

### Question 1 Minimax

Figure 1 shows a game tree to be searched based on the minimax algorithm. The root of the tree corresponds to Max's turn.

- Fill in the minimax value at each node.
- If *alpha-beta pruning* is to be used to search the game tree, clearly circle the branches that are pruned (i.e, branches that do *not* need to be searched).  
Note: at each node, search the child nodes from left to right.

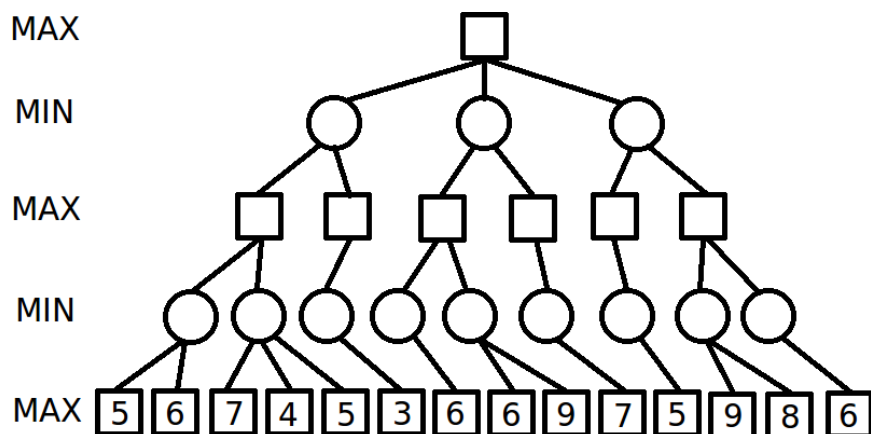


Figure 1: Game tree

### Question 2 Games (Question 6.1 of AIMA 2ed)

Consider the game of tic-tac-toe. Draw the game tree to a depth of two (shows the starting board at root and then has two more levels). *Use symmetry to avoid drawing states that are essentially equivalent.* Now annotate the “leaf nodes” (leaves of this partial tree) with utility according to the following. We define  $X_n$  as the number of rows, columns or diagonals with nothing else in them but  $n$  X's. Likewise define  $O_n$  for the noughts.

For a complete tree, the utility function assigns +1 to any position with  $X_3 = 1$  (the game has been won by the X player) and -1 to any position with  $O_3 = 1$  (the game has been won by the O player). All other terminal nodes would have utility zero.

Non-terminal leaf nodes (i.e., leaves of a partial tree for a certain lookahead) are given a utility by a linear relation  $Eval(s) = 3X_2(s) + X_1(s) - (3O_2(s) + O_1(s))$ .

Annotate your leaf nodes with their cost and then use Min-Max analysis to annotate all other nodes.

- What is the best starting move according to this analysis?
- Circle the nodes that would not be evaluated using alpha-beta pruning *assuming an ordering of the leaf nodes that maximises the benefit of such pruning*.

### Question 3 Nearest neighbour classification

Figure 2 shows a scatter plot of points  $p_1, p_2, \dots, p_{10}$  with their class labels. The plot also includes the testing point  $z$ .

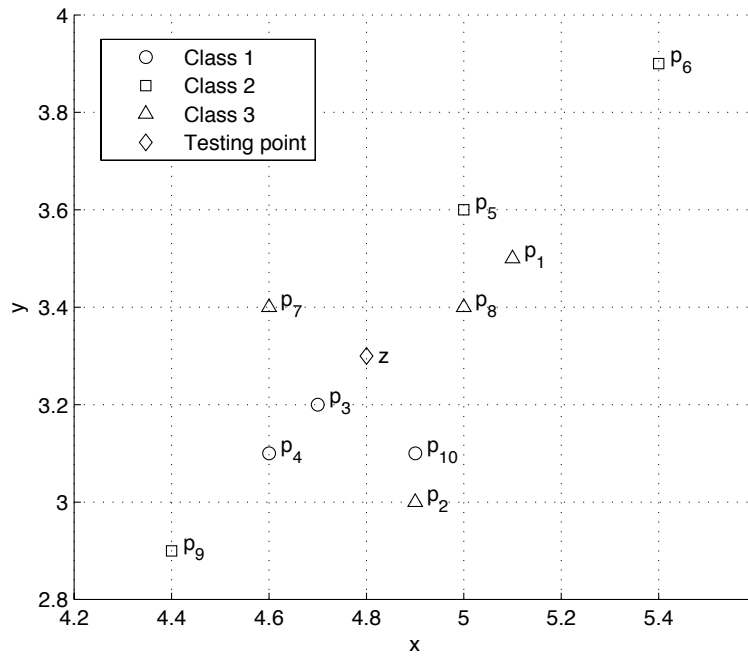


Figure 2: Scatter plot of points with class labels.

The precise coordinates of all the points are as follows:

	$p_1$	$p_2$	$p_3$	$p_4$	$p_5$	$p_6$	$p_7$	$p_8$	$p_9$	$p_{10}$	$z$
x	5.1	4.9	4.7	4.6	5.0	5.4	4.6	5.0	4.4	4.9	4.8
y	3.5	3.0	3.2	3.1	3.6	3.9	3.4	3.4	2.9	3.1	3.3

Classify the testing point  $z$  using  $K$  nearest neighbours with  $K = 1, 3, 4, 5$  and  $7$ .

### Question 4 Kd-trees

Using the training data  $X = \{p_1, p_2, \dots, p_{10}\}$  in Figure 2,

1. Construct a Kd-tree with a bucket-size of 1, i.e., each leaf node must contain at least 1 point. Indicate clearly the branching criterion at each node.
2. In Figure 2, show clearly the spatial partitioning corresponding to the constructed Kd-tree.
3. Search the Kd-tree for the nearest neighbour of testing point  $z$ , indicating clearly on the Kd-tree the nodes that are visited.

### Question 5

A compilation of the playing conditions and outcomes of matches between tennis players Federer and Nadal is given in Table 1, where the time of the match is either Morning (M), Afternoon (A) or Night (N); the type of match is either Grand Slam (G), Masters (M) or Friendly; the type of court is either Grass (G), Hard (H), Clay (C) or Mixed (M); and the outcome is a Federer (F) win or a Nadal (N) win.

Time	Match	Surface	Outcome
M	M	G	F
A	G	C	F
N	F	H	F
A	F	M	N
A	M	C	N
A	G	G	F
A	G	H	F
A	G	H	F
M	M	G	F
A	G	C	N
N	F	H	F
N	M	M	N
A	M	C	N
A	M	G	F
A	G	H	F
A	G	C	F

Table 1: A record of previous tennis matches between Federer and Nadal.

Build a decision tree that can predict the outcome of a new match, given information about the time, type of match, and surface. Show all your working to demonstrate that you are correctly using information gain as the splitting criterion.

### Question 6

Figure 3 shows data pertaining to two species of Irises (Setosa and Virginica). The data contains measurements of two attributes, sepal length and width. Our goal is to construct a decision tree based on this set of training data to predict the species of a given instance of an Iris flower.

- a. What is the best value for attribute sepal length to split the data (consider only values on the grid, i.e. 4, 4.25, 4.5, ...)? What is the information gain associated with this value?
- b. Following the first split using attribute sepal length, we now choose to split using attribute sepal width. What are the best values for attribute sepal width to split the *remaining* data (again, consider only values on the grid, i.e. 2.25, 2.5, 2.75, ...)? What is the information gain of each split?
- c. If we alternately choose between the two attributes to split, how many splits are required in total so that the data is cleanly separated (i.e. all leaf nodes are pure)? Mark in Figure 3 the regions corresponding to the leaf nodes and their labels.

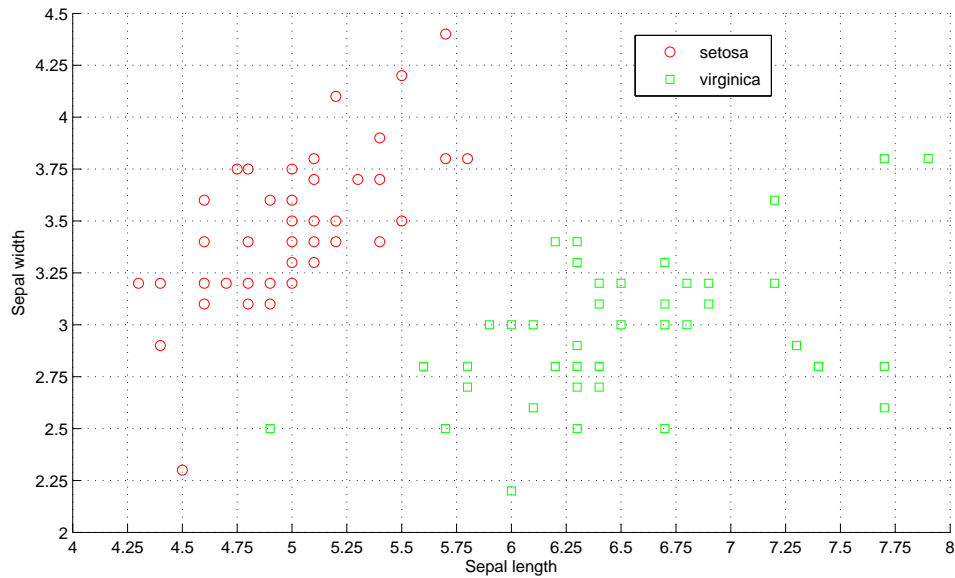


Figure 3: Iris data with two attributes.