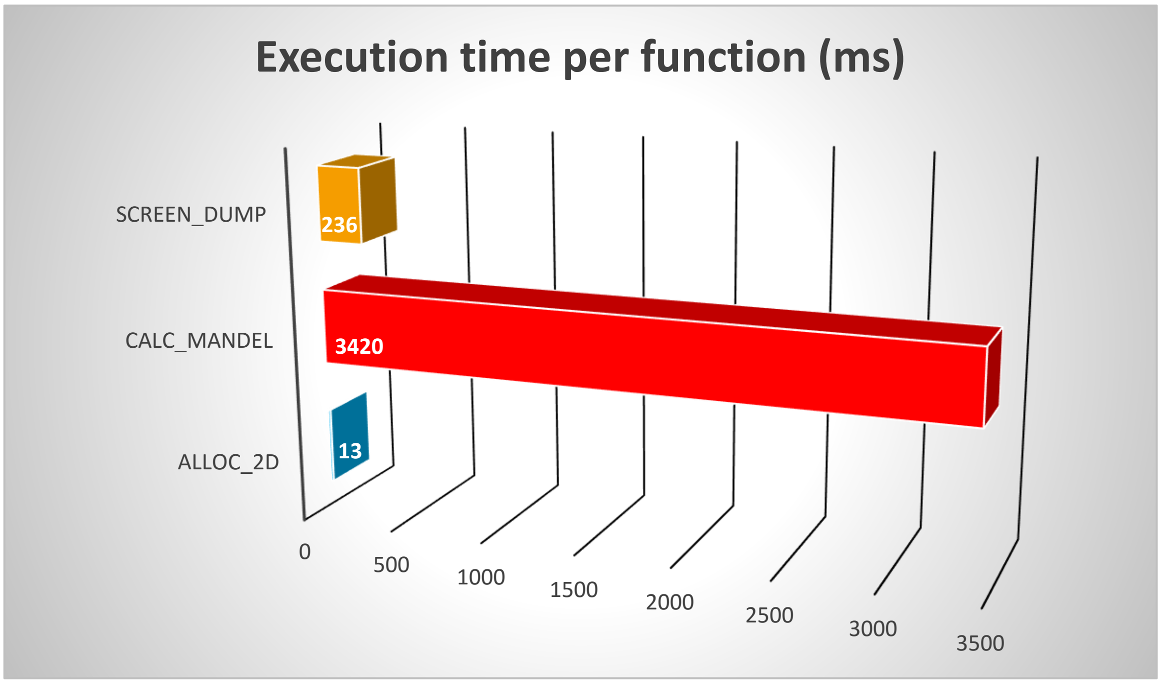


Figure 3. Execution time for each function in CPU-based solution

During the execution, *calc_mandel* calls another function *map_colour.* This is the code of the function:

```
void map_colour(rgb_t * const px)
{
  const uchar num_shades = 16;
  const rgb_t mapping[num_shades] =
    {{66,30,15},    {25,7,26},     {9,1,47},      {4,4,73},      {0,7,100},
     {12,44,138},   {24,82,177},   {57,125,209},  {134,181,229},{211,236,248},
     {241,233,191},{248,201,95},   {255,170,0},   {204,128,0},   {153,87,0},
     {106,52,3}};

  if (px->r == max_iter || px->r == 0) {
    px->r = 0; px->g = 0; px->b = 0;
  } else {
    const uchar uc = px->r % num_shades;
    *px = mapping[uc];
  }
}
```

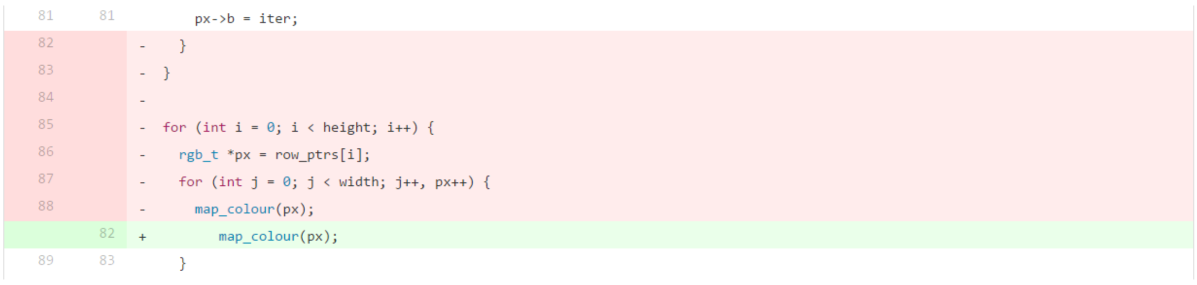
With italic bold is highlighted the code that slows the execution time.

This code is executed in the *calc_mandel* function for height *x* width iterations, and with each iteration we create the shades in the body of the map_colour, and with each iteraration a pixel is evaluated and given a colour.

During the execution of the *calc_mandel* we already traverse all the pixels, so the shade attribution for each pixel could be done inside the for loops belonging to the *calc_mandel*.

Sample code optimisation

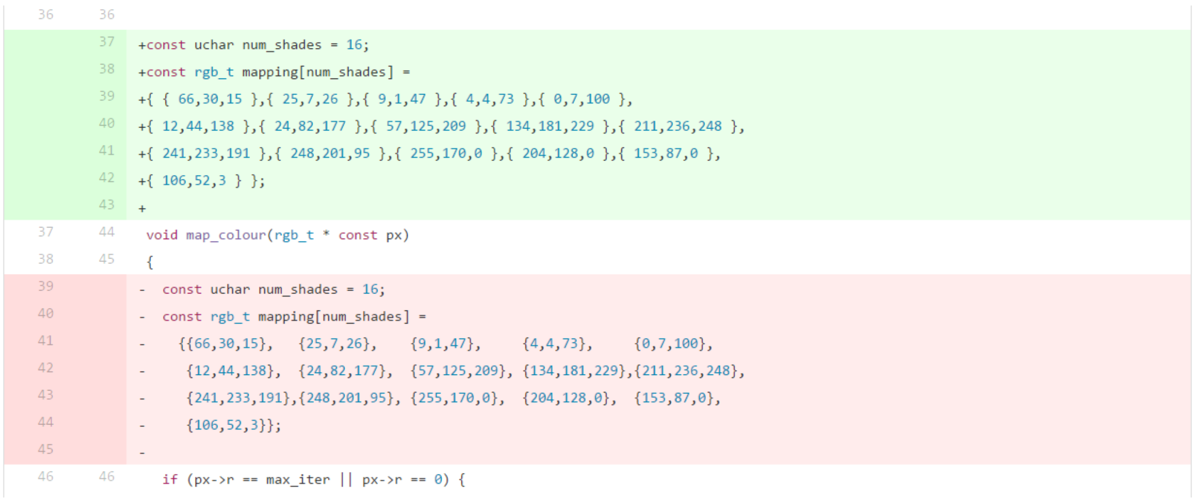
Before optimising the code on the GPU, improvements can be done to the code running on the CPU. To highlight the performance gain, the code was executed 5 times and the average execution time was measured with each improvement. The measurement was done with a high_resolution_clock.



```
      px->b = iter;
-    }
-  }
-
-  for (int i = 0; i < height; i++) {
-    rgb_t *px = row_ptrs[i];
-    for (int j = 0; j < width; j++, px++) {
-      map_colour(px);
+         map_colour(px);
    }
```

Figure 4. Default Code optimisation - Attempt 1

The first attempt was to move the map_colour in the for loops from the calc_mandel function as shown in Figure 3. Interestingly after doing this, the performance dropped and the execution time grew from 3630 milliseconds to 3789 milliseconds. Seeing that this was not a good result the code was reverted.

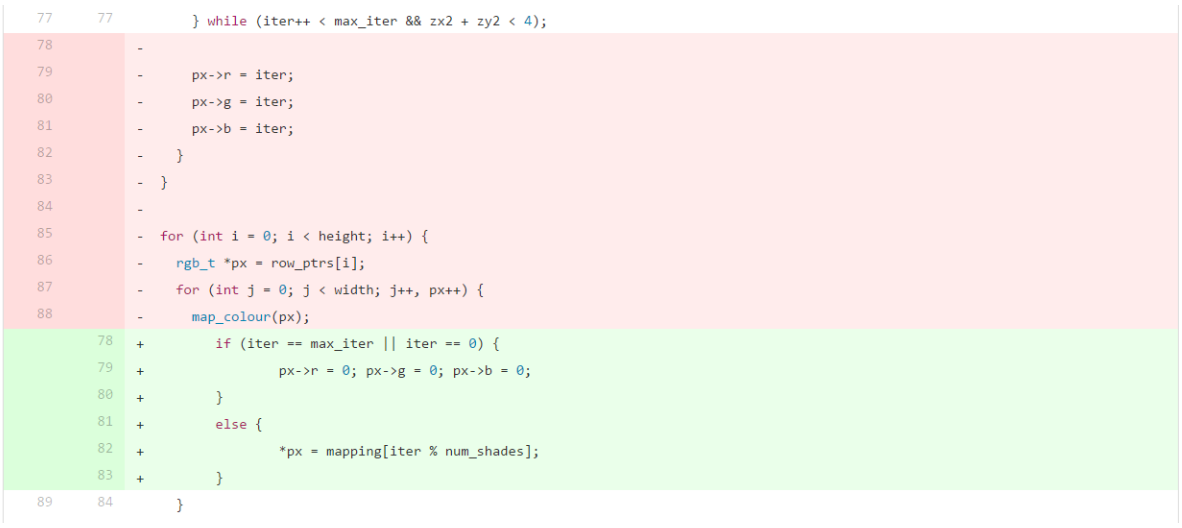


```

+const uchar num_shades = 16;
+const rgb_t mapping[num_shades] =
+{ { 66,30,15 },{ 25,7,26 },{ 9,1,47 },{ 4,4,73 },{ 0,7,100 },
+{ 12,44,138 },{ 24,82,177 },{ 57,125,209 },{ 134,181,229 },{ 211,236,248 },
+{ 241,233,191 },{ 248,201,95 },{ 255,170,0 },{ 204,128,0 },{ 153,87,0 },
+{ 106,52,3 } };
+
 void map_colour(rgb_t * const px)
 {
-  const uchar num_shades = 16;
-  const rgb_t mapping[num_shades] =
-    {{66,30,15},  {25,7,26},    {9,1,47},     {4,4,73},     {0,7,100},
-     {12,44,138}, {24,82,177},  {57,125,209}, {134,181,229},{211,236,248},
-     {241,233,191},{248,201,95}, {255,170,0},  {204,128,0},  {153,87,0},
-     {106,52,3}};
-
   if (px->r == max_iter || px->r == 0) {
```

Figure 5. Default Code optimisation - Attempt 2

The second attempt was to move the declaration of the shades outside of the map_colour function as seen in Figure 4. This means that the shades would only be declared only one time, even though they would sit in the global scope. The results show a performance gain, with the execution time dropping from 3630 milliseconds to 3325 milliseconds.



```
        } while (iter++ < max_iter && zx2 + zy2 < 4);
-
-       px->r = iter;
-       px->g = iter;
-       px->b = iter;
-     }
-   }
-
-   for (int i = 0; i < height; i++) {
-     rgb_t *px = row_ptrs[i];
-     for (int j = 0; j < width; j++, px++) {
-       map_colour(px);
+         if (iter == max_iter || iter == 0) {
+                 px->r = 0; px->g = 0; px->b = 0;
+         }
+         else {
+                 *px = mapping[iter % num_shades];
+         }
      }
```

Figure 6. Default Code optimisation - Attempt 3

The last step was to try again to move the colouring step in the calc_mandel for loop, but this time by eliminating the map_colour function, and to evaluate the pixel using the value of iter, as shown in Figure 5. Also, to calculate the right index of the shade, the local declaration of the uc has been removed. The results highlight another performance gain, with the execution time dropping from the previous result of 3325 milliseconds to 2959 milliseconds.

Overall with the current changes, the code runs faster, on average, with 671.4 milliseconds, a 18% performance gain.

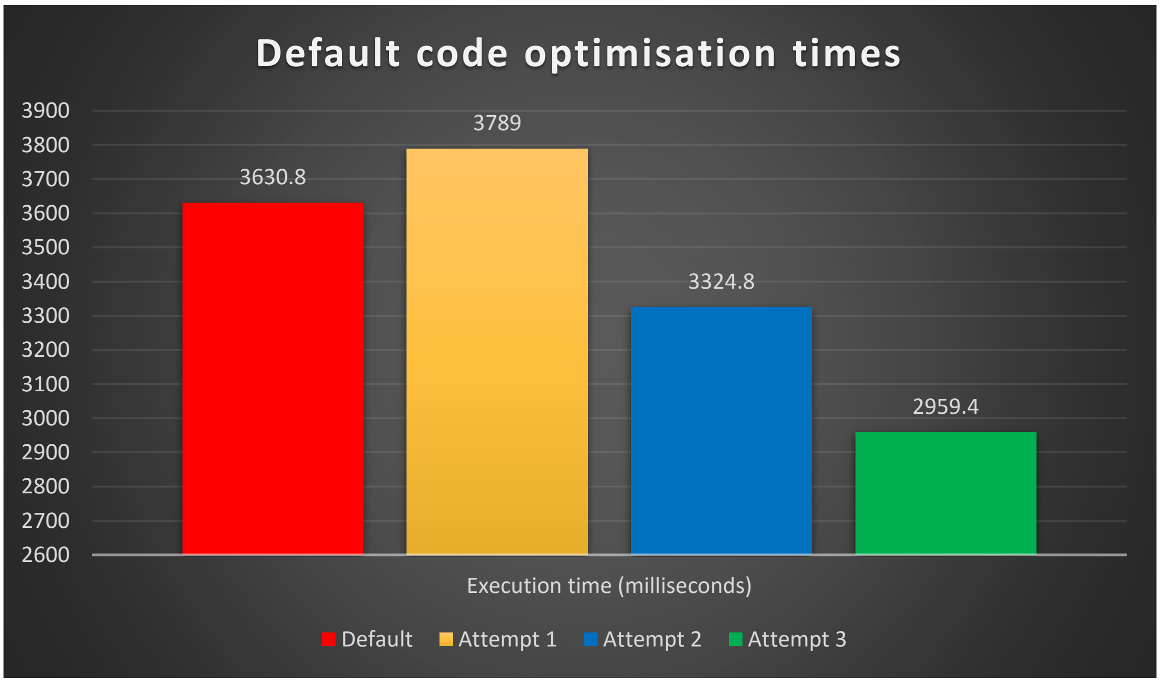


Figure 7. Default code optimisation graph (lower is better)

Cuda version

With CUDA, *host* CPU code can launch GPU *kernels* by calling *device* functions that execute on the GPU.

The necessary steps to convert the serial code into code running with the CUDA API are the following:

- create handles for the image data for both device and host
- allocate device memory for the device image data
- prepare kernel
    - find grid size and block size
    - calculate index (make sure index is calculated in reverse order)
    - run Mandelbrot algorithm
- retrieve image from device
- output image to file
- clean-up

To abstract away the functionality related to the creation of a Mandelbrot set, the code related to it has been encapsulated in a separated class. The class is defined by the width, height and the starting complex number which is tested if it is inside a such set. Also, two pointers are being declared to easy identify the image data on the host and on the device.

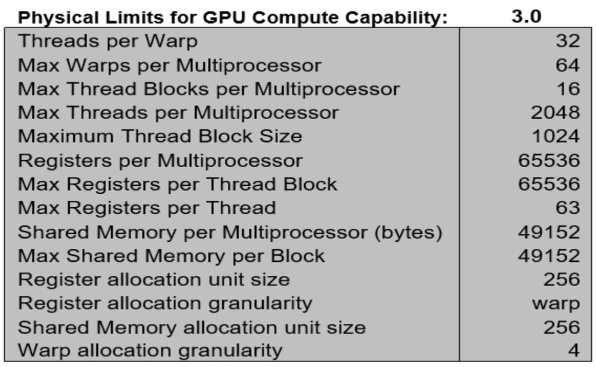
The constructor is responsible for instant allocation of memory on the device using an init() function, but also an array of Pixels is reserved on the host. The destructor is responsible for cleaning up the memory. Memory from both host and device is freed when the destructor is called.

Other function is fetch() which syncs the GPU processing units and copies the image data from device to host. The last useful function (ignoring the getters and setters) is the saveAs() function, which takes a string as a parameter to use as a name when writing the image to disk. This function does not check if there is any data on the host data, so it should be used only after the data has been fetched from the device.

The Util.h header file holds directives and definitions for some constants and structs. A check() function improves logging the cudaError_t. Another useful utility is the measure template which records the execution time for any method passed to it. It also contains the CUDA Occupancy API which allows the calculation of the best kernel configuration.

The hardware

The specification of the used GPU with Compute Capability of 3.0 is visible on the right in Figure 8 (NV_Quadro_K4000, 2013). The device has a maximum block size of 1024 threads, a warp size of 32, maximum registers 63, max shared memory per block 49152 bytes and the maximum grid size is 16. These limitations will reduce the number of combinations that can be used to output the best performance.



Physical Limits for GPU Compute Capability:	3.0
Threads per Warp	32
Max Warps per Multiprocessor	64
Max Thread Blocks per Multiprocessor	16
Max Threads per Multiprocessor	2048
Maximum Thread Block Size	1024
Registers per Multiprocessor	65536
Max Registers per Thread Block	65536
Max Registers per Thread	63
Shared Memory per Multiprocessor (bytes)	49152
Max Shared Memory per Block	49152
Register allocation unit size	256
Register allocation granularity	warp
Shared Memory allocation unit size	256
Warp allocation granularity	4

Figure 8. Hardware limitations for Quadro K4000

Running the first CUDA version

There are two important aspects to be considered before starting to use the CUDA API: index calculation and kernel parameters identification. Because the files are saved in reverse order on the CPU, we will directly flip the image data on the CPU, by assigning the index to go in reverse order from the lowest, most bottom line.

```
int row = blockIdx.y * blockDim.y + threadIdx.y;

int column = blockIdx.x * blockDim.x + threadIdx.x;
```

int index = (height -1 – row) * width + column;

The other aspect involved calculating the grid size and block size. A 2D grid will be used to calculate each direction (weight or height) in its own dimension. To try to find the right values, we need to consider the warp size of 32 to avoid padding, so for the block sizes, a multiple of 32 seems to be a good candidate (talonmies, 2012), but using our limitations it should be maximum 1024. Also, the grid can have maximum 16 blocks. Once a block size has been identified, the number of blocks, the grid size will be computed by dividing the image data to the number of threads activated for each block.

So, for example, to obtain a measure the execution time for a 4096x4096 image, the following experiment was done ignoring the hardware limits using the following blockSize values:

Blocks	4096x4096	2048x204	1024x1024	512x512	256x256	128x128
Threads	1x1	2x2	4x4	8x8	16x16	32x32

Figure 9. Estimated good configurations. All values multiplied should return 16777216 pixels

. . .   .

```
dim3 blockSize(32, 32);

dim3 gridSize(128, 128);

calc_mandel << < gridSize, blockSize >> >();
```

. . .   .

Another aspect was to make sure that the shades used at each iteration are immediately available to be used on each block. For this, the array of shades has been globally declared as __constant__.

The application was executed with a maximum of 32 registers per thread. Later these values will be tweaked to improve performance.

Averaging and measuring – Methodology

To try to find the best execution times for a piece of code, micro-benchmarking has been performed using the following procedure:

Given a target code which requires its execution time to be measured, we surround it between a start and end timer, record the difference and repeat this process for an arbitrary chosen number of cycles and average the differences. This process is again repeated for different kernel configurations and each average time is saved along with the configuration it has been used. The lowest average should highlight the best kernel configuration. The process is again repeated for different image resolutions. All timings are recorded in different files (one file with timings for each resolution) and through observation, the fastest times will be highlighted.

```
For each cycle
        Assign current time instance to start
        Run block of code
        Assign current time to end
        Add difference to total cycle time
End for
Record time/cycles as average execution time.
```

Cuda Occupancy API

To ease the optimisation, Harris (2014) suggests to use the Occupancy API if a CUDA 6.5 or higher is available. The API makes it possible to compute an efficient execution configuration for a kernel.

Using two variables for the block and grid size and the cudaOccupancyMaxPotentialBlockSize() we can calculate the grid size depending on the width (or height) of the image:

- a variable minGridSize and the block size is passed by reference to the above-mentioned function, together with the kernel and the maximum block size is obtained.
- based on the block size, gridSize is calculated using (width + blockSize -1) / blockSize
- this way a rounded (integer) number of blocks is used.
- these two values are stored in a KernelProperties struct and can be further used.

Just for logging, the theoretical occupancy is also calculated after the deviceProperties have been retrieved:

occupancy = (maxActiveBlocks * blockSize / warpSize) / (maxThreadsPerMultiProcessor / warpSize);

This can be further simplified to:

occupancy = maxActiveBlocks * blockSize / maxThreadsPerMultiProcessor;

Even though Luitjens & Rennich (2011) suggest that 66% occupancy is enough to saturate the bandwidth, our kernel configuration will aim to be configured for 100% theoretical occupancy. During the measuring process, the recommended configuration will be also timed and recorded.

Data collection

To collect minimum, maximum and the average execution time for each kernel configuration we run the application via command line passing the following arguments:

```
gpu.exe width height shouldCompare realPart imaginaryPart
```

- **width** represents the value of the width of the image in pixels
- **height** represents the value of the height of the image in pixels
- **shouldCompare** enables/disables file comparison at the end of the program
- **realPart** is the real component of the complex number to check if it is in the set
- **imaginaryPart** is the imaginary component of the complex number to check if it is in the set

All arguments have default values so when we want to run the program for a certain image size, we just pass the width at height:

```
gpu.exe 128 128
```

Images will be generated carrying the output, and in the filename the image will have the kernel configuration and the execution time. Also, same data is collected in an excel file with these headers:

Width	Height				
Blocks	Grid Layout	Threads	Block Layout	Millis	Nanos

Figure 10. The headers of the Excel generated tables

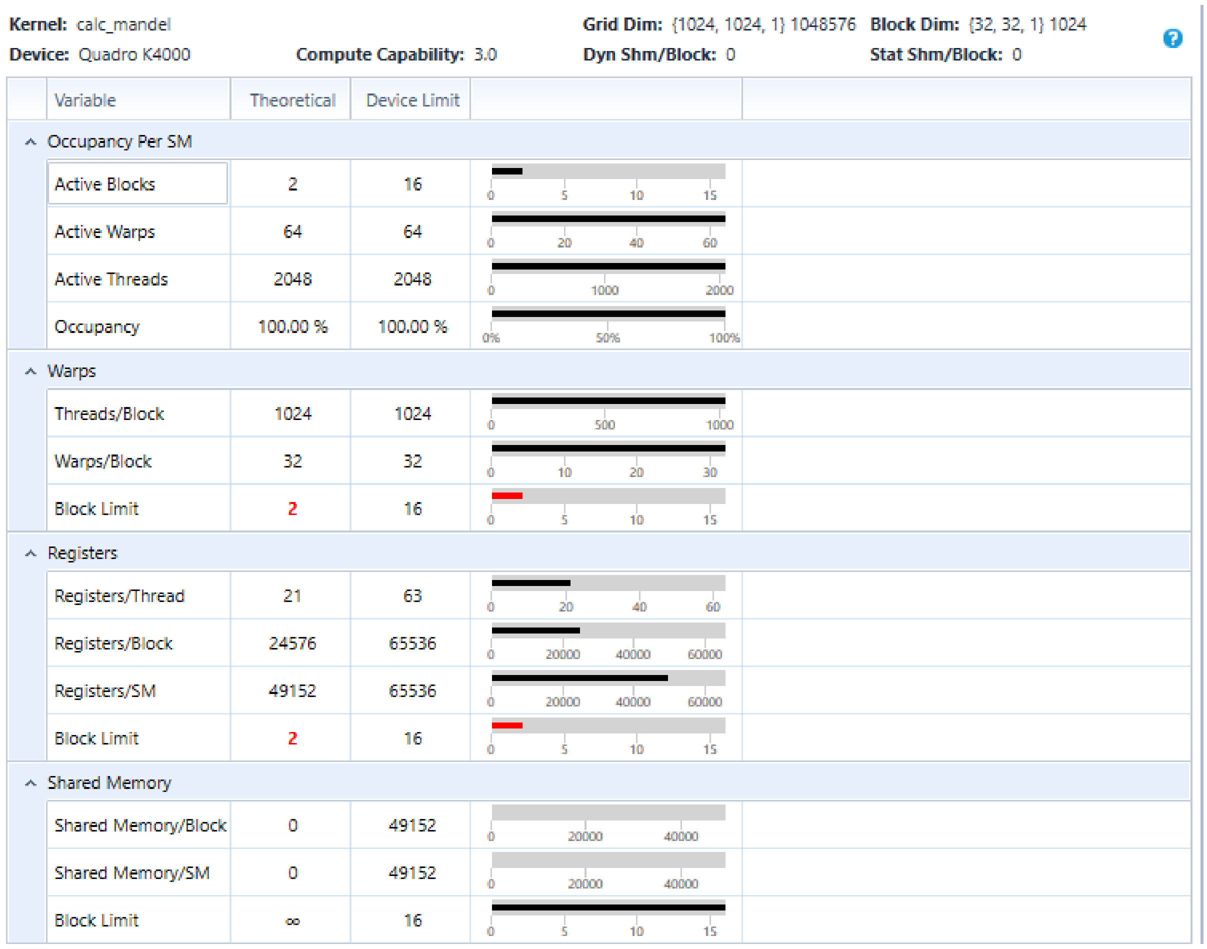
Repeating the execution of the program for different images sizes, enough data is generated that can be further processed to reveal the minimum, maximum and average execution time.

Results

The most results are stored in the appendix section which will have charts that display the **minimum**, **maximum** and **average** execution time needed to generate Mandelbrot sets of different sizes, for various kernel configurations in format blocks x threads, visible on the table-legend at the bottom of each chart. Always the topmost entry in the chart, displayed with red text, will represent the recommended configuration (using the CUDA occupancy API). We normally care about the average time as it is more representative. The fastest average time is highlighted with green text in the data table, the recommended configuration time has a red outline, the fastest configuration that does not use the CUDA occupancy API is displayed using a green outline, while the lowest execution times are represented with a blue outline. The application generates a lot of data, even irrelevant data (data collected using more than 1024 threads per block). Other image sizes execution times for each configuration can be found in the appendix.

4096x4096

For the 4096x4096 set, CUDA suggested to use 4 blocks of 1024 threads each, or a 2x2x32x32. These settings unfortunately did not create a correct output. So the configuration with full occupancy was a grid of 1024,1024,1 blocks with 32,32,1 threads per block as visible in Figure 11:



Kernel: calc_mandel **Grid Dim:** {1024, 1024, 1} 1048576 **Block Dim:** {32, 32, 1} 1024
Device: Quadro K4000 **Compute Capability:** 3.0 **Dyn Shm/Block:** 0 **Stat Shm/Block:** 0

Variable	Theoretical	Device Limit
Occupancy Per SM		
Active Blocks	2	16
Active Warps	64	64
Active Threads	2048	2048
Occupancy	100.00 %	100.00 %
Warps		
Threads/Block	1024	1024
Warps/Block	32	32
Block Limit	2	16
Registers		
Registers/Thread	21	63
Registers/Block	24576	65536
Registers/SM	49152	65536
Block Limit	2	16
Shared Memory		
Shared Memory/Block	0	49152
Shared Memory/SM	0	49152
Block Limit	∞	16

Figure 11. Full occupancy for a 4096x4096 image with a 1024,1024,1 grid and 32,32,1 threads for each block

The execution time for this configuration was 168 milliseconds. According to the Nsight profiler, the *Block Limit Reason* is: "*warps, registers*". Another similar configuration with full occupancy, with a smaller grid is the grid with 128,128,1 blocks (16384) again with 32,32,1 (1024) threads per block. This configuration executed in 75 milliseconds, so in theory further reducing the blocks used, will improve performance. The conclusion seems to lead to the CUDA Occupancy API suggested solution, but unfortunately any grid with less blocks of 1024 threads will not output the image correctly. The occupancy graphs obtained from the Profiler are the same with the ones from the CUDA Occupancy calculator:

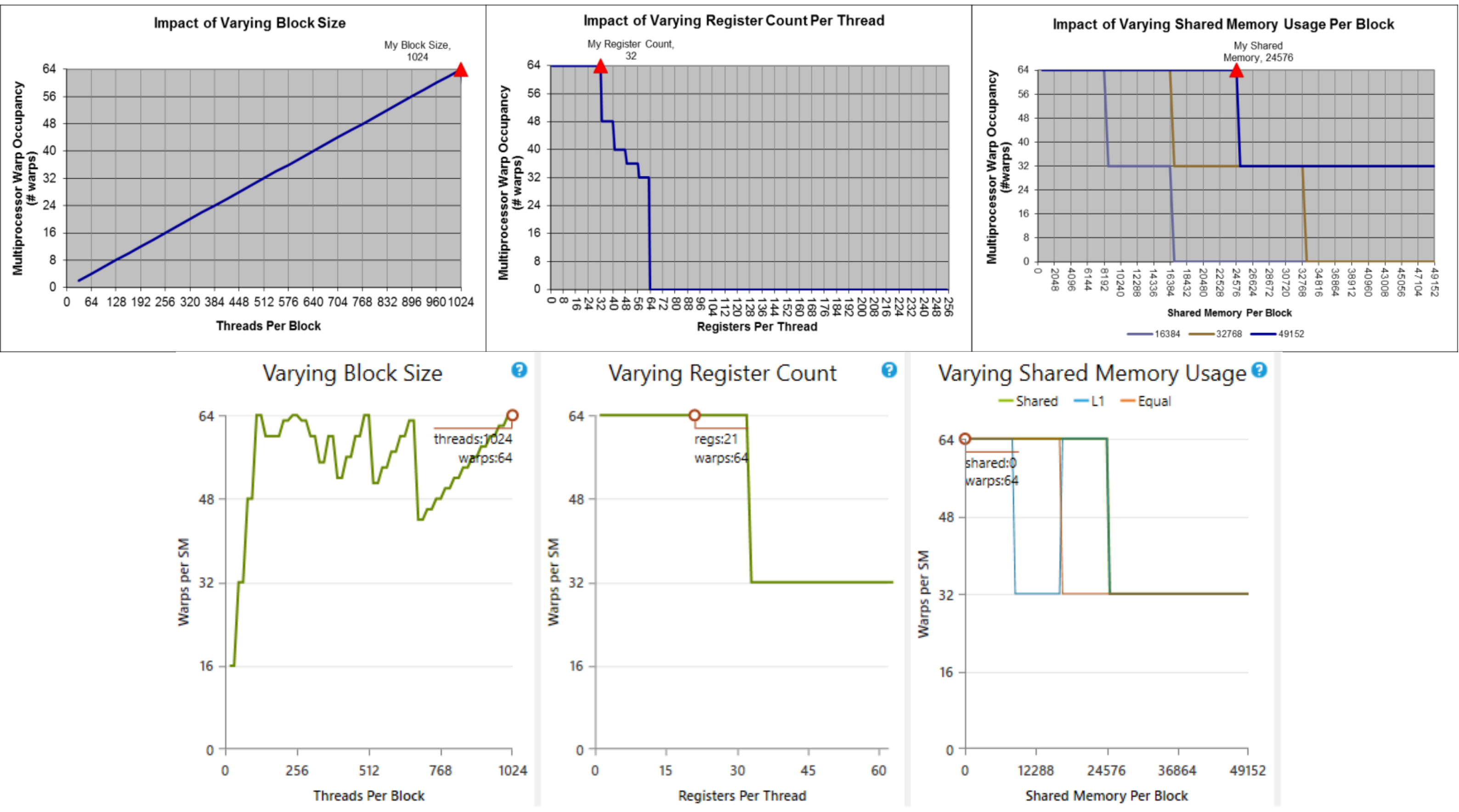
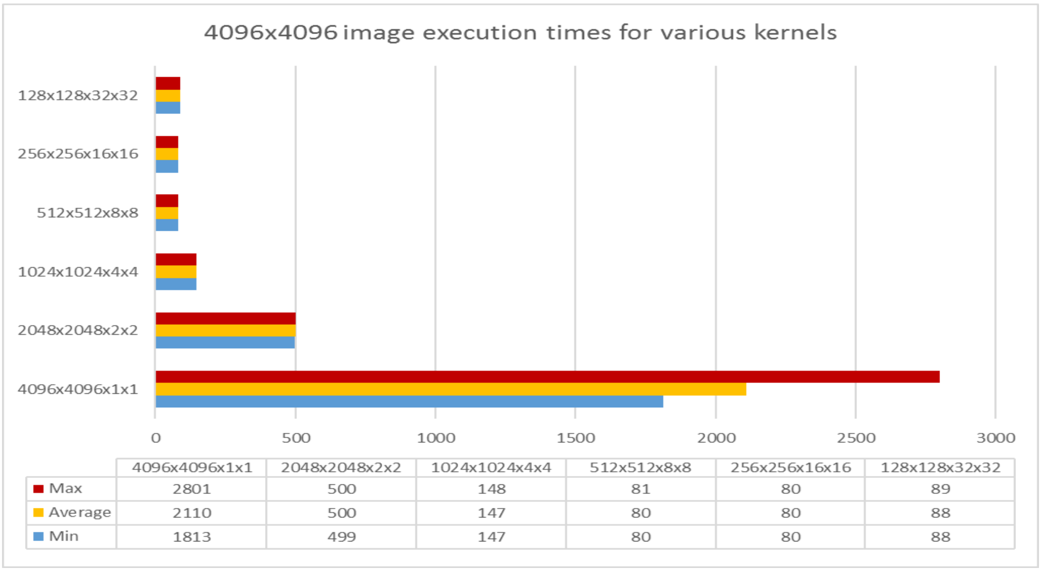


Figure 12. Optimizations available for 128x128x32x32 kernel configurations

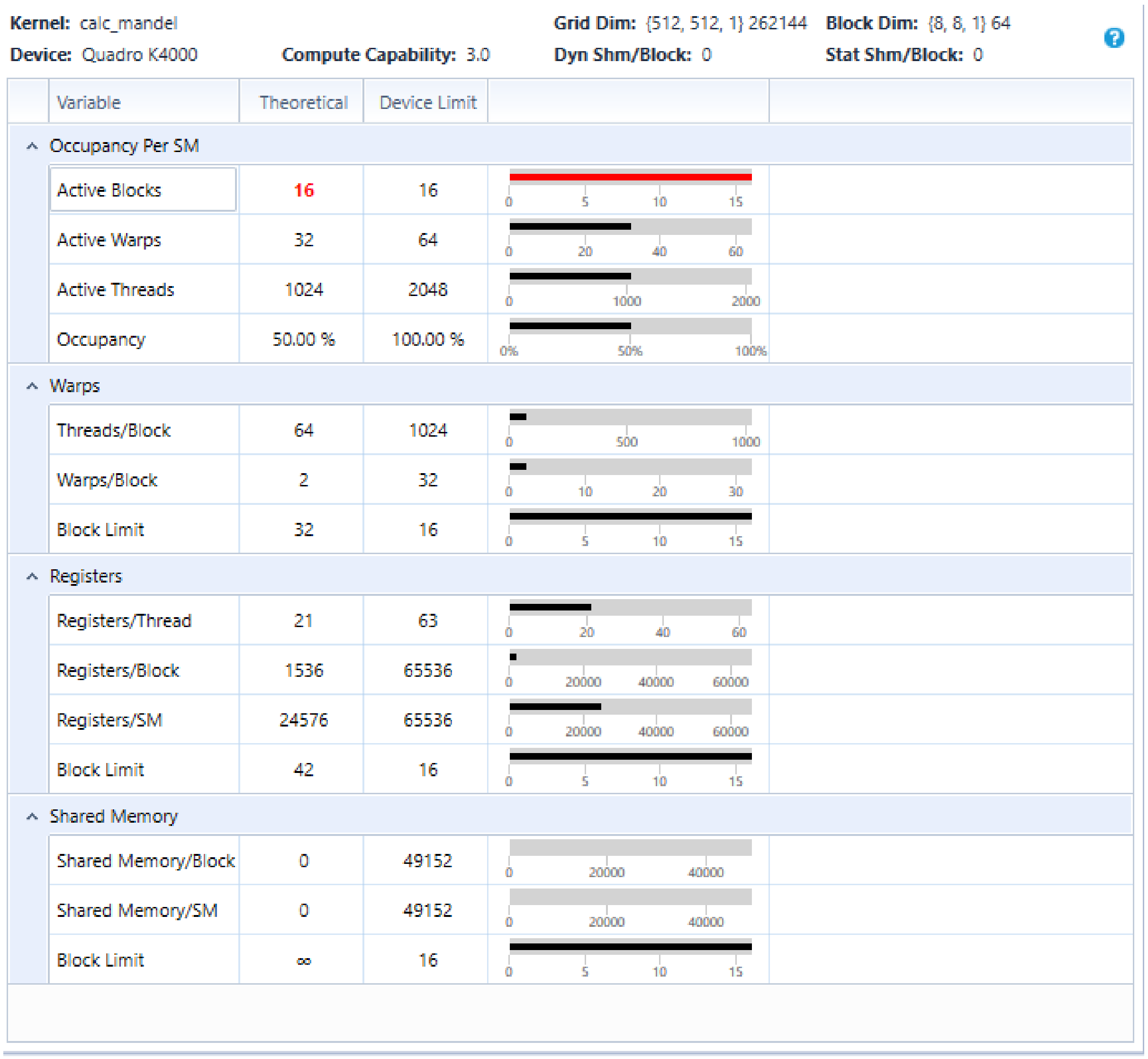
The difference between the two graphs highlight the possible improvements: the shared memory could be increased to 24576 bytes per block; regarding to the block size, the configuration is already at maximum and another improvement could be achieved by raising the maximum register count to 32. Any superior value will have an impact on the occupancy. Other performant configurations are:



	4096x4096x1x1	2048x2048x2x2	1024x1024x4x4	512x512x8x8	256x256x16x16	128x128x32x32
🟥 Max	2801	500	148	81	80	89
🟨 Average	2110	500	147	80	80	88
🟦 Min	1813	499	147	80	80	88

Figure 13. Other working configurations.

What is interesting to observe is that there are other configurations which do not have 100% theoretical occupancy but which run faster a 512,512,1 grid with 64 threads per block and a 50% theoretical occupancy is executed in 68 milliseconds. This time the profiler reveals that the *Block Limit Reason* is related to the number of active blocks.



Kernel: calc_mandel | **Grid Dim:** {512, 512, 1} 262144 | **Block Dim:** {8, 8, 1} 64

Device: Quadro K4000 | **Compute Capability:** 3.0 | **Dyn Shm/Block:** 0 | **Stat Shm/Block:** 0

Variable	Theoretical	Device Limit
Occupancy Per SM		
Active Blocks	16	16
Active Warps	32	64
Active Threads	1024	2048
Occupancy	50.00 %	100.00 %
Warps		
Threads/Block	64	1024
Warps/Block	2	32
Block Limit	32	16
Registers		
Registers/Thread	21	63
Registers/Block	1536	65536
Registers/SM	24576	65536
Block Limit	42	16
Shared Memory		
Shared Memory/Block	0	49152
Shared Memory/SM	0	49152
Block Limit	∞	16

Figure 14. 50% occupancy for a 4096x4096 image with a 512,512,1 grid and 8,8,1 threads for each block

Checking the Nsight Occupancy Graph against the CUDA Occupancy Calculator, it is noticeable that the next improvement is to increase the number of threads. After changing the kernel to 512,512,1 with 16,16,1 (256) threads per block, a theoretical occupancy of 100% has been obtained, but the *Block Limit* now is the only related to the only 8 active blocks out of 16, even though the reported reason is *Warps.* This configuration completed the application in 71 milliseconds.

A preliminary conclusion can already be observed: higher theoretical occupancy does not mean that the application will actually be faster.

Gathering the minimum, maximum and average time for three different sizes already starts to show a pattern. Given the following table with the average execution time for each image size using each of the working configurations, a chart can be generated to highlight that the performance gain is directly proportional to the image size. Also, based on the slower kernel configuration, the fastest kernel configuration represents almost the same percent of the slowest obtained time, indifferently of the image size. It can also be noted in the table that the best execution times are obtained in the area between 8x8 or 16x16 number of threads per block.

Kernel \ Size	4096	2048	1024	512
4096x4096x1x1	118	202	529	2110
2048x2048x2x2	31	55	147	500
1024x1024x4x4	9	16	43	147
512x512x8x8	3	7	23	80
256x256x16x16	3	7	23	80
128x128x32x32	4	8	26	88

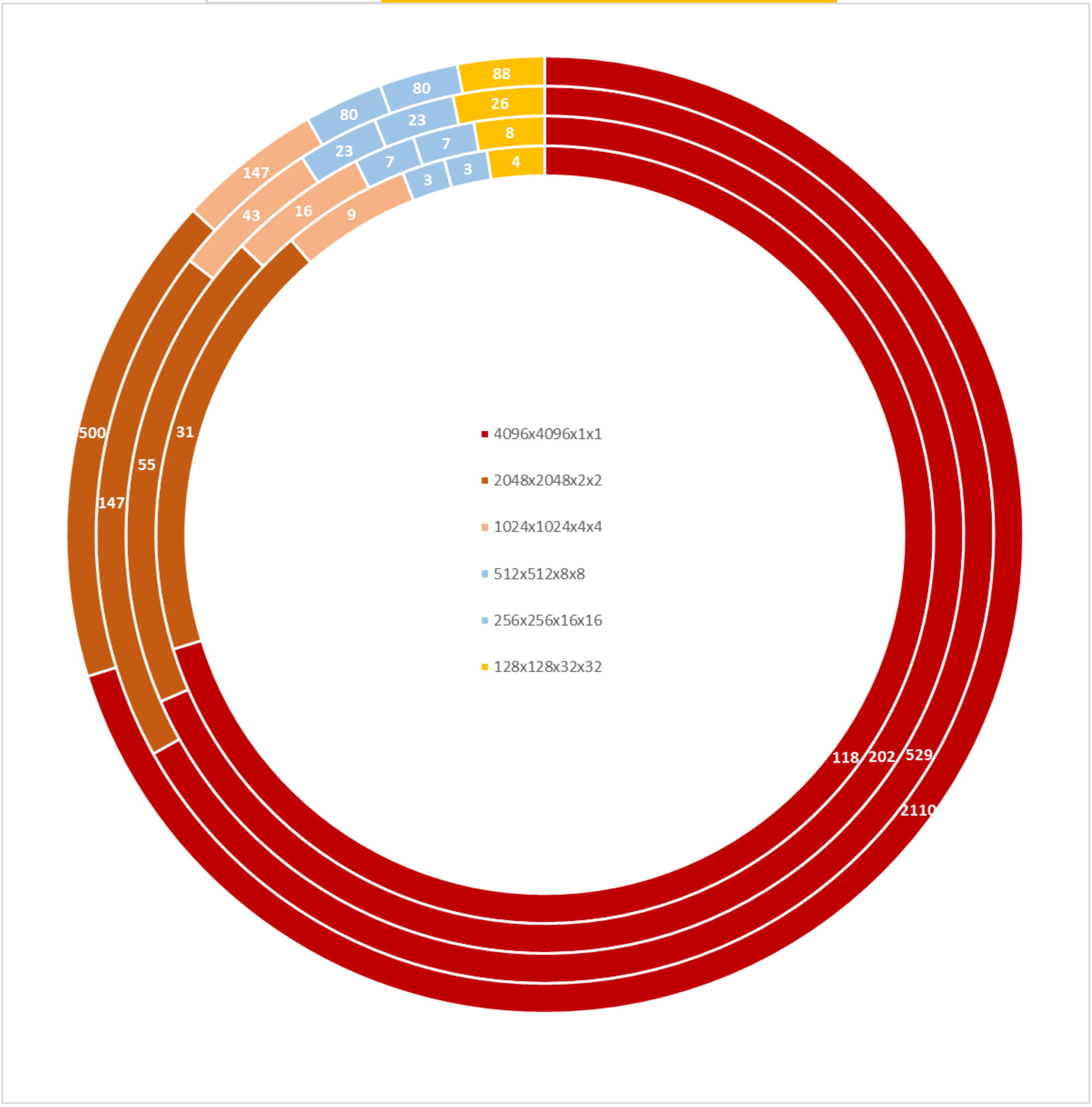


Figure 15. Kernel Performance gain area and quasi constant proportion, for various image sizes. Times are in milliseconds

Overall comparing the best average GPU execution time against the average CPU execution time, it can be noted a performance improvement of 2890 milliseconds, making the GPU application 40+ times faster, which is quite impressive.

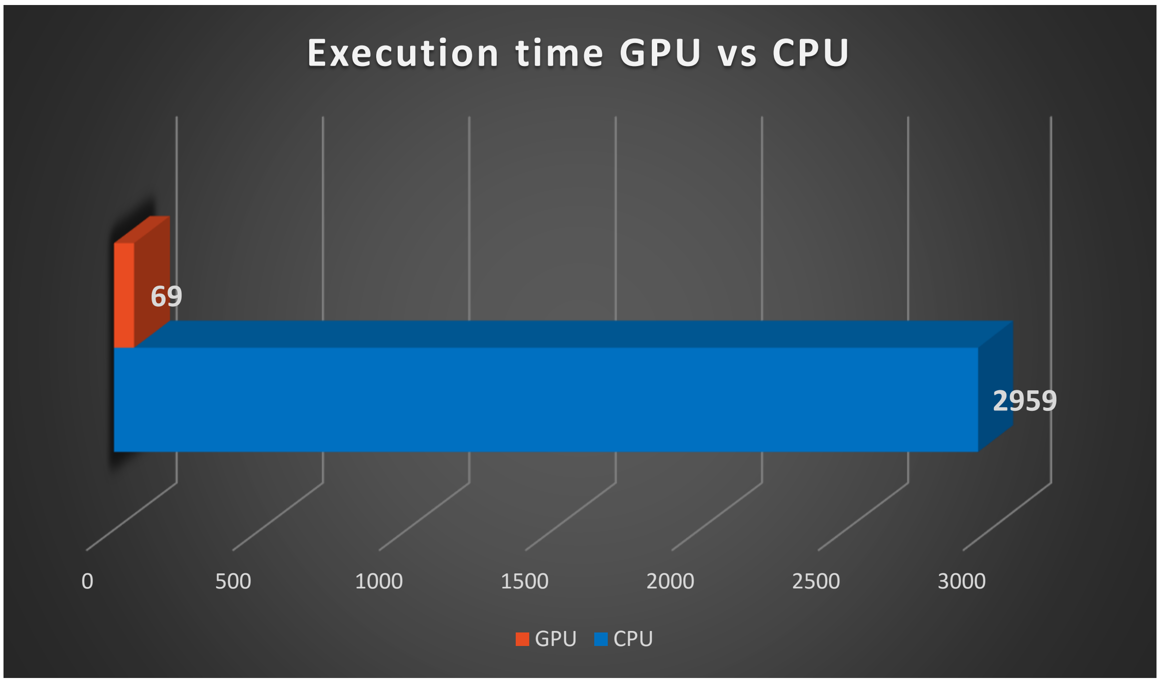


Figure 16. GPU vs CPU execution time

Other configurations have been checked for different image size, and in a few cases (512x512 image and 1024x1024 image), incompatible configurations (1x2048) obtained better times than the CUDA recommended configuration.

Also, it can be noted that in all suggested configurations, CUDA recommended 1024 threads, only varying the number of blocks. The next image shows the recommended number of blocks for different image sizes:

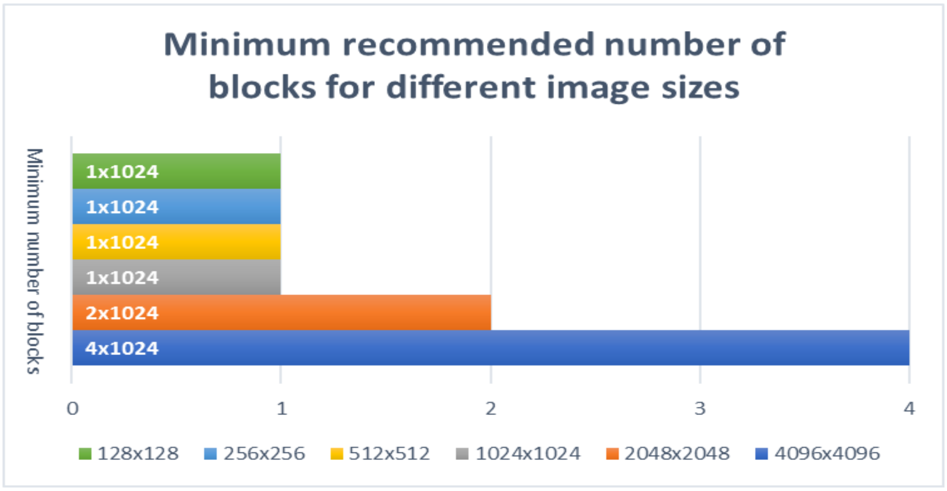


Figure 17. Minimum number of blocks suggested for different image sizes

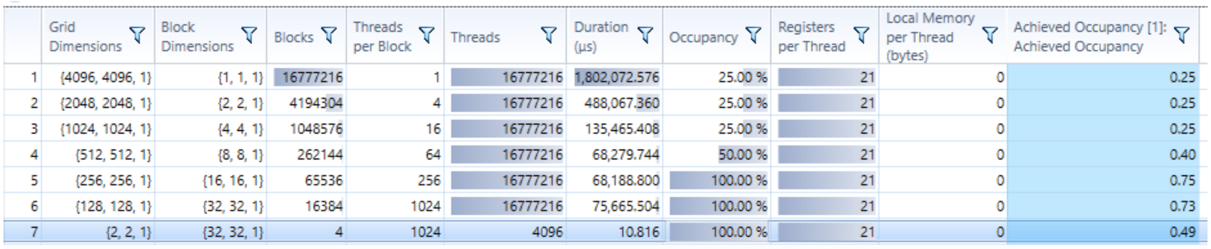
To generalise a better configuration for all different sizes, the minimum number of active blocks should be 4. Because it is a minimum, it should work efficiently with configurations that used to be 1x1024 or 2x1024.

Comparing the output

A wrongly calculated index, or a misused shade, could totally corrupt the resulted Mandelbrot set. To verify the validity of the image, on request, using the third command line argument set to “true” the application can be executed normally and before the last step a CPU generated Mandelbrot Set can be compared against the GPU set generated with CUDA recommended kernel settings.

The comparison checks if the two files are identical. The comparison is done at binary level. It can be noticed that for images up to 2048x2048, the two outputs were completely identical. With the 4096x4096 image, 2 pixels were identified to be different against the CPU image. After locating the pixels and performing a visual comparison, the images still look identical. But on a binary level, 2 pixels had two shades differing, which means they had 6 values different (2 reds, 2 greens and 2 blues). This is because of the denormalized (qdot, 2013) numbers and because of the fused multiply add of the compute 2.0 chipsets (Sayan, 2012). Setting the compiler -use_fast_math -fmad=false will help remove the accuracy problem and with fast math, will make the performance loss less noticeable.

If the average sets that need to be generated are smaller than 4096x4096, or if losing the file accuracy of two pixels (visually not affected) is tolerable, it would be better to use the compiler flag fmad enabled.



	Grid Dimensions	Block Dimensions	Blocks	Threads per Block	Threads	Duration (µs)	Occupancy	Registers per Thread	Local Memory per Thread (bytes)	Achieved Occupancy [1]: Achieved Occupancy
1	{4096, 4096, 1}	{1, 1, 1}	16777216	1	16777216	1,802,072.576	25.00 %	21	0	0.25
2	{2048, 2048, 1}	{2, 2, 1}	4194304	4	16777216	488,067.360	25.00 %	21	0	0.25
3	{1024, 1024, 1}	{4, 4, 1}	1048576	16	16777216	135,465.408	25.00 %	21	0	0.25
4	{512, 512, 1}	{8, 8, 1}	262144	64	16777216	68,279.744	50.00 %	21	0	0.40
5	{256, 256, 1}	{16, 16, 1}	65536	256	16777216	68,188.800	100.00 %	21	0	0.75
6	{128, 128, 1}	{32, 32, 1}	16384	1024	16777216	75,665.504	100.00 %	21	0	0.73
7	{2, 2, 1}	{32, 32, 1}	4	1024	4096	10.816	100.00 %	21	0	0.49

Figure 18. Various execution times and theoretical occupancies in profiler

Conclusions

Obviously using CUDA API efficiently can significantly increase performance and should be the best choice when looking to find a generalised kernel configuration (even though in reality it generates a specific configuration for each kernel). Figure 13 shows that a higher theoretical occupancy does not always result in the fastest times. If we compare the time from execution 4 against time against execution 5, it can be observed that they are similar, even though the execution 5 had higher theoretical and achieved occupancy. In most cases, efficient results are returned when theoretical occupancy is 100%, except for the accepted cases when the data has finished processing in 1 cycle (smaller images).

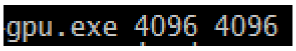
One aspect that can be observed, is that even if the maximum blocks per SM is 16, CUDA had no problems generating output for grids with more than 16 blocks, even though in most cases it was a lot slower. Also, it is interesting to observe that the CUDA API did not always have the best average time and, for some of the outputs, the API failed to provide the best performance.

It is important to saturate the hardware in the best possible way. Using the CUDA Occupancy Calculator it is possible to observe for which kernel configurations the theoretical 100% occupancy can be achieved. Setting a maximum number of registers and maximum amount of share memory as shown in the Occupancy Calculator are other two optional optimisations that could be applied. Generalising a configuration to work for different image sizes can be performed with an insignificant, tolerable performance loss for some of the images.

A performance achievement like the one obtained when comparing the GPU against CPU execution time, is outstanding, as the Cuda application is 269 times faster, and probably can be even further optimised.

How to reproduce the results:

In the application folder, run the gpu.exe in command line using the following arguments:



```
gpu.exe 4096 4096
```

Repeat the command for the remaining image sizes: 2048, 1024, 512, 256, 128.

To compare GPU output against CPU output run the application with the following arguments:

```
gpu.exe 4096 4096 true
```

This will compare the GPU output generated by the kernel using the recommended configuration against the CPU output.

To manually enter the complex number value, run the application with the following arguments:

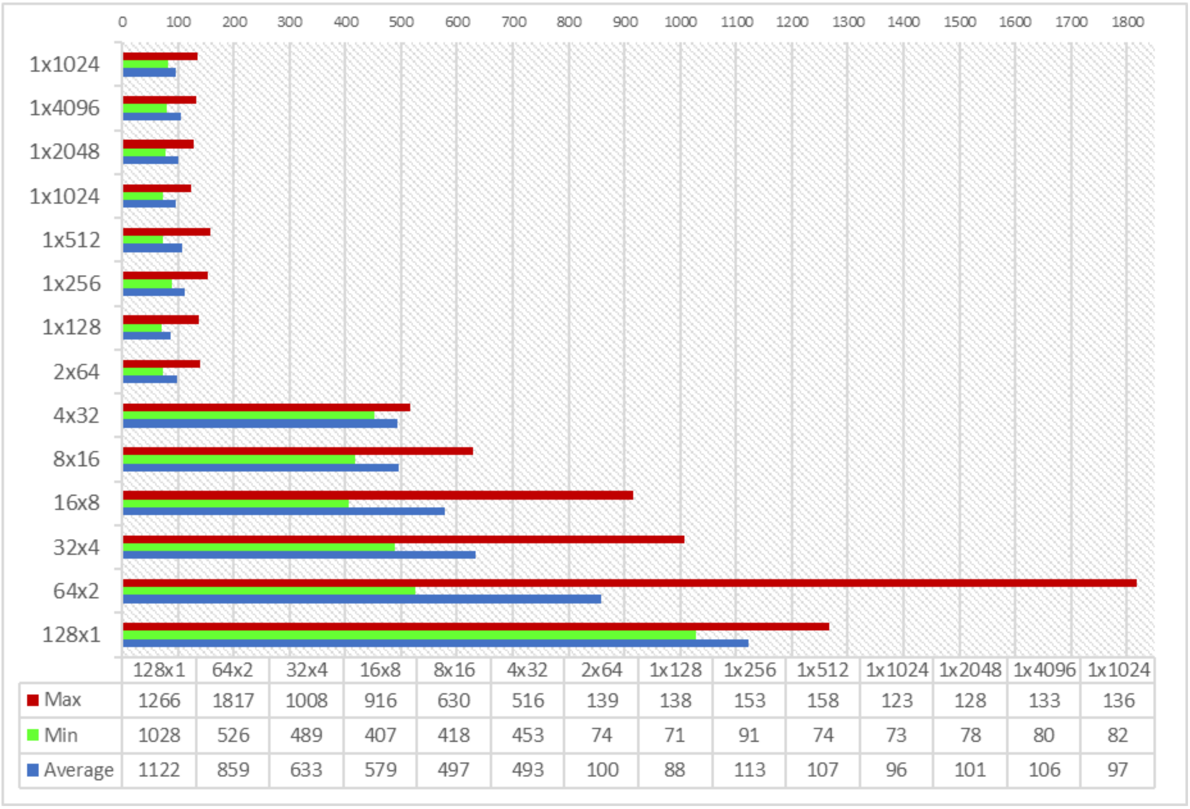
```
gpu.exe 4096 4096 false -0.6 0
```

-0.6 is the real part and 0 is the imaginary part of the complex number.

To try and experiment with custom kernel configurations, use the application in the folder customKernelConfigWithCompare. Instructions are included.

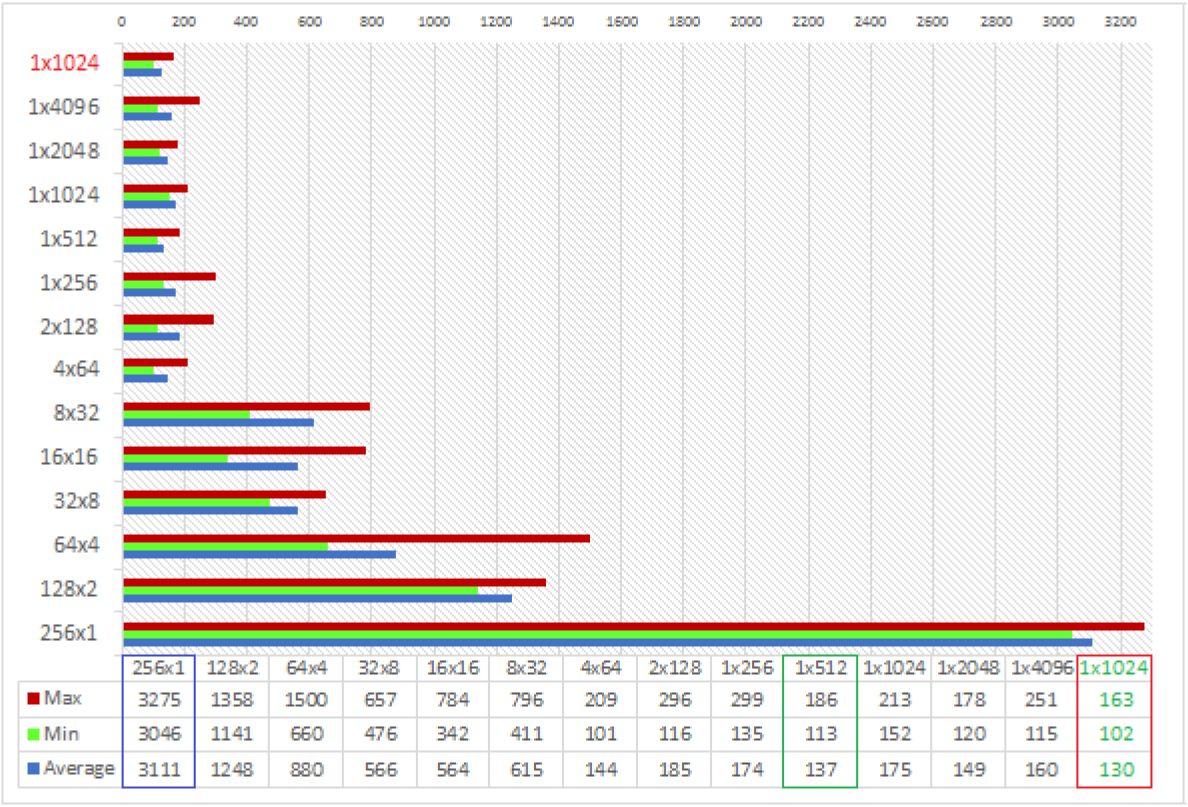
Appendix (charts for various kernel configuration and their obtained timings (invalid data with red fill)

128x128



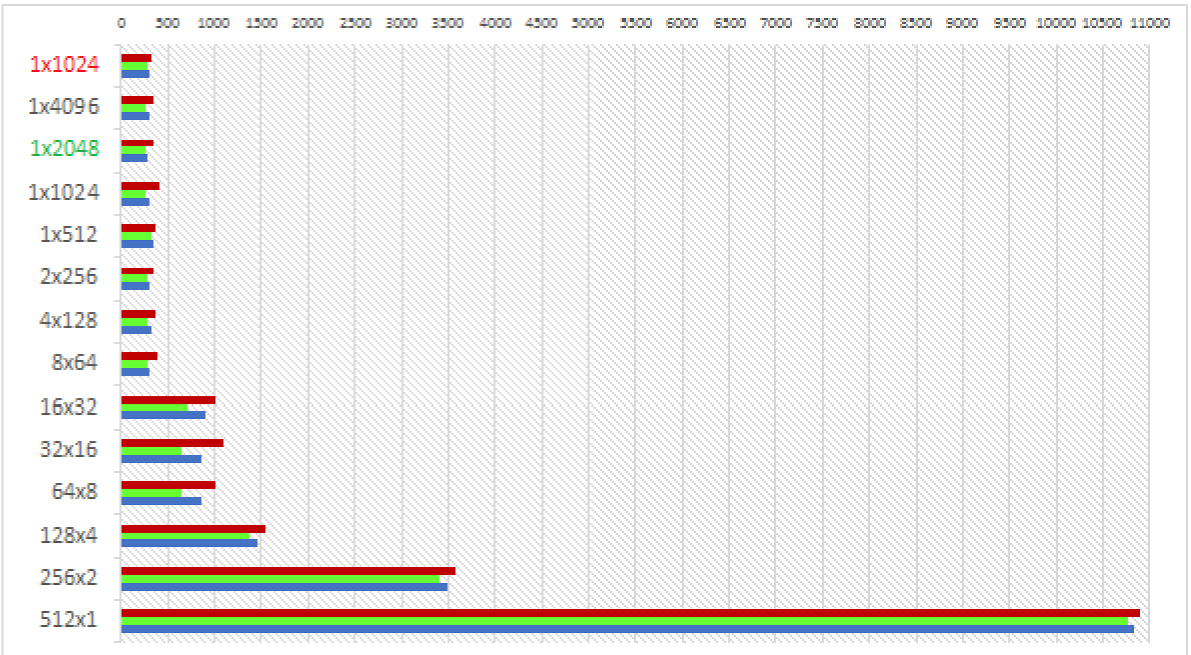
	128x1	64x2	32x4	16x8	8x16	4x32	2x64	1x128	1x256	1x512	1x1024	1x2048	1x4096	1x1024
🟥 Max	1266	1817	1008	916	630	516	139	138	153	158	123	128	133	136
🟩 Min	1028	526	489	407	418	453	74	71	91	74	73	78	80	82
🟦 Average	1122	859	633	579	497	493	100	88	113	107	96	101	106	97

256x256



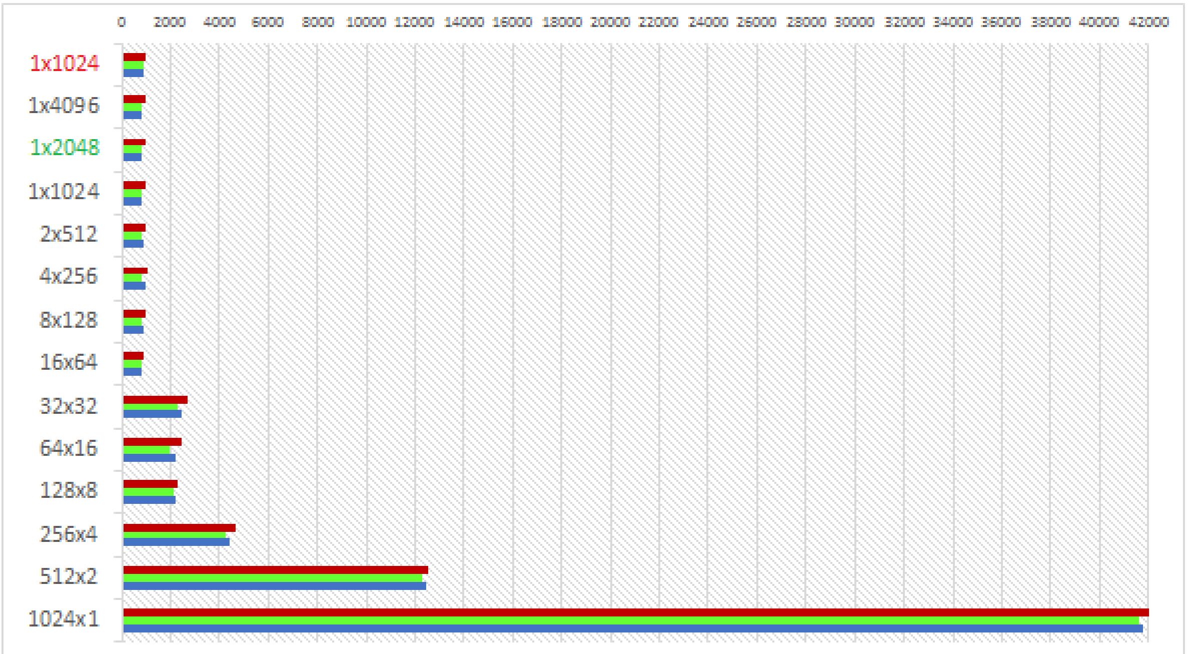
	256x1	128x2	64x4	32x8	16x16	8x32	4x64	2x128	1x256	1x512	1x1024	1x2048	1x4096	1x1024
🟥 Max	3275	1358	1500	657	784	796	209	296	299	186	213	178	251	163
🟩 Min	3046	1141	660	476	342	411	101	116	135	113	152	120	115	102
🟦 Average	3111	1248	880	566	564	615	144	185	174	137	175	149	160	130

512x512



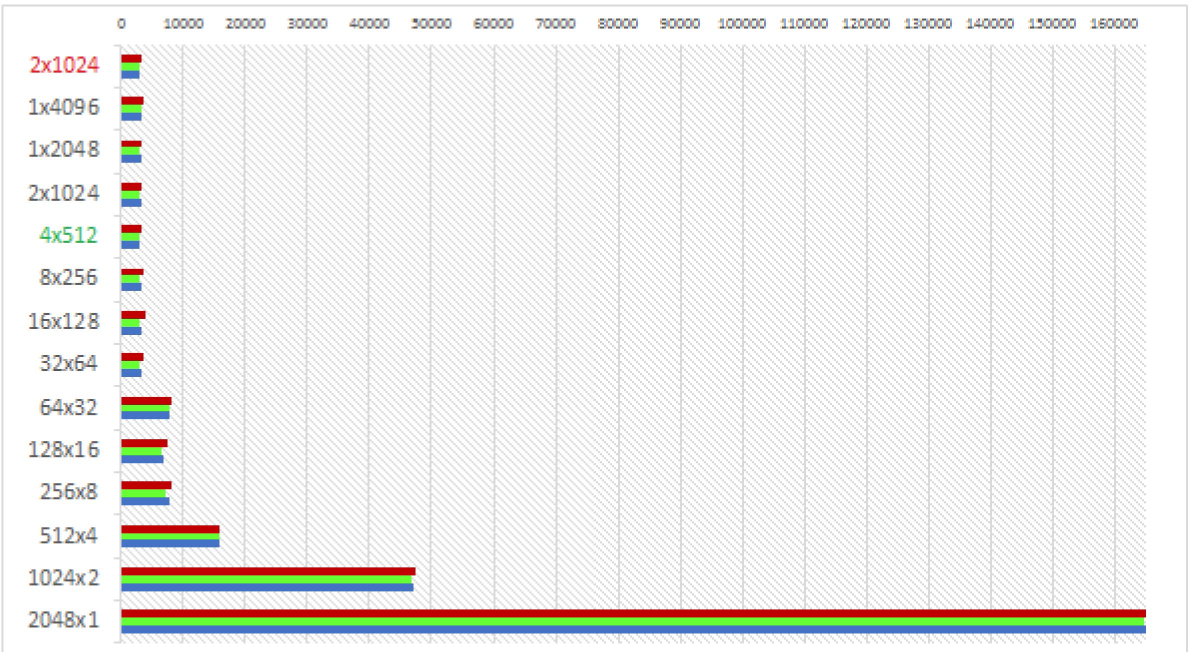
	512x1	256x2	128x4	64x8	32x16	16x32	8x64	4x128	2x256	1x512	1x1024	1x2048	1x4096	1x1024
🟥 Max	10885	3577	1540	1000	1096	1005	381	365	339	369	405	343	338	328
🟩 Min	10758	3411	1372	646	647	705	278	281	290	321	265	264	271	277
🟦 Average	10828	3481	1458	856	849	897	304	321	314	342	309	286	300	298

1024x1024



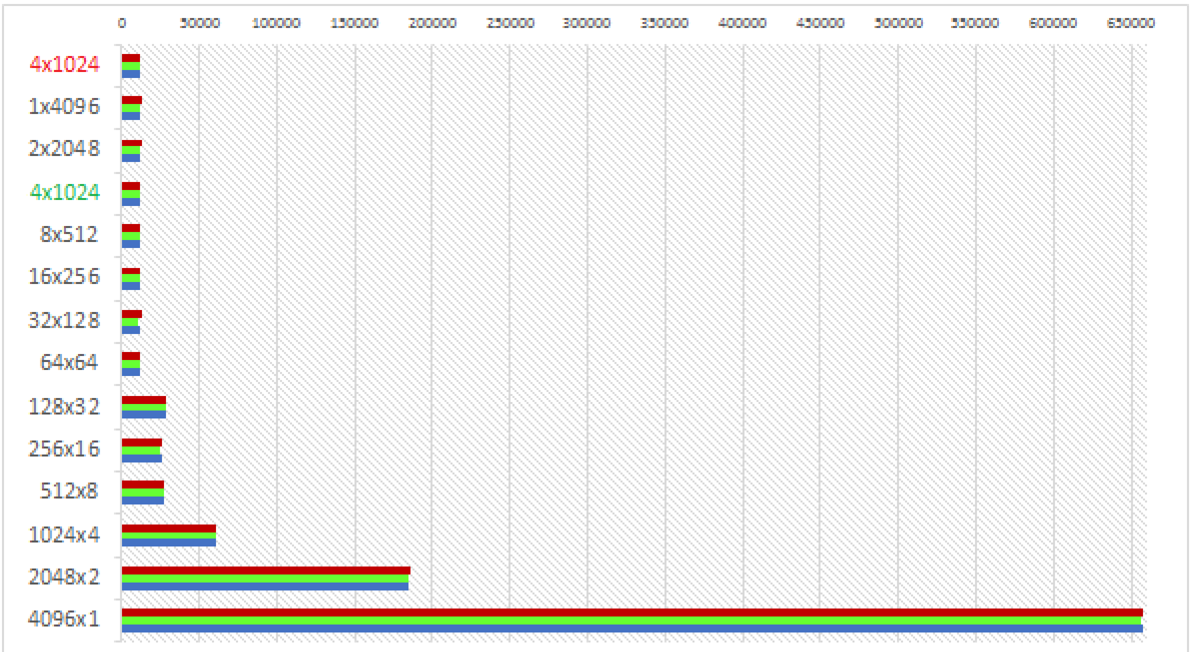
	1024x1	512x2	256x4	128x8	64x16	32x32	16x64	8x128	4x256	2x512	1x1024	1x2048	1x4096	1x1024
🟥 Max	42246	12549	4709	2282	2480	2664	929	1010	1087	991	954	947	949	991
🟩 Min	41580	12326	4259	2145	1965	2283	793	794	843	811	791	779	791	868
🟦 Average	41815	12429	4450	2209	2246	2461	835	879	971	874	852	831	836	917

2048x2048



	2048x1	1024x2	512x4	256x8	128x16	64x32	32x64	16x128	8x256	4x512	2x1024	1x2048	1x4096	2x1024
🟥 Max	165317	47395	15794	8083	7307	7956	3521	3890	3542	3389	3359	3363	3593	3356
🟩 Min	164632	46833	15693	7170	6482	7710	2991	2917	3094	2831	2980	2989	3124	2915
🟦 Average	164982	47114	15744	7655	6797	7813	3231	3423	3300	3021	3202	3199	3272	3089

4096x4096



	4096x1	2048x2	1024x4	512x8	256x16	128x32	64x64	32x128	16x256	8x512	4x1024	2x2048	1x4096	4x1024
🟥 Max	657748	185418	61023	27276	26331	28913	11959	13382	12246	12393	11858	12829	12904	12397
🟩 Min	656842	184888	60348	26779	24888	28021	11339	11011	11335	11233	11243	11538	11258	11331
🟦 Average	657308	185112	60755	27013	25635	28391	11653	11734	11655	11721	11526	12121	11854	11748

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Extra reading materials

http://www.stuffedcow.net/research/cudabmk

http://docs.nvidia.com/cuda/cuda-driver-api/group__CUDA__OCCUPANCY.html#group__CUDA__OCCUPANCY_1g04c0bb65630f82d9b99a5ca0203ee5aa

https://devblogs.nvidia.com/parallelforall/cuda-pro-tip-occupancy-api-simplifies-launch-configuration/