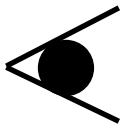


Advanced image synthesis



Ray-casting basics

- For each pixel
 - Compute pixel ray
 - For each object
 - Check ray-object intersection
 - Get closest intersection



camera

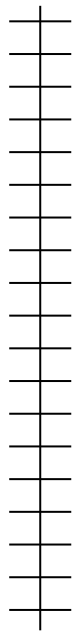
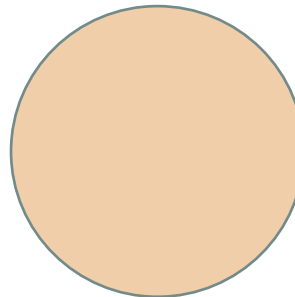
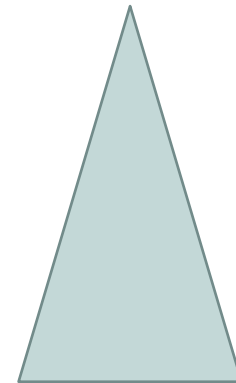


Image plane

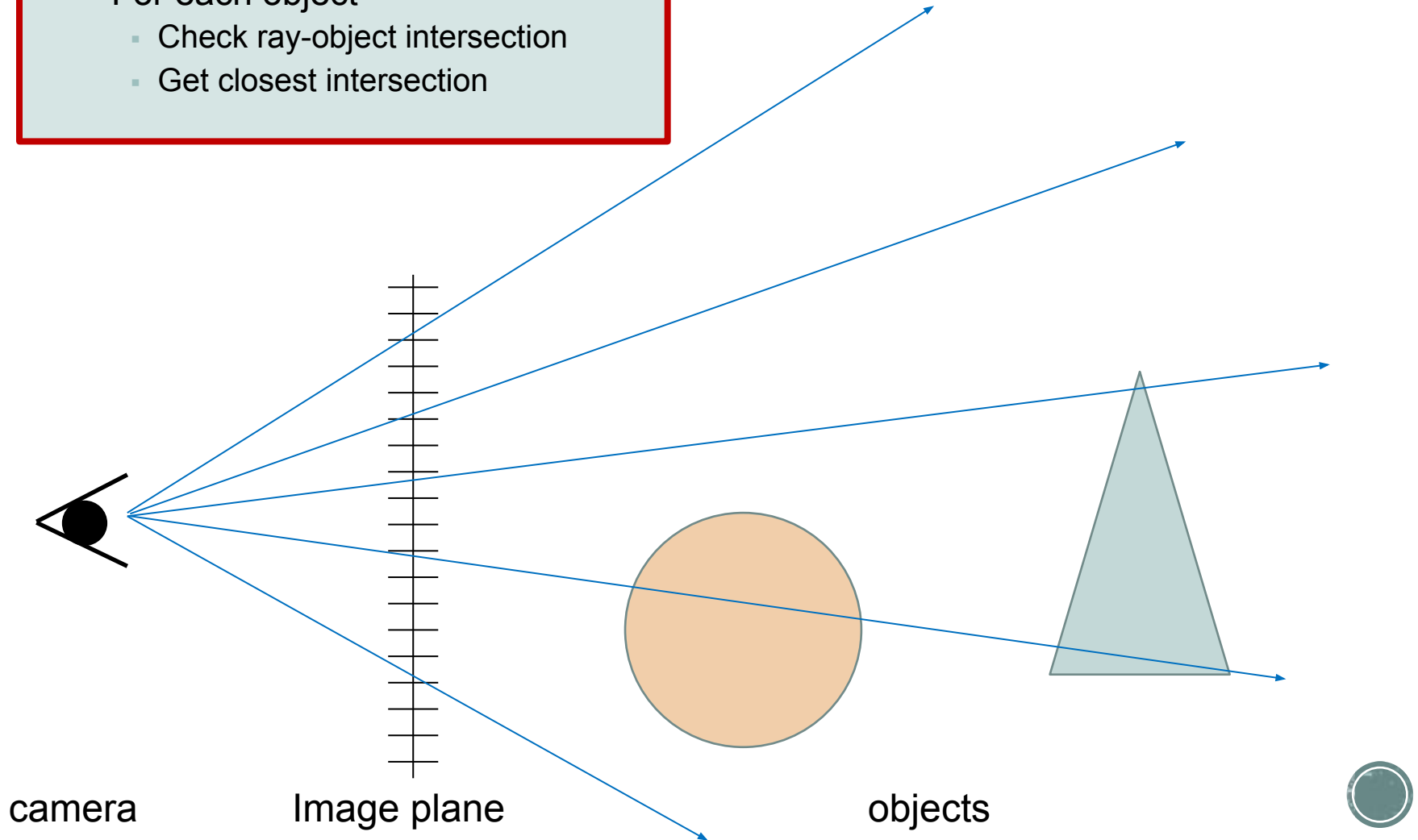


objects



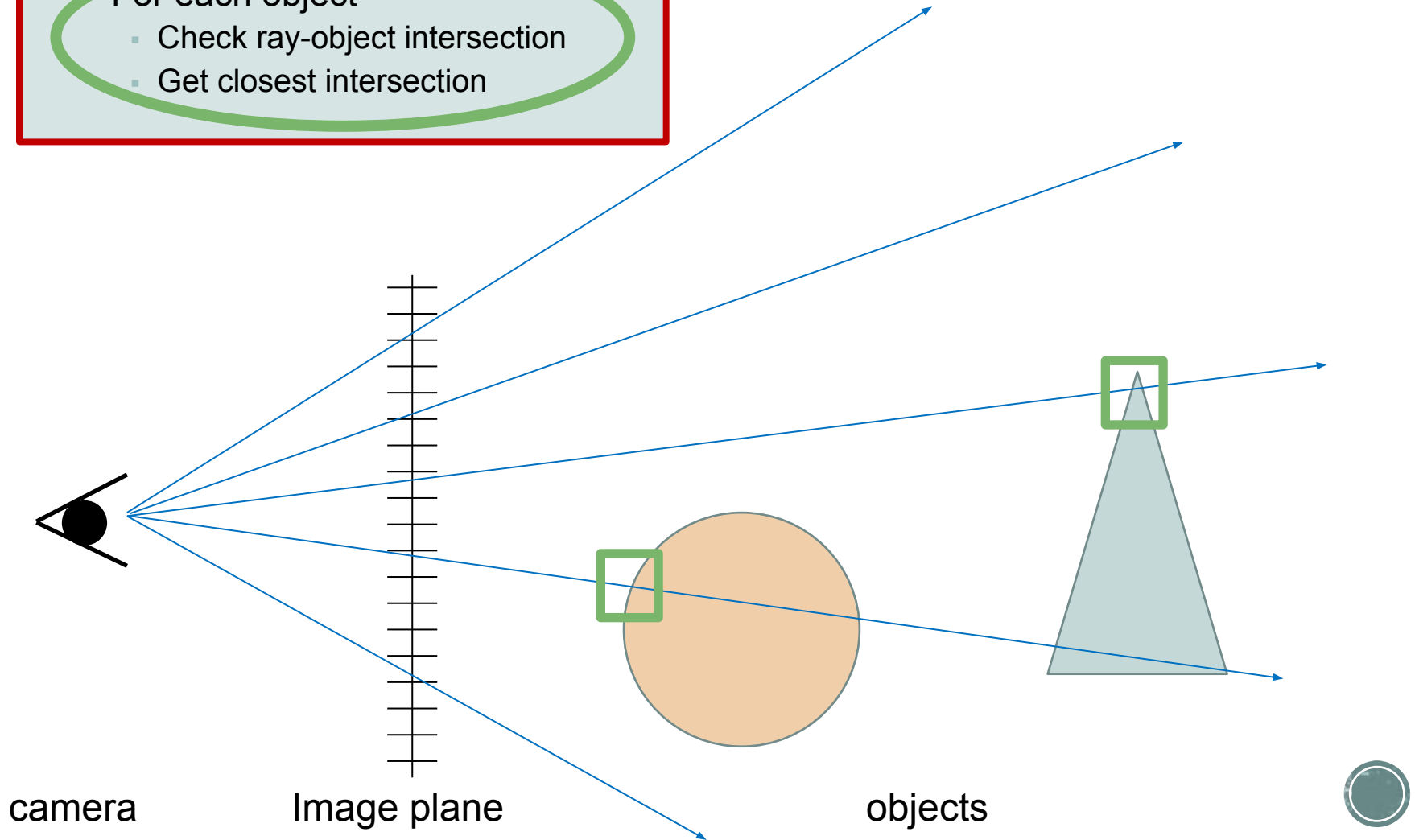
Ray-casting basics

- For each pixel
 - Compute pixel ray
 - For each object
 - Check ray-object intersection
 - Get closest intersection



Ray-casting basics

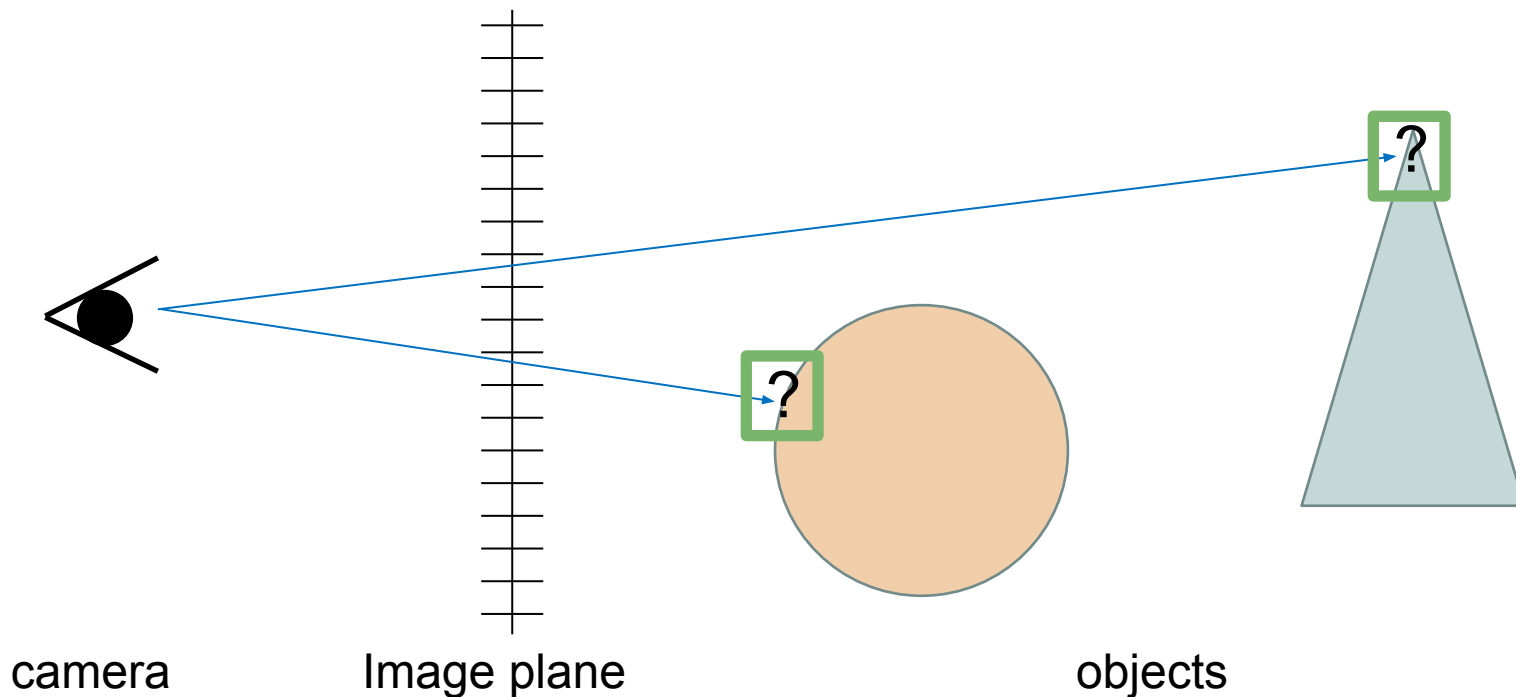
- For each pixel
 - Compute pixel ray
- For each object
 - Check ray-object intersection
 - Get closest intersection



Ray-casting basics

- For each pixel
 - Compute pixel ray
 - For each object
 - Check ray-object intersection
 - Get closest intersection

And then?

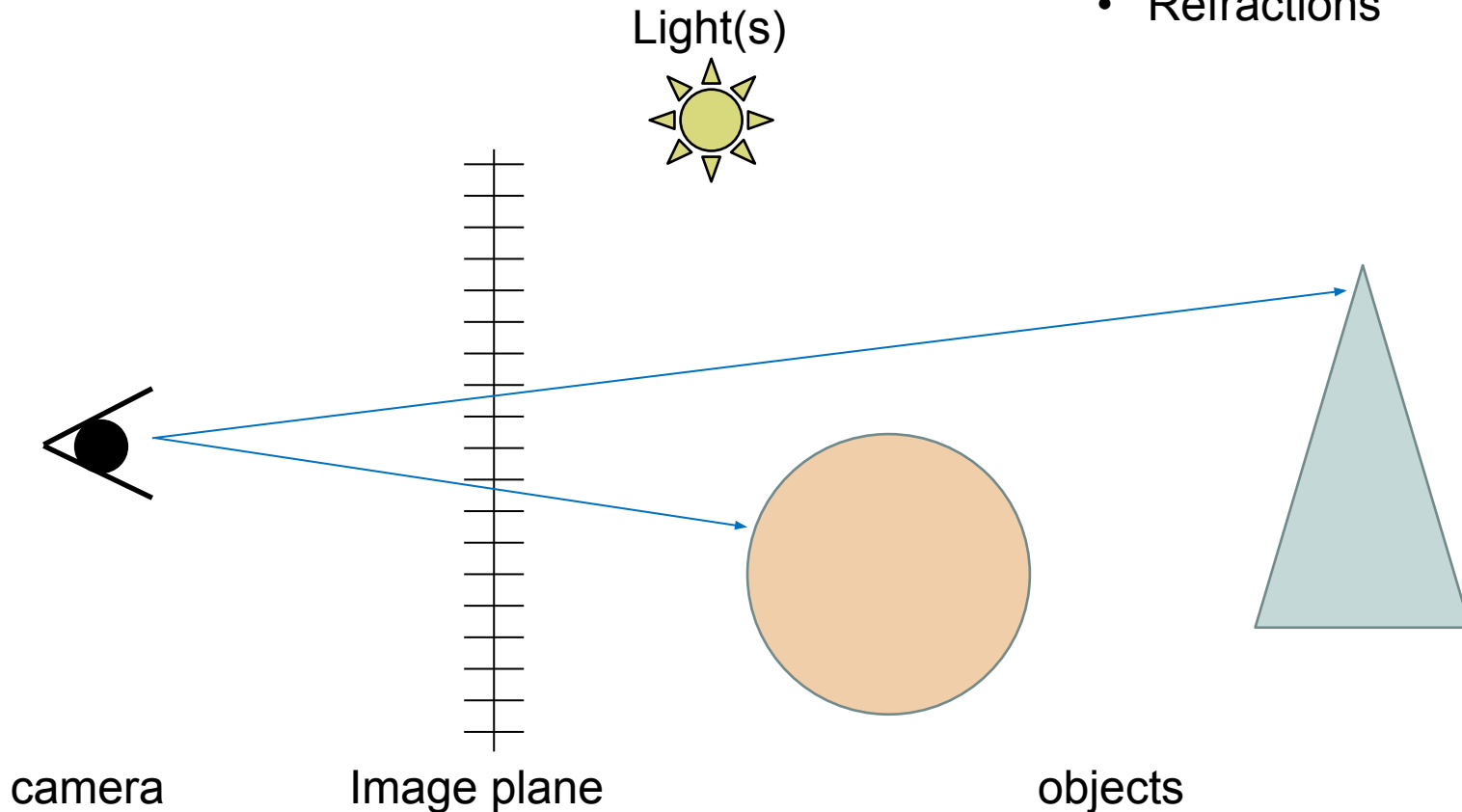


Ray-casting basics

- For each pixel
 - Compute pixel ray
 - For each object
 - Check ray-object intersection
 - Get closest intersection

And then?
Shade!

- Shadow rays
- Reflections
- Refractions

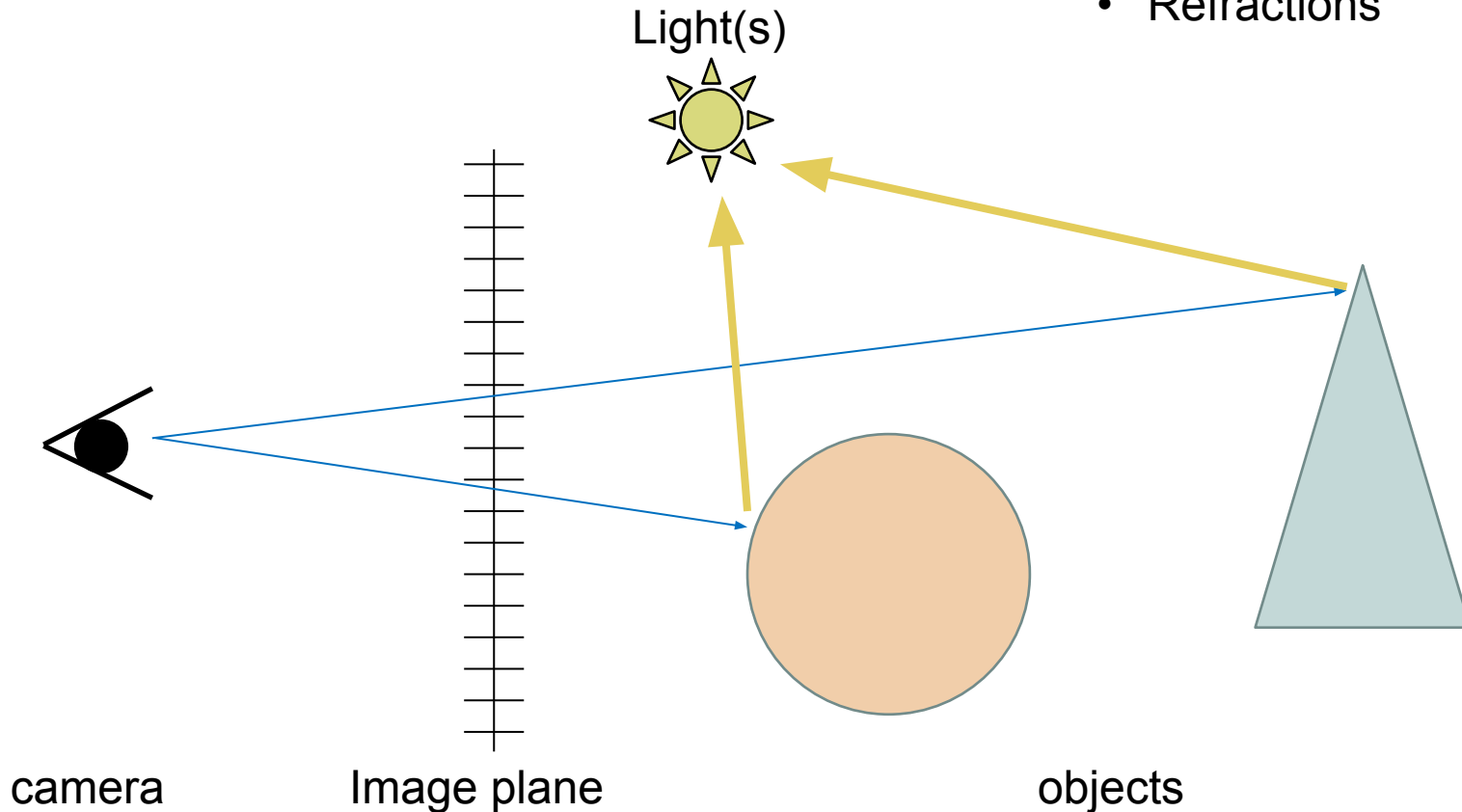


Ray-casting basics

- For each pixel
 - Compute pixel ray
 - For each object
 - Check ray-object intersection
 - Get closest intersection

And then?
Shade!

- Shadow rays
- Reflections
- Refractions

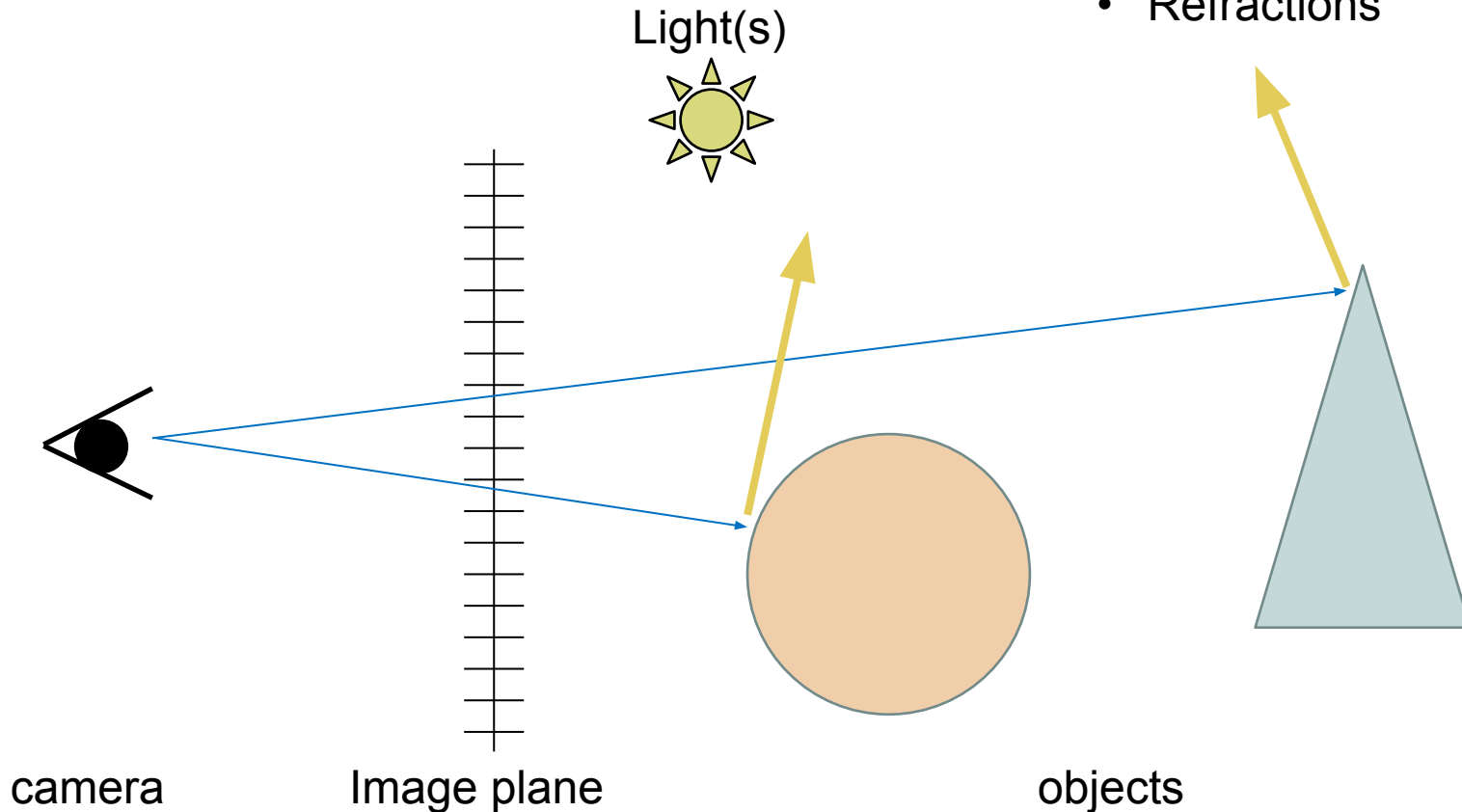


Ray-casting basics

- For each pixel
 - Compute pixel ray
 - For each object
 - Check ray-object intersection
 - Get closest intersection

And then?
Shade!

- Shadow rays
- Reflections
- Refractions

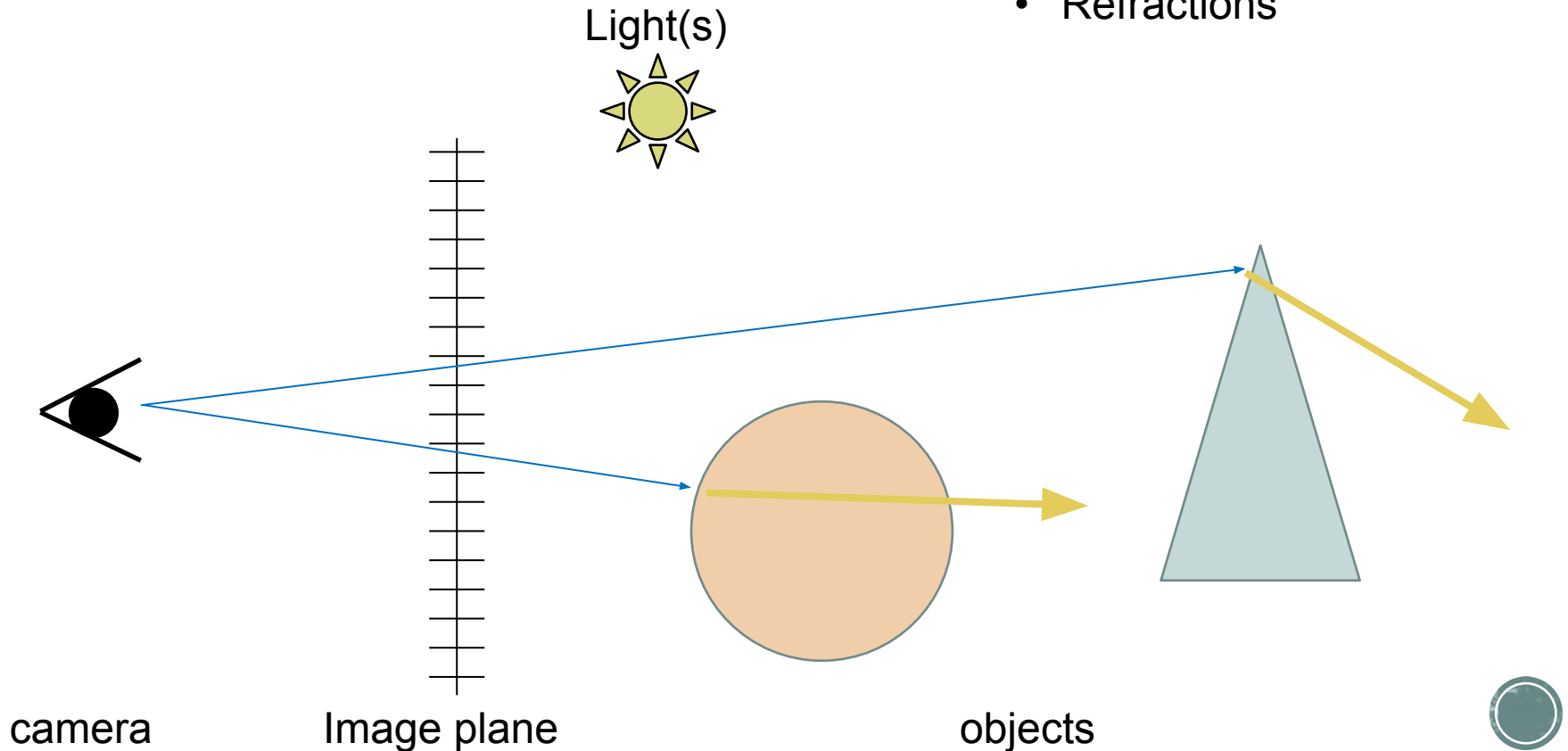


Ray-casting basics

- For each pixel
 - Compute pixel ray
 - For each object
 - Check ray-object intersection
 - Get closest intersection

And then?
Shade!

- Shadow rays
- Reflections
- Refractions



Ray-casting: summary

- For each pixel
 - Compute eye ray
 - For each object
 - Check ray-object intersection
 - Get closest intersection
 - Shade depending on light and normal vector

Finding intersection point and normal is the central part of ray-casting!



Ray tracing



Ray tracing

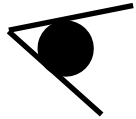
```
#include <stdlib.h>    // card > aek.ppm
#include <stdio.h>
#include <math.h>
typedef int i;typedef float f;struct v{
f x,y,z;v operator+(v r){return v(x+r.x
,y+r.y,z+r.z);}v operator*(f r){ return
v(x*r,y*r,z*r);}f operator%(v r){return
x*r.x+y*r.y+z*r.z;}v(){}v operator^(v r
){return v(y*r.z-z*r.y,z*r.x-x*r.z,x*r.
y-y*r.x);}v(f a,f b,f c){x=a;y=b;z=c;}v
operator!(){return*this*(1/sqrt(*this*
this));}};i G[]={133022, 133266,133266,
133022, 254096, 131216, 131984, 131072,
258048,};f R(){return(f)rand()/RAND_MAX
};i T(v o,v d,f&t,v&n){t=1e9;i m=0;f p=
-o.z/d.z; if(.01<p)t=p, n=v(0,0,1),m=1;
for(i k=19;k--;)for(i j=9;j--;)if(G[j]&
1<<k){v p=o+v(-k,0,-j-4);f b=p&d,c=p&p-
1,q=b*b-c;if(q>0){f s=-b-sqrt(q);if(s<t
&&s>.01)t=s,n=(p+d*t),m=2;}}return m;}
v S(v o,v d){f t;v n;i m=T(o,d,t,n);if(
!m)return v(.7,.6,1)*pow(1-d.z,4);v h=o
+d*t,l=(v(9+R(),9+R(),16)+h*-1),r=d+n*
(n&d*-2);f b=l&n;if(b<0||T(h,l,t,n))b=0
;f p=pow(1&r*(b>0),99);if(m&1){h=h*.2;
return((i)(ceil(h.x)+ceil(h.y))&1?v(3,1
,1):v(3,3,3))*(b*.2+.1);}return v(p,p,p
)+S(h,r)*.5;};i main(){printf("P6 512 "
"512 255 ");v g=!v(-6,-16,0),a=!v(0,0,
1)^g*.002,b=!g^a*.002,c=(a+b)*-256+g
;for(i y=512;y--;)for(i x=512;x--;){v p
(9,9,9);for(i r=64;r--;){v t=a*(R()-.5)
*99+b*(R()-.5)*99;p=S(v(17,16,8)+t,! (t*
-1+(a*(R()+x)+b*(y+R()+c)*16))*3.5+p;}
printf("%c%c%c", (i)p.x, (i)p.y, (i)p.z);}}
```



Ray tracing

Basically 2 functions:

- trace
 - find intersection with an object
- direct illumination
 - Direct lighting at a given point



camera

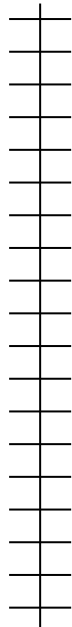
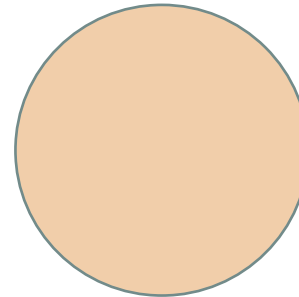
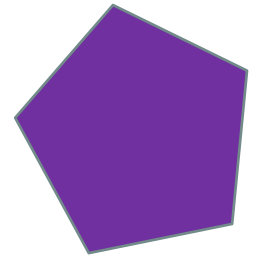
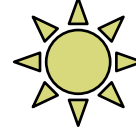


Image plane



objects

Light(s)

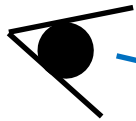
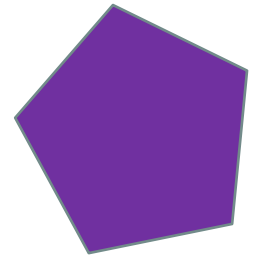
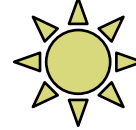


Ray tracing

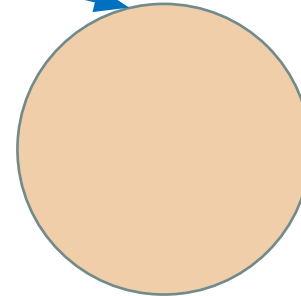
Basically 2 functions:

- trace
 - find intersection with an object
- directIllumination
 - Direct lighting at a given point

Light(s)



trace



camera

Image plane

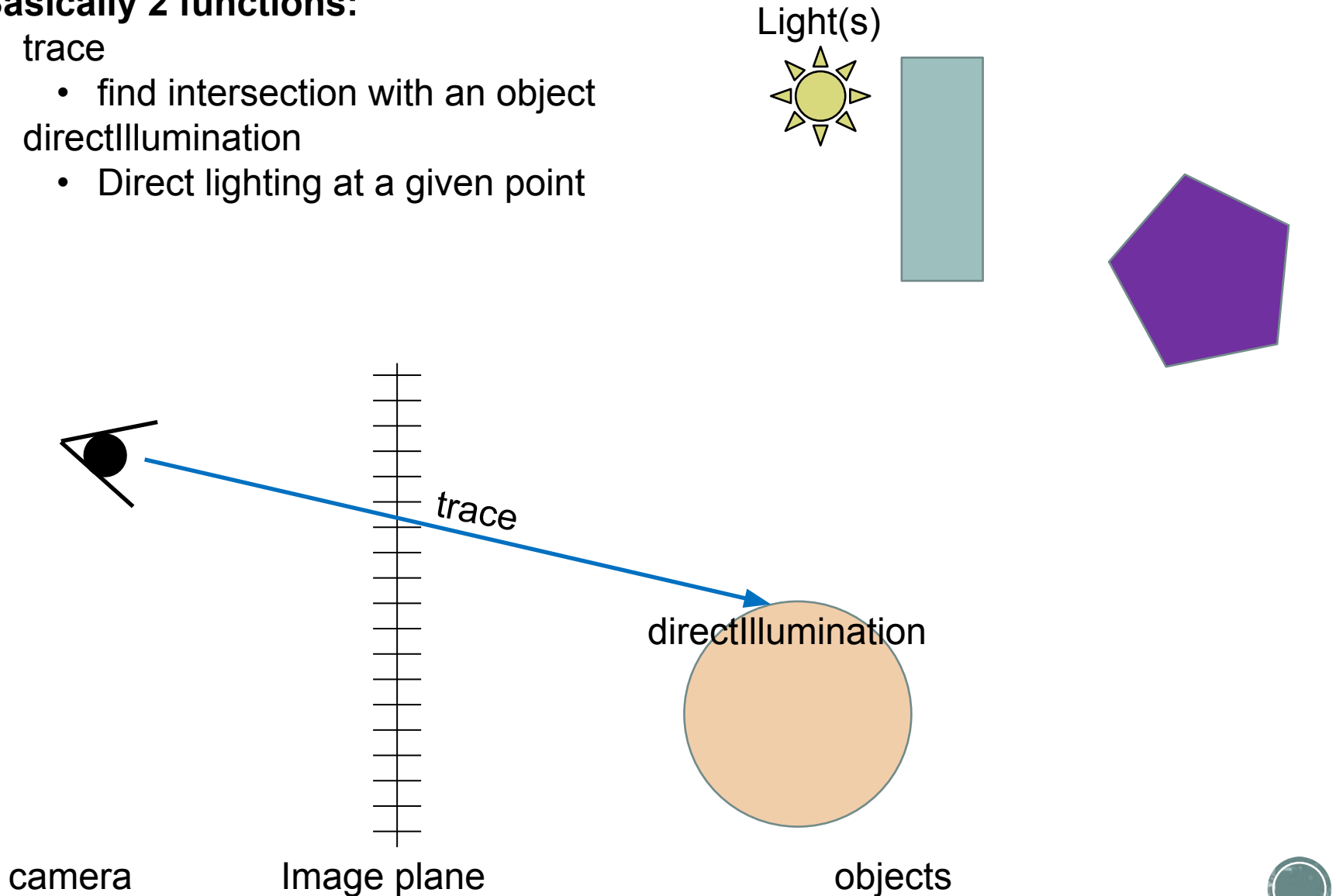
objects



Ray tracing

Basically 2 functions:

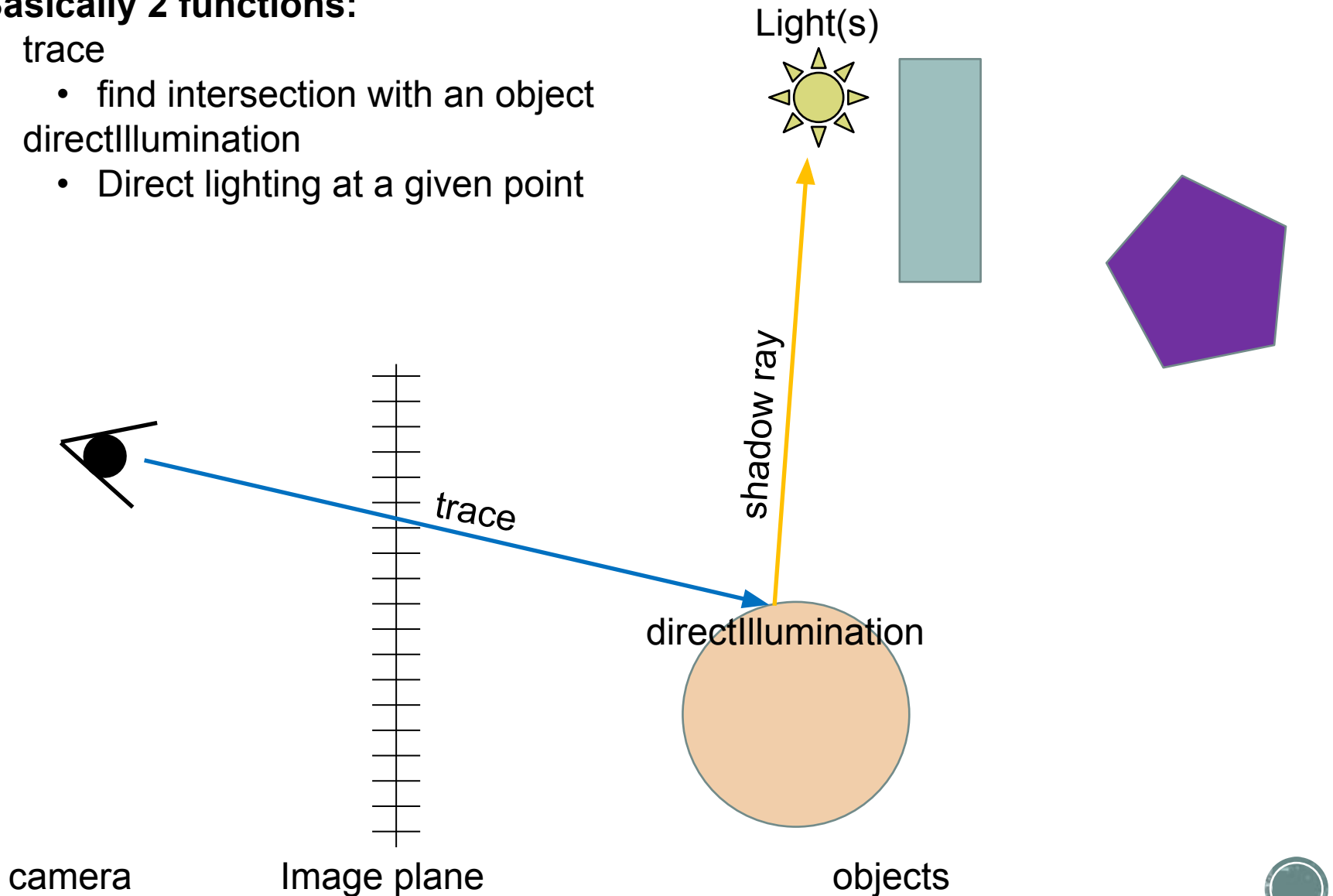
- trace
 - find intersection with an object
- directIllumination
 - Direct lighting at a given point



Ray tracing

Basically 2 functions:

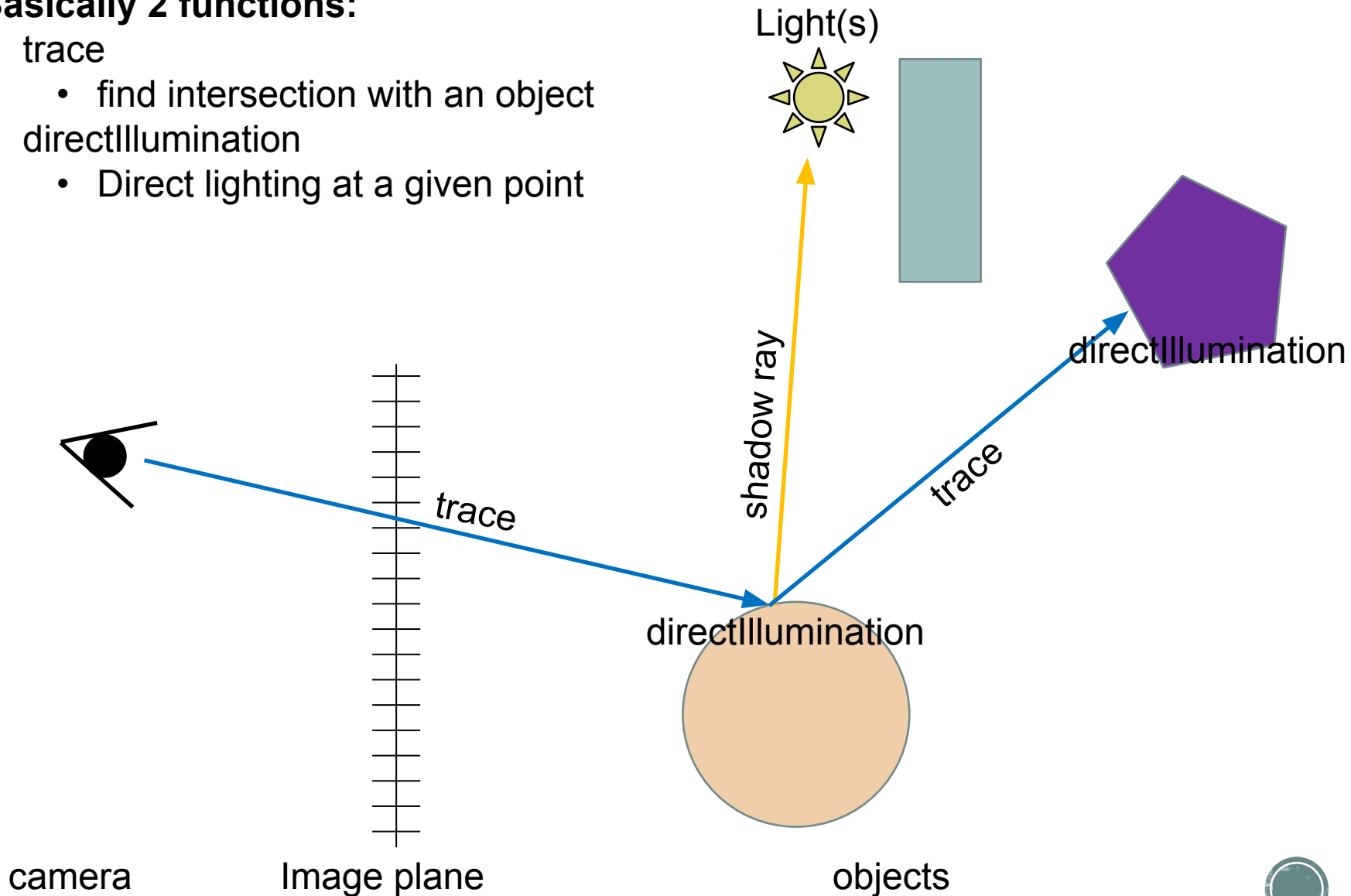
- trace
 - find intersection with an object
- directIllumination
 - Direct lighting at a given point



Ray tracing

Basically 2 functions:

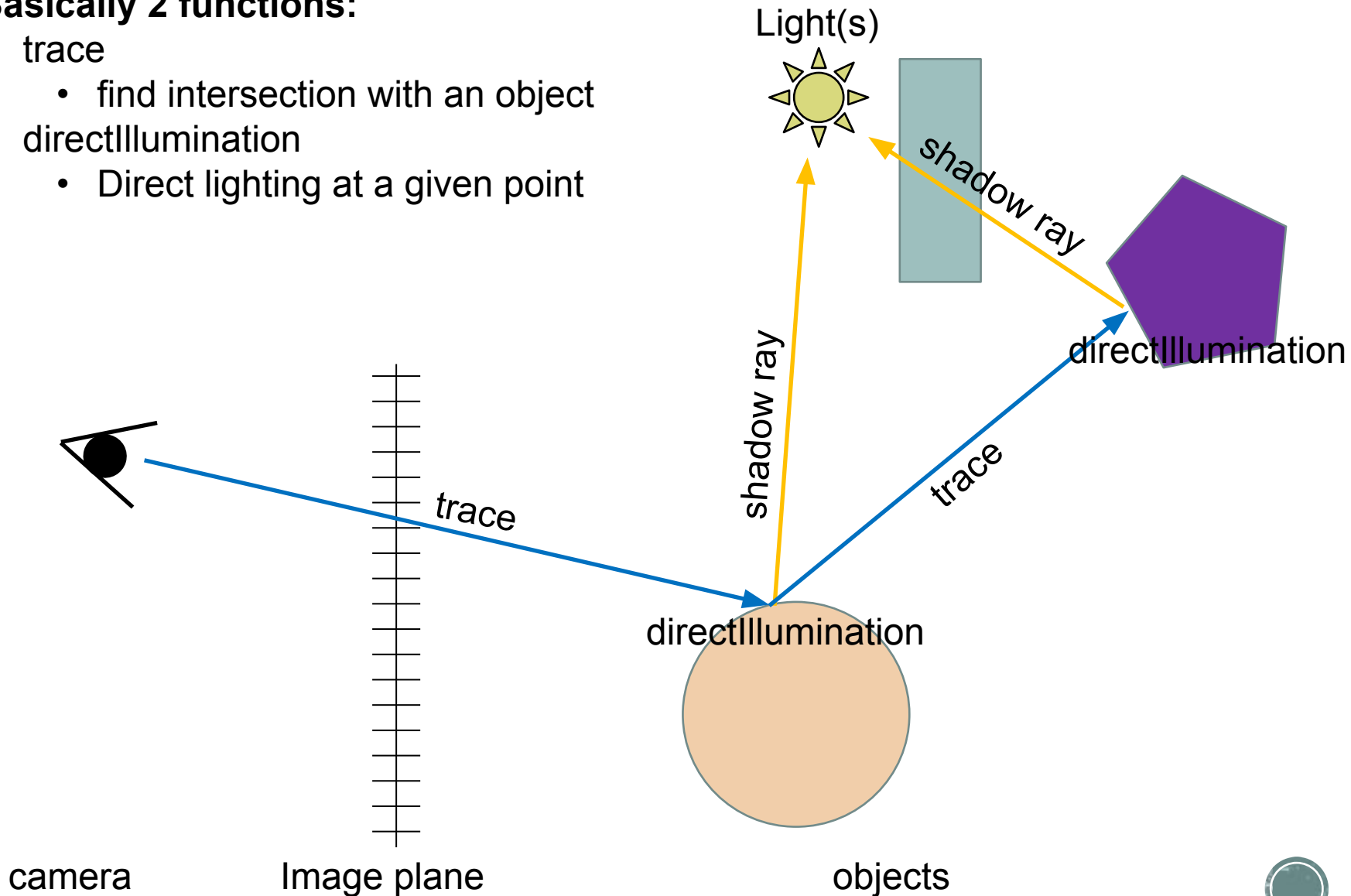
- trace
 - find intersection with an object
- directIllumination
 - Direct lighting at a given point



Ray tracing

Basically 2 functions:

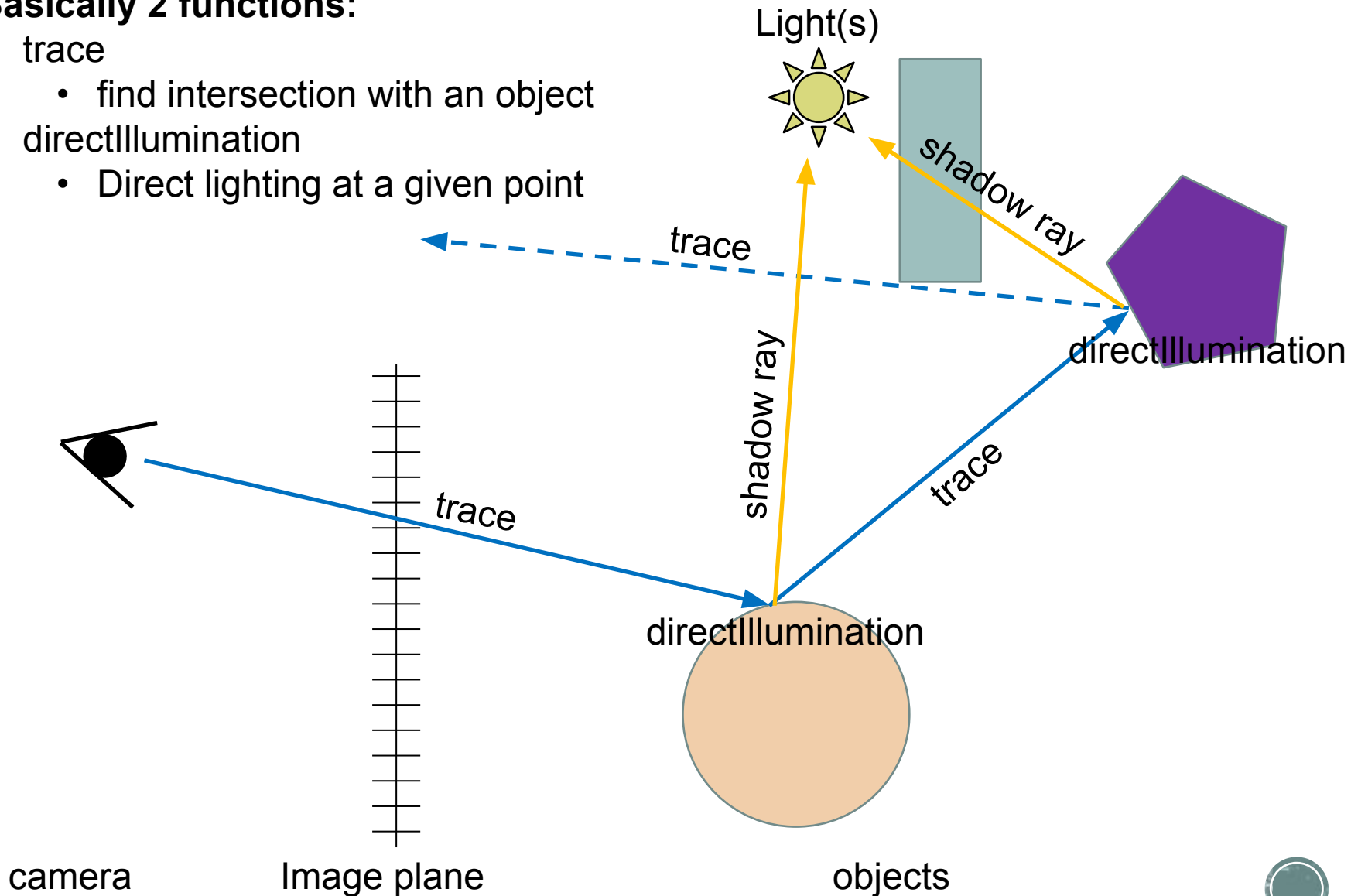
- trace
 - find intersection with an object
- directIllumination
 - Direct lighting at a given point



Ray tracing

Basically 2 functions:

- trace
 - find intersection with an object
- directIllumination
 - Direct lighting at a given point



Ray tracing

- `color trace(ray) {`
 - `hit = intersectScene(ray)`
 - `if(hit) {`
 - `color = directIllumination(hit)`
 - `if hit is reflective`
 - `color += c_refl * trace(reflected ray)`
 - `if hit is transmissive`
 - `color += c_trans * trace(refracted ray)`
 - `} else`
 - `color = background_color`
 - `return color`
- `}`



Ray tracing

- `color trace(ray) {`
 - `hit = intersectScene(ray)` OK
 - `if(hit) {`
 - `color = directIllumination(hit)`
 - `if hit is reflective`
 - `color += c_refl * trace(reflected ray)`
 - `if hit is transmissive`
 - `color += c_trans * trace(refracted ray)`
 - `} else`
 - `color = background_color`
 - `return color`
- `}`



Ray tracing

- `color trace(ray) {`
 - `hit = intersectScene(ray)`
 - `if(hit) {`
 - `color = directIllumination(hit)` ?
 - ~~`if hit is reflective`~~
 - `color += c_refl * trace(reflected ray)`
 - `if hit is transmissive`
 - `color += c_trans * trace(refracted ray)`
 - `} else`
 - `color = background_color`
 - `return color`
- `}`



Ray tracing

- `color directIllumination(hit) {`
 - `color = (0,0,0)`
 - `for each light L {`
 - `T = cast shadow ray to L`
 - `if hit is not shadowed by L`
 - `color += Ambient+diffuse+specular terms(L, hit)`
 - `}`
 - `return color`
- `}`



Ray tracing

- `color directIllumination(hit) {`
 - `color = (0,0,0)`
 - `for each light L {`
 - `I = cast shadow ray to L` ?
 - `if hit is not shadowed by L`
 - `color += Ambient+diffuse+specular terms(L, hit)`
 - `}`
 - `return color`
- `}`

Difference between eye ray and shadow ray ?



Ray tracing

- `color directIllumination(hit) {`
 - `color = (0,0,0)`
 - `for each light L {`
 - `T = cast shadow ray to L`
 - `if hit is not shadowed by L`
 - `color += Ambient+diffuse+specular terms(L, hit)`
 - `}`
 - `return color`
- `}`

?

Material properties (we will see it soon in details)



Ray tracing

- color trace(ray) {
 - hit = intersectScene(ray)
 - if(hit) {
 - color = directIllumination(hit) OK
 - if hit is reflective
 - color += c_refl * trace(reflected ray)
 - if hit is transmissive
 - color += c_trans * trace(refracted ray)
 - } else
 - color = background_color
 - return color
- }



Ray tracing

```
■ color trace(ray) {  
  ■ hit = intersectScene(ray)  
  ■ if(hit) {  
    ■ color = directIllumination(hit)  
    ■ if hit is reflective  
      ■ color += c_refl * trace(reflected ray)  
    ■ if hit is transmissive  
      ■ color += c_trans * trace(refracted ray)  
  ■ } else  
    ■ color = background_color  
  ■ return color  
■ }
```

Recursive!



Ray tracing

```
■ color trace(ray) {  
  ■ hit = intersectScene(ray)  
  ■ if(hit) {  
    ■ color = directIllumination(hit)  
    ■ if hit is reflective  
      ■ color += c_refl * trace(reflected ray)  
    ■ if hit is transmissive  
      ■ color += c_trans * trace(refracted ray)  
  ■ } else  
    ■ color = background_color  
  ■ return color  
■ }
```

Recursive!

How to stop?

- Recursion depth
 - After a number of bounces
- Ray contribution
 - Reflected/transmitted contribution becomes too small



Ray tracing

```
■ color trace(ray) {  
  ■ hit = intersectScene(ray)  
  ■ if(hit) {  
    ■ color = directIllumination(hit)  
    ■ if hit is reflective  
      ■ color += c_refl * trace(reflected ray)  
    ■ if hit is transmissive  
      ■ color += c_trans * trace(refracted ray)  
  ■ } else  
    ■ color = background_color  
  ■ return color  
■ }
```

Recursive!

How to stop?



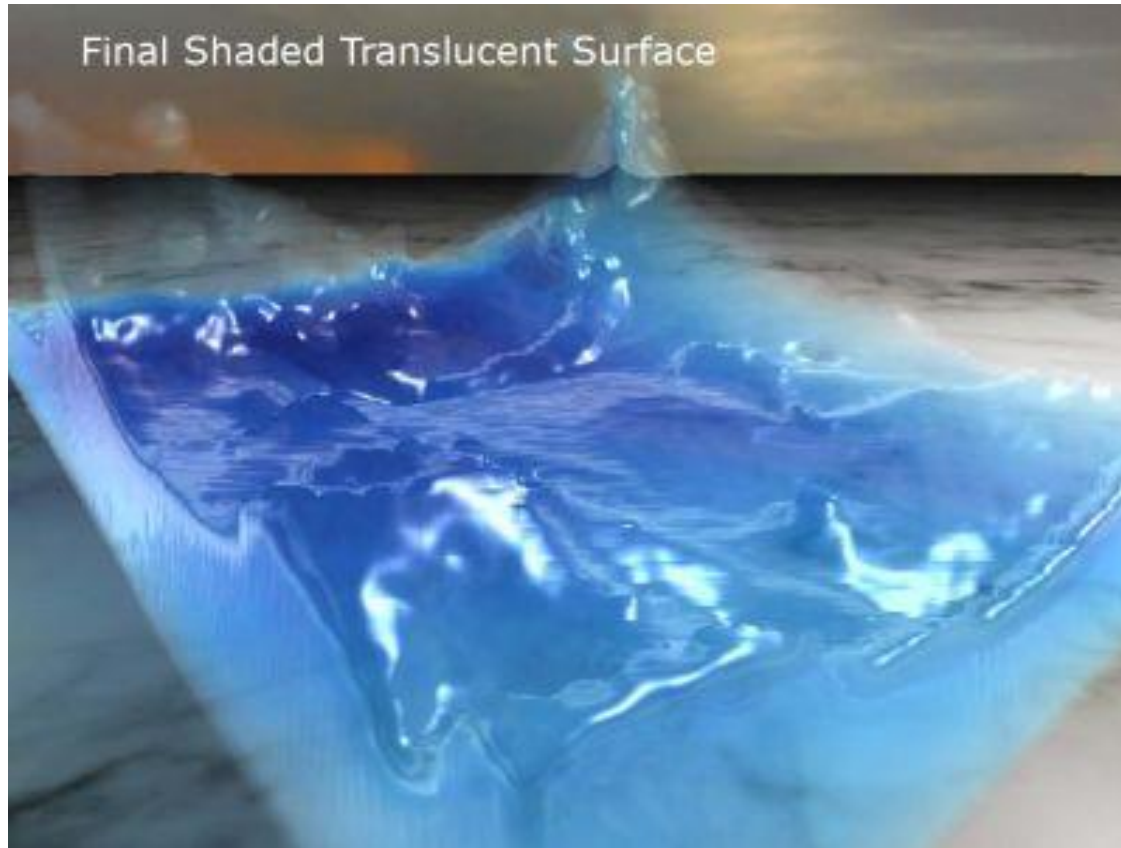
glossary

- Ray casting:
 - eye ray only
- Ray tracing:
 - all secondary rays
- Shadow ray:
 - surface to light ray (shadow test)
- Ray marching:
 - step by step surface intersection test



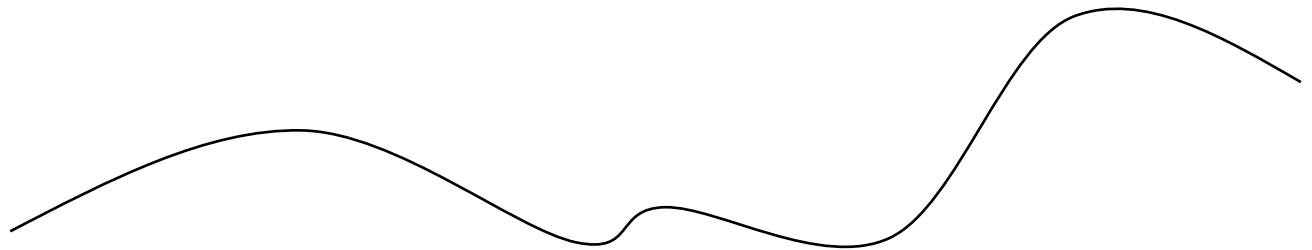
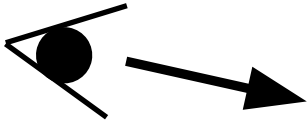
Ray-tracing

What if intersection cannot be computed analytically?



Ray marching height-fields

$$P(t) = r_o + r_d * t$$



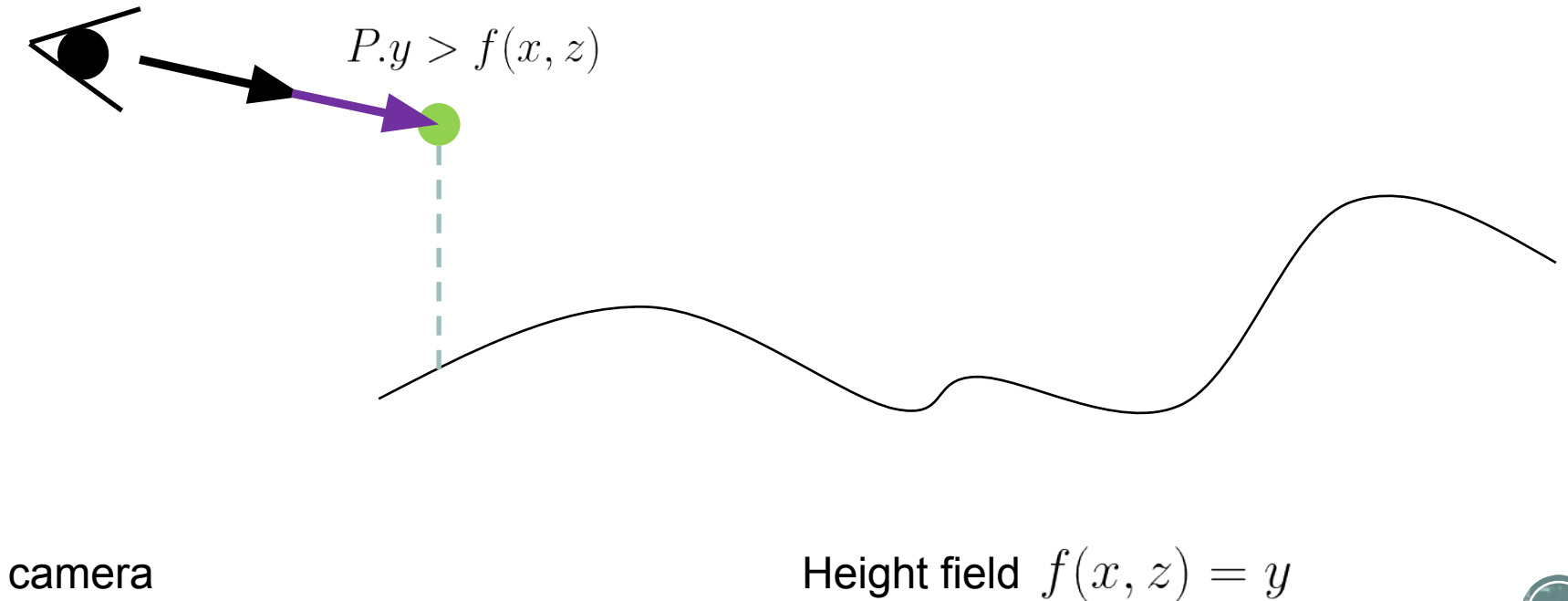
camera

Height field $f(x, z) = y$



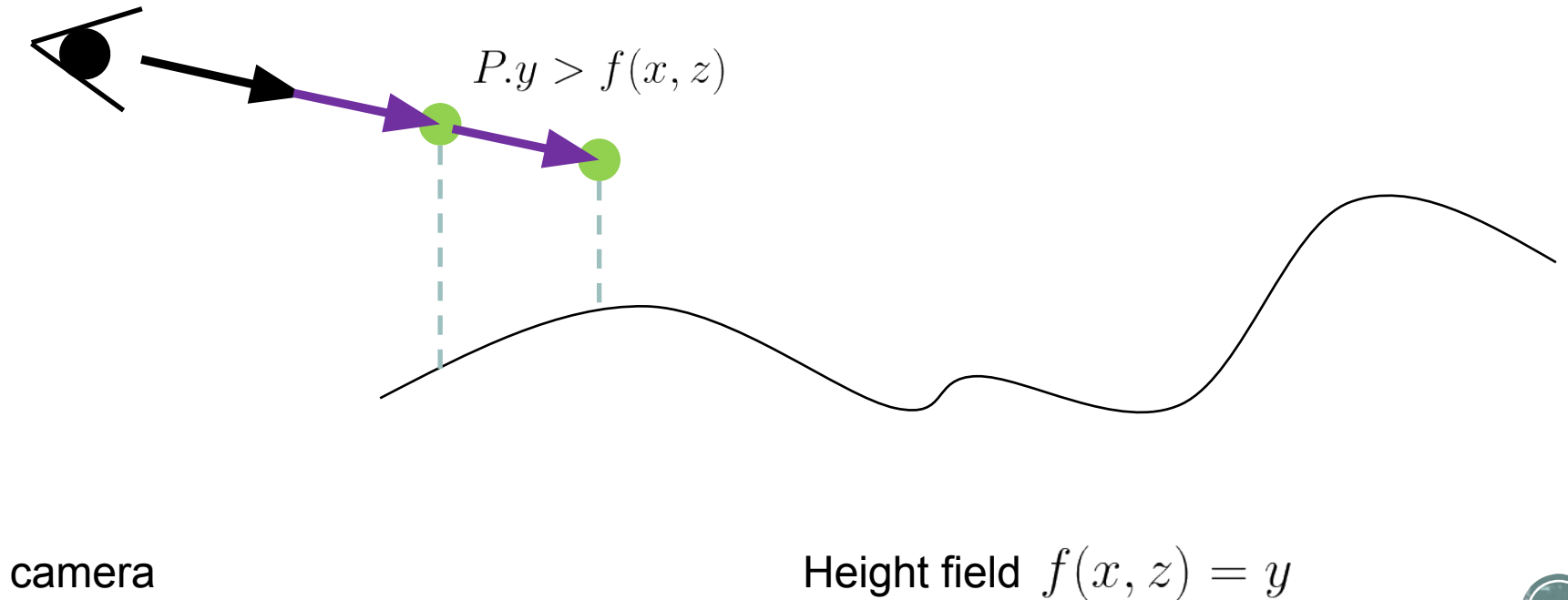
Ray marching height-fields

$$P(t) = r_o + r_d * t$$



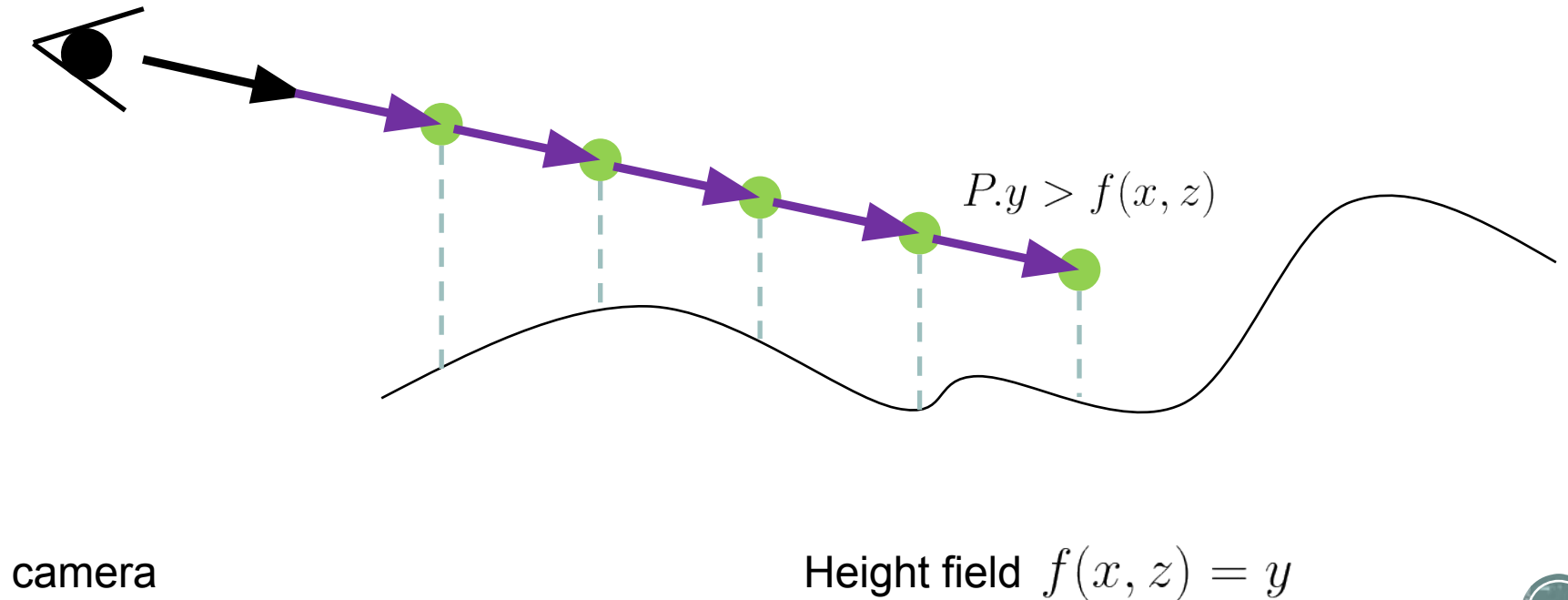
Ray marching height-fields

$$P(t) = r_o + r_d * t$$



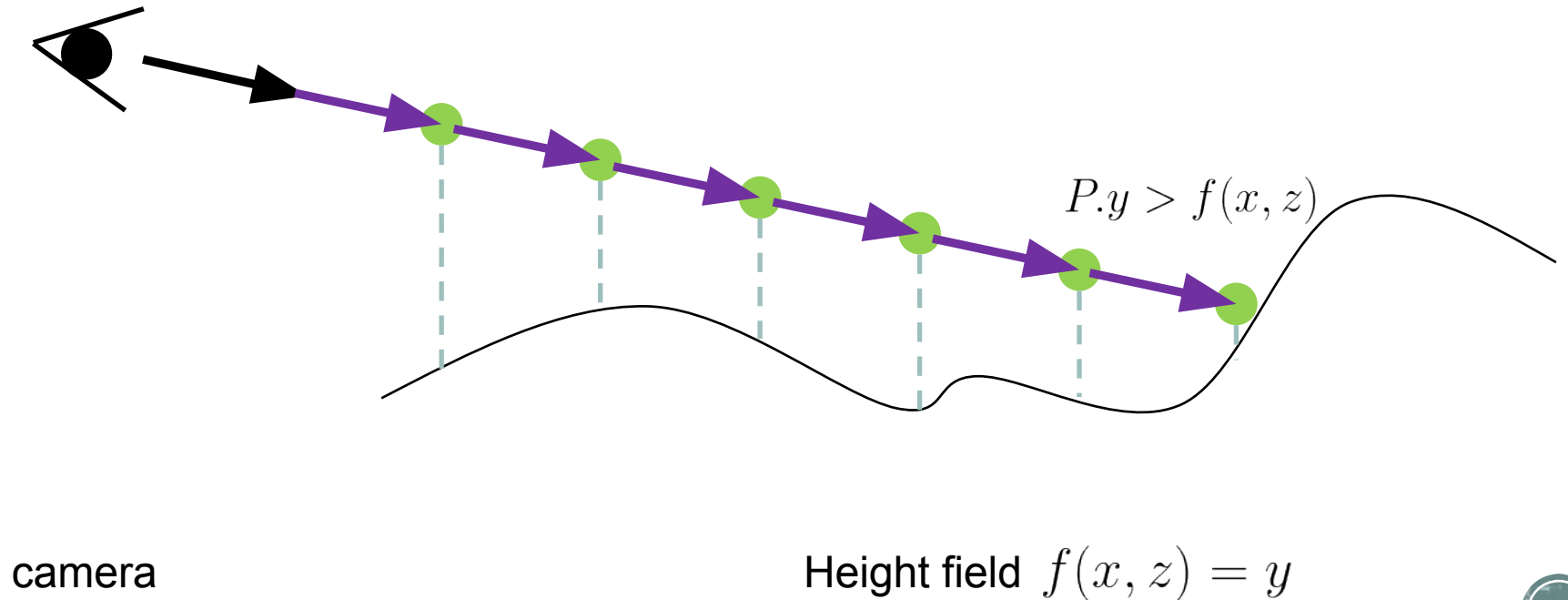
Ray marching height-fields

$$P(t) = r_o + r_d * t$$



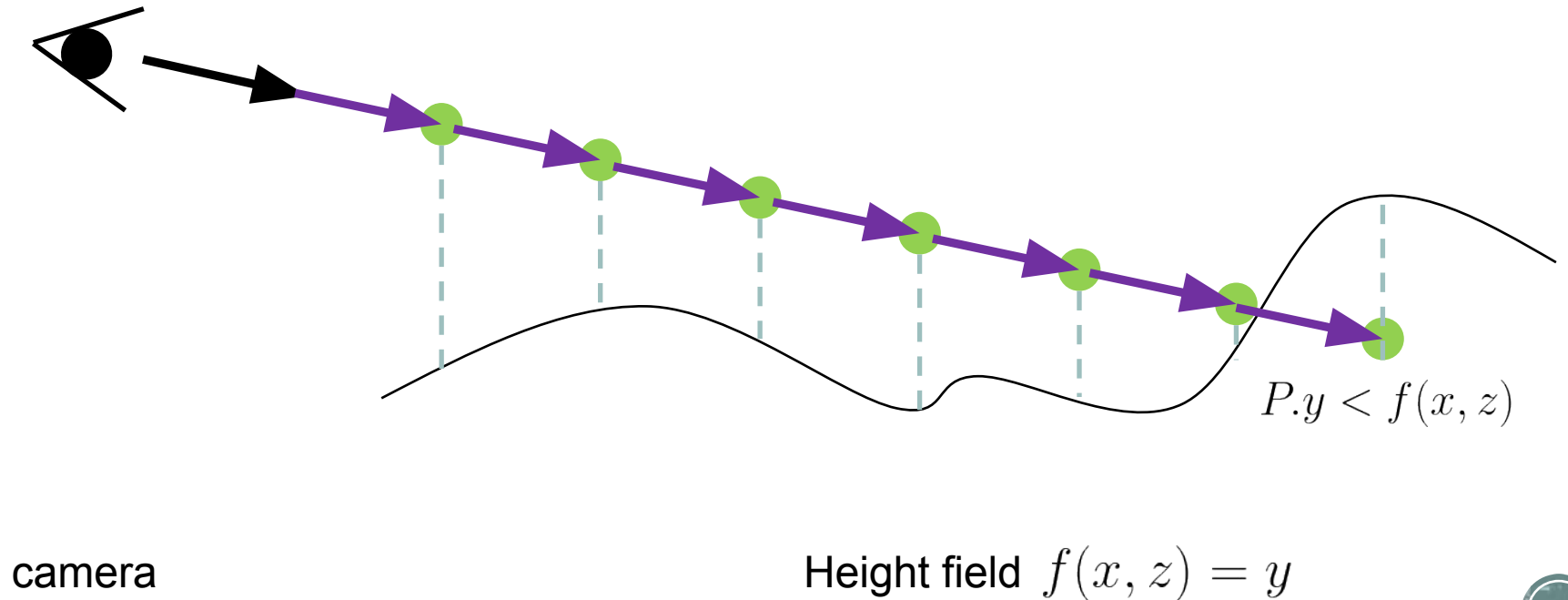
Ray marching height-fields

$$P(t) = r_o + r_d * t$$



Ray marching height-fields

$$P(t) = r_o + r_d * t$$

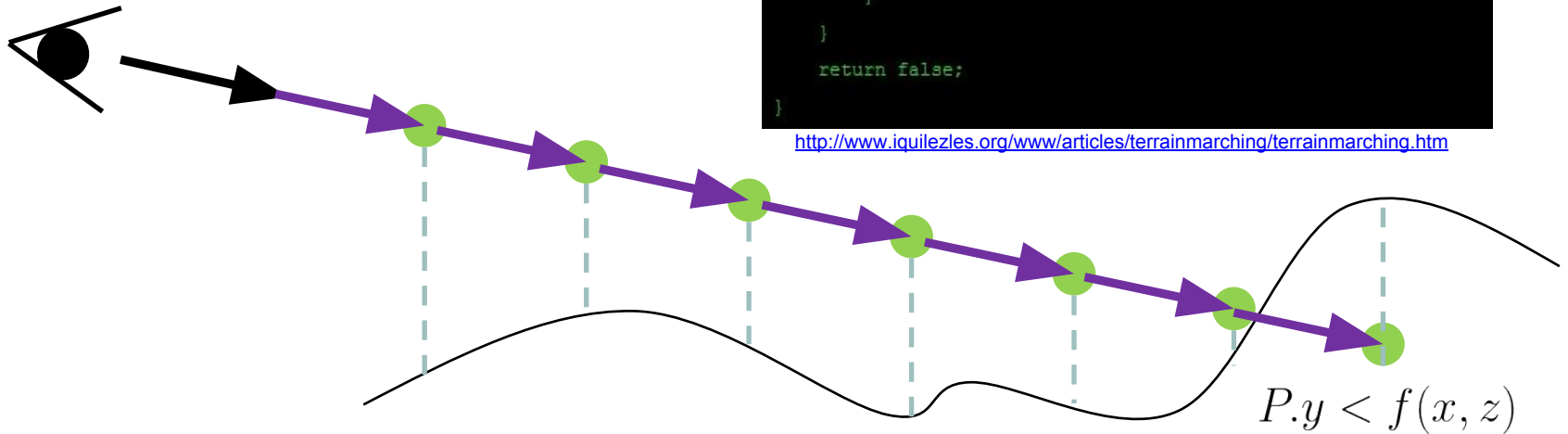


Ray marching height-fields

$$P(t) = r_o + r_d * t$$

```
bool castRay( const vec3 & ro, const vec3 & rd, float & resT )
{
    const float delt = 0.01f;
    const float mint = 0.001f;
    const float maxt = 10.0f;
    for( float t = mint; t < maxt; t += delt )
    {
        const vec3 p = ro + rd*t;
        if( p.y < f( p.x, p.z ) )
        {
            resT = t - 0.5f*delt;
            return true;
        }
    }
    return false;
}
```

<http://www.iquilezles.org/www/articles/terrainmarching/terrainmarching.htm>



camera

Height field $f(x, z) = y$

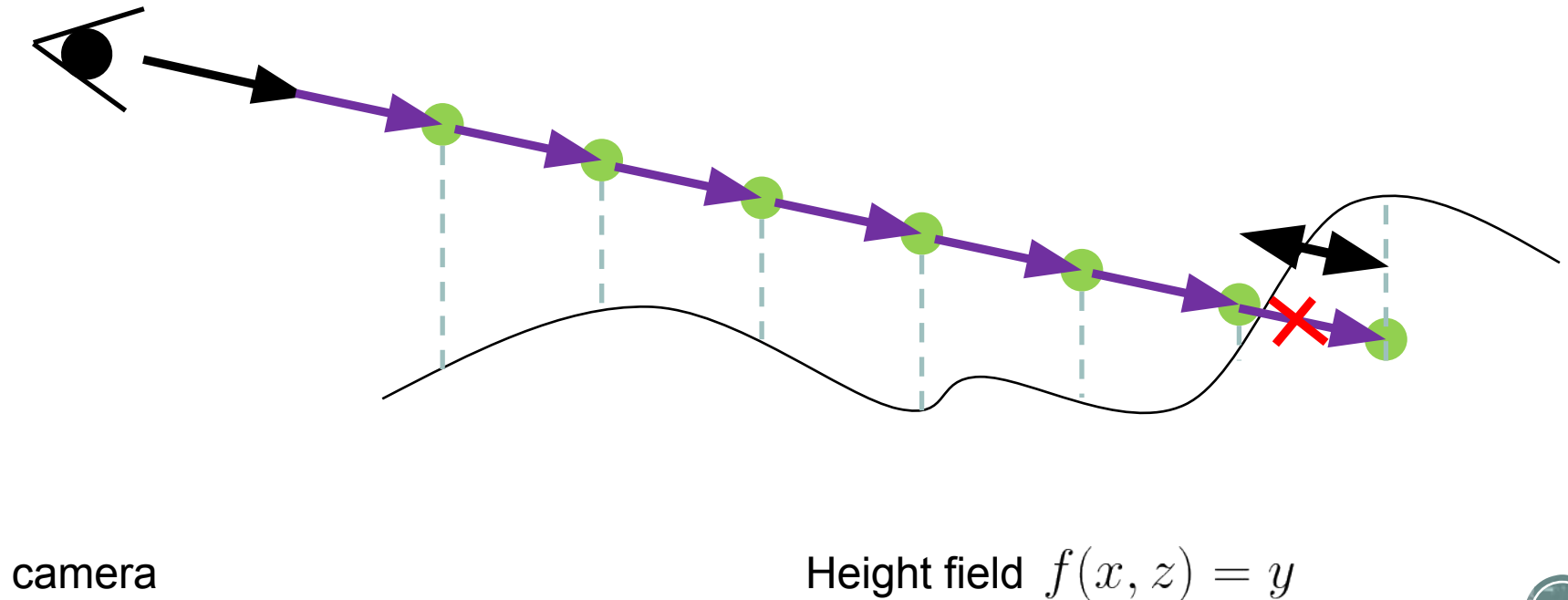


Ray marching height-fields

$$P(t) = r_o + r_d * t$$

Optimizations:

- Interpolate between the 2 last positions

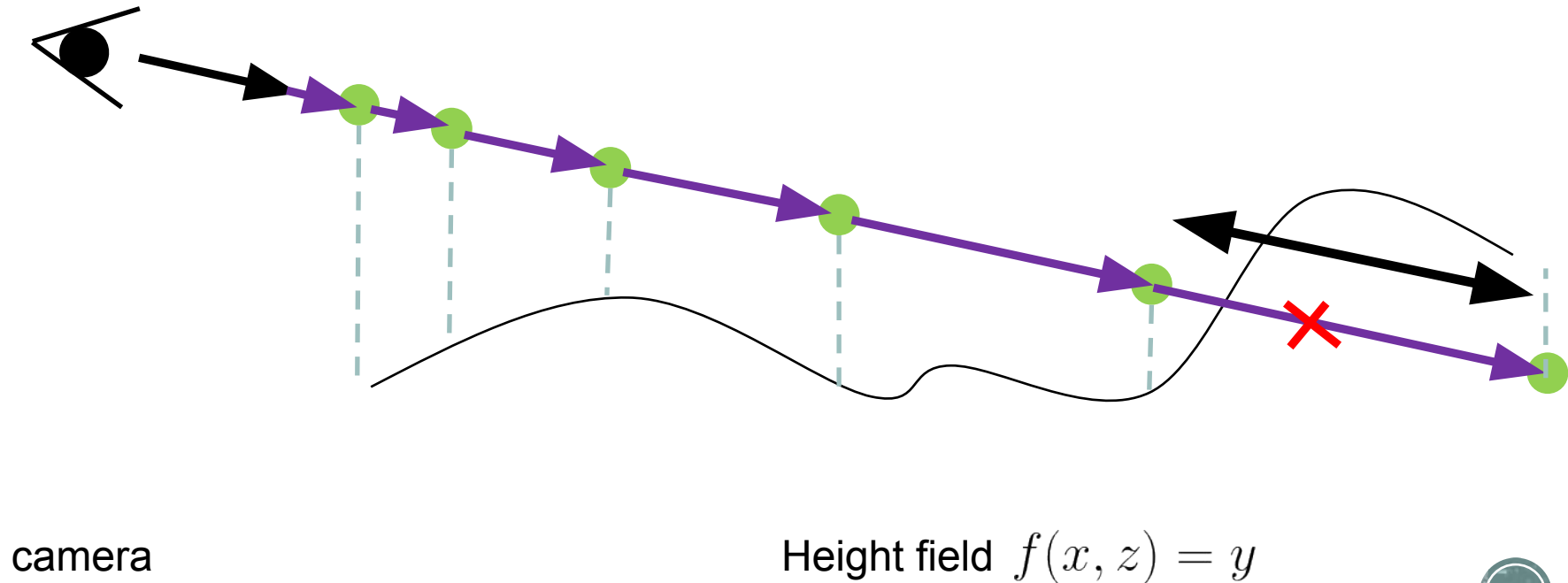


Ray marching height-fields

$$P(t) = r_o + r_d * t$$

Optimizations:

- Interpolate between the 2 last positions
- Increase deltaT with distance from eye

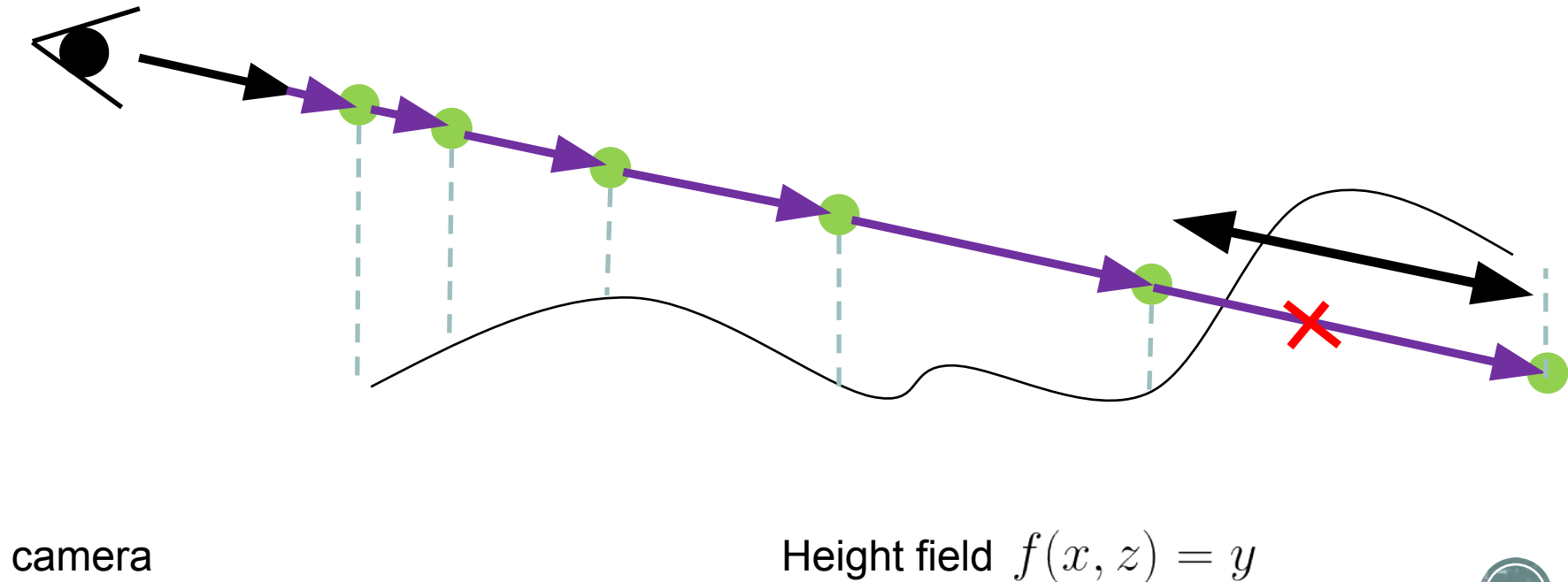


Ray marching height-fields

$$P(t) = r_o + r_d * t$$

Optimizations:

- Interpolate between the 2 last positions
- Increase deltaT with distance from eye
- See: <http://www.iquilezles.org>



Ray marching height-fields

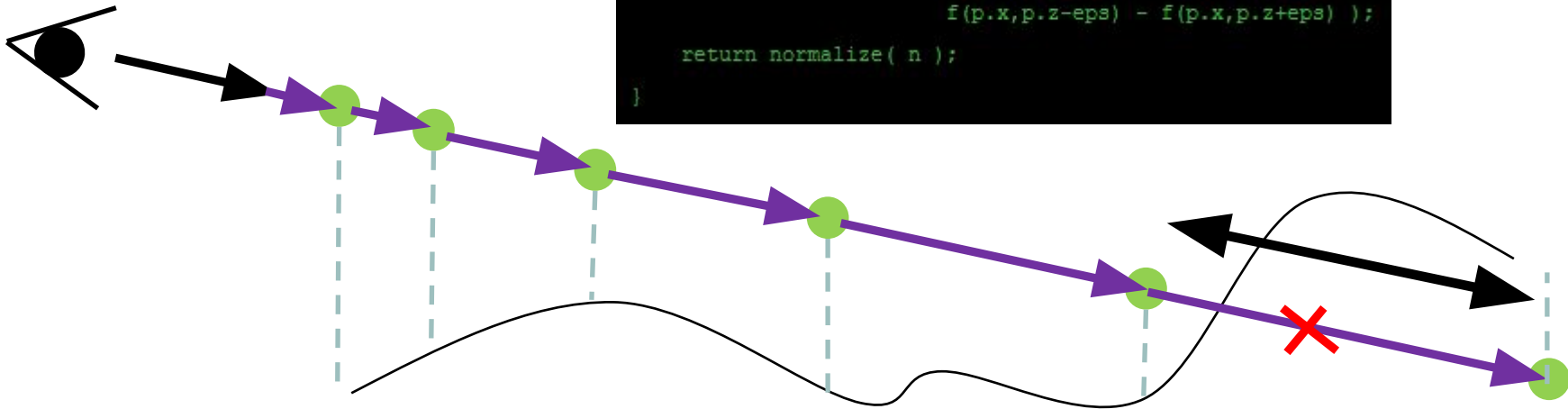
$$P(t) = r_o + r_d * t$$

Optimizations:

- Interpolate between the 2 last positions
- Increase deltaT with distance from eye
- See: <http://www.iquilezles.org>

Normal computation:

```
vec3 getNormal( const vec3 & p )  
{  
    const vec3 n = vec3( f(p.x-eps,p.z) - f(p.x+eps,p.z),  
                        2.0f*eps,  
                        f(p.x,p.z-eps) - f(p.x,p.z+eps) );  
    return normalize( n );  
}
```



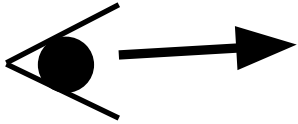
camera

Height field $f(x, z) = y$

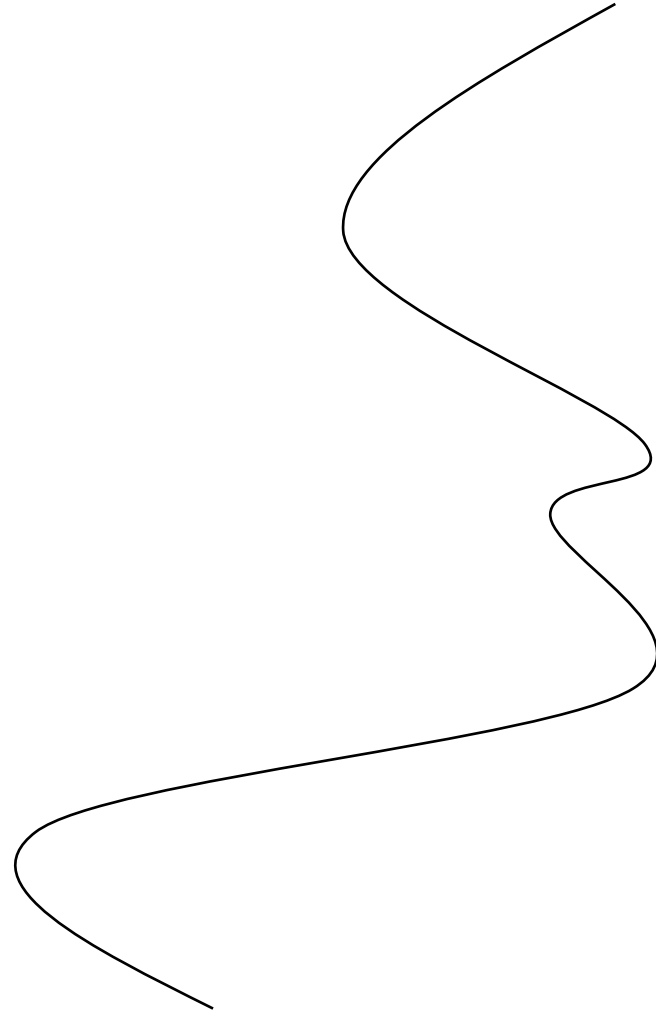


Ray marching implicit surfaces

$$P(t) = r_o + r_d * t$$



camera



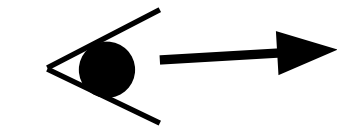
Implicit surface



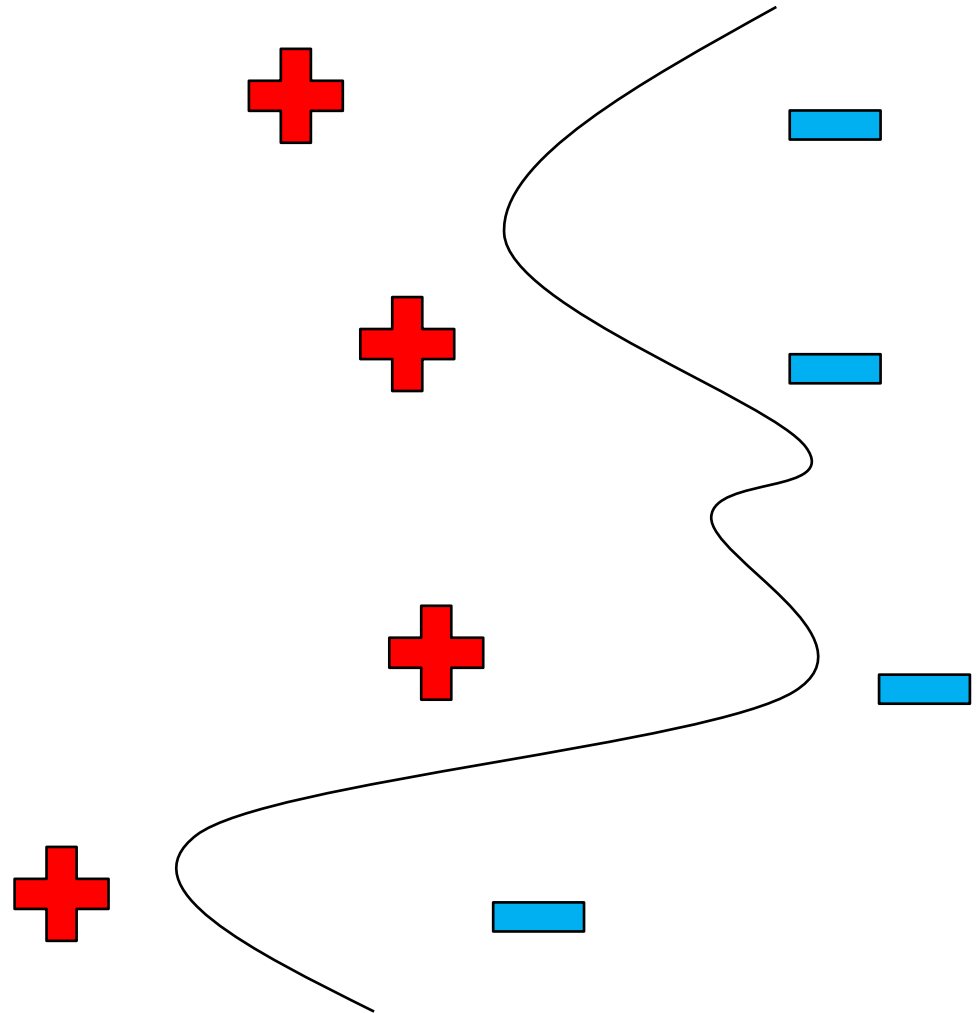
Ray marching implicit surfaces

$$P(t) = r_o + r_d * t$$

$$f(x, y, z) = 0$$



camera



Implicit surface

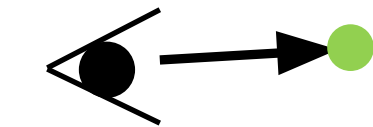


Ray marching implicit surfaces

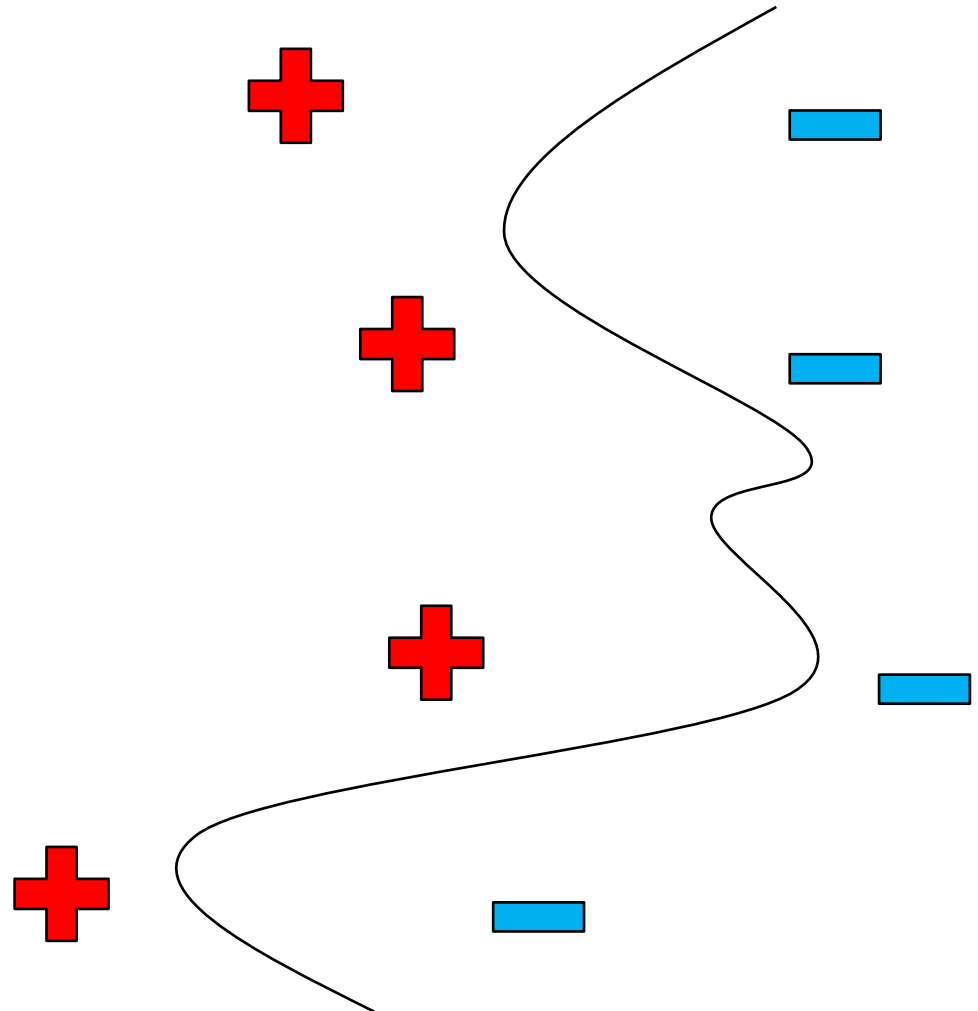
$$P(t) = r_o + r_d * t$$

$$f(x, y, z) = 0$$

$$f(x, y, z) > 0$$



camera



Implicit surface

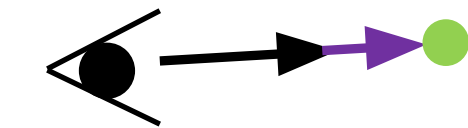


Ray marching implicit surfaces

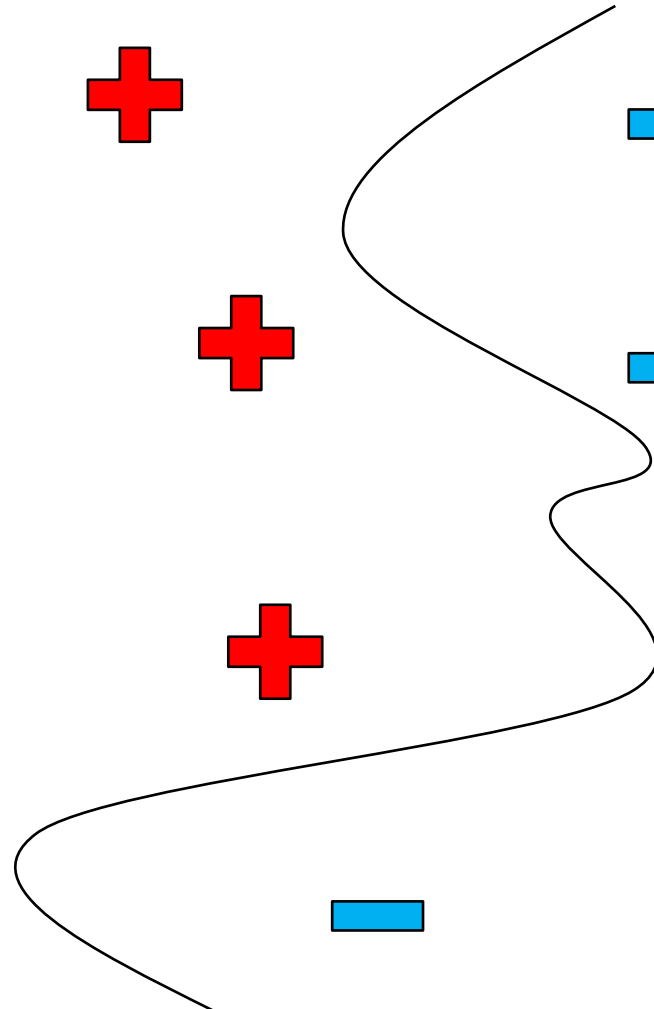
$$P(t) = r_o + r_d * t$$

$$f(x, y, z) = 0$$

$$f(x, y, z) > 0$$



camera



Implicit surface

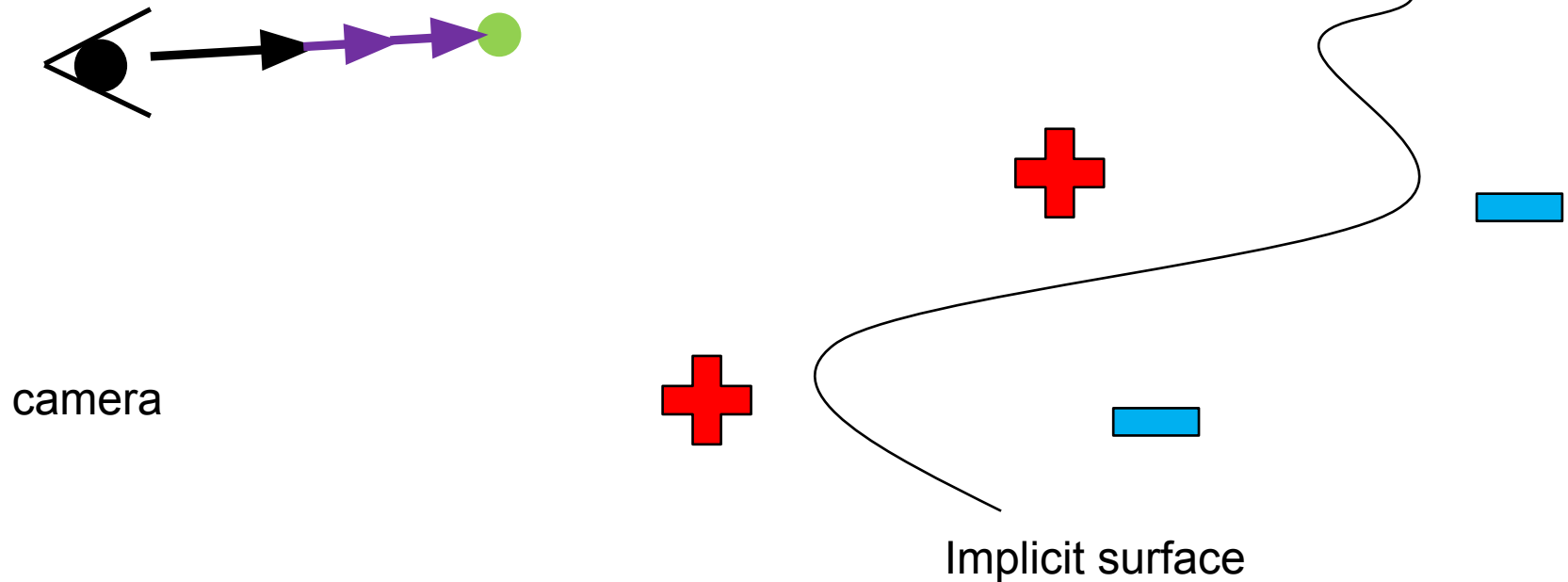


Ray marching implicit surfaces

$$P(t) = r_o + r_d * t$$

$$f(x, y, z) = 0$$

$$f(x, y, z) > 0$$

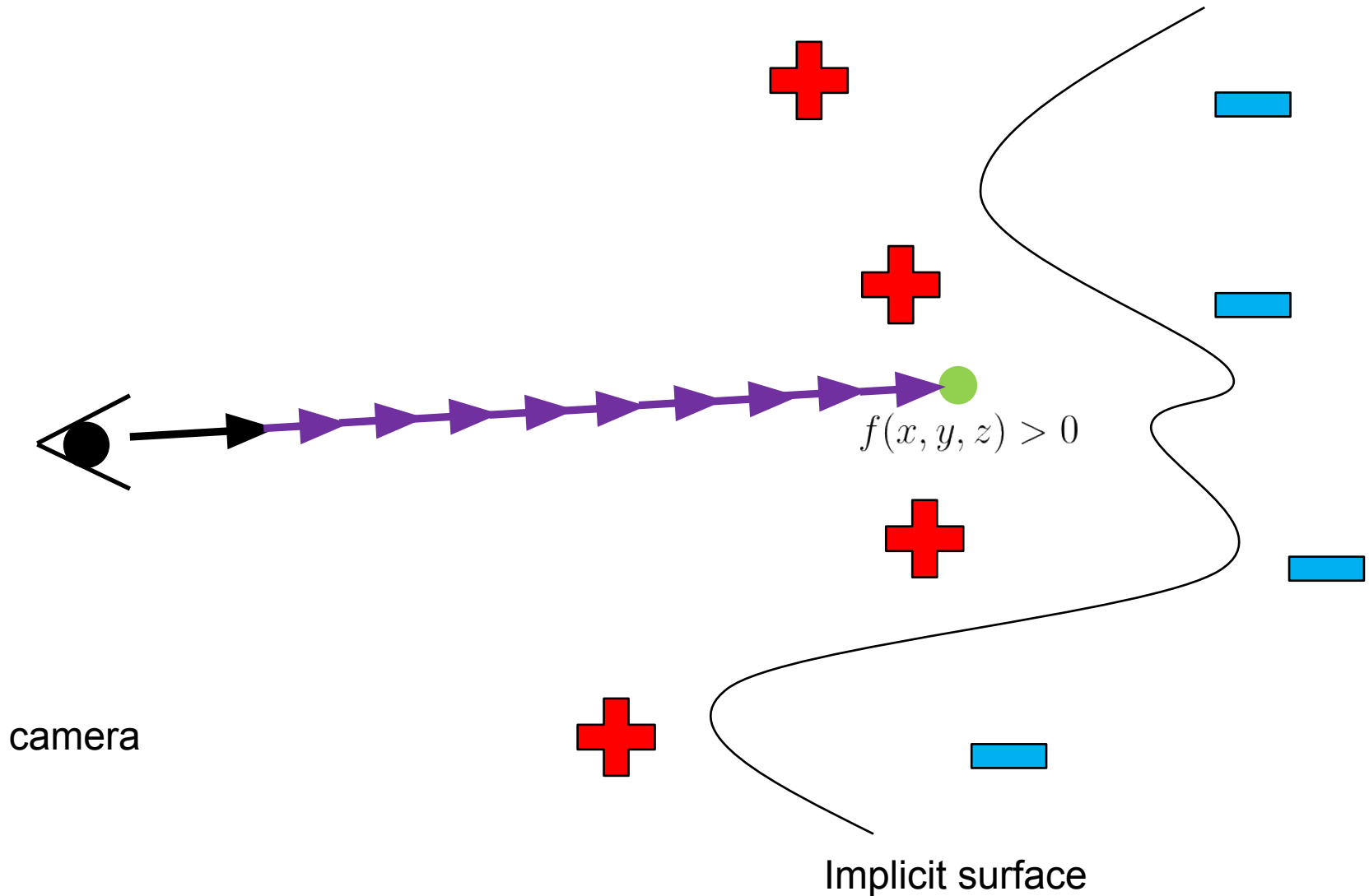


Ray marching implicit surfaces

$$P(t) = r_o + r_d * t$$

$$f(x, y, z) = 0$$

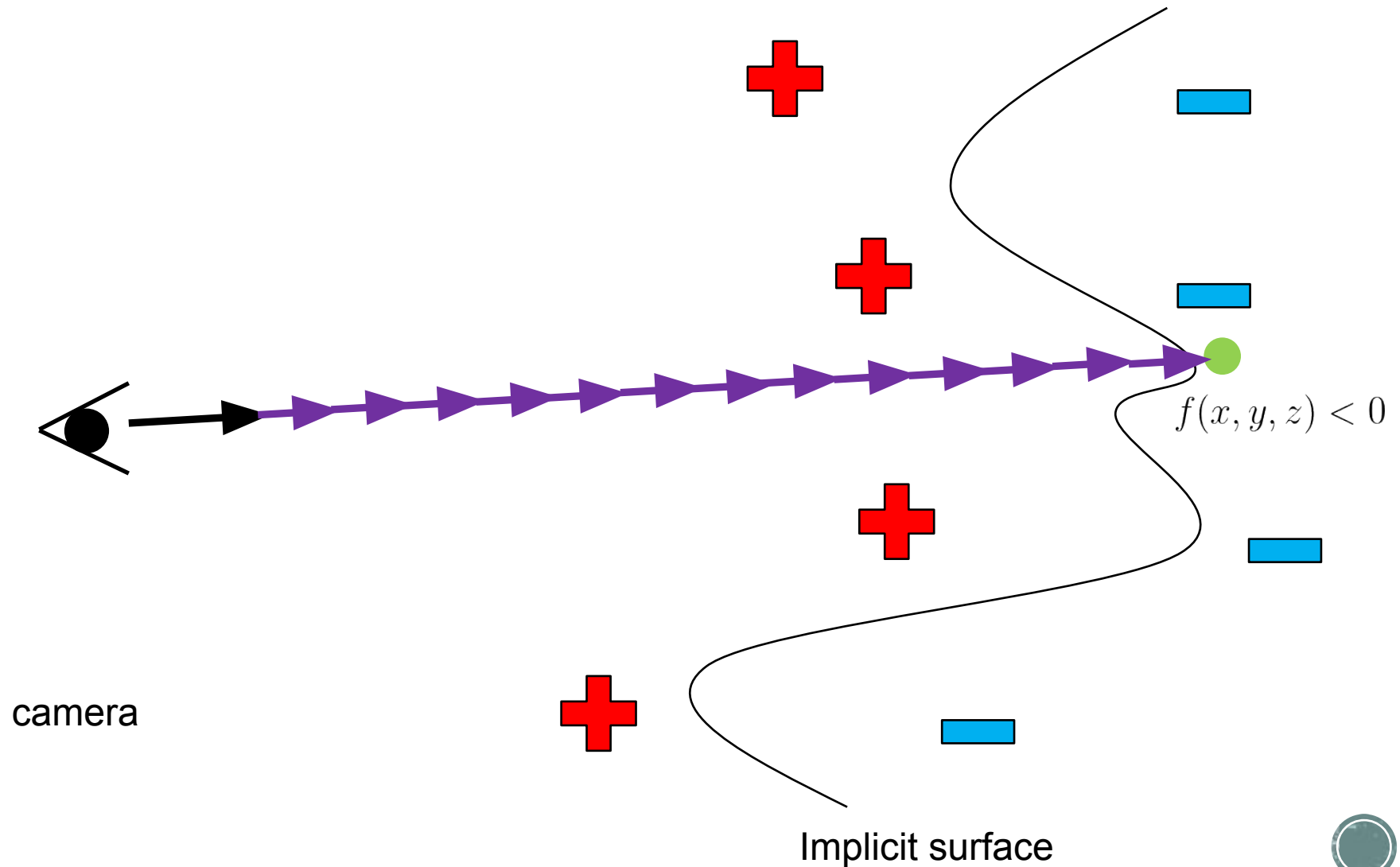
$$f(x, y, z) > 0$$



Ray marching implicit surfaces

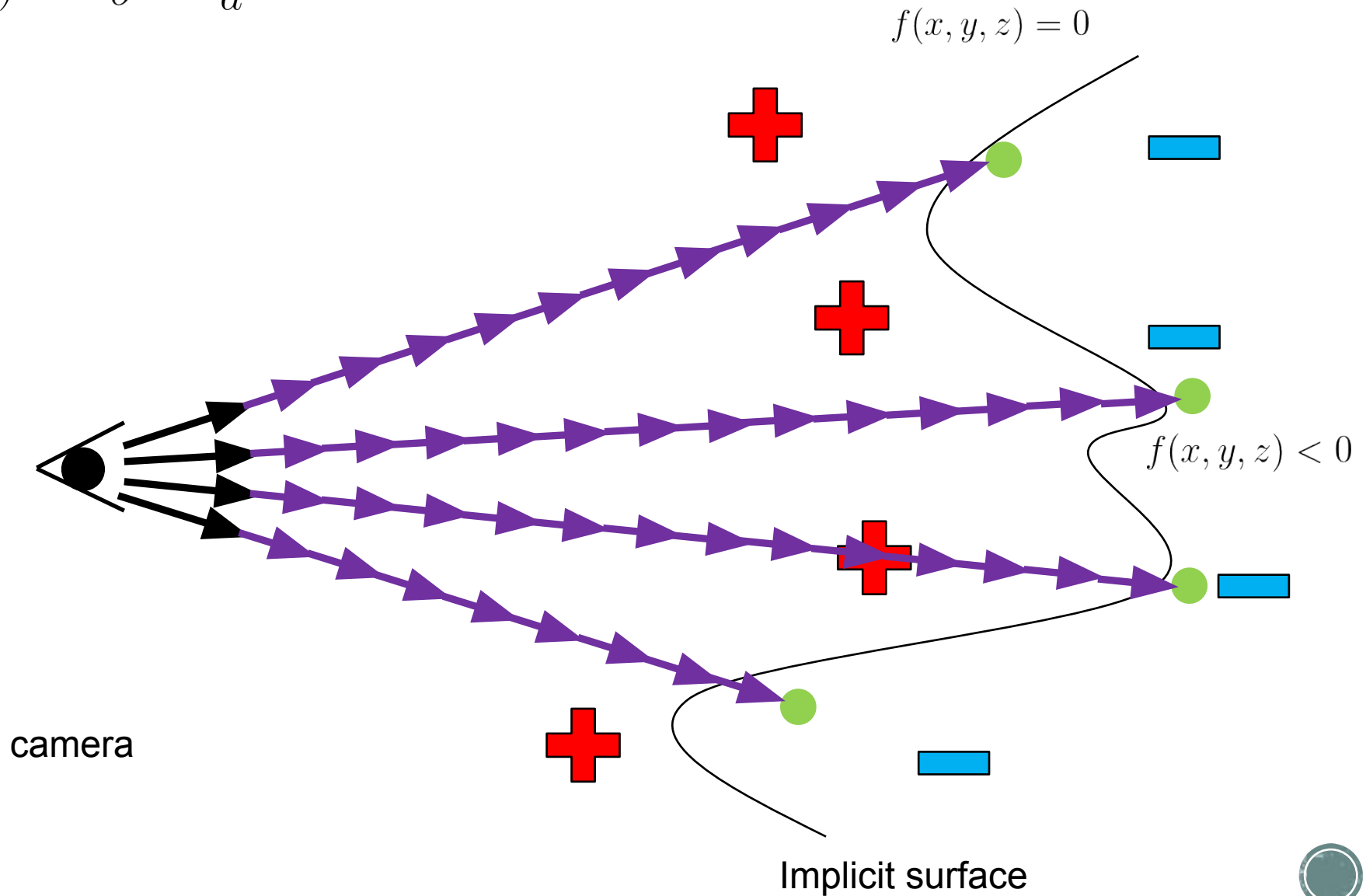
$$P(t) = r_o + r_d * t$$

$$f(x, y, z) = 0$$



Ray marching implicit surfaces

$$P(t) = r_o + r_d * t$$

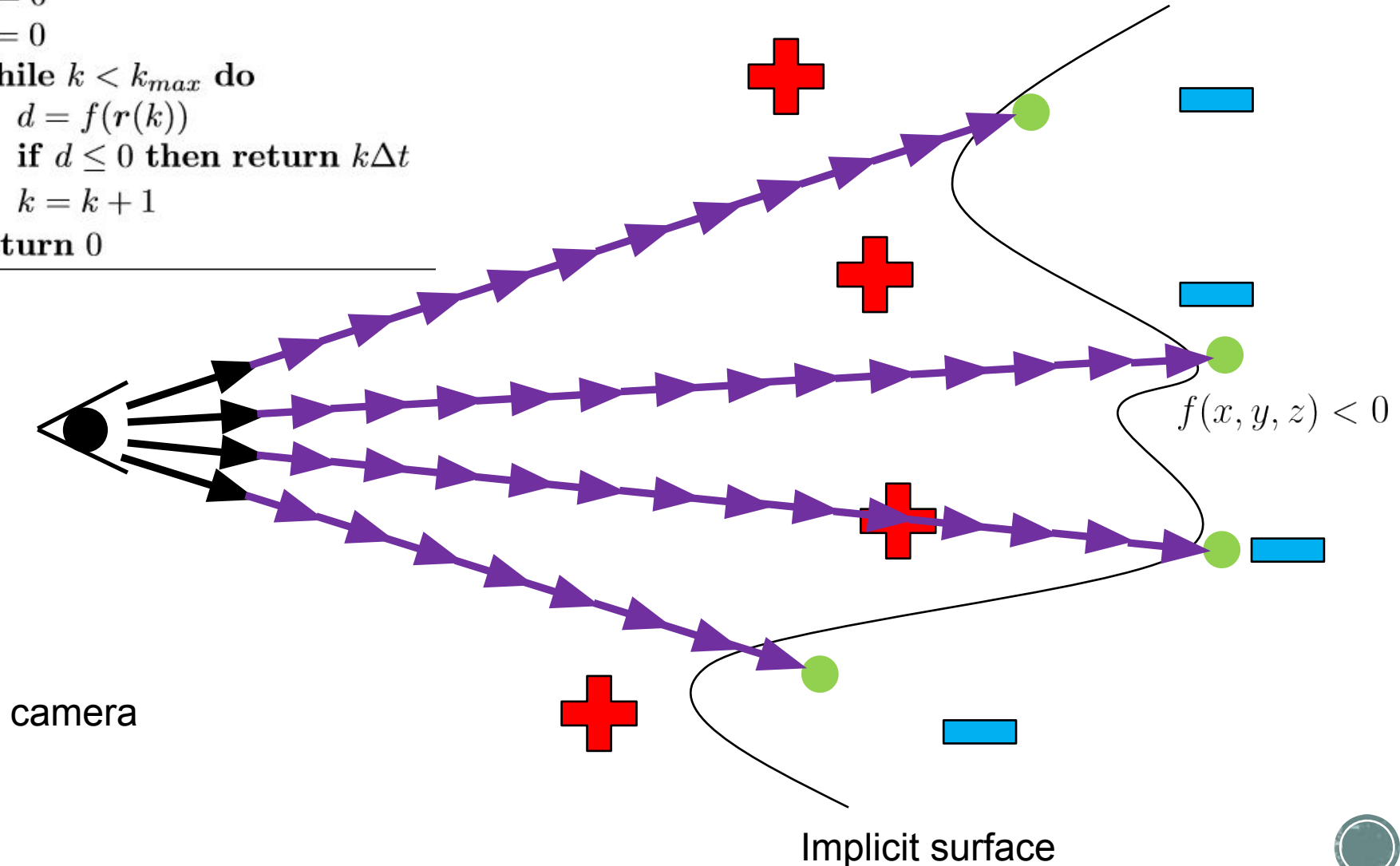


Ray marching implicit surfaces

$$P(t) = r_o + r_d * t$$

```
1:  $k = 0$   
2:  $d = 0$   
3: while  $k < k_{max}$  do  
4:    $d = f(r(k))$   
5:   if  $d \leq 0$  then return  $k\Delta t$   
6:    $k = k + 1$   
7: return 0
```

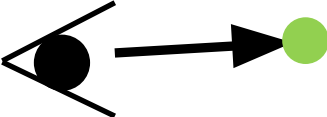
$$f(x, y, z) = 0$$



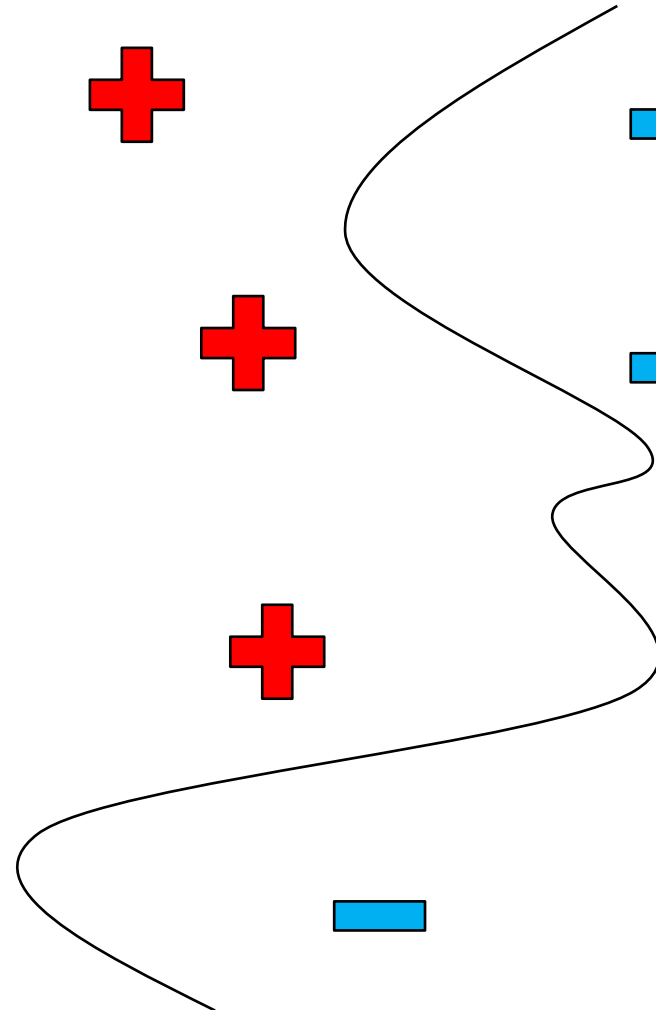
Ray marching implicit surfaces

$$P(t) = r_o + r_d * t$$

$$f(x, y, z) = 0$$

$$f(x, y, z) = d$$
A diagram showing a camera on the left, represented by a black circle with a triangle pointing right. A black arrow points from the camera to a green circle on the right, representing a point on the ray.

camera



Implicit surface



Ray marching implicit surfaces

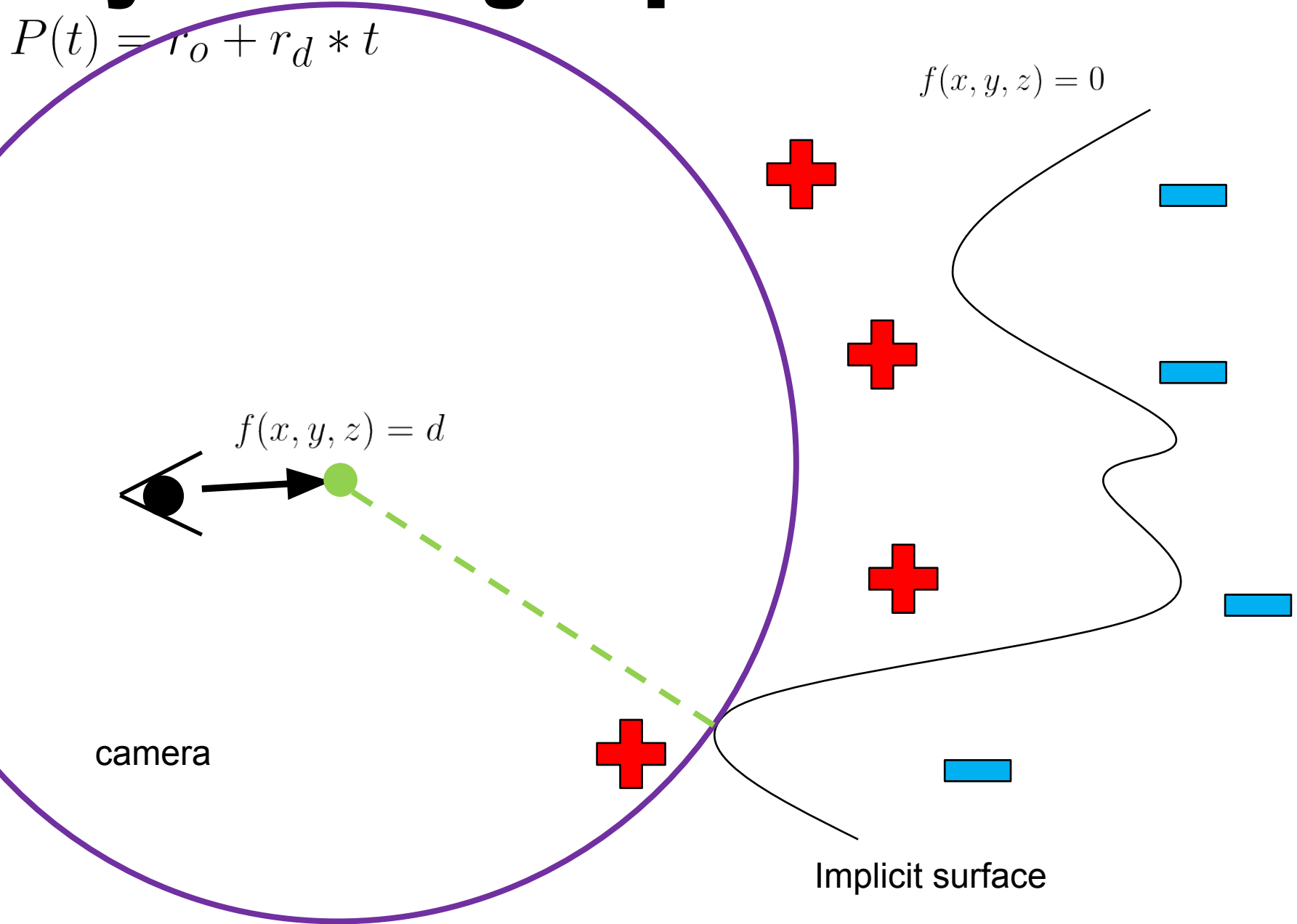
$$P(t) = r_o + r_d * t$$

$$f(x, y, z) = 0$$

$$f(x, y, z) = d$$

camera

Implicit surface



Ray marching implicit surfaces

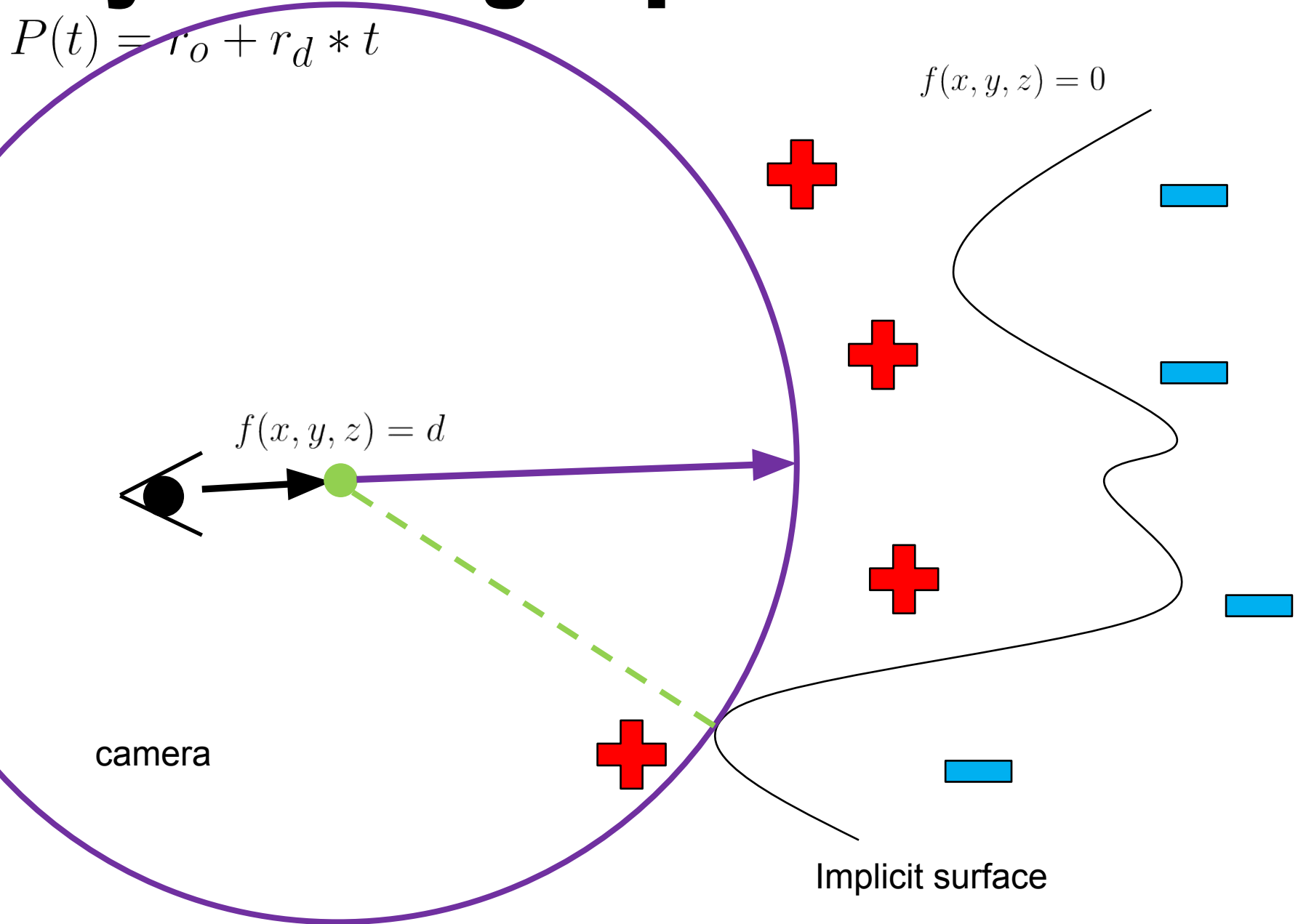
$$P(t) = r_o + r_d * t$$

$$f(x, y, z) = 0$$

$$f(x, y, z) = d$$

camera

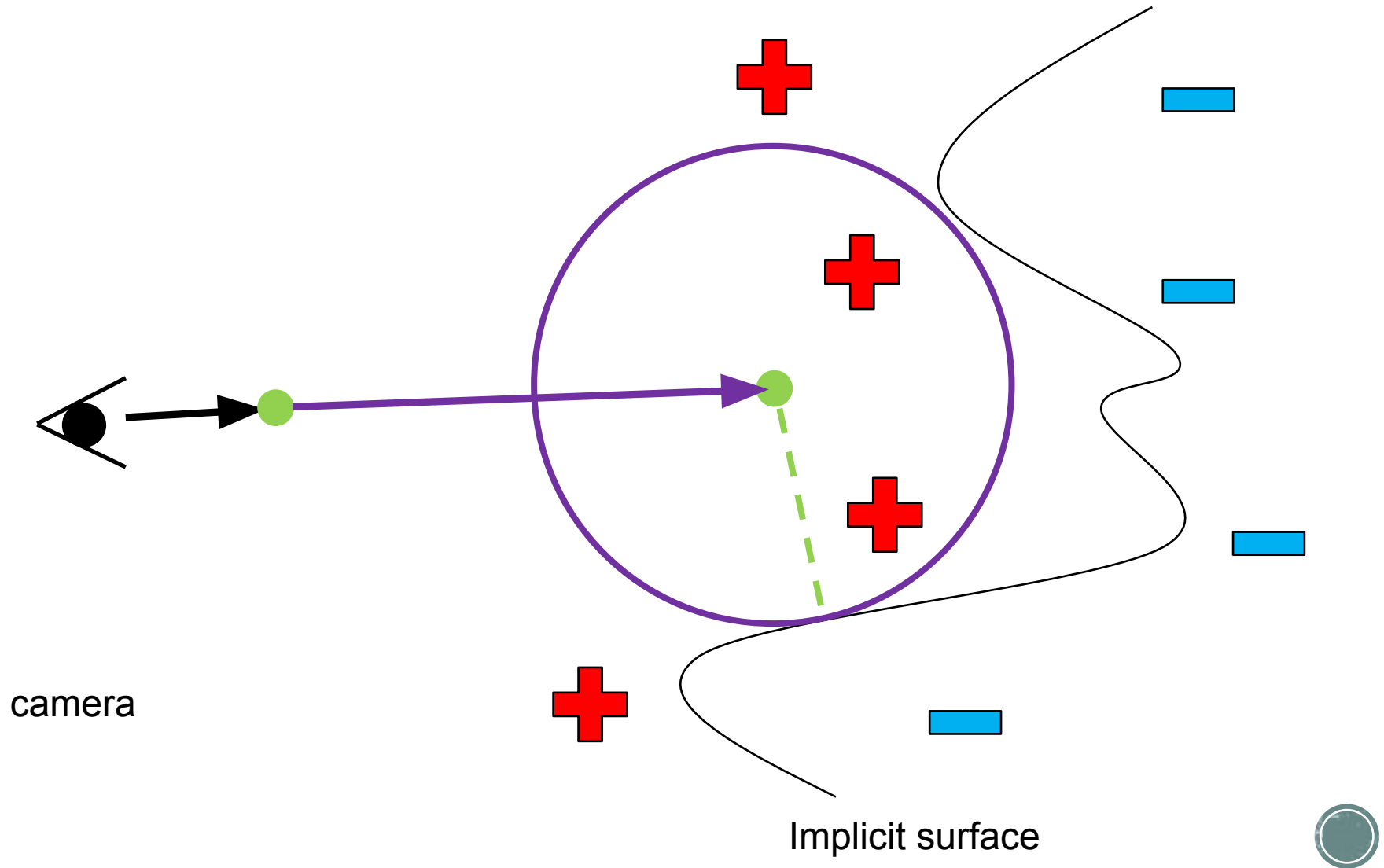
Implicit surface



Ray marching implicit surfaces

$$P(t) = r_o + r_d * t$$

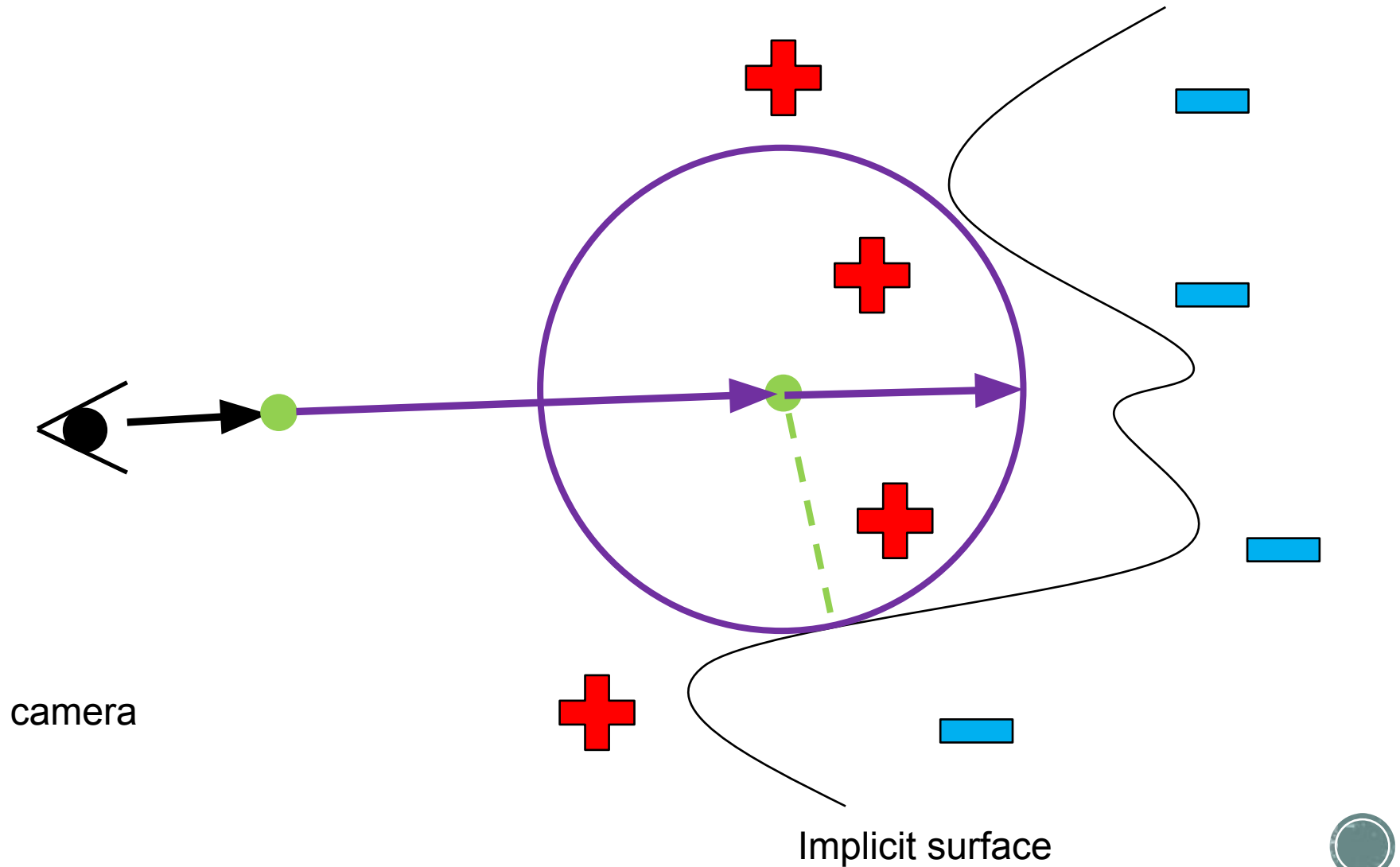
$$f(x, y, z) = 0$$



Ray marching implicit surfaces

$$P(t) = r_o + r_d * t$$

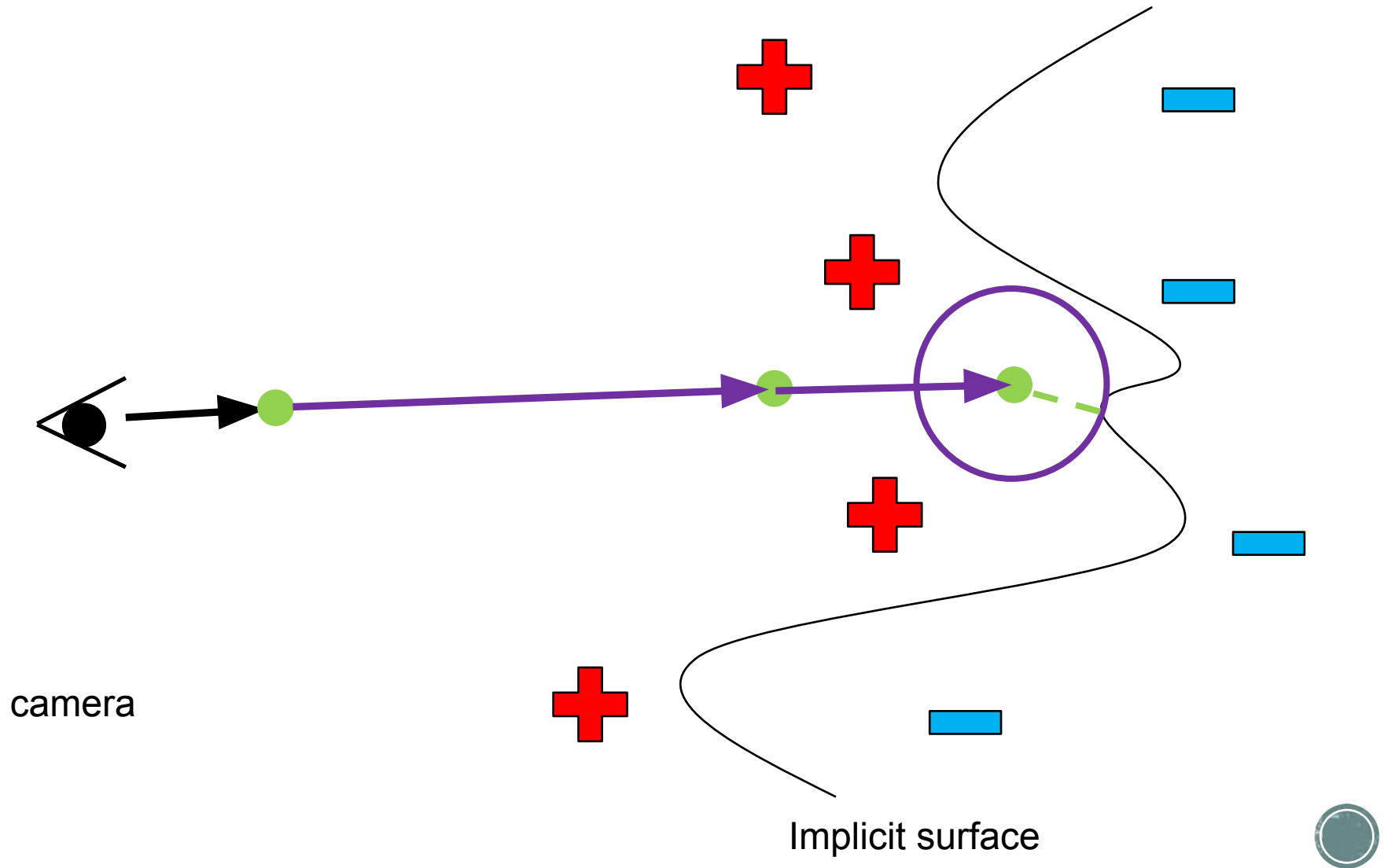
$$f(x, y, z) = 0$$



Ray marching implicit surfaces

$$P(t) = r_o + r_d * t$$

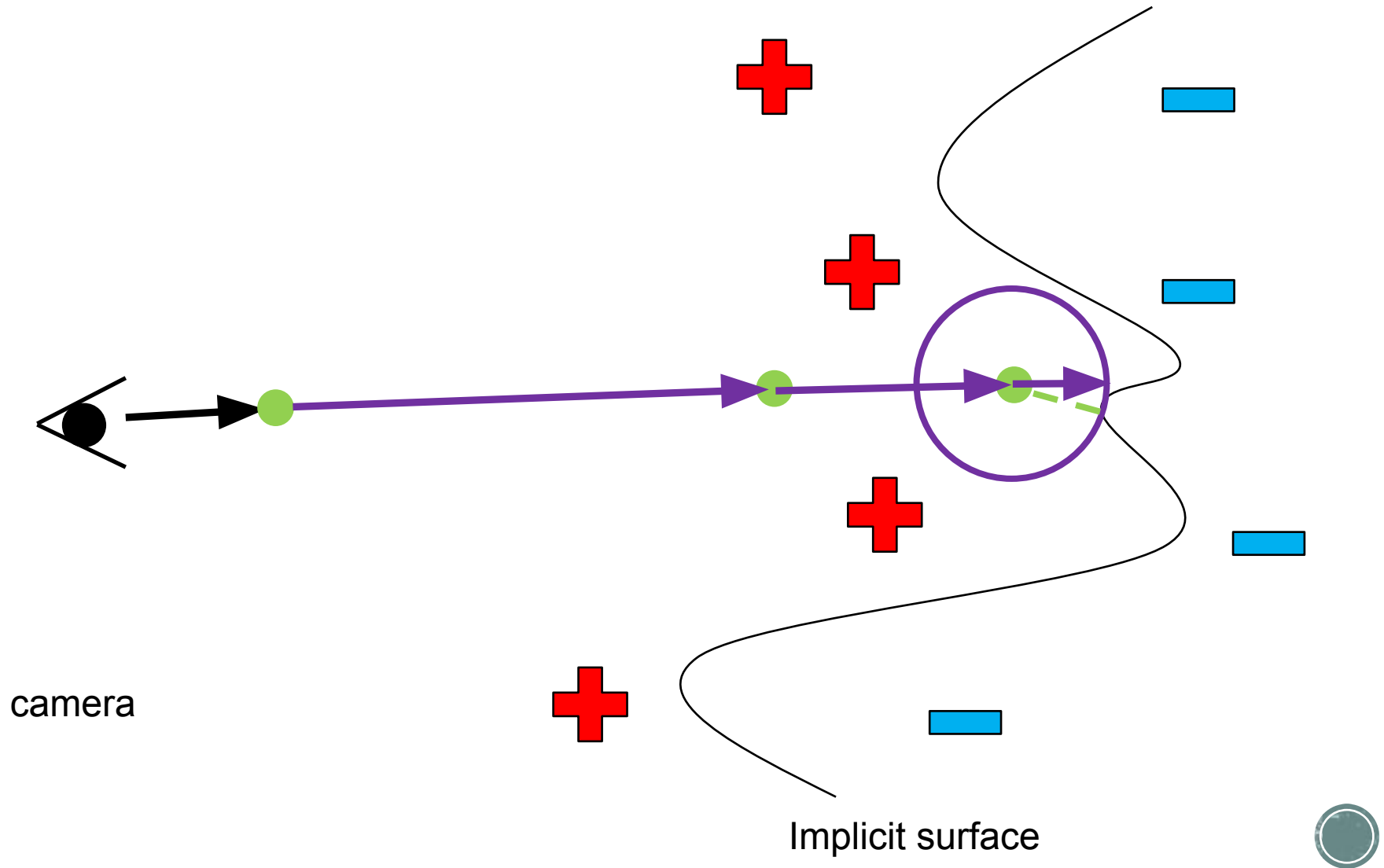
$$f(x, y, z) = 0$$



Ray marching implicit surfaces

$$P(t) = r_o + r_d * t$$

$$f(x, y, z) = 0$$



$$P(t) = r_o + r_d * t$$

$$P(t) = r_o + r_d * t$$

$$f(x, y, z) = 0$$



Implicit surface



$$P(t) = r_o + r_d * t$$

$$P(t) = r_o + r_d * t$$

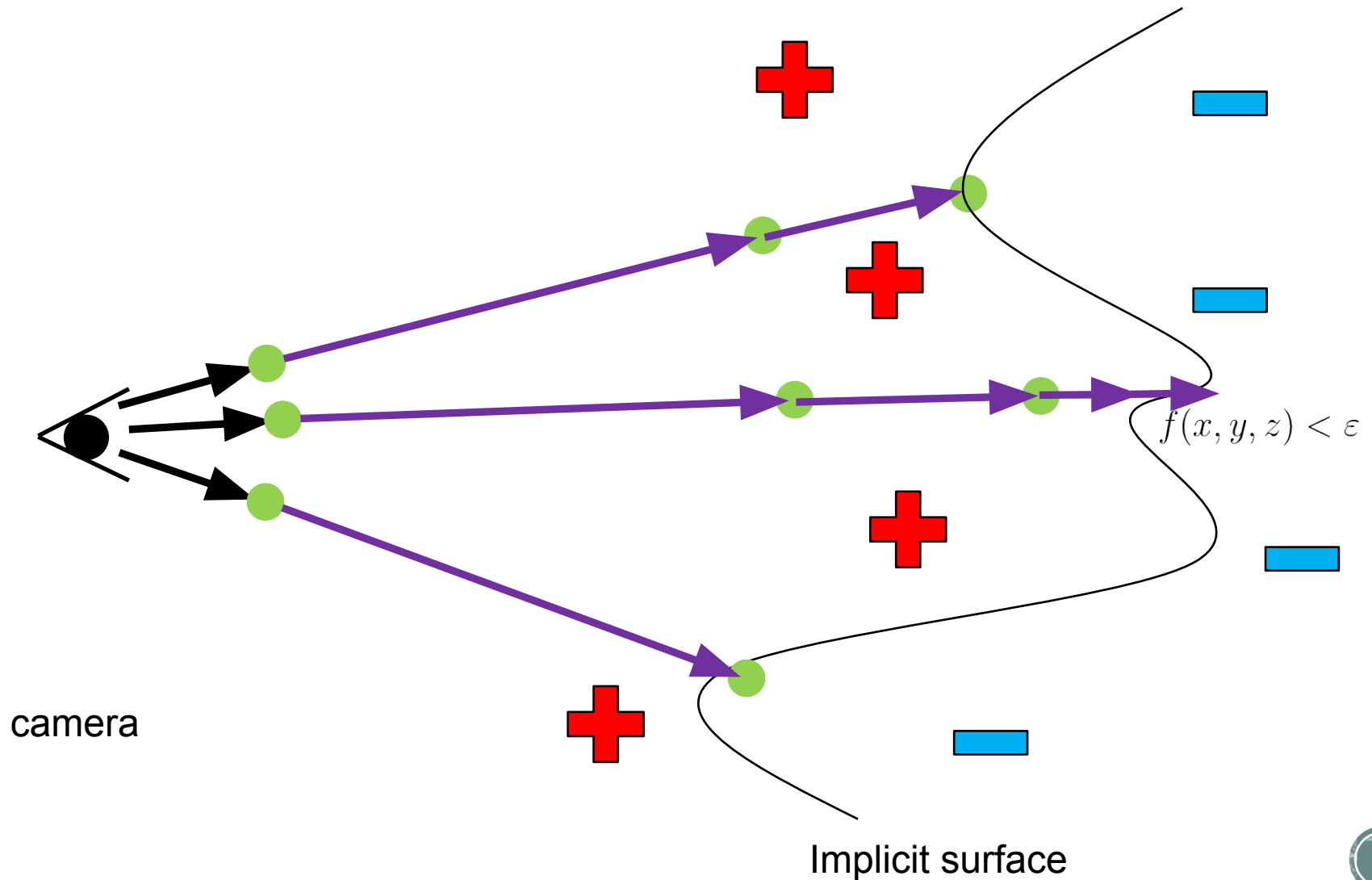
$$f(x, y, z) = 0$$



Ray marching implicit surfaces

$$P(t) = r_o + r_d * t$$

$$f(x, y, z) = 0$$

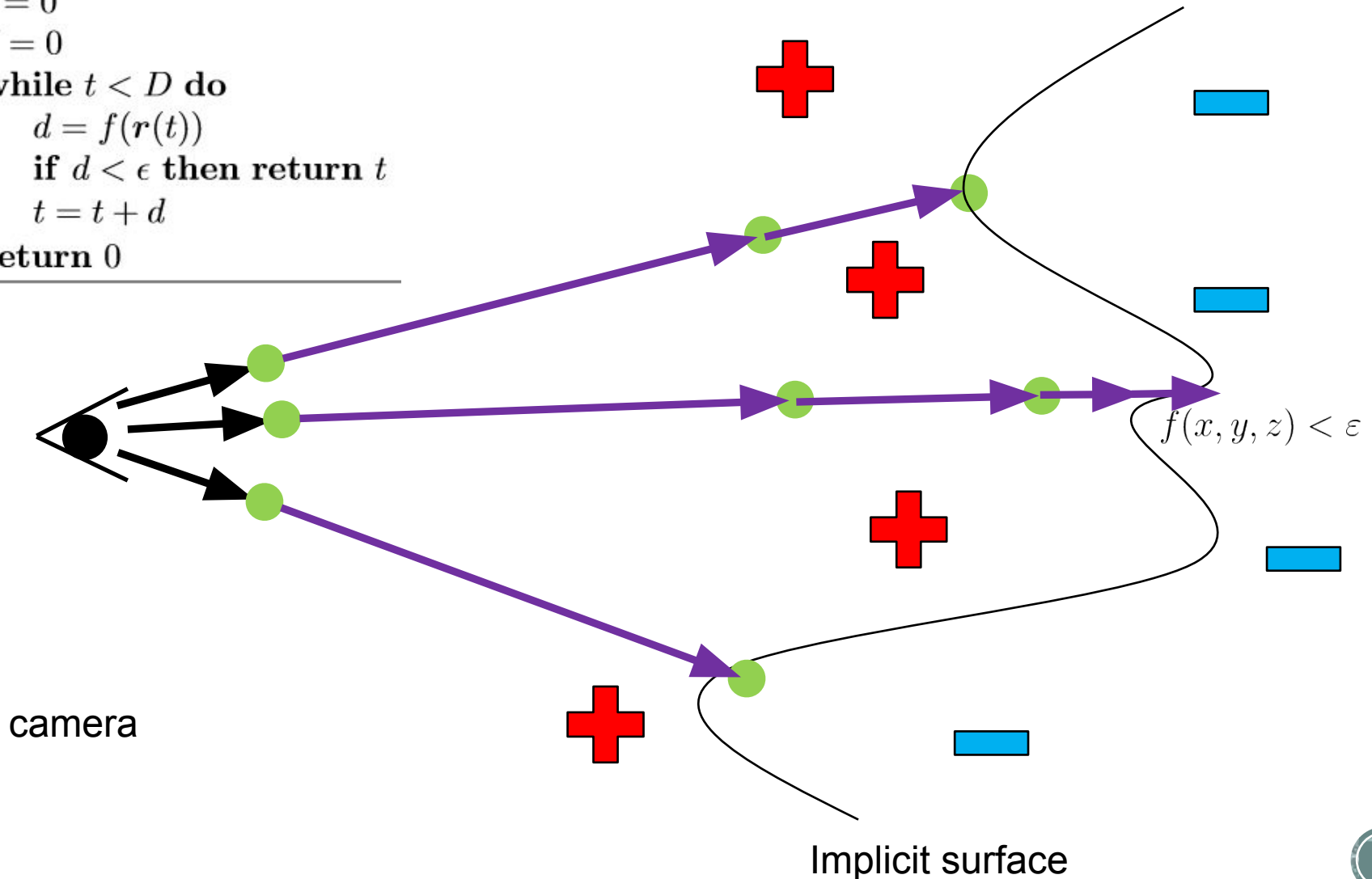


Ray marching implicit surfaces

$$P(t) = r_o + r_d * t$$

```
1:  $t = 0$   
2:  $d = 0$   
3: while  $t < D$  do  
4:    $d = f(r(t))$   
5:   if  $d < \epsilon$  then return  $t$   
6:    $t = t + d$   
7: return 0
```

$$f(x, y, z) = 0$$



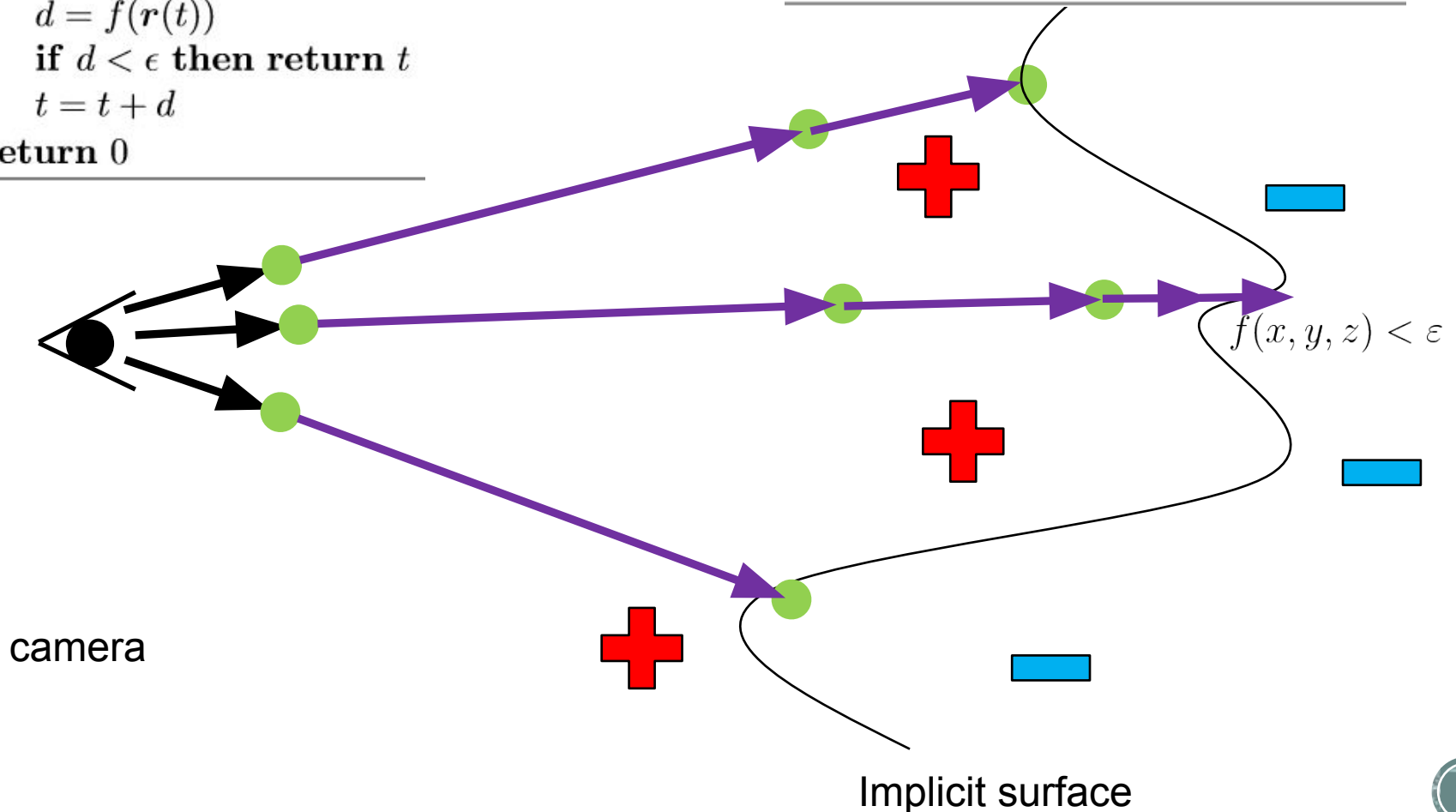
Ray marching implicit surfaces

$$P(t) = r_o + r_d * t$$

```
1:  $t = 0$   
2:  $d = 0$   
3: while  $t < D$  do  
4:    $d = f(r(t))$   
5:   if  $d < \epsilon$  then return  $t$   
6:    $t = t + d$   
7: return 0
```

Normal computation:

$$\begin{aligned} n_x &= f(x + \epsilon, y, z) - f(x - \epsilon, y, z) \\ n_y &= f(x, y + \epsilon, z) - f(x, y - \epsilon, z) \\ n_z &= f(x, y, z + \epsilon) - f(x, y, z - \epsilon) \end{aligned}$$



Common implicit surfaces

Sphere - signed

```
float sdSphere( vec3 p, float s )  
{  
    return length(p)-s;  
}
```



Common implicit surfaces

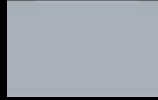
Sphere - signed

```
float sdSphere( vec3 p, float s )  
{  
    return length(p)-s;  
}
```



Plane - signed

```
float sdPlane( vec3 p, vec4 n )  
{  
    // n must be normalized  
    return dot(p,n.xyz) + n.w;  
}
```



Common implicit surfaces

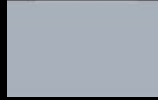
Sphere - signed

```
float sdSphere( vec3 p, float s )  
{  
    return length(p)-s;  
}
```



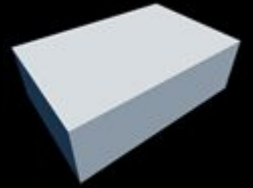
Plane - signed

```
float sdPlane( vec3 p, vec4 n )  
{  
    // n must be normalized  
    return dot(p,n.xyz) + n.w;  
}
```



Box - signed

```
float sdBox( vec3 p, vec3 b )  
{  
    vec3 d = abs(p) - b;  
    return min(max(d.x,max(d.y,d.z)),0.0) +  
        length(max(d,0.0));  
}
```



Common implicit surfaces

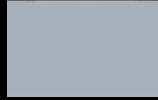
Sphere - signed

```
float sdSphere( vec3 p, float s )  
{  
    return length(p)-s;  
}
```



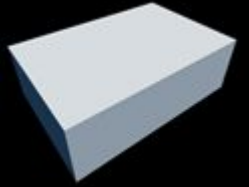
Plane - signed

```
float sdPlane( vec3 p, vec4 n )  
{  
    // n must be normalized  
    return dot(p,n.xyz) + n.w;  
}
```



Box - signed

```
float sdBox( vec3 p, vec3 b )  
{  
    vec3 d = abs(p) - b;  
    return min(max(d.x,max(d.y,d.z)),0.0) +  
           length(max(d,0.0));  
}
```



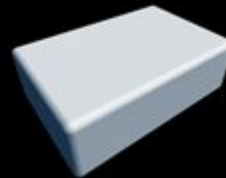
Torus - signed

```
float sdTorus( vec3 p, vec2 t )  
{  
    vec2 q = vec2(length(p.xz)-t.x,p.y);  
    return length(q)-t.y;  
}
```



Round Box - unsigned

```
float udRoundBox( vec3 p, vec3 b, float r )  
{  
    return length(max(abs(p)-b,0.0))-r;  
}
```



Cone - signed

```
float sdCone( vec3 p, vec2 c )  
{  
    // c must be normalized  
    float q = length(p.xy);  
    return dot(c,vec2(q,p.z));  
}
```



Common implicit surfaces

Sphere - signed

```
float sdSphere( vec3 p, float s )
{
    return length(p)-s;
}
```



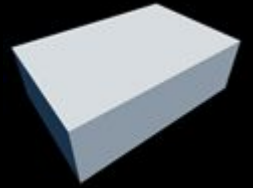
Plane - signed

```
float sdPlane( vec3 p, vec4 n )
{
    // n must be normalized
    return dot(p,n.xyz) + n.w;
}
```



Box - signed

```
float sdBox( vec3 p, vec3 b )
{
    vec3 d = abs(p) - b;
    return min(max(d.x,max(d.y,d.z)),0.0) +
        length(max(d,0.0));
}
```



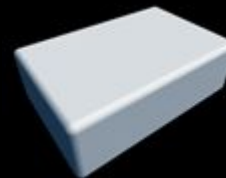
Torus - signed

```
float sdTorus( vec3 p, vec2 t )
{
    vec2 q = vec2(length(p.xz)-t.x,p.y);
    return length(q)-t.y;
}
```



Round Box - unsigned

```
float udRoundBox( vec3 p, vec3 b, float r )
{
    return length(max(abs(p)-b,0.0))-r;
}
```



Cone - signed

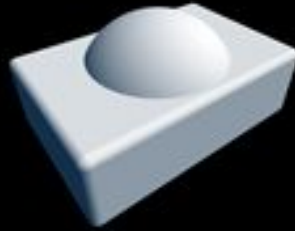
```
float sdCone( vec3 p, vec2 c )
{
    // c must be normalized
    float q = length(p.xy);
    return dot(c,vec2(q,p.z));
}
```



Complete list: <http://www.iquilezles.org/www/articles/distfunctions/distfunctions.htm>

Distance operations

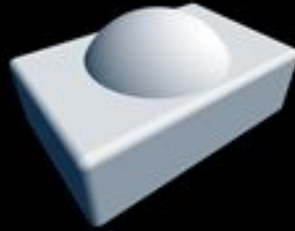
Union



```
float opU( float d1, float d2 )  
{  
    return min(d1,d2);  
}
```

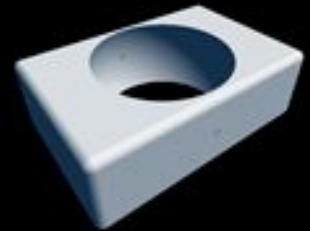
Distance operations

Union



```
float opU( float d1, float d2 )  
{  
    return min(d1,d2);  
}
```

Substraction

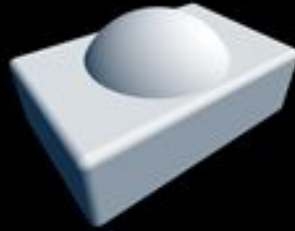


```
float opS( float d1, float d2 )  
{  
    return max(-d1,d2);  
}
```

Distance operations

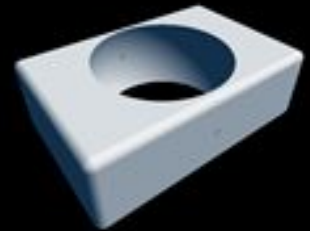
Union

```
float opU( float d1, float d2 )  
{  
    return min(d1,d2);  
}
```



Substraction

```
float opS( float d1, float d2 )  
{  
    return max(-d1,d2);  
}
```



Intersection

```
float opI( float d1, float d2 )  
{  
    return max(d1,d2);  
}
```



Complete list: <http://www.iquilezles.org/www/articles/distfunctions/distfunctions.htm>

Domain operations

Repetition

```
float opRep( vec3 p, vec3 c )  
{  
    vec3 q = mod(p,c)-0.5*c;  
    return primitive( q );  
}
```



Complete list: <http://www.iquilezles.org/www/articles/distfunctions/distfunctions.htm>

Domain operations

Repetition

```
float opRep( vec3 p, vec3 c )  
{  
    vec3 q = mod(p,c)-0.5*c;  
    return primitive( q );  
}
```



Scale

```
float opScale( vec3 p, float s )  
{  
    return primitive(p/s)*s;  
}
```



Domain operations

Repetition

```
float opRep( vec3 p, vec3 c )
{
    vec3 q = mod(p,c)-0.5*c;
    return primitive( q );
}
```



Scale

```
float opScale( vec3 p, float s )
{
    return primitive(p/s)*s;
}
```



Rotation/Translation

```
vec3 opTx( vec3 p, mat4 m )
{
    vec3 q = invert(m)*p;
    return primitive(q);
}
```

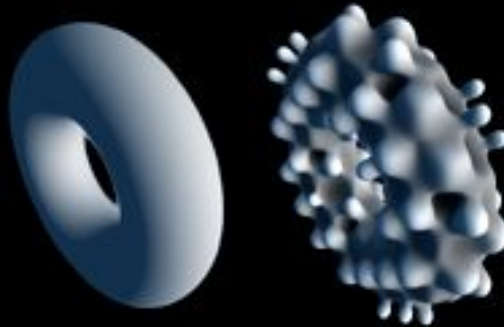


Complete list: <http://www.iquilezles.org/www/articles/distfunctions/distfunctions.htm>

Distance deformations

Displacement

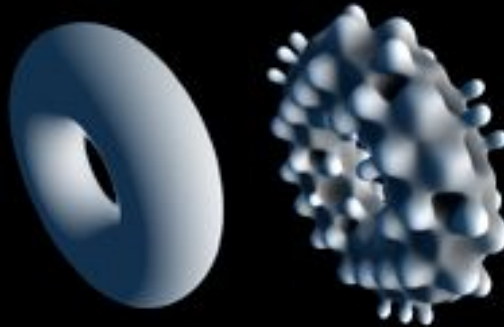
```
float opDisplace( vec3 p )  
{  
    float d1 = primitive(p);  
    float d2 = displacement(p);  
    return d1+d2;  
}
```



Distance deformations

Displacement

```
float opDisplace( vec3 p )
{
    float d1 = primitive(p);
    float d2 = displacement(p);
    return d1+d2;
}
```



Blend

```
float opBlend( vec3 p )
{
    float d1 = primitiveA(p);
    float d2 = primitiveB(p);
    return smin( d1, d2 );
}
```



```
// polynomial smooth min (k = 0.1);
float smin( float a, float b, float k )
{
    float h = clamp( 0.5+0.5*(b-a)/k, 0.0, 1.0 );
    return mix( b, a, h ) - k*h*(1.0-h);
}
```

for instance...

Domain deformations

Twist

```
float opTwist( vec3 p )
{
    float c = cos(20.0*p.y);
    float s = sin(20.0*p.y);
    mat2 m = mat2(c,-s,s,c);
    vec3 q = vec3(m*p.xz,p.y);
    return primitive(q);
}
```



Domain deformations

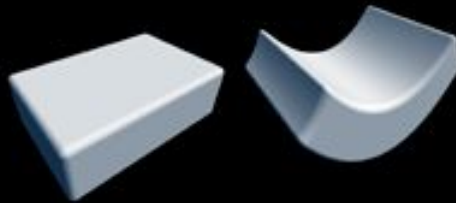
Twist

```
float opTwist( vec3 p )
{
    float c = cos(20.0*p.y);
    float s = sin(20.0*p.y);
    mat2 m = mat2(c,-s,s,c);
    vec3 q = vec3(m*p.xz,p.y);
    return primitive(q);
}
```



Cheap Bend

```
float opCheapBend( vec3 p )
{
    float c = cos(20.0*p.y);
    float s = sin(20.0*p.y);
    mat2 m = mat2(c,-s,s,c);
    vec3 q = vec3(m*p.xy,p.z);
    return primitive(q);
}
```



Complete list: <http://www.iquilezles.org/www/articles/distfunctions/distfunctions.htm>

Domain deformations

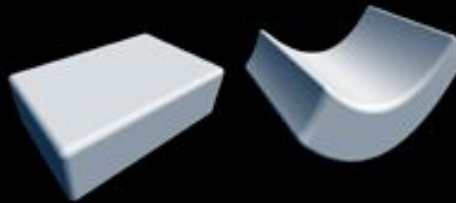
Twist

```
float opTwist( vec3 p )
{
    float c = cos(20.0*p.y);
    float s = sin(20.0*p.y);
    mat2 m = mat2(c,-s,s,c);
    vec3 q = vec3(m*p.xz,p.y);
    return primitive(q);
}
```



Cheap Bend

```
float opCheapBend( vec3 p )
{
    float c = cos(20.0*p.y);
    float s = sin(20.0*p.y);
    mat2 m = mat2(c,-s,s,c);
    vec3 q = vec3(m*p.xy,p.z);
    return primitive(q);
}
```



Demos:

<https://www.shadertoy.com/view/Xds3zN>

<https://www.shadertoy.com/view/Mss3zM>

Complete list: <http://www.iquilezles.org/www/articles/distfunctions/distfunctions.htm>

References

- MIT:
 - <http://ocw.mit.edu/courses/electrical-engineering-and-computer-science/6-837-computer-graphics-fall-2012/lecture-notes/>
- Stanford:
 - <http://candela.stanford.edu/cs348b-14/doku.php>
- Siggraph:
 - <http://blog.selfshadow.com/publications/s2014-shading-course/>
 - <http://blog.selfshadow.com/publications/s2013-shading-course/>
- Image synthesis & OpenGL:
 - http://romain.vergne.free.fr/blog/?page_id=97
- Path tracing and global illum:
 - <http://www.graphics.stanford.edu/courses/cs348b-01/course29.hanrahan.pdf>
 - http://web.cs.wpi.edu/~emmanuel/courses/cs563/write_ups/zackw/realistic_raytracing.html
- GLSL / Shadertoy:
 - <https://www.opengl.org/documentation/glsl/>
 - <https://www.shadertoy.com/>
 - <http://www.iquilezles.org/>
- <http://fileadmin.cs.lth.se/cs/Education/EDAN30/lectures/L2-rt.pdf>
- <http://csokavar.hu/raytrace/imm6392.pdf>

