$$\dfrac{\dfrac{}{(= (p(x))(3))}\,\text{definition}}{}\,\text{asserted} \qquad \dfrac{\dfrac{}{(= (3)(p(x)))}\,\text{definition}}{}\,\text{symm}$$

Proof/Definition of a!1

$$\dfrac{\dfrac{}{(\mathit{or}(= (x)(2))(\mathit{not}(\leq (x)(2)))(\mathit{not}(\geq (x)(2))))}\,\text{definition}}{}\,\text{th-lemma}$$

Proof/Definition of a!2

$$\dfrac{\dfrac{}{(\mathit{or}(= (1)(x))(\mathit{not}(\leq (x)(1)))(\mathit{not}(\geq (x)(1))))}\,\text{definition}}{}\,\text{th-lemma}$$

Proof/Definition of a!3

$$\dfrac{\dfrac{\dfrac{}{(\leq (1)(x))}\,\text{definition}}{}\,\text{asserted} \qquad \dfrac{\dfrac{}{(= (\leq (1)(x))(\geq (x)(1)))}\,\text{definition}}{}\,\text{rewrite} \qquad \dfrac{}{(\geq (x)(1))}\,\text{definition}}{}\,\text{mp}$$

Proof/Definition of a!4

$$\dfrac{\dfrac{\dfrac{\dfrac{}{(= (1)(x))}\,\text{definition}}{}\,\text{hypothesis} \qquad \dfrac{}{(= (x)(1))}\,\text{definition}}{}\,\text{symm} \qquad \dfrac{}{(= (p(x))(p(1)))}\,\text{definition}}{}\,\text{monotonicity}$$

Proof/Definition of a!6

$$\dfrac{\dfrac{\dfrac{\dfrac{}{(= (a)(1))}\,\text{definition}}{}\,\text{asserted} \qquad \dfrac{}{(= (p(a))(p(1)))}\,\text{definition}}{}\,\text{monotonicity} \qquad \dfrac{}{(= (= (p(a))(4))(= (p(1))(4)))}\,\text{definition}}{}\,\text{monotonicity}$$

Proof/Definition of a!7

$$\dfrac{\dfrac{\dfrac{\dfrac{}{(= (b)(2))}\,\text{definition}}{}\,\text{asserted} \qquad \dfrac{}{(= (p(b))(p(2)))}\,\text{definition}}{}\,\text{monotonicity} \qquad \dfrac{}{(= (= (p(b))(5))(= (p(2))(5)))}\,\text{definition}}{}\,\text{monotonicity}$$

Proof/Definition of a!13

$$\dfrac{\dfrac{}{(a!3)}\,\text{definition} \qquad \dfrac{}{(a!4)}\,\text{definition} \qquad \dfrac{}{(\mathit{or}(= (1)(x))(\mathit{not}(\leq (x)(1))))}\,\text{definition}}{}\,\text{unit-resolution}$$

Proof/Definition of a!5

$$\dfrac{\dfrac{\dfrac{}{(= (p(a))(4))}\,\text{definition}}{}\,\text{asserted} \qquad \dfrac{}{(a!7)}\,\text{definition} \qquad \dfrac{}{(= (p(1))(4))}\,\text{definition}}{}\,\text{mp}$$

Proof/Definition of a!8

$$\dfrac{\dfrac{\dfrac{}{(= (p(b))(5))}\,\text{definition}}{}\,\text{asserted} \qquad \dfrac{}{(a!13)}\,\text{definition} \qquad \dfrac{}{(= (p(2))(5))}\,\text{definition}}{}\,\text{mp}$$

Proof/Definition of a!14

$$\dfrac{\dfrac{\dfrac{}{(a!1)}\,\text{definition} \qquad \dfrac{}{(a!6)}\,\text{definition} \qquad \dfrac{}{(a!8)}\,\text{definition} \qquad \dfrac{}{(= (3)(4))}\,\text{definition}}{}\,\text{trans*} \qquad \dfrac{\dfrac{}{(= (= (3)(4))(\mathit{false}))}\,\text{definition}}{}\,\text{rewrite} \qquad \dfrac{}{(\mathit{false})}\,\text{definition}}{}\,\text{mp}$$

Proof/Definition of a!9

$$\dfrac{\dfrac{}{(a!5)}\,\text{definition} \qquad \dfrac{\dfrac{}{(a!9)}\,\text{definition} \qquad \dfrac{}{(\mathit{not}(= (1)(x)))}\,\text{definition}}{}\,\text{lemma} \qquad \dfrac{}{(\mathit{not}(\leq (x)(1)))}\,\text{definition}}{}\,\text{unit-resolution}$$

Proof/Definition of a!10

$$\dfrac{\dfrac{\dfrac{}{(\mathit{or}(\geq (x)(2))(\leq (x)(1)))}\,\text{definition}}{}\,\text{th-lemma} \qquad \dfrac{}{(a!10)}\,\text{definition} \qquad \dfrac{}{(\geq (x)(2))}\,\text{definition}}{}\,\text{unit-resolution}$$

Proof/Definition of a!11

$$\dfrac{\dfrac{\dfrac{}{(a!2)}\,\text{definition} \qquad \dfrac{\dfrac{}{(\leq (x)(2))}\,\text{definition}}{}\,\text{asserted} \qquad \dfrac{}{(a!11)}\,\text{definition} \qquad \dfrac{}{(= (x)(2))}\,\text{definition}}{}\,\text{unit-resolution} \qquad \dfrac{}{(= (p(x))(p(2)))}\,\text{definition}}{}\,\text{monotonicity}$$

Proof/Definition of a!12

$$\dfrac{\dfrac{\dfrac{}{(a!1)}\,\text{definition} \qquad \dfrac{}{(a!12)}\,\text{definition} \qquad \dfrac{}{(a!14)}\,\text{definition} \qquad \dfrac{}{(= (3)(5))}\,\text{definition}}{}\,\text{trans*} \qquad \dfrac{\dfrac{}{(= (= (3)(5))(\mathit{false}))}\,\text{definition}}{}\,\text{rewrite} \qquad \dfrac{}{(\mathit{false})}\,\text{definition}}{}\,\text{mp}$$