Efficient implementation of interpolation algorithms for the theories of equality with uninterpreted functions, unit two variable per inequality, and its combination

by

José Abel Castellanos Joo

B.Tech., Universidad de las Américas Puebla, 2015

THESIS

Submitted in Partial Fulfillment of the
Requirements for the Degree of

Master of Science
Computer Science

The University of New Mexico

Albuquerque, New Mexico

July, 2020

Dedication

TODO

Acknowledgments

TODO

Efficient implementation of interpolation algorithms for the theories of equality with uninterpreted functions, unit two variable per inequality, and its combination

by

José Abel Castellanos Joo

B.Tech., Universidad de las Américas Puebla, 2015

M.S., Computer Science, University of New Mexico, 2020

Abstract

TODO

Contents

List of Figures

List of Tables

Glossary

TODO

Chapter 1

Introduction

1.1 Overview

A lot of work to do...

1.2 Conclusions

I conclude that this is a really short thesis... Heheh

Chapter 2

Related Work

Chapter 3

Background

Chapter 4

The Theory of EUF

Theorem 4.0.1. *Let t be an uncommon term and let H be a collection of Horn equations. Assume $antecedent(H) \neq \{\}$. If $consequent(t) = \{\}$, then we cannot conditionality eliminate t from any Horn equation $h \in H$.*

Proof. Suppose, by contradiction, that we can conditionally eliminate the term t from H. Then, there exists $h^{'} \in H$ such that t appears in the consequent of $h^{'}$. But $consequent(t) = \{\}$, contradiction. $\square$

Corollary 4.0.1.1. *We cannot eliminate an uncommont term t unconditionally once $consequent(t) = \{\}$.*

Chapter 5

The Theory of Octagonal Formulas

Theorem 5.0.1. *Given a mutually contradictory pair* (α, β)*, where* α, β *are finite conjunctions of octagonal atoms, the above algorithm terminates with an interpolant* I_α *that is a finite conjunction of octagonal atoms and is equivalent to* $\exists \vec{x}.\alpha$*, where* $\vec{x}$ *is the symbols in* α *which are not in* β*. Further* I_α *is he strongest interpolant for* (α, β)*.*

Proof. First we will prove that $\models \alpha \rightarrow I_\alpha$. The latter follows since the algorithm produces a conjunction of octagonal formulas using the Elim rule, which is a truth-preserving rule of inference, eliminating conjuncts with uncommon symbols.

Now, we will prove that $\not\models I_\alpha \wedge \beta$. We will prove the latter by induction on the number of variables to eliminate (k).

- Base case: $k = 0$. Then the algorithm outputs α. Then the statement holds since (α, β) is unsatisfiable.

- Inductive case: $k = n + 1$. Since the set S of variables to eliminate is non-empty, we just take any variable $x \in S$ and apply the above algorithm to eliminate such variable. Let X be the set of octagonal inequalities of (α, β) and $X^{'}$ be the set of octagonal inequalities $(\alpha^{'}, \beta)$ where $\alpha^{'}$ is the conjunct obtained after removing the variable x. We know X and $X^{'}$ are equisatisfiable using a similar argument as in the Fourier-Motzkin elimination method [**?**]. Let us suppose $(\alpha^{'}, \beta)$ is satisfiable, hence (α, β) is unsatisfiable as well. But the latter entails a contradiction since (α, β) is assumed to be unsatisfiable. Hence, $(\alpha^{'}, \beta)$ is unsatisfiable. Since $(\alpha^{'}, \beta)$ is an unsatisfiable formula with n variables to eliminate, using the Inductive Hypothesis we conclude that (I_α, β) is unsatisfiable as well.

$\square$

Chapter 6

Combining Theories: EUF and Octagonal Formulas

A relevant comment from the paper A combination method for generating interpolants.

The question is how to split it inot two formulas, A' and B'. The condition for splitting is that the common symbols for ¡A', B'¿ should be (a subset of) AB-common symbols, because we would like to use the resultant interpolant for ¡A', B'¿ as a part of an interpolant for the original A and B.

Suppose that Eq contains only AB-pure equalities ...

The latter captures the idea behind equality-interpolating theories. The core definition guarantees that the Nelson-Oppen combination framework is able to propagate AB-pure equalities so the splitting part is not a concern for the algorithm.

Chapter 7

Future Work

I'm sure my future work will consist of lots of other famous stuff.

Appendices