

# lymath package: semantic low level math formulas

Sources available on <https://github.com/bc-latex/ly-math>.

Version 0.1.0-beta developped and tested on Mac OS X.

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# 1 Introduction

L<sup>A</sup>T<sub>E</sub>X is a great tool for writing mathematics, maybe it is the best, but the power of L<sup>A</sup>T<sub>E</sub>X allows very poor semantic writings. The modest purpose of the package `lymath` is to give some semantic macros to write elementary mathematic formulas. Let's consider the following standard L<sup>A</sup>T<sub>E</sub>X code

```
Knowing that  $\frac{df}{dx}(x) = 4 \cos(x^2)$  on  $[a ; b]$ , we have:
```

```
 $\int_a^b \cos(x^2) dx = \left[ \frac{1}{4} f(x) \right]_a^b.$ 
```

With `lymath`, you can write the following code.

```
Knowing that  $\frac{df}{dx}(x) = 4 \cos(x^2)$  on  $\text{intervalC}\{a\}\{b\}$ , we have:
```

```
 $\int_a^b \cos(x^2) \, dd{x} = \hook{\frac{1}{4} f(x)}{a}\{b\}.$ 
```

Even if some commands are longest to write than direct L<sup>A</sup>T<sub>E</sub>X commands, there are two benefits.

1. The formatting inside your document is consistent.
2. `lymath` resolves some "complex" problems automatically for you.

## 2 Semi-colon and spacing with the french option of babel

Only if you use `babel` with the option `francais`, then you will see the same spacing around the semi-colon in  $A(x;y)$ . Que c'est beau !

## 3 Sets

### 3.1 Different kind of sets

#### 3.1.1 Sets vs braces

Example of use #1

```
A set of beautiful numbers :  $\text{geneset}\{1 ; 3 ; 5\}.$ 
```

```
A set of beautiful numbers :  $\{1;3;5\}.$ 
```

Example of use #2

```
Choose your team :
```

```
 $\displaystyle \text{geneset}\{\frac{1}{3} ; \frac{5}{7} ; \frac{9}{11}\}$ 
```

```
or
```

```
 $\displaystyle \text{geneset}*\{\frac{1}{3} ; \frac{5}{7} ; \frac{9}{11}\}.$ 
```

```
Choose your team :  $\left\{\frac{1}{3}; \frac{5}{7}; \frac{9}{11}\right\}$  or  $\left\{\frac{1}{3}; \frac{5}{7}; \frac{9}{11}\right\}.$ 
```

## Technical IDs

`\geneset` (1 Argument)

`\geneset*` (1 Argument)

— Argument: the definition of the set.

### 3.1.2 Sets for geometry

#### Example of use #1

You can semantically write `\geoset{C}`, `\geoset{D}` and `\geoset{d}` but you can not write things like `\verb+\geoset{ABC}`.

You can semantically write  $\mathcal{C}$ ,  $\mathcal{D}$  and  $\mathcal{d}$  but you can not write things like `\geoset{ABC}`.

#### Example of use #2

Subscripts can be used like in `\geoset*{C}{1}`, `\geoset*{C}{2}`, `\dots`

Subscripts can be used like in  $\mathcal{C}_1$ ,  $\mathcal{C}_2$ ,  $\dots$

## Technical IDs

`\geoset` (1 Argument)

— Argument: one single ASCII letter indicating a geometrical set.

`\geoset*` (2 Arguments)

— Argument #1: one single ASCII letter indicating  $\mathcal{U}$  in the name  $\mathcal{U}_d$  of a geometrical set.

— Argument #2: one text indicating  $d$  in the name  $\mathcal{U}_d$  of a geometrical set.

### 3.1.3 Sets for probability

#### Example of use #1

You can semantically write `\probaset{E}` and `\probaset{G}` but you can not write things like `\verb+\probaset{ABC}`.

You can semantically write  $\mathcal{E}$  and  $\mathcal{G}$  but you can not write things like `\probaset{ABC}`.

#### Example of use #2

Subscripts can be used like in `\probaset*{E}{1}`, `\probaset*{E}{2}`, `\dots`

Subscripts can be used like in  $\mathcal{E}_1$ ,  $\mathcal{E}_2$ ,  $\dots$

## Technical IDs

`\probaset` (1 Argument)

— Argument: one single upper ASCII upper letter indicating a probabilistic set.

`\probaset*` (2 Arguments)

— Argument #1: one single ASCII letter indicating  $\mathcal{U}$  in the name  $\mathcal{U}_d$  of a geometrical set.

— Argument #2: one text indicating  $d$  in the name  $\mathcal{U}_d$  of a geometrical set.

### 3.1.4 Sets for rings and fields theory

#### Example of use #1

You can semantically write `\fieldset{A}`, `\fieldset{K}`, `\fieldset{h}` and `\fieldset{k}`, but you can't write things like `\verb+\fieldset{ABC}+`.

You can semantically write  $\mathbb{A}$ ,  $\mathbb{K}$ ,  $\mathbb{h}$  and  $\mathbb{k}$ , but you can't write things like `\fieldset{ABC}`.

#### Example of use #2

Subscripts can be used like in `\fieldset*{k}{1}`, `\fieldset*{k}{2}`, `\dots`

Subscripts can be used like in  $k_1$ ,  $k_2$ ,  $\dots$

## Technical IDs

`\fieldset` (1 Argument)

— Argument: either one of the letters  $\mathbb{h}$  and  $\mathbb{k}$ , or one single upper ASCII letter indicating a field or ring like set.

`\fieldset*` (2 Arguments)

— Argument #1: one single ASCII letter indicating  $\mathbb{U}$  in the name  $\mathbb{U}_d$  of a geometrical set.

— Argument #2: one text indicating  $d$  in the name  $\mathbb{U}_d$  of a geometrical set.

### 3.1.5 Classical sets

You can directly use `\nullset`, `\NN`, `\ZZ`, `\DD`, `\QQ`, `\RR`, `\CC`, `\HH` and `\OO`.

You can directly use  $\emptyset$ ,  $\mathbb{N}$ ,  $\mathbb{Z}$ ,  $\mathbb{D}$ ,  $\mathbb{Q}$ ,  $\mathbb{R}$ ,  $\mathbb{C}$ ,  $\mathbb{H}$  and  $\mathbb{O}$ .

### 3.1.6 Classical sets with suffixes

It is easy to type `\RRn`, `\RRp`, `\RRs`, `\RRsn` and `\RRsp`.

It is easy to type  $\mathbb{R}_-$ ,  $\mathbb{R}_+$ ,  $\mathbb{R}^*$ ,  $\mathbb{R}_-^*$  and  $\mathbb{R}_+^*$ .

We have used suffixes **n** for **Negative**, **p** for **Positive**, and **s** for **@star** with the additional composite suffixes **sn** et **sp**.

Note that you can't use  $\mathbb{C}_n$  for  $\mathbb{C}_-$  because the set  $\mathbb{C}$  doesn't have any standard powerful ordered structure. Take a look at the next section to see how to write  $\mathbb{C}_-$  if you need it.

**Remark.** The table 1, see on this page, shows legal associations between classical sets and suffixes.

Table 1: Suffixes

	n	p	s	sn	sp
N			×		
Z	×	×	×	×	×
D	×	×	×	×	×
Q	×	×	×	×	×
R	×	×	×	×	×
C			×		
H			×		
O			×		

### 3.1.7 Suffixes on demand

#### Example of use

You can indeed write things like  $\mathbb{C}_n$  or  $\mathbb{H}_{sp}$ . There is also  $\mathbb{P}_n$  with another formatting.

You can indeed write things like  $\mathbb{C}_-$  or  $\mathbb{H}_+^*$ . There is also  $\mathcal{P}_{\leq 0}$  with another formatting.

#### Technical IDs

`\specialset` (2 Arguments)

`\specialset*` (2 Arguments)

— Argument #1: the set to be "suffixed".

— Argument #2: one of the suffixes **n**, **p**, **s**, **sn** and **sp**.

## 3.2 Intervals

### 3.2.1 Real intervals - French (?) notation

#### Example of use #1

In this example, the syntax refers to **O**-pened and **C**-losed, and we will see that **CC** and **OO** are reduced to **C** and **O**.

In  $I = ]a ; b] = \text{intervalOC}\{a\}\{b\}$ , you can see that the macro used solves a spacing problem.

In  $I = ]a; b] = ]a; b]$ , you can see that the macro used solves a spacing problem.

## Example of use #2

The delimiters automatically stretches vertically, but you can use the @star version of a macro if you don't want this feature. In that case, the delimiters are a little bigger than traditional hooks. Here is an example.

```

$$\begin{aligned} & \backslash\displaystyle \backslash\intervalC{\frac{1}{2}}{1^{2^3}} \\ & = [\frac{1}{2}; 1^{2^3}] \\ & = \backslash\intervalC*{\frac{1}{2}}{1^{2^3}} \end{aligned}$$

```

$$\left[\frac{1}{2}; 1^{2^3}\right] = \left[\frac{1}{2}; 1^{2^3}\right] = \left[\frac{1}{2}; 1^{2^3}\right]$$

## Technical IDs

For all the macros above, the @star version produces intervals with delimiters that fit vertically with the bounds of the interval.

$\backslash\intervalC0$  (2 Arguments)  
 $\backslash\intervalC0*$  (2 Arguments)

- Argument #1: lower bound  $a$  of the interval  $[a; b[$ .
- Argument #2: upper bound  $b$  of the interval  $[a; b[$ .

$\backslash\intervalC$  (2 Arguments)  
 $\backslash\intervalC*$  (2 Arguments)

- Argument #1: lower bound  $a$  of the interval  $[a; b]$ .
- Argument #2: upper bound  $b$  of the interval  $[a; b]$ .

$\backslash\interval0$  (2 Arguments)  
 $\backslash\interval0*$  (2 Arguments)

- Argument #1: lower bound  $a$  of the interval  $]a; b[$ .
- Argument #2: upper bound  $b$  of the interval  $]a; b[$ .

$\backslash\interval0C$  (2 Arguments)  
 $\backslash\interval0C*$  (2 Arguments)

- Argument #1: lower bound  $a$  of the interval  $]a; b]$ .
- Argument #2: upper bound  $b$  of the interval  $]a; b]$ .

### 3.2.2 Real intervals - American notation

#### Example of use

In this example, the syntax refers to **P**-arenthesis.

In America, we write a semi-closed interval  $\backslash\intervalPC{a}{b} = (a ; b]$  and an opened one  $\backslash\intervalP{a}{b} = (a ; b)$ .

In America, we write a semi-closed interval  $(a; b] = (a; b]$  and an opened one  $(a; b) = (a; b)$ .

## Technical IDs

For all the macros above, the @star version produces intervals with delimiters that fit vertically with the bounds of the interval.

`\intervalCP (2 Arguments)`

`\intervalCP* (2 Arguments)`

— Argument #1: lower bound  $a$  of the interval  $[a; b)$ .

— Argument #2: upper bound  $b$  of the interval  $[a; b)$ .

`\intervalP (2 Arguments)`

`\intervalP* (2 Arguments)`

— Argument #1: lower bound  $a$  of the interval  $(a; b)$ .

— Argument #2: upper bound  $b$  of the interval  $(a; b)$ .

`\intervalPC (2 Arguments)`

`\intervalPC* (2 Arguments)`

— Argument #1: lower bound  $a$  of the interval  $(a; b]$ .

— Argument #2: upper bound  $b$  of the interval  $(a; b]$ .

### 3.2.3 Discrete intervals of integers

#### Example of use

In this example, the syntax refers to  $\mathbb{Z}$  the set of integers.

By definition,  $\mathbb{Z}_{\text{intervalC}\{-1\}\{4\}} = \{-1; 0; 1; 2; 3; 4\}$ . So we also have  $\mathbb{Z}_{\text{intervalC}\{-1\}\{4\}} = \mathbb{Z}_{\text{interval0}\{-2\}\{5\}}$ .

By definition,  $\llbracket -1; 4 \rrbracket = \{-1; 0; 1; 2; 3; 4\}$ . So we also have  $\llbracket -1; 4 \rrbracket = \llbracket -2; 5 \rrbracket$ .

## Technical IDs

For all the macros above, the @star version produces intervals with delimiters that fit vertically with the bounds of the interval.

`\ZintervalC0 (2 Arguments)`

`\ZintervalC0* (2 Arguments)`

— Argument #1: lower bound  $a$  of the interval  $\llbracket a; b \rrbracket$ .

— Argument #2: upper bound  $b$  of the interval  $\llbracket a; b \rrbracket$ .

`\ZintervalC (2 Arguments)`

`\ZintervalC* (2 Arguments)`

— Argument #1: lower bound  $a$  of the interval  $\llbracket a; b \rrbracket$ .

— Argument #2: upper bound  $b$  of the interval  $\llbracket a; b \rrbracket$ .

`\Zinterval0 (2 Arguments)`

`\Zinterval0* (2 Arguments)`

— Argument #1: lower bound  $a$  of the interval  $\llbracket a; b \rrbracket$ .



— Argument #2: upper bound  $b$  of the interval  $\llbracket a; b \rrbracket$ .

`\ZintervalOC` (2 Arguments)

`\ZintervalOC*` (2 Arguments)

— Argument #1: lower bound  $a$  of the interval  $\llbracket a; b \rrbracket$ .

— Argument #2: upper bound  $b$  of the interval  $\llbracket a; b \rrbracket$ .

## 4 Analysis

### 4.1 Constants

#### 4.1.1 Classical constants

##### Complete list

List of all classical constants where  $\tau = 2\pi$  is the youngest one:  
`\gamma`, `\pi`, `\tau`, `e`, `i`, `j` and `k`.

List of all classical constants where  $\tau = 2\pi$  is the youngest one:  $\gamma$ ,  $\pi$ ,  $\tau$ ,  $e$ ,  $i$ ,  $j$  and  $k$ .

**Remark.** Take care that `\Large \pi \neq \pi` produces  $\pi \neq \pi$ . As you can see, the symbols are not the same. Indeed, this is true for all the greek constants.

#### 4.1.2 User's latine constants

##### Example of use

It is easy to write `\ct{a} x^2 + \ct{b} x + \ct{c}` instead of `a x^2 + b x + c` such as to stress the fact that `\ct{a}`, `\ct{b}` and `\ct{c}` are constants.

It is easy to write  $\mathbf{a}x^2 + \mathbf{b}x + \mathbf{c}$  instead of  $ax^2 + bx + c$  such as to stress the fact that  $\mathbf{a}$ ,  $\mathbf{b}$  and  $\mathbf{c}$  are constants.

##### Technical ID

`\ct` (1 Argument)

— Argument: a latine text, and not a formula, indicating one constant.

### 4.2 Absolute value function

##### Example of use

It is easy to write `\abs{2}` or `\displaystyle \abs{\frac{3}{5}}` or even `\displaystyle \abs*{\frac{3}{5}}` if you prefer or need little vertical rules.

It is easy to write  $|2|$  or  $\left|\frac{3}{5}\right|$  or even  $\left|\frac{3}{5}\right|$  if you prefer or need little vertical rules.

**Remark.** The L<sup>A</sup>T<sub>E</sub>X code comes directly from this post: <https://tex.stackexchange.com/a/43009/6880>.

## Technical IDs

`\abs` (1 Argument)  
`\abs*` (1 Argument)

— **Argument:** the number on which we apply the absolute value function

## 4.3 Special named functions

### 4.3.1 Some examples of use

Some additional special named functions :  
`\ch x \neq ch x`, `\ppcm(x;y)`, `\lg x = \logb{2} x` and `\expb{6} y`.

Some additional special named functions :  $ch\,x \neq chx$ ,  $ppcm(x;y)$ ,  $\lg x = \log_2 x$  and  $\exp_6 y$ .

### 4.3.2 Named functions without parameter

All the following macros don't have any parameter.

<code>\pgcd</code>	<code>\ppcm</code>	<code>\ch</code>	<code>\sh</code>
<code>\th</code>	<code>\ach</code>	<code>\ash</code>	<code>\ath</code>
<code>\arccosh</code>	<code>\arcsinh</code>	<code>\arctanh</code>	<code>\acos</code>
<code>\asin</code>	<code>\atan</code>		

### 4.3.3 Named functions with parameters

#### The complete list

All the following macros have at least one parameter.

`\expb` (1 parameter)                      `\logb` (1 parameter)

## Technical IDs

`\expb` (1 Argument)  
`\logb` (1 Argument)

— **Argument:** the base of the exponential or the logarithm

## 4.4 Extended notations for special sequences

### Example of use

Sometimes we need to write `\seqplus{F}{1}{2}` or `\hypergeo{F}{1}{2}` and crazy -wo-men really (?) love `\suprgeo{F}{1}{2}{3}{4}`.

Sometimes we need to write  $F_1^2$  or  ${}_1F_2$  and crazy -wo-men really (?) love  ${}_1F_2^3$ .

## Technical IDs

`\seqplus` (2 Arguments)

- Argument #1: the right exponent like expression.
- Argument #2: the right indice.

`\hypergeo` (2 Arguments)

- Argument #1: the left indice.
- Argument #2: the right indice.

`\suprageo` (4 Arguments)

- Argument #1: the left indice.
- Argument #2: the right indice.
- Argument #3: the right exponent.
- Argument #4: the left exponent.

## 4.5 Differential calculus

### 4.5.1 The $\partial$ and $d$ operators

#### Example of use

You can write `\dd{x}` and `\pp{t}` and also `\dd[5]{x}` or `\pp[n]{x}`.

You can write  $dx$  and  $\partial t$  and also  $d^5x$  or  $\partial^n x$ .

## Technical IDs

`\dd` [1 Option] (1 Argument)

`\pp` [1 Option] (1 Argument)

- Option: if used, this argument will be the exponent of the symbol  $\partial$  or  $d$ .
- Argument: the variable of differentiation at the right of the symbol  $\partial$  or  $d$ .

### 4.5.2 Total derivation

#### Example of use #1

```
\displaystyle f'(a)
= \derpow{f} (a)
= \derfrac{f}{x} (a)
= \dersub{f}{x} (a)
```

$$f'(a) = f^{(1)}(a) = \frac{df}{dx}(a) = d_x f(a)$$

## Example of use #2

```

$$f'''(a)$$

$$= \operatorname{derpow}[3]{f}(a)$$

$$= \operatorname{derfrac}[3]{f}{x}(a)$$

$$= \operatorname{dersub}[3]{f}{x}(a)$$
and
$$\cos''' a = \operatorname{derfrac}[3]{\cos}{x}(a).$$

```

---

$$f'''(a) = f^{(3)}(a) = \frac{d^3 f}{dx^3}(a) = d_x^3 f(a) \text{ and } \cos''' a = \frac{d^3 \cos}{dx^3}(a).$$

## Example of use #3

```
If 
$$f(x) = \frac{1}{x^2+3}$$
, then we can write :
$$\operatorname{derpow}[3]{f}(a)$$

$$= \operatorname{derfrac*}[3]{\left( \frac{1}{x^2+3} \right)}{x}(a).$$

```

---

If  $f(x) = \frac{1}{x^2+3}$ , then we can write :  $f^{(3)}(a) = \frac{d^3}{dx^3} \left( \frac{1}{x^2+3} \right) (a).$

## Technical IDs

$\operatorname{derpow}$  [1 Option] (1 Argument)

- Option: if used, this argument will be the exponent of derivation put inside braces.
- Argument: the function to be differentiated.

$\operatorname{derfrac}$  [1 Option] (2 Arguments)

$\operatorname{derfrac*}$  [1 Option] (2 Arguments)

$\operatorname{dersub}$  [1 Option] (2 Arguments)

- Option: if used, the exponent of derivation.
- Argument #1: the function to be differentiated.
- Argument #2: the variable used for the derivation.

### 4.5.3 Partial derivation

#### Example of use #1

```

$$\operatorname{partialfrac}{f}{x}(a;b)$$

$$= \operatorname{partialsub}{f}{x}(a;b)$$

$$= \operatorname{partialprime}{f}{x}(a;b)$$

```

---

$$\frac{\partial f}{\partial x}(a;b) = \partial_x f(a;b) = f'_x(a;b)$$

## Example of use #2

```
\displaystyle \partialfrac[3]{G}{f^2 // v} (a;b)
= \partialfrac{G}{f^2 // v} (a;b)
= \partialsub{G}{f^2 // v} (a;b)
= \partialprime{G}{f^2 // v} (a;b)$
```

---


$$\frac{\partial^3 G}{\partial f^2 \partial v}(a; b) = \frac{\partial G}{\partial f^2 \partial v}(a; b) = \partial_{f(2) v} G(a; b) = G'_{f(2) v}(a; b)$$

## Example of use #3

```
If \displaystyle f(x;y) = \frac{\cos(x y)}{x^2+y^2}$, then we can study
\displaystyle \partialfrac[2]{f}{x // y}
= \partialfrac*[2]{\left( \frac{\cos(x y)}{x^2 + y^2} \right)}{x // y}$.
```

---


$$\text{If } f(x; y) = \frac{\cos(xy)}{x^2 + y^2}, \text{ then we can study } \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2}{\partial x \partial y} \left( \frac{\cos(xy)}{x^2 + y^2} \right).$$

## Technical IDs

`\partialfrac` [1 Option] (2 Arguments)  
`\partialfrac*` [1 Option] (2 Arguments)

- Option: if used, the exponent of  $\partial$  associated to the function differenciated.
- Argument #1: the function to be partially differenciated.
- Argument #2: the variables used for the partial derivation. The syntax is particular : for example, `x // y^3 // ...` indicates regarding to the variables  $x$  one time,  $y$  three times... and so on.

`\partialsub` (2 Arguments)  
`\partialprime` (2 Arguments)

- Argument #1: the function to be partially differenciated.
- Argument #2: the variables used for the partial derivation. The syntax is particular : for example, `x // y^3 // ...` indicates regarding to the variables  $x$  one time,  $y$  three times... and so on.

## 4.6 Integral calculus

### 4.6.1 The hook operator - 1<sup>st</sup> version

#### Example of use #1

```
By definition, \displaystyle \int_{a}^b f(x) \, \mathrm{d}x = \hook{F(x)}{a}{b}$ where
\hook{F(x)}{a}{b} = F(b) - F(a)$.
```

---


$$\text{By definition, } \int_a^b f(x) \, dx = [F(x)]_a^b \text{ where } [F(x)]_a^b = F(b) - F(a).$$

## Example of use #2

By default, the hooks stretch vertically, but if you don't want this you just have to use the @star version as in the example below.

```


$$\hookrightarrow \frac{x-1}{5+x^2} \Big|_a^b = \hookrightarrow^* \frac{x-1}{5+x^2} \Big|_a^b.$$


```

---

$$\left[ \frac{x-1}{5+x^2} \right]_a^b = \left[ \frac{x-1}{5+x^2} \right]_a^b.$$

## Technical IDs

`\hook` (3 Arguments)

`\hook*` (3 Arguments)

- **Argument #1:** the content inside the hooks.
- **Argument #2:** the lower bound displayed as an index.
- **Argument #3:** the upper bound displayed as an exponent.

### 4.6.2 The hook operator - 2<sup>nd</sup> version

#### Example of use #1

You can use `\vhook{F(x)}{a}{b}` instead of `\hook{F(x)}{a}{b}`.

You can use  $F(x)|_a^b$  instead of  $\left[F(x)\right]_a^b$ .

## Example of use #2

Like with first version of the hook operator, you can use a @star version to not have the default behavior of vertical stretch. Here is an example.

```


$$\vhook{\frac{x-1}{5+x^2}}{a}{b} = \vhook^*{\frac{x-1}{5+x^2}}{a}{b}.$$


```

---

$$\frac{x-1}{5+x^2} \Big|_a^b = \frac{x-1}{5+x^2} \Big|_a^b.$$

## Technical IDs

`\vhook` (3 Arguments)

`\vhook*` (3 Arguments)

- **Argument #1:** the content before the vertical line | .
- **Argument #2:** the lower bound displayed as an index.
- **Argument #3:** the upper bound displayed as an exponent.

### 4.6.3 Several integrals

The package minimizes spacings between consecutive symbols of integration. Here is an example.

```

$$\int_a^b \int_c^d \int_e^f F(x;y;z) \, dx \, dy \, dz = \int_a^b \int_c^d \int_e^f F(x;y;z) \, dx \, dy \, dz$$

```

$$\int_a^b \int_c^d \int_e^f F(x;y;z) \, dx \, dy \, dz = \int_a^b \int_c^d \int_e^f F(x;y;z) \, dx \, dy \, dz$$

**Remark.** By default, L<sup>A</sup>T<sub>E</sub>X prints  $\int_a^b \int_c^d \int_e^f F(x;y;z) \, dx \, dy \, dz = \int_a^b \int_c^d \int_e^f F(x;y;z) \, dx \, dy \, dz$ .

## 4.7 Asymptotic comparisons of sequences and functions

### 4.7.1 The $\mathcal{O}$ and $\mathcal{o}$ notations

Example of use #1

You can use the symbols  $\mathcal{O}$  and  $\mathcal{o}$  created by Landau.

You can use the symbols  $\mathcal{O}$  and  $\mathcal{o}$  created by Landau.

Example of use #2

You can write  $\mathcal{O}(x) \neq \mathcal{o}(x)$  and  $e^{t + \mathcal{o}(t)} = e^{\mathcal{O}(t)}$ .

You can write  $\mathcal{O}(x) \neq \mathcal{o}(x)$  and  $e^{t + \mathcal{o}(t)} = e^{\mathcal{O}(t)}$ .

### Technical IDs

$\backslash\mathrm{bigO}$  (1 Argument)

$\backslash\mathrm{smallO}$  (1 Argument)

— **Argument:** the content inside the braces after the symbol  $\mathcal{O}$  or  $\mathcal{o}$ .

### 4.7.2 The $\Omega$ notation

Example of use #1

You can use the symbol  $\Omega$  created by Hardy and Littlewood.

You can use the symbol  $\Omega$  created by Hardy and Littlewood.

Example of use #2

$f(n) = \Omega(g(n))$  means:  $\exists (m, n_0)$  such that  $n \geq n_0$  implies  $f(n) \geq mg(n)$ .

$f(n) = \Omega(g(n))$  means:  $\exists (m, n_0)$  such that  $n \geq n_0$  implies  $f(n) \geq mg(n)$ .

## Technical ID

`\bigomega` (1 Argument)

— **Argument**: the content inside the braces after the symbol  $\Omega$ .

### 4.7.3 The $\Theta$ notation

#### Example of use #1

Here is the last symbol `\bigtheta{}` that can be helpful.

Here is the last symbol  $\Theta$  that can be helpful.

#### Example of use #2

`$f(n) = \bigtheta{g(n)}$` means: `$\exists (m, M, n_0)$` such that `$n \geqslant n_0$` implies `$m g(n) \leqslant f(n) \leqslant M g(n)$`.

$f(n) = \Theta(g(n))$  means:  $\exists(m, M, n_0)$  such that  $n \geq n_0$  implies  $mg(n) \leq f(n) \leq Mg(n)$ .

## Technical ID

`\bigtheta` (1 Argument)

— **Argument**: the content inside the braces after the symbol  $\Theta$ .

## 5 Geometry

### 5.1 Points

#### Example of use #1

`$\gpt{I}$` indicates a point named "I".

I indicates a point named "I".

**Remark.** `\gpt` is for "geometric point". This name has been chosen so as to avoid a conflict with `lyxam` another project of the author of `lymath`.

#### Example of use #2

A list of points : `$\gpt*{I}{1}$`, `$\gpt*{I}{2}$`, `\dots`

A list of points :  $I_1, I_2, \dots$



## Technical ID

`\gpt` (1 Argument) where `gpt` = G-eometric P-oin-T

— Argument: one text indicating the name of a point.

`\gpt*` (2 Arguments)

— Argument #1: one text indicating Up in the name  $Up_{down}$  of a point.

— Argument #2: one text indicating *down* in the name  $Up_{down}$  of a point.

## 5.2 Writing vectors

### Example of use #1

Here is one vector  $\vec{ABCDEFG}$  using a lot of letters and you can write  $\vec{e_{rot}}$  instead of  $\vec{e_{rot}}$ .

Here is one vector  $\overrightarrow{ABCDEFG}$  using a lot of letters and you can write  $\vec{e_{rot}}$  instead of  $\overrightarrow{e_{rot}}$ .

### Example of use #2

You can also simply write  $\vec{i}$  and  $\vec{j_2}$  without points below the arrows only for vectors named "i" or "j".

You can also simply write  $\vec{i}$  and  $\vec{j_2}$  without points below the arrows only for vectors named "i" or "j".

## Technical IDs

`\vect` (1 Argument)

— Argument: one text indicating the name of a vector.

`\vect*` (2 Arguments)

— Argument #1: one text indicating *up* in the name  $\vec{up_{down}}$  of a vector.

— Argument #2: one text indicating *down* in the name  $\vec{up_{down}}$  of a vector.

## 5.3 Norm of a vector

### Example of use

Let's write  $\|\vec{i}\|$ ,  $\|\frac{2}{7}\vec{e_k}\|$ , or  $\|\frac{2}{7}\vec{e_k}\|$  with little vertical rules.

Let's write  $\|\vec{i}\|$ ,  $\|\frac{2}{7}\vec{e_k}\|$ , or  $\|\frac{2}{7}\vec{e_k}\|$  with little vertical rules.

**Remark.** The L<sup>A</sup>T<sub>E</sub>X code comes directly from this post: <https://tex.stackexchange.com/a/43009/6880>.

## Technical IDs

`\norm (1 Argument)`

`\norm* (1 Argument)`

— **Argument:** the vector on which we apply the norm

## 5.4 Inner angles

### Example of use #1

Here is one inner angle `\anglein{ABCDEF}` using a lot of letters, and you can write `\anglein*{A}{rot}` instead of `\anglein{A_{rot}}`.

Here is one inner angle  $\widehat{ABCDEF}$  using a lot of letters, and you can write  $\widehat{A}_{rot}$  instead of  $\widehat{A_{rot}}$ .

### Example of use #2

You can also simply write `\anglein{i}` and `\anglein*{j}{2}` without points below the angle symbol only for inner angles named "i" or "j".

You can also simply write  $\hat{i}$  and  $\hat{j}_2$  without points below the angle symbol only for inner angles named "i" or "j".

## Technical IDs

`\anglein (1 Argument)`

— **Argument:** one text indicating the name of an inner angle.

`\anglein* (2 Arguments)`

— **Argument #1:** one text indicating *up* in the name  $\widehat{up}_{down}$  of an inner angle.

— **Argument #2:** one text indicating *down* in the name  $\widehat{up}_{down}$  of an inner angle.

## 5.5 Circular arcs

### Example of use #1

Here is one arc `\arc{ABCDEF}` using a lot of letters, and you can write `\arc*{A}{rot}` instead of `\arc{A_{rot}}`.

Here is one arc  $\widehat{ABCDEF}$  using a lot of letters, and you can write  $\widehat{A}_{rot}$  instead of  $\widehat{A_{rot}}$ .

### Example of use #2

You can also simply write `\arc{i}` and `\arc*{j}{2}` without points below the arc only for arcs named "i" or "j".

You can also simply write  $\hat{i}$  and  $\hat{j}_2$  without points below the arc only for arcs named "i" or "j".

## Technical IDs

`\arc (1 Argument)`

— **Argument**: one text indicating the name of a circular arc.

`\arc* (2 Arguments)`

— **Argument #1**: one text indicating *up* in the name  $\widehat{up}_{down}$  of a circular arc.

— **Argument #2**: one text indicating *down* in the name  $\widehat{up}_{down}$  of a circular arc.

## 5.6 Naming axes

### Example of use #1

This example is not the best way to do as we will see later.

In a plane, three points  $\text{\gpt{O}}$ ,  $\text{\gpt{I}}$  and  $\text{\gpt{J}}$  not aligned define a cartesian system of coordinates  $\text{\axis{\gpt{O}} // \gpt{I} // \gpt{J}}$ .

In a plane, three points O, I and J not aligned define a cartesian system of coordinates (O; I, J).

### Example of use #2

$$\text{\axis{\gpt{O}} // \frac{7}{3} \text{\vect{i}} // \text{\vect{j}}}$$
or
$$\text{\axis*{\gpt{O}} // \frac{7}{3} \text{\vect{i}} // \text{\vect{j}}}$$

$(O; \frac{7}{3} \vec{i}, \vec{j})$  or  $(O; \frac{7}{3} \vec{i}, \vec{j})$

### Example of use #3

Here you must at least use two "pieces" separated by // but there is no maximum !

$$\text{\axis{\gpt{O}} // \text{\vect*{i}{1}} // \text{\vect*{i}{2}} // \text{\vect*{i}{3}} // \dots // \text{\vect*{i}{9}} // \text{\vect*{i}{10}}}$$

$(O; \vec{i}_1, \vec{i}_2, \vec{i}_3, \dots, \vec{i}_9, \vec{i}_{10})$

### Example of use #4 - No star version here

In this example, the prefix **gp** is for **g**-eometric **p**-oint.

$$\text{\gpaxis{O // I // J // K}}$$
 is just the same than
$$\text{\axis{\gpt{O}} // \gpt{I} // \gpt{J} // \gpt{K}}.$$

(O; I, J, K) is just the same than (O; I, J, K).

### Example of use #5 - No star version here

In this example, the prefix **v** is for **v**-ector.

`$\vaxis{\gpt{0} // i // j}$` is just the same than  
`$\axis{\gpt{0} // \vect{i} // \vect{j}}$`.

$(O; \vec{i}, \vec{j})$  is just the same than  $(O; \vec{i}, \vec{j})$ .

### Example of use #6 - No star version here

In this example, the prefix **gpv** adds the features of the prefixes **gp** and **v**.

`$\gpvaxis{0 // i // j}$` is just the same than  
`$\axis{\gpt{0} // \vect{i} // \vect{j}}$`.

$(O; \vec{i}, \vec{j})$  is just the same than  $(O; \vec{i}, \vec{j})$ .

### Technical IDs

`\axis (1 Argument)`

`\axis* (1 Argument)`

— **Argument:** the argument is made of formulas separated by `//` with the following meanings.

- The first one is the origin of the cartesian system of coordinates.
- Then there are points or vectors which "define" each axis.

`\gpaxis (1 Argument)` where **gp** = g-eometric p-oint

— **Argument:** the argument is made of formulas separated by `//` with the following meanings.

- The first one is the origin of the cartesian system of coordinates on which the macro `\gpt` will be automatically applied.
- Then there are points which "define" each axis, and on each of this points the macro `\gpt` will be automatically applied.

`\vaxis (1 Argument)` where **v** = v-ector

— **Argument:** the argument is made of formulas separated by `//` with the following meanings.

- The first one is the origin of the cartesian system of coordinates.
- Then there are vectors which "define" each axis, and on each of this vectors the macro `\vect` will be automatically applied.

`\gpvaxis (3 Arguments)` where **gpv** = **gp** + **v**

— **Argument:** the argument is made of formulas separated by `//` with the following meanings.

- The first one is the origin of the cartesian system of coordinates on which the macro `\vect` will be automatically applied.
- Then there are vectors which "define" each axis, and on each of this vectors the macro `\vect` will be automatically applied.

## 6 Continued fractions

### 6.1 Standard continued fractions

#### Example of use

In the example, the inline notation seems to have been introduced by Alfred Pringsheim. The left notation is always space consuming for a better readability.

```
$ \contfrac{u_0 // u_1 // u_2 // \dots // u_n}
= \contfrac*{u_0 // u_1 // u_2 // \dots // u_n}$
```

$$u_0 + \frac{1}{u_1 + \frac{1}{u_2 + \frac{1}{\dots + \frac{1}{u_n}}}} = u_0 + \left| \frac{1}{u_1} \right| + \left| \frac{1}{u_2} \right| + \left| \frac{1}{\dots} \right| + \left| \frac{1}{u_n} \right|$$

#### Technical IDs

`\contfrac` (1 Argument)

`\contfrac*` (1 Argument)

— **Argument**: all the elements of the continued fraction separated by `//`.

### 6.2 Generalized continued fractions

#### Example of use

Here is how to write generalized continued fractions.

```
$\displaystyle \contfracgene{a // b // c // d // e // f // \dots // y // z}
= \contfracgene*{a // b // c // d // e // f // \dots // y // z}$
```

$$a + \frac{b}{c + \frac{d}{e + \frac{f}{\dots + \frac{y}{z}}}} = a + \left| \frac{b}{c} \right| + \left| \frac{d}{e} \right| + \left| \frac{f}{\dots} \right| + \left| \frac{y}{z} \right|$$

#### Technical IDs

`\contfracgene` (1 Argument)

`\contfracgene*` (1 Argument)

— **Argument**: all the elements of the generalized continued fraction separated by `//`.

### 6.3 Single like continued fraction

#### Example of use

The existence of the macro below just comes from its use internally.

Crazy men really (?) need to write things like `\singlecontfrac{a}{b}`.

Crazy men really (?) need to write things like  $\frac{a}{b}$ .

## Technical ID

`\singlecontfrac` (2 Arguments)

— Argument #1: the pseudo numerator

— Argument #2: the pseudo denominator

## 6.4 The $\mathcal{K}$ operator

### Example of use #1

The following notation is very closed to the one used by Carl Friedrich Gauss.

```
\displaystyle
\contfracope_{k=1}^n (b_k:c_k)
= \cfrac{b_1}{\contfracgene{c_1 // b_2 // c_2 // b_3 // \dots // b_n // c_n}}
```

$$\mathcal{K}_{k=1}^n(b_k : c_k) = \frac{b_1}{c_1 + \frac{b_2}{c_2 + \frac{b_3}{\dots + \frac{b_n}{c_n}}}}$$

**Remark.** The letter  $\mathcal{K}$  comes from "kettenbruch" which means "continued fraction" in german.

### Example of use #2

```
\displaystyle
u_0 + \contfracope_{k=1}^n (1:u_k)
= \contfrac{u_0 // u_1 // u_2 // \dots // u_n}
```

$$u_0 + \mathcal{K}_{k=1}^n(1 : u_k) = u_0 + \frac{1}{u_1 + \frac{1}{u_2 + \frac{1}{\dots + \frac{1}{u_n}}}}$$

## 7 Change log

All the changes are described inside the folders `change_log` : see the sources of `lymath` on [github](#). Here we just give a very short history of `lymath`.

- 2017-11-01** New minor version **0.1.0-beta** : changes and additional tools for sets, functions and geometry.
- 2017-10-21** Little history of `lymath` will be indicated from now in this PDF documentation.
- 2017-10-18** New patch version **0.0.2-beta** : additional tools for differential calculus.
- 2017-10-06** New patch version **0.0.1-beta** : additional tools for arithmetic, geometry, integral calculus and differential calculus.
- 2017-10-02** First version **0.0.0-beta** of the package.