Efficient Implementation of Interpolation Algorithms for EUF and Octagonal Theories

by

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B.Tech., Universidad de las Américas Puebla, 2015

THESIS

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Dedication

To my family and friends, and love.

“Lol” – Anonymous

Acknowledgments

I would like to thank my advisor, Professor Martin Sheen, for his support and some great action movies. I would also like to thank my dog, Spot, who only ate my homework two or three times. I have several other people I would like to thank, as well.[1]

[1] To my brother and sister, who are really cool.

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Abstract

The theory of relativity is a real "toughie" to prove, but with the help of my family and my great grandpa Al, this paper presents the proof in its entirety. Most of the math is correct, and the part about "warp speed" and "parallel universe" sounds very high-tech.

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Glossary

a_{lm}	Taylor series coefficients, where $l, m = \{0..2\}$
$A_{\mathbf{p}}$	Complex-valued scalar denoting the amplitude and phase.
A^T	Transpose of some relativity matrix.

Chapter 1

Introduction

1.1 Overview

What???

1.2 Conclusions

I conclude that this is a really short thesis.

Chapter 2

Related Work

Chapter 3

Background

Chapter 4

The Theory of EUF

Theorem 4.0.1. *Let t be an uncommon term and let H be a collection of Horn equations. Assume $antecedent(H) \neq \{\}$. If $consequent(t) = \{\}$, then we cannot conditionality eliminate t from any Horn equation $h \in H$.*

Proof. Suppose, by contradiction, that we can conditionally eliminate the term t from H. Then, there exists $h^{'} \in H$ such that t appears in the consequent of $h^{'}$. But $consequent(t) = \{\}$, contradiction. $\square$

Corollary 4.0.1.1. *We cannot eliminate an uncommont term t unconditionally once $consequent(t) = \{\}$.*

Chapter 5

The Theory of Octagonal Formulas

Theorem 5.0.1. *Given a mutually contradictory pair* (α, β)*, where* α, β *are finite conjunctions of octagonal atoms, the above algorithm terminates with an interpolant* I_α *that is a finite conjunction of octagonal atoms and is equivalent to* $\exists \vec{x}.\alpha$*, where* $\vec{x}$ *is the symbols in* α *which are not in* β*. Further* I_α *is he strongest interpolant for* (α, β)*.*

Proof. First we will prove that $\models \alpha \rightarrow I_\alpha$. The latter follows since the algorithm produces a conjunction of octagonal formulas using the Elim rule, which is a truth-preserving rule of inference, eliminating conjuncts with uncommon symbols.

Now, we will prove that $\not\models I_\alpha \wedge \beta$. We will prove the latter by induction on the number of variables to eliminate (k).

- Base case: $k = 0$. Then the algorithm outputs α. Then the statement holds since (α, β) is unsatisfiable.

- Inductive case: $k = n + 1$. Since the set S of variables to eliminate is non-empty, we just take any variable $x \in S$ and apply the above algorithm to eliminate such variable. Let X be the set of octagonal inequalities of (α, β) and $X^{'}$ be the set of octagonal inequalities $(\alpha^{'}, \beta)$ where $\alpha^{'}$ is the conjunct obtained after removing the variable x. We know X and $X^{'}$ are equisatisfiable using a similar argument as in the Fourier-Motzkin elimination method [1]. Let us suppose $(\alpha^{'}, \beta)$ is satisfiable, hence (α, β) is unsatisfiable as well. But the latter entails a contradiction since (α, β) is assumed to be unsatisfiable. Hence, $(\alpha^{'}, \beta)$ is unsatisfiable. Since $(\alpha^{'}, \beta)$ is an unsatisfiable formula with n variables to eliminate, using the Inductive Hypothesis we conclude that (I_α, β) is unsatisfiable as well.

$\square$

Chapter 6

Combining Theories: EUF and Octagonal Formulas

A relevant comment from the paper A combination method for generating interpolants.

The question is how to split it inot two formulas, A' and B'. The condition for splitting is that the common symbols for ¡A', B'¿ should be (a subset of) AB-common symbols, because we would like to use the resultant interpolant for ¡A', B'¿ as a part of an interpolant for the original A and B.

Suppose that Eq contains only AB-pure equalities ...

The latter captures the idea behind equality-interpolating theories. The core definition guarantees that the Nelson-Oppen combination framework is able to propagate AB-pure equalities so the splitting part is not a concern for the algorithm.

Chapter 7

Future Work

I’m sure my future work will consist of lots of other famous stuff.

Appendices

Bibliography

[1] Alexander Schrijver. *Theory of Linear and Integer Programming.* John Wiley & Sons, Inc., New York, NY, USA, 1986.