# lymath package: semantic low level math formulas

(developped and tested on Mac OS X)

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## 1 Introduction

LATEX is a great tool for writing mathematics, maybe it is the best, but the power of LATEX allows very poor semantic writings. The modest purpose of the package lymath is to give some semantic macros to write elementary mathematic formulas. Here is an example of LATEX code that you can use with lymath.

```
% Without lyxam

Knowing that \frac{df}{dx}(x) = 4 \cos(x^2) on a : b, we have: \frac{df}{dx}(x) = 4 \cos(x^2) on a : b, we have: \frac{df}{dx}(x) = \frac{1}{4} f(x) \right]_a^b.

% With lyxam

Knowing that \frac{df}{dx}(x) = 4 \cos(x^2) on \frac{df}{dx}(x), we have: \frac{df}{dx}(x) = \frac{df}{dx}(x) = \frac{df}{dx}(x).
```

Even if some commands are longest to write than direct LATEX commands, there are two benefits.

- 1. The formatting inside your document is consistent.
- 2. lymath resolves some "complex" problems automatically for you.

## 2 Miscellaneous

# 2.1 Semi-colon and spacing with the french option of babel

```
Only \text{only } \text{onl
```

Only if you use babel with the option francais, then you will see the same spacing around the semi-colon in A(x; y). Que c'est beau!

## 3 Sets

## 3.1 Different kind of sets

## 3.1.1 Sets for geometry

## Example of use

You can semantically write  $\geoset{C}\$ ,  $\geoset{D}\$  and  $\geoset{d}\$  but you can not write things like  $\$  verb+ $\geoset{ABC}\$ +.

You can semantically write  $\mathscr{C}$ ,  $\mathscr{D}$  and  $\mathscr{d}$  but you can not write things like  $\c \$ 

#### Technical ID

\geoset (1 Argument)

— Argument: one single ASCII letter indicating a geometrical set.

## 3.1.2 Sets for probability

## Example of use

You can semantically write  $\operatorname{S}$  and  $\operatorname{G}$  but you can not write things like  $\operatorname{ABC}$ .

You can semantically write  $\mathcal{E}$  and  $\mathcal{G}$  but you can not write things like  $\frac{ABC}{s}$ .

#### Technical ID

\probaset (1 Argument)

— Argument: one single upper ASCII upper letter indicating a probabilistic set.

## 3.1.3 Sets for rings and fields theory

## Example of use

You can semantically write \$\fieldset{A}\$, \$\fieldset{K}\$, \$\fieldset{h}\$ and \$\fieldset{k}\$, but you can't write things like \verb+\$\fieldset{ABC}\$+.

You can semantically write  $\mathbb{A}$ ,  $\mathbb{K}$ ,  $\mathbb{h}$  and  $\mathbb{k}$ , but you can't write things like  $fieldset{ABC}$ .

## Technical ID

\fieldset (1 Argument)

— Argument: either one of the letters h and k, or one single upper ASCII letter indicating a field or ring like set.

#### 3.1.4 Classical sets

```
You can directly use \nullset, \NN, \ZZ, \DD, \QQ, \RR, \CC, \HH and \OO.
```

You can directly use  $\emptyset$ ,  $\mathbb{N}$ ,  $\mathbb{Z}$ ,  $\mathbb{D}$ ,  $\mathbb{Q}$ ,  $\mathbb{R}$ ,  $\mathbb{C}$ ,  $\mathbb{H}$  and  $\mathbb{O}$ .

## 3.1.5 Classical sets with suffixes

```
It is easy to type $\RRn$, $\RRp$, $\RRs$, $\RRsn$ and $\RRsp$.   
It is easy to type \mathbb{R}_-, \mathbb{R}_+, \mathbb{R}^*, \mathbb{R}_-^* and \mathbb{R}_+^*.
```

We have used suffixes n for Negative, p for Positive, and s for Star with the additional composite suffixes sn et sp.

Note that you can't use  $\CCn$  for  $C_$  because the set C doesn't have any standard powerful ordered structure. Take a look at the next section to see how to write  $C_$  if you need it.

The following table shows when you can add one of the suffixes n, p, s, sn and sp.

р s sn sp  $\times$ Z  $\times$  $\times$  $\times$  $\times$  $\times$ D X X X X X Q  $\times$  $\times$ X X R  $\times$ × X X  $\times$ С  $\times$ Η  $\times$ 0 X

Table 1: Suffixes

## 3.1.6 Suffixes on demand

## Example of use

You can indeed write things like  $\mathbb{C}_{-}$  or  $\mathbb{H}_{+}^{*}$ . There is also  $\mathcal{P}_{\leq 0}$  with another formatting.

## Technical IDs

\specialset (2 Arguments) \specialset\* (2 Arguments)

- Argument #1: the set to be "suffixed".
- Argument #2: one of the suffixes n, p, s, sn and sp.

## 3.2 Intervals

## 3.2.1 Real intervals - French (?) notation

## Example of use #1

In  $\hat{b} = a ; b = \int C\{a\}\{b\}$ , you can see that the macro used solves a spacing problem, and that the delimiters are a little bigger.

 $\mbox{\%}$  The syntax refers to O-pened and C-losed but CC and OO are reduced to C and O.

In ]a;b] = ]a;b] = ]a;b], you can see that the macro used solves a spacing problem, and that the delimiters are a little bigger.

## Example of use #2

You can use the star version of a macro if you want the delimiters to stretch vertically.

 $\displaystyle \frac{1}{2} }{ 1^{2^{3}} }$ 

- = [ \frac{1}{2} ; 1^{2^{3}} ]
- =  $\int \frac{1}{2} ^{2^{3}}$

You can use the star version of a macro if you want the delimiters to stretch vertically.

$$\left[\frac{1}{2}; 1^{2^3}\right] = \left[\frac{1}{2}; 1^{2^3}\right] = \left[\frac{1}{2}; 1^{2^3}\right]$$

#### Technical IDs

For all the macros above, the star version produces intervals with delimiters that fit vertically with the bounds of the interval.

\intervalCO (2 Arguments)

\intervalCO\* (2 Arguments)

- Argument #1: lower bound a of the interval [a; b[.
- Argument #2: upper bound b of the interval [a; b].

\intervalC (2 Arguments)

\intervalC\* (2 Arguments)

- Argument #1: lower bound a of the interval [a; b].
- Argument #2: upper bound b of the interval [a; b].

\intervalO (2 Arguments)

\intervalO\* (2 Arguments)

```
Argument #1: lower bound a of the interval ]a; b[.
Argument #2: upper bound b of the interval ]a; b[.
\intervalOC (2 Arguments)
Argument #1: lower bound a of the interval ]a; b].
Argument #2: upper bound b of the interval ]a; b].
```

## 3.2.2 Real intervals - American notation

## Example of use

```
A semi-closed interval \pi interval \pi and an opened one \pi interval \pi and \pi opened one \pi interval \pi and \pi opened one \pi and \pi are interval \pi and \pi opened one \pi are \pi and \pi opened one \pi and \pi opened one \pi and \pi opened one \pi are \pi and \pi opened one \pi and \pi opened one \pi and \pi opened one \pi are \pi and \pi opened one \pi and \pi opened one \pi are \pi are \pi and \pi opened one \pi are \pi and \pi opened one \pi are \pi are \pi and \pi opened one \pi are \pi ar
```

## Technical IDs

For all the macros above, the star version produces intervals with delimiters that fit vertically with the bounds of the interval.

```
\intervalCP (2 Arguments)
\intervalCP* (2 Arguments)

— Argument #1: lower bound a of the interval [a; b).

— Argument #2: upper bound b of the interval [a; b).

\intervalP (2 Arguments)
\intervalP* (2 Arguments)

— Argument #1: lower bound a of the interval (a; b).

— Argument #2: upper bound b of the interval (a; b).

\intervalPC (2 Arguments)
\intervalPC* (2 Arguments)

— Argument #1: lower bound a of the interval (a; b].

— Argument #2: upper bound b of the interval (a; b].
```

## 3.2.3 Discrete intervals of integers

## Example of use

## Technical IDs

For all the macros above, the star version produces intervals with delimiters that fit vertically with the bounds of the interval.

```
\ZintervalCO (2 Arguments)
\ZintervalCO* (2 Arguments)
— Argument #1: lower bound a of the interval ||a;b||.
— Argument #2: upper bound b of the interval ||a;b||.
\ZintervalC (2 Arguments)
\ZintervalC* (2 Arguments)
— Argument #1: lower bound a of the interval ||a;b||.
— Argument #2: upper bound b of the interval ||a;b||.
\ZintervalO (2 Arguments)
\ZintervalO* (2 Arguments)
— Argument #1: lower bound a of the interval ||a;b||.
— Argument #2: upper bound b of the interval ||a;b||.
\ZintervalOC (2 Arguments)
\ZintervalOC* (2 Arguments)
— Argument #1: lower bound a of the interval ||a;b||.
— Argument #2: upper bound b of the interval ||a;b||.
```

# 4 Analysis

## 4.1 Constants

## 4.1.1 Classical constants

## Complete list

```
List of all classical constants where \hat \tau = \frac{\pi}{2} is the youngest one: \eta, \pi, \tau, e, i, j and k.
```

**Remark.** Take care that {\Large \$\ppi \neq \pi\$} produces  $\pi \neq \pi$ . As you can see, the symbols are not the same. Indeed, this is true for all the greek constants.

## 4.1.2 User's latine constants

## Example of use

```
It is easy to write ct{a} x^2 + ct{b} x + ct{c}\ instead of a x^2 + b x + c such as to stress the fact that ct{a}\, ct{b}\ and c are constants.
```

## Technical ID

\ct (1 Argument)

— Argument: a latine text, and not a formula, indicating one constant.

## 4.2 Special functions

## 4.2.1 Some examples of use

## 4.2.2 Functions without parameter

All the following macros don't have any parameter.

\pgcd	\ppcm	\ch	\sh
\th	\ach	\ash	\ath
\arccosh	\arcsinh	\arctanh	\acos
\asin	\atan		

## 4.2.3 Functions with parameters

## The complete list

All the following macros have at least one parameter.

\expb (1 parameter) \logb (1 parameter)

#### Technical IDs

```
\expb (1 Argument) \logb (1 Argument)
```

— Argument: the base of the exponential or the logarithm

## 4.3 Extended notations for special sequences

## Example of use

```
Sometimes we need to write \simeq F_{1}_{2} or \simeq F_{1}_{2} and crazy -wo-men really (?) love \simeq F_{1}_{2}_{3}_{4}.
```

Sometimes we need to write  $F_1^2$  or  ${}_1F_2$  and crazy -wo-men really (?) love  ${}_1^4F_2^3$ .

## Technical IDs

```
\seqplus (2 Arguments)
```

- Argument #1: the right exponent like expression.
- Argument #2: the right indice.

\hypergeo (2 Arguments)

- Argument #1: the left indice.
- Argument #2: the right indice.

\suprageo (4 Arguments)

- Argument #1: the left indice.
- Argument #2: the right indice.
- Argument #3: the right exponent.
- Argument #4: the left exponent.

## 4.4 Differential calculus

## 4.4.1 The $\partial$ and d operators

## Example of use

```
You can write dd{x}\ and pp{t}\ and also dd[5]{x}\ or p[n]{x}\.
```

You can write dx and  $\partial t$  and also  $d^5x$  or  $\partial^n x$ .

## Technical IDs

```
\dd [1 Option] (1 Argument)
\pp [1 Option] (1 Argument)
```

- Option: if used, this argument will be the exponent of the symbol  $\partial$  or d.
- Argument: the variable of differentiation at the right of the symbol  $\partial$  or d.

## 4.4.2 Total derivation

## Example of use #1

## Example of use #2

## Example of use #3

```
If $\displaystyle f(x) = \frac{1}{x^2+3}$, then we can write : $\displaystyle \derpow[3]{f} (a) = \derfrac*[3]{\left( \frac{1}{x^2+3} \right)}{x} (a)$. If f(x) = \frac{1}{x^2+3}, then we can write : f^{(3)}(a) = \frac{\mathrm{d}^3}{\mathrm{d}x^3} \left(\frac{1}{x^2+3}\right)(a).
```

## Technical IDs

\derpow [1 Option] (1 Argument)

- Option: if used, this argument will be the exponent of derivation put inside braces.
- Argument: the function to be differenciated.

\derfrac [1 Option] (2 Arguments)

```
\derfrac* [1 Option] (2 Arguments) \dersub [1 Option] (2 Arguments)
```

- Option: if used, the exponent of derivation.
- Argument #1: the function to be differenciated.
- Argument #2: the variable used for the derivation.

#### 4.4.3 Partial derivation

## Example of use #1

## Example of use #2

## Example of use #3

```
If \alpha(x;y) = \frac{\cos(xy)}{x^2 + y^2}, then we can study \beta(x;y) = \frac{\cos(xy)}{x^2 + y^2}, then we can study \beta(x;y) = \frac{\cos(xy)}{x^2 + y^2}, then we can study \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2}{\partial x \partial y} \left(\frac{\cos(xy)}{x^2 + y^2}\right).
```

## Technical IDs

```
\partialfrac [1 Option] (2 Arguments)
\partialfrac* [1 Option] (2 Arguments)
```

- Option: if used, the exponent of  $\partial$  associated to the function differenciated.
- Argument #1: the function to be partially differenciated.
- Argument #2: the variables used for the partial derivation. The syntax is particular: for example,  $x // y^3 // \ldots$  indicates regarding to the variables x one time, y three times... and so on.

```
\partialsub (2 Arguments) \partialprime (2 Arguments)
```

- Argument #1: the function to be partially differenciated.
- Argument #2: the variables used for the partial derivation. The syntax is particular: for example,  $x // y^3 // \ldots$  indicates regarding to the variables x one time, y three times... and so on.

## 4.5 Integral calculus

## 4.5.1 The hook operator - 1st version

## Example of use #1

```
By definition, \displaystyle \int_{a}^{b} f(x) dd\{x\} = \displaystyle \int_{a}^{b} \sinh(x) dd\{x\} = \frac{F(x)}{a}{b} \ where \displaystyle \int_{a}^{b} f(x) dd\{x\} = \frac{F(x)}{a}{b} = F(b) - F(a).
```

By definition, 
$$\int_a^b f(x) dx = [F(x)]_a^b$$
 where  $[F(x)]_a^b = F(b) - F(a)$ .

## Example of use #2

With the star version, the hooks stretch vertically like in  $\frac{x - 1}{5 + x^2}{a}{b}$ =  $\frac{x - 1}{5 + x^2}{a}{b}$ .

With the star version, the hooks stretch vertically like in  $\left[\frac{x-1}{5+x^2}\right]_a^b = \left[\frac{x-1}{5+x^2}\right]_a^b$ .

## Technical IDs

\hook (3 Arguments) \hook\* (3 Arguments)

- Argument #1: the content inside the hooks.
- Argument #2: the lower bound displayed as an index.
- Argument #3: the upper bound displayed as an exponent.

## 4.5.2 The hook operator - 2nd version

## Example of use #1

You can use  $\ \$  instead of  $\$  hook $\{F(x)\}\{a\}\{b\}$ \$.

You can use  $F(x)|_a^b$  instead of  $[F(x)]_a^b$ .

## Example of use #2

```
With the star version, the rule at the right stretches vertically like in \displaystyle \frac{x - 1}{5 + x^2}{a}{b} = \\ \displaystyle \frac{x - 1}{5 + x^2}{a}{b}.
```

With the star version, the rule at the right stretches vertically like in  $\frac{x-1}{5+x^2}\Big|_a^b = \frac{x-1}{5+x^2}\Big|_a^b$ .

## Technical IDs

\vhook (3 Arguments)
\vhook\* (3 Arguments)

- Argument #1: the content before the vertical line | .
- Argument #2: the lower bound displayed as an index.
- Argument #3: the upper bound displayed as an exponent.

## 4.5.3 Several integrals

The package minimizes spacings between consecutive symbols of integration. Here is an example.

**Remark.** By default, LATEX prints  $\int \int \int F(x;y;z) dx dy dz = \int_a^b \int_c^d \int_e^f F(x;y;z) dx dy dz$ .

## 4.6 Asymptotic comparisons of sequences and functions

#### 4.6.1 The $\mathcal{O}$ and $\phi$ notations

## Example of use

Let's see how to use the symbols  $\sigma = 0$  and  $\mbox{mallo{}}\$  created by Landau.

\medskip

You can write  $\frac{x}{x} \neq \frac{t + \mallo{t}} = e^{\big0{t}}$ .

Let's see how to use the symbols  $\mathcal{O}$  and  $\sigma$  created by Landau.

You can write  $\mathcal{O}(x) \neq \mathcal{O}(x)$  and  $e^{t+\mathcal{O}(t)} = e^{\mathcal{O}(t)}$ .

## Technical IDs

```
\bigO (1 Argument)
\smallO (1 Argument)
```

— Argument: the content inside the braces after the symbol  $\mathcal{O}$  or  $\mathcal{O}$ .

## 4.6.2 The $\Omega$ notation

## Example of use

```
Let's see how to use the symbol \sigma_{s}\ created by Hardy and Littlewood. 

\medskip

\f(n) = \bigomega{g(n)}\$ means: \exists (m, n_0)\$ such that \\
\f(n) = \geqslant n_0\$ implies \f(n) \geqslant m g(n)\$.

Let's see how to use the symbol \Omega created by Hardy and Littlewood. \\

f(n) = \Omega(g(n)) means: \exists (m, n_0) such that n \ge n_0 implies f(n) \ge mg(n).
```

#### Technical ID

\bigomega (1 Argument)

— Argument: the content inside the braces after the symbol  $\Omega$ .

## 4.6.3 The $\Theta$ notation

## Example of use

```
Let's see how to use the symbol \bullet bigtheta{}$. $$ \mathbb{g}(n) = \bigoplus_{g \in \mathcal{B}} \mathbb{G}(n) \le \mathbb{G}(n) \le
```

## Technical ID

\bigtheta (1 Argument)

— Argument: the content inside the braces after the symbol  $\Theta$ .

# 5 Geometry

## 5.1 Points

## 5.1.1 Example of use

We can call \$\pt{I}\$ the middle of \$[\pt{A}\pt{B}]\$.

We can call I the middle of [AB].

## 5.1.2 Technical ID

\pt (1 Argument)

— Argument: one text indicating the name of a point.

## 5.2 Vectors

## 5.2.1 Example of use

Here is one vector  $\c ABCDEF$  using a lot of letters and you can write  $\c E_{rot}$  instead of  $\c E_{rot}$ .

Here is one vector  $\overrightarrow{ABCDEF}$  using a lot of letters and you can write  $\overrightarrow{e}_{rot}$  instead of  $\overrightarrow{e_{rot}}$ .

## 5.2.2 Technical IDs

\vect (1 Argument)

— Argument: one text indicating the name of a vector.

\vect\* (2 Arguments)

- Argument #1: one text indicating up in the name  $\overrightarrow{up}_{down}$  of a vector.
- Argument #2: one text indicating down in the name  $\overrightarrow{up}_{down}$  of a vector.

# 5.3 Inner angles

## 5.3.1 Example of use

Here is one inner angle  $\alpha = 1000$  using a lot of letters, and you can write  $\alpha = 1000$  instead of  $\alpha = 1000$ .

Here is one inner angle  $\widehat{ABCDEF}$  using a lot of letters, and you can write  $\widehat{A}_{rot}$  instead of  $\widehat{A}_{rot}$ .

## 5.3.2 Technical IDs

## \anglein (1 Argument)

— Argument: one text indicating the name of an inner angle.

## \anglein\* (2 Arguments)

- Argument #1: one text indicating up in the name  $\widehat{up}_{down}$  of an inner angle.
- Argument #2: one text indicating down in the name  $\widehat{up}_{down}$  of an inner angle.

## 5.4 Circular arcs

## 5.4.1 Example of use

Here is one arc  $\arc{ABCDEF}$  using a lot of letters, and you can write  $\arc*{A}{rot}$  instead of  $\arc{A_{rot}}$ .

Here is one arc  $\widehat{ABCDEF}$  using a lot of letters, and you can write  $\widehat{A}_{rot}$  instead of  $\widehat{A}_{rot}$ .

## 5.4.2 Technical IDs

## \arc (1 Argument)

— Argument: one text indicating the name of a circular arc.

## \arc\* (2 Arguments)

- Argument #1: one text indicating up in the name  $\widehat{up}_{down}$  of a circular arc.
- Argument #2: one text indicating down in the name  $\widehat{up}_{down}$  of a circular arc.

## 6 Arithmetic

## 6.1 Continued fractions

## 6.1.1 Standard continued fractions

## Example of use

It is easy to write what is just after where the inline notation seems to have been introduced by Alfred Pringsheim (the left notation is always space consuming for a better readability).

- $\contfrac{u_0 // u_1 // u_2 // \dots // u_n}$
- =  $\contfrac*{u_0 // u_1 // u_2 // \dots // u_n}$ \$

It is easy to write what is just after where the inline notation seems to have been introduced by Alfred Pringsheim (the left notation is always space consuming for a better readability).

by Alfred Pringsneim (the left notation is always space con 
$$u_0 + \frac{1}{u_1 + \frac{1}{u_2 + \frac{1}{u_1}}} = u_0 + \frac{1}{u_1} + \frac{1}{u_2} + \frac{1}{u_2} + \frac{1}{u_n} + \frac{1}{u_n}$$

$$\dots + \frac{1}{u_n}$$

## Technical IDs

\contfrac (1 Argument)
\contfrac\* (1 Argument)

— Argument: all the elements of the continued fraction separated by //.

## 6.1.2 Generalized continued fractions

## Example of use

You can use similar notations for generalized continued fractions :

You can use similar notations for generalized continued fractions:

$$a + \frac{b}{c + \frac{d}{e + \frac{f}{c}}} = a + \frac{b}{c} + \frac{d}{e} + \frac{f}{c} + \frac{y}{z}$$

## Technical IDs

\contfracgene (1 Argument)

\contfracgene\* (1 Argument)

— Argument: all the elements of the generalized continued fraction separated by //.

## 6.1.3 Single like continued fraction

## Example of use

Crazy men really (?) need to write things like \$\singlecontfrac{a}{b}\$.

% The existence of this macro just comes from its use internally.

Crazy men really (?) need to write things like  $\begin{bmatrix} a \\ b \end{bmatrix}$ .

## Technical ID

\singlecontfrac (2 Arguments)

— Argument #1: the pseudo numerator

— Argument #2: the pseudo denominator

## 6.1.4 The $\mathcal{K}$ operator

Example of use #1

The following notation is very closed to the one used by Carl Friedrich Gauss:

\$\displaystyle

 $\contfracope_{k=1}^{n} (b_k:c_k)$ 

=  $\cfrac{b_1}{\contfracgene{c_1 // b_2 // c_2 // b_3 // \dots // b_n // c_n}}$ 

The following notation is very closed to the one used by Carl Friedrich Gauss:

The following notation is very closed 
$$\sum_{k=1}^{n} (b_k : c_k) = \frac{b_1}{c_1 + \frac{b_2}{c_2 + \frac{b_3}{c_n}}}$$

**Remark.** The letter  $\mathcal{K}$  comes from "kettenbruch" which means "continued fraction" in german.

## Example of use #2

# 7 Change log

2017-10-21 Major changes are now added in this PDF documentation.

2017-10-18 New minor version 0.0.2-beta: for differential calculus, two new star versions of \derfrac and \partialfrac which use an operator like notation.

2017-10-06 New minor version 0.0.1-beta: additional tools for arithmetic, geometry, integral calculus and differential calculus.

2017-10-02 First version of the package.