

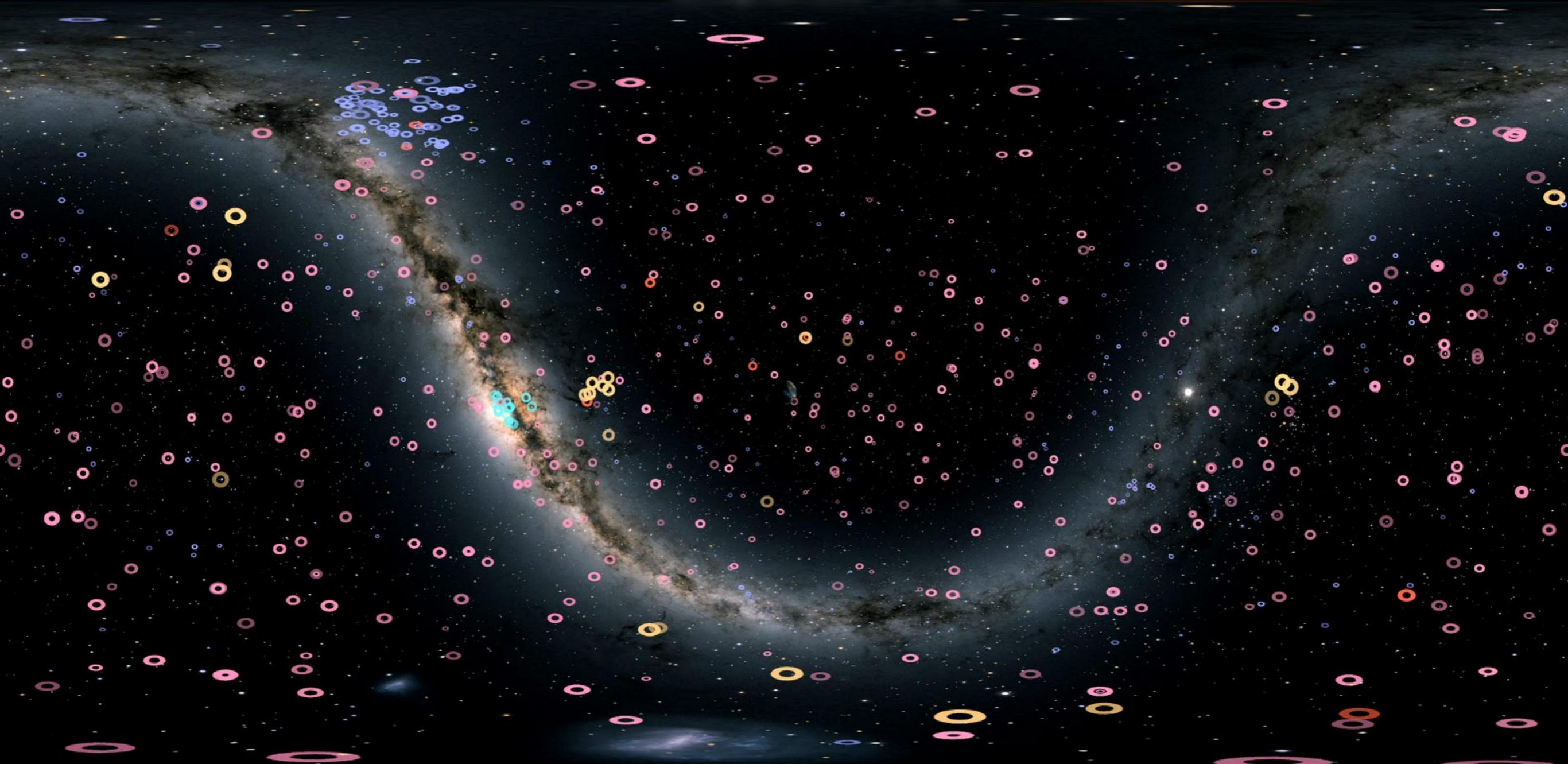
# **Exoplanet Research**

**Empirical Predictions for the Period Distribution of Planets  
to be Discovered by TESS**

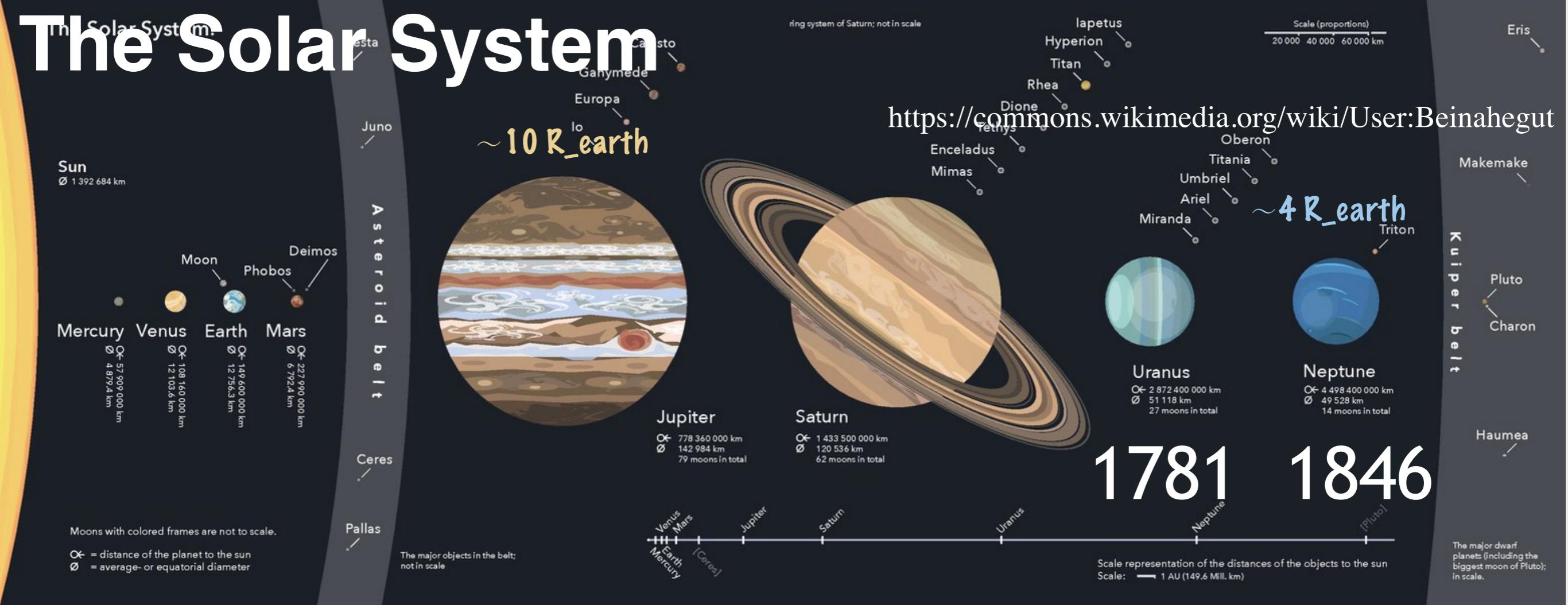
Radial Velocity  
Transit  
Imaging  
Microlensing

Year: 2013  
Exoplanets: 776

Timing Variations  
Brightness Modulation  
Astrometry



# The Solar System

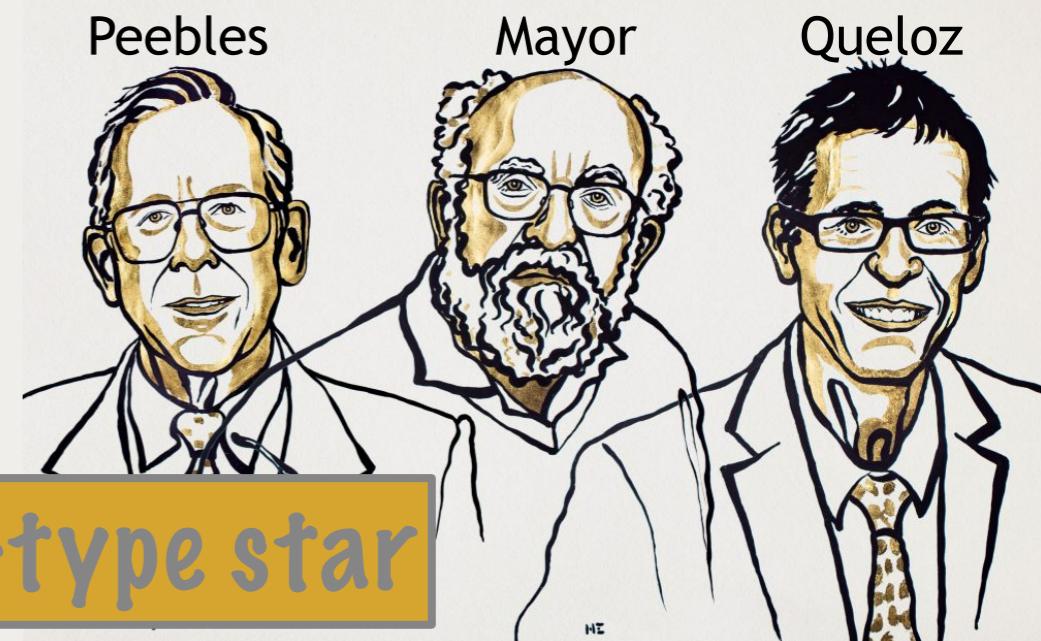


## The Nobel Prize in Physics 2019

## New perspectives on our place in the universe

October 1995,  
Michel Mayor and Didier Queloz  
Haute-Provence Observatory in France

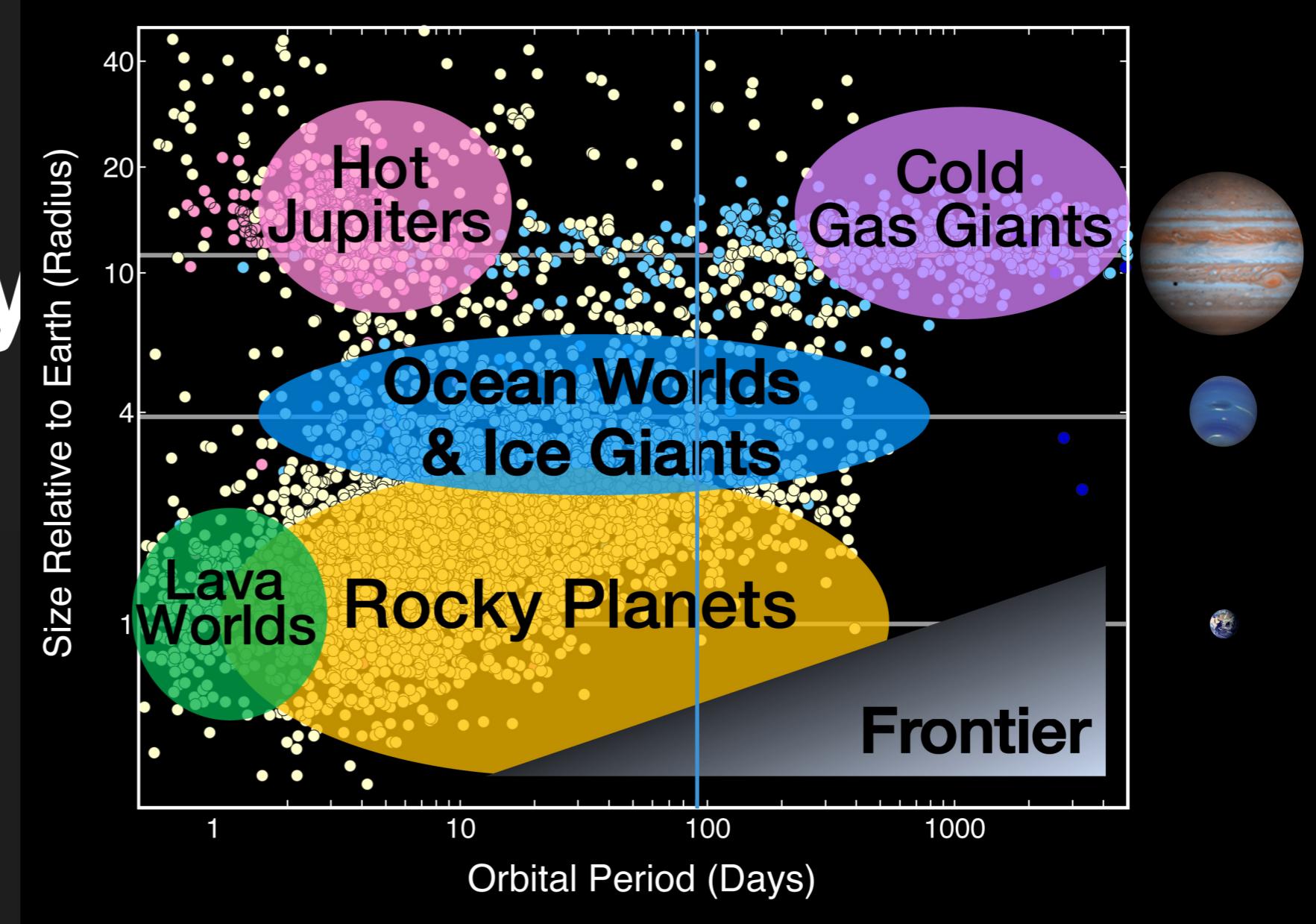
51 Pegasi b: Exoplanet Orbiting Solar-type star



# Exoplanets Demography



4104  
CONFIRMED  
EXOPLANETS

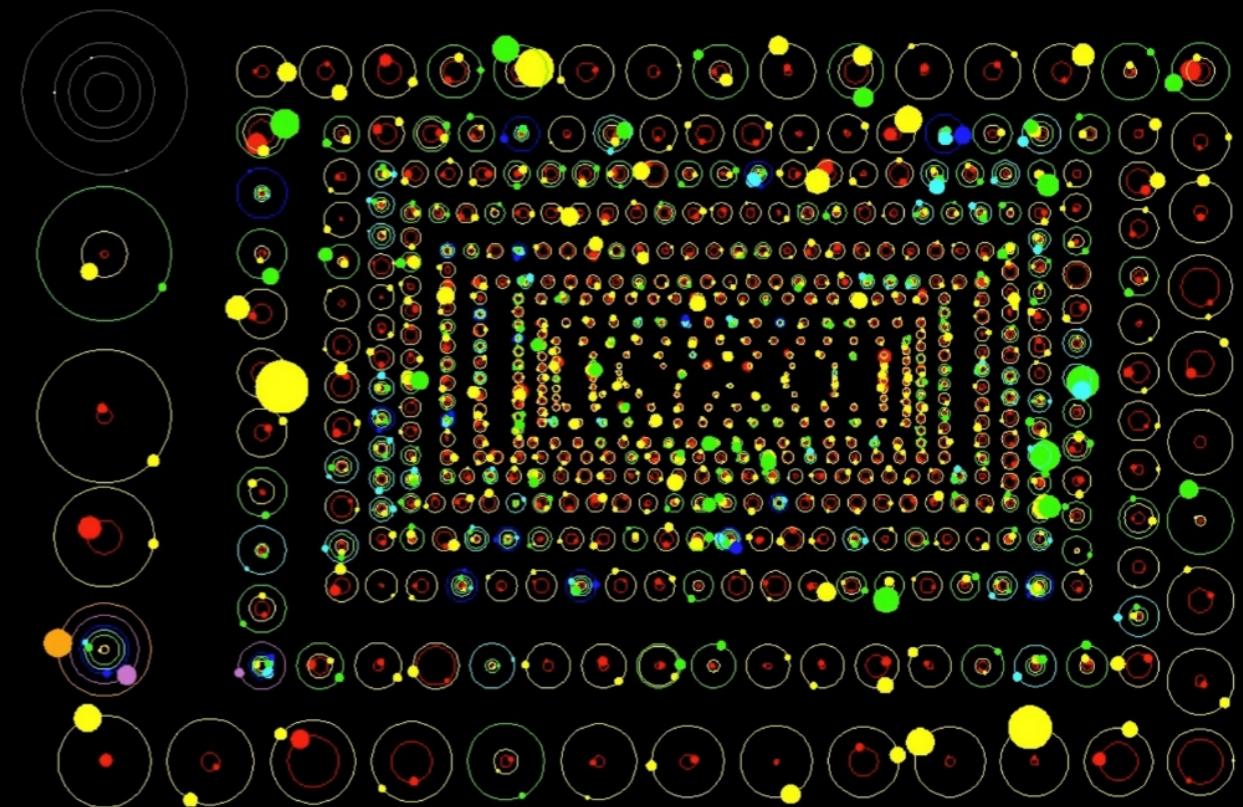
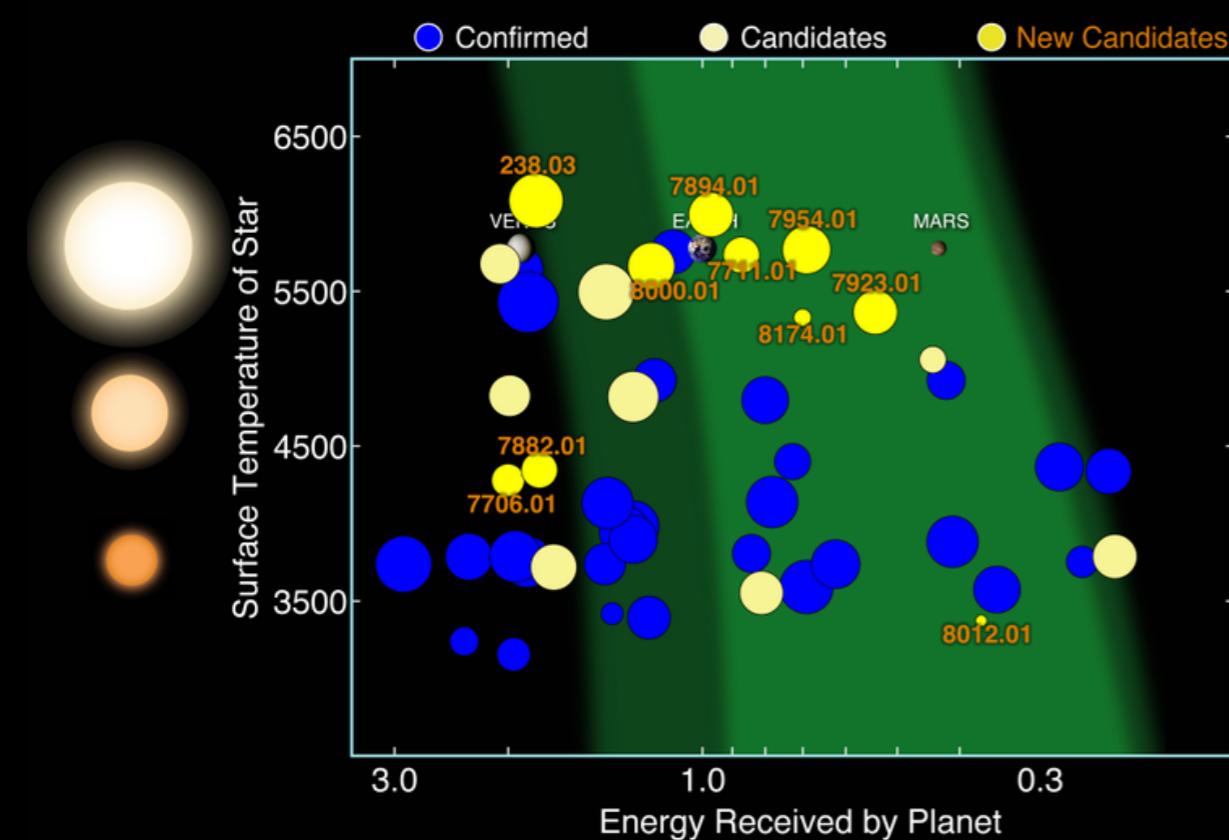


NASA Ames Research Center/Natalie  
Batalha/Wendy Stenzel

Neptune-like  
Gas Giant  
Super Earth  
Terrestrial  
Unknown

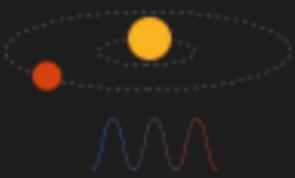
# What can we learn from exoplanets?

- 1) Life outside the Solar System
- 2) planetary formation and evolution





76.4% Transit



19.1% Radial Velocity



2.1% Microlensing



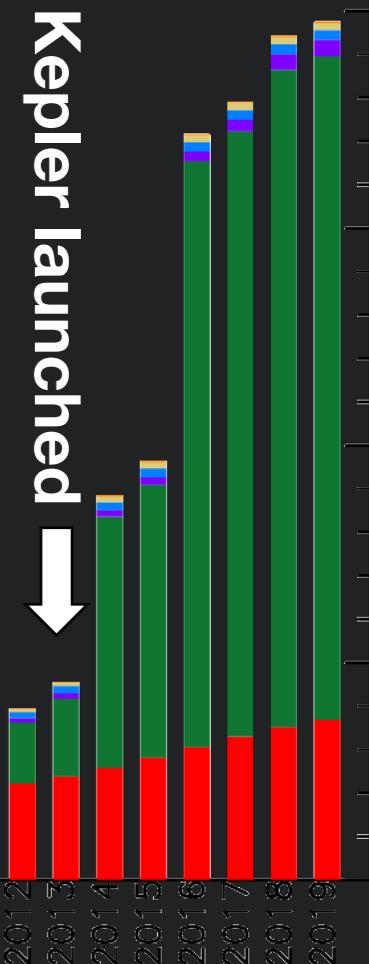
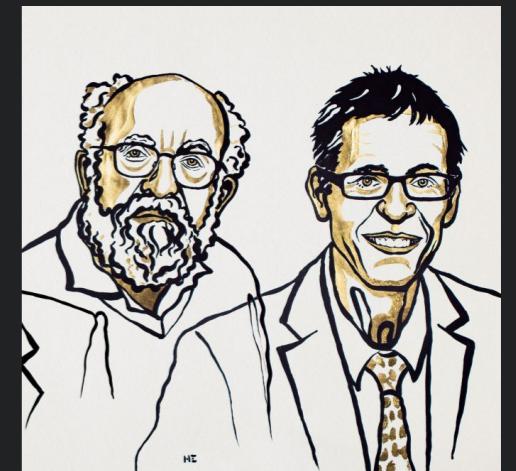
1.1% Imaging

0.51% Transit Timing Variations,  
0.39% Eclipse Timing Variations,  
0.17% Pulsar Timing,  
0.15% Orbital Brightness  
Modulation,  
0.05% Pulsation Timing Variations,  
0.02% Disk Kinematics,  
0.02% Astrometry

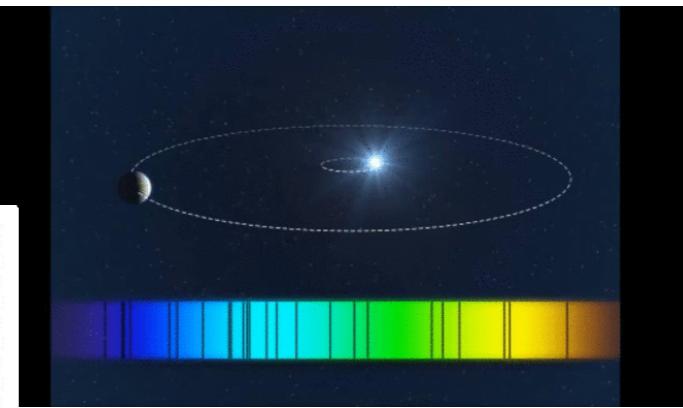
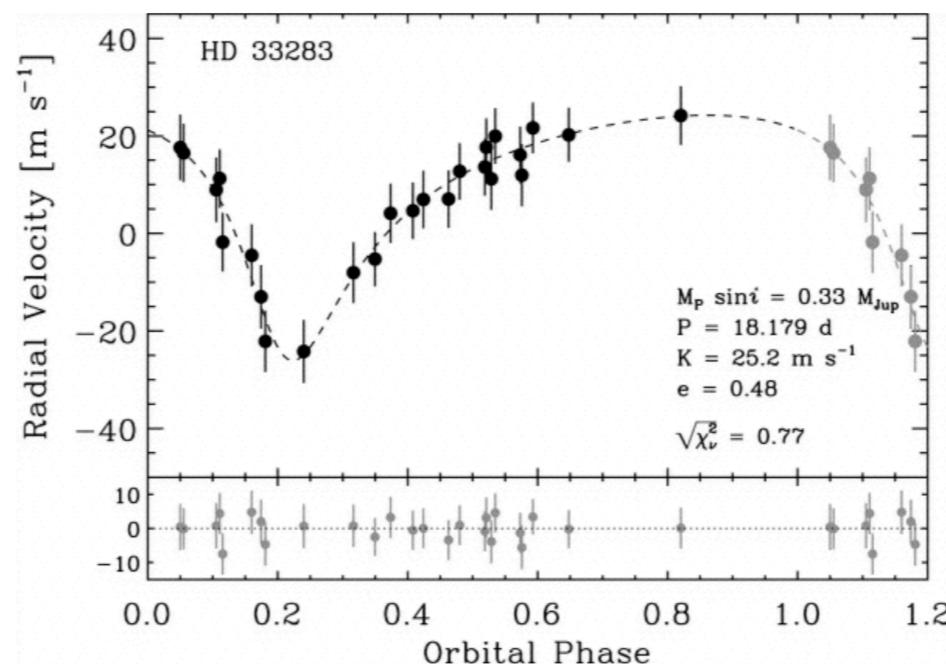
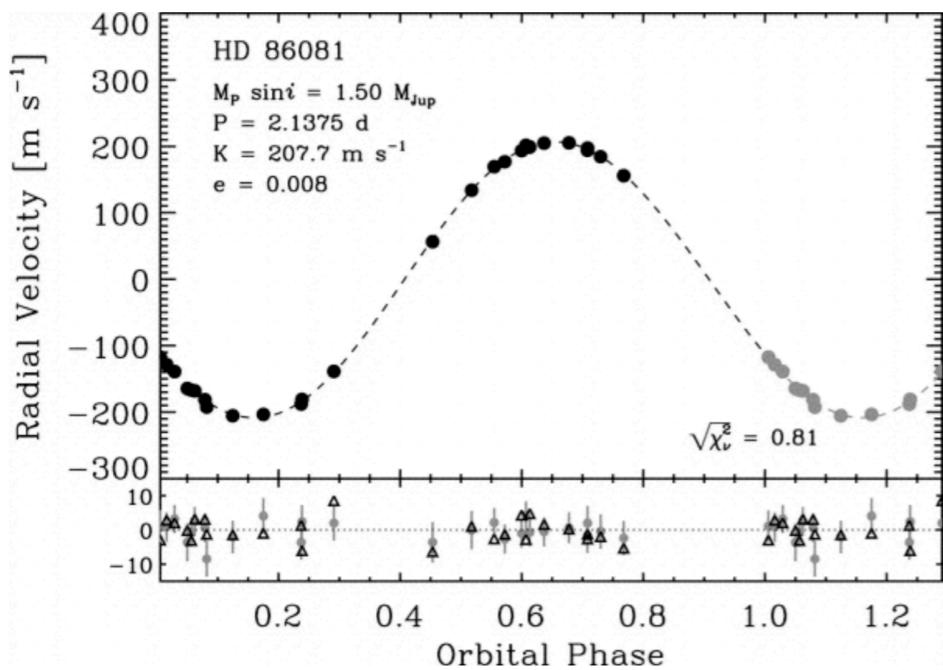
Radial Velocity  
Transits  
Microlensing  
Imaging  
Timing Variations  
Orbital Brightness  
Modulation  
Astrometry



In 1995, Mayor and Queloz discovered the 1<sup>st</sup> exoplanet orbiting a sun-like star.  
Haute-Provence Observatory, France



# Radial Velocity



$$\Delta V_{\max} = \left( \frac{2\pi G}{P} \right)^{1/3} \frac{M_P \sin I}{(M_P + M_S)^{2/3}}$$

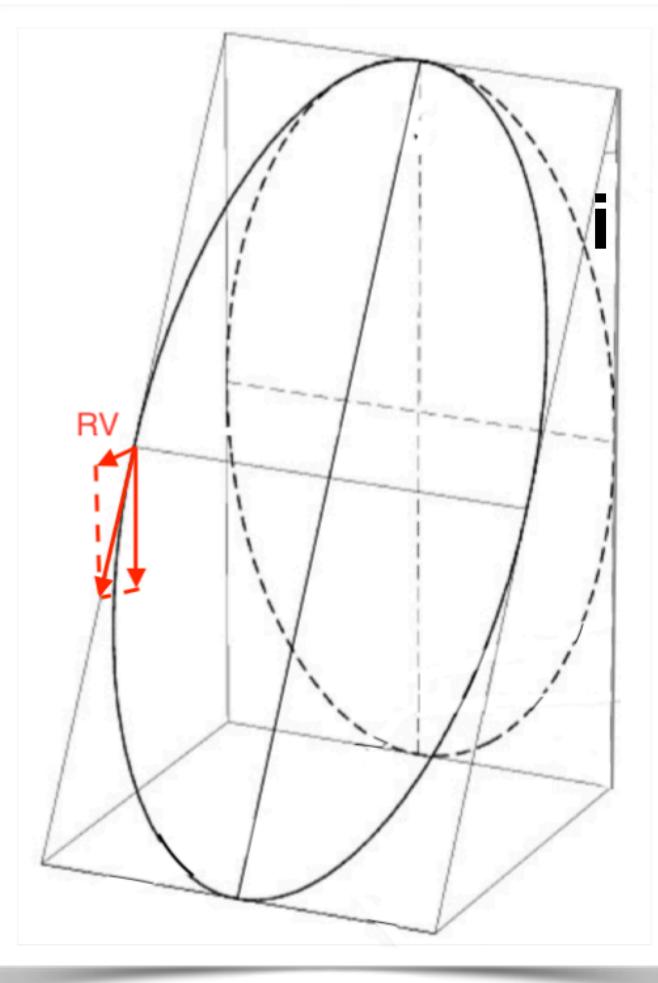
$$\approx (12 \text{ m s}^{-1}) \left( \frac{P}{12 \text{ yr}} \right)^{-1/3} \left( \frac{M_P \sin I}{M_{Jup}} \right) \left( \frac{M_S}{M_{Sun}} \right)^{-2/3}$$

\* 限制

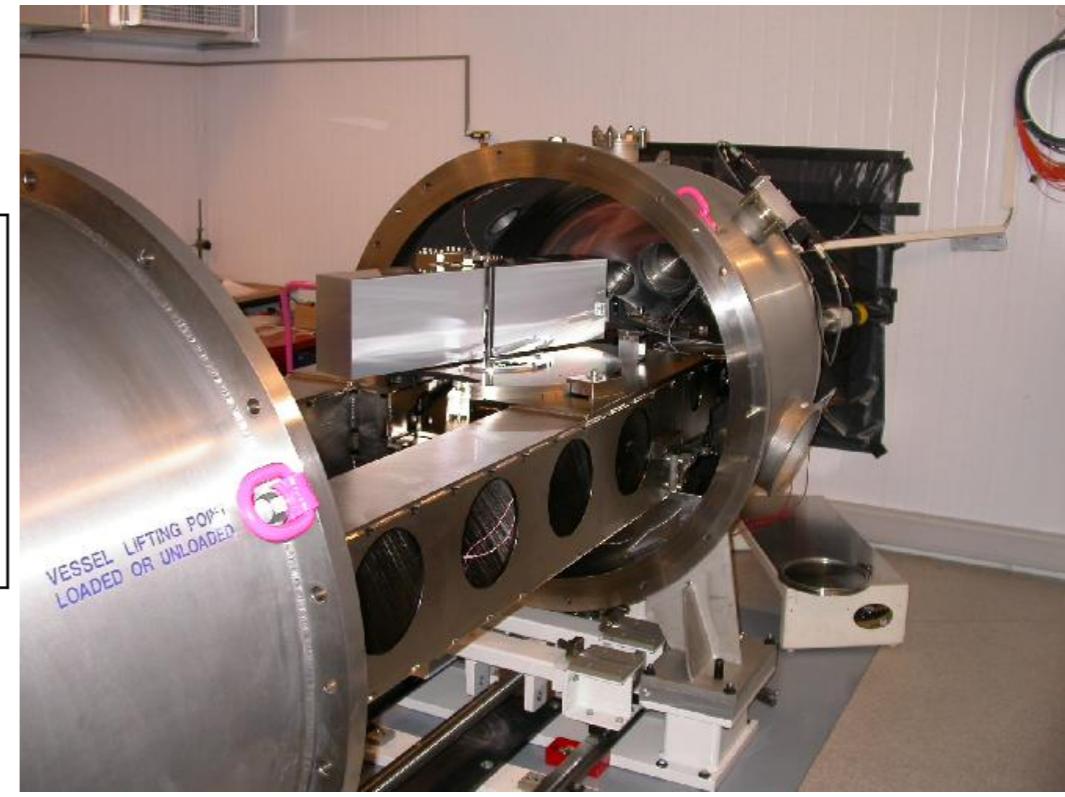
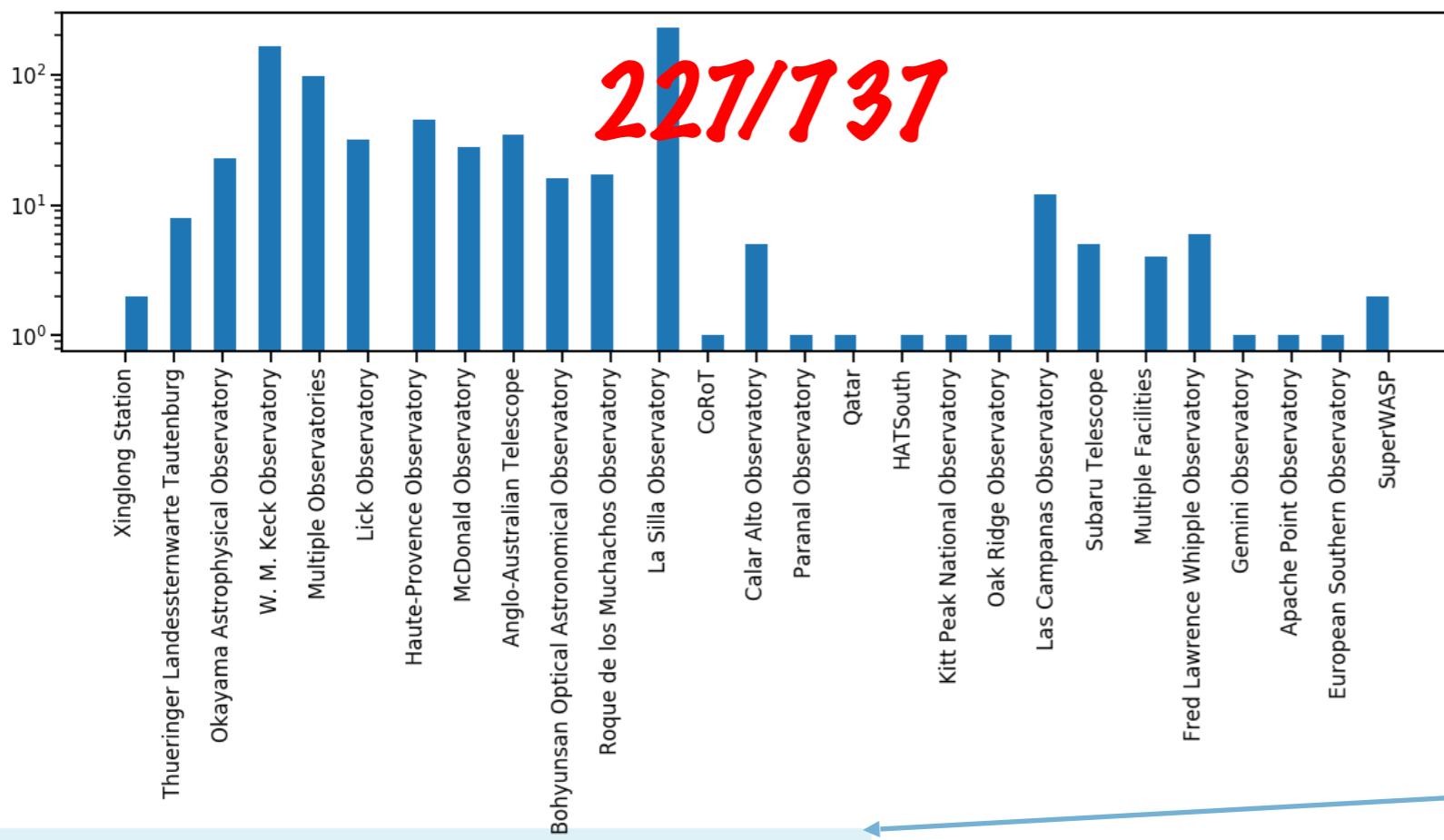
\*  $\sin i$  ambiguity

\* 半径未知

\* 假信号, 恒星脉动



# Radial Velocity



\* 精度

\* 1 m/s HARPS, 3.6m, ESO, 智利La Silla天文台

\* 13 m/s 太阳 – 木星

\* 0.09 m/s 太阳 – 地球

\* 稳定性

\* 真空 压力 < 0.01 mb

\* 温度 保持 17°C, 0.01°C

\* 空间劣势

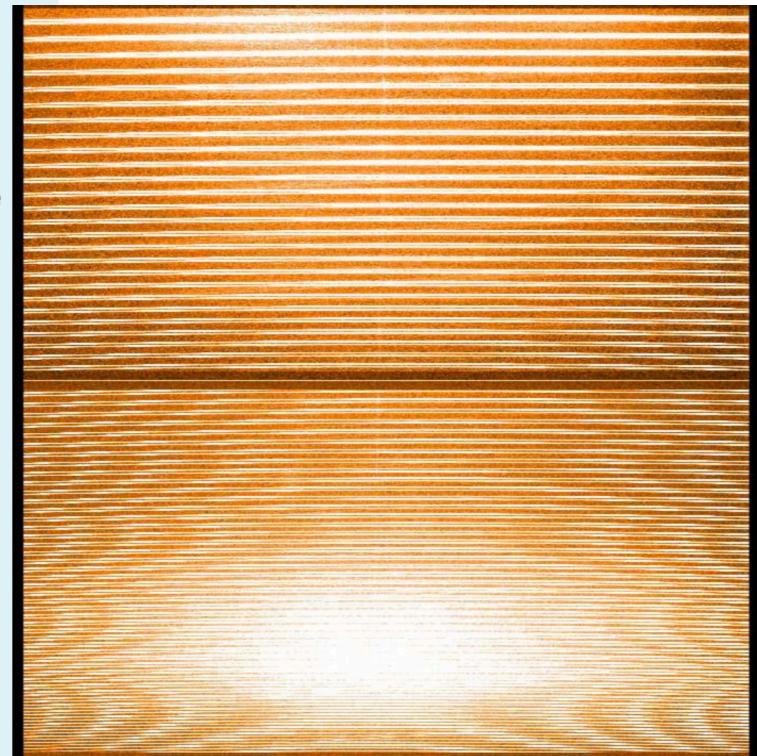
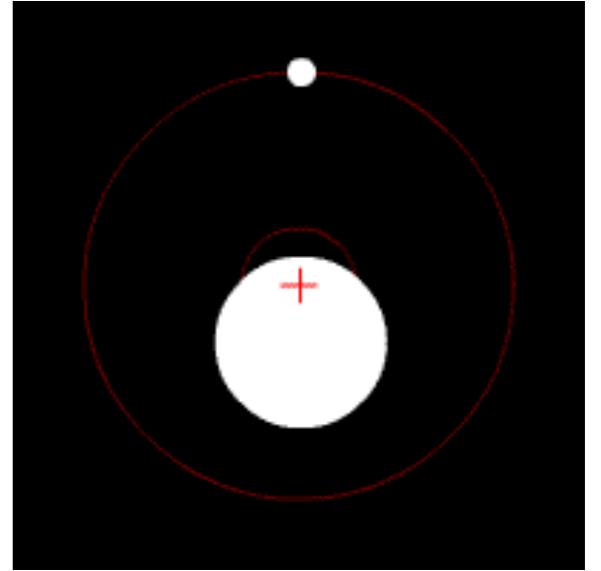


Table 1: HARPS spectrograph characteristics

|                          |                                                 |
|--------------------------|-------------------------------------------------|
| Optical design           | fibre-fed, cross-dispersed echelle spectrograph |
| Technique                | simultaneous ThAr Reference                     |
| Number of fibres         | 2                                               |
| Fibre aperture on sky    | 1 arcsec                                        |
| Collimated beam diameter | 208 mm                                          |
| Covered spectral range   | 380 nm to 690 nm                                |
| Spectral resolution      | R=115,000                                       |
| Spectral format          | 72 echelle orders<br>61.44 x 62.74 mm           |
| CCD chip                 | mosaic, 2xEEV2k4<br>pixel size=15μm             |
| Sampling                 | 3.2 pixels/SE                                   |
| Min. inter-order         | 33 pixels                                       |

# Astrometry ——古老的方法



- \* 18世纪~2009年：0颗
- \* 2010年10月：HD\_176051 b Jovian planet
- \* ——Palomar Testbed Interferometer
- \* 精度要求：~0.05 mas precision

$$\begin{aligned}\Delta\theta_{\max} &= \left(\frac{M_P/M_S}{d}\right) \left(\frac{G(M_P + M_S)P^2}{4\pi^2}\right)^{1/3} \\ &\approx 0.5 \text{ mas} \left(\frac{P}{12 \text{ yr}}\right)^{2/3} \left(\frac{M_P}{M_{\text{Jup}}}\right) \left(\frac{M_S}{M_{\text{Sun}}}\right)^{-2/3} \left(\frac{d}{10 \text{ pc}}\right)^{-1}\end{aligned}$$

GAIA?  
干涉?

- \* 优势：对长轨道行星敏感、精确的轨道参数、质量可知
- \* 劣势：大气未知、行星半径未知

# Imaging

\* 难点1：恒星和行星光度的极端对比

\* 解决方法

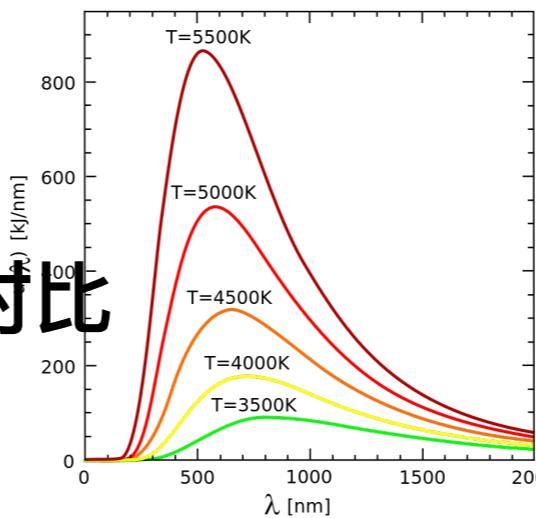
\* 可见光、近红外

$$L_{\text{sun}}/L_{\text{Jupiter}} = 10^8$$

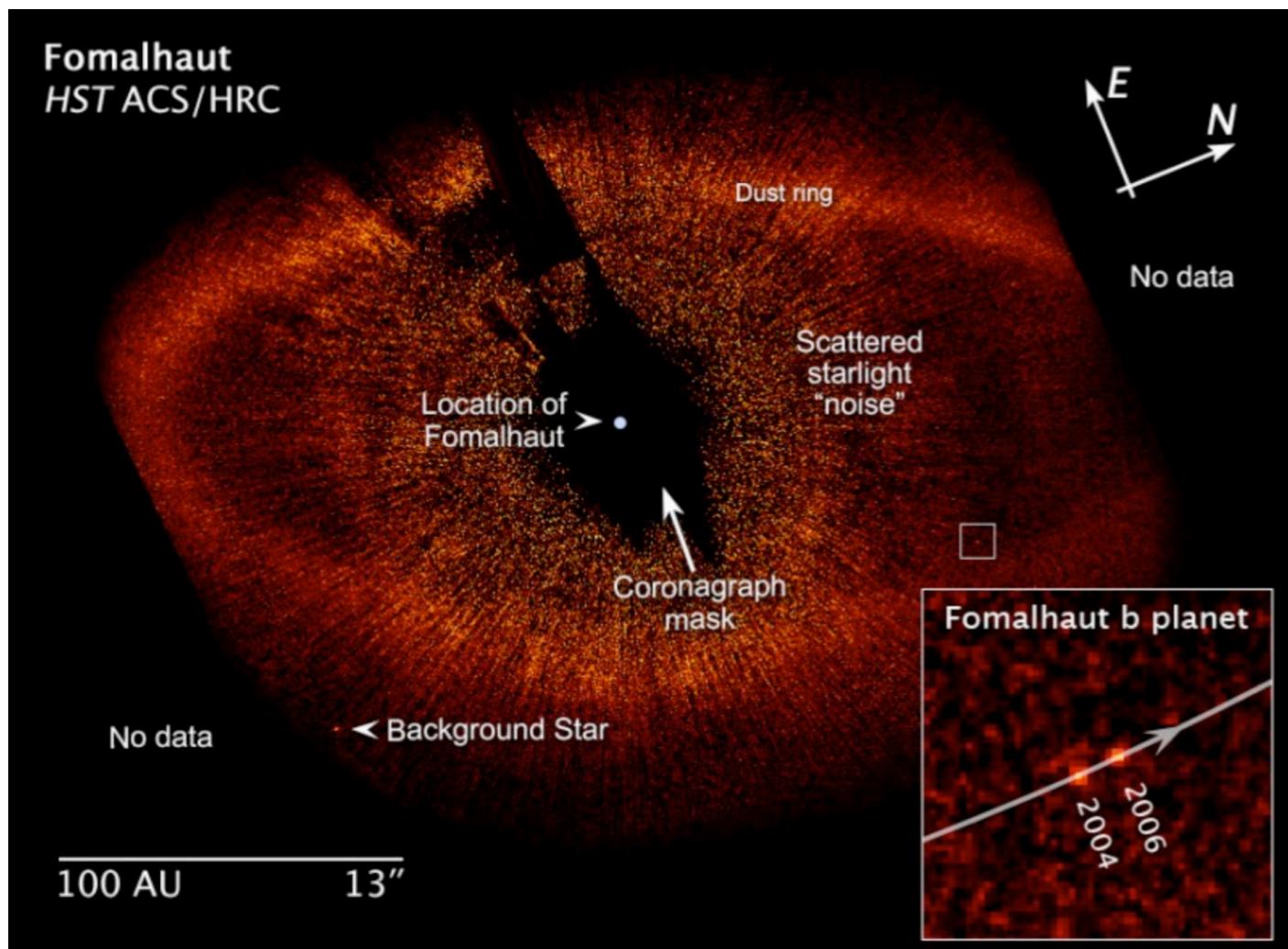
\* 中红外，空间！

$$L_{\text{sun}}/L_{\text{Jupiter}} = 10^4$$

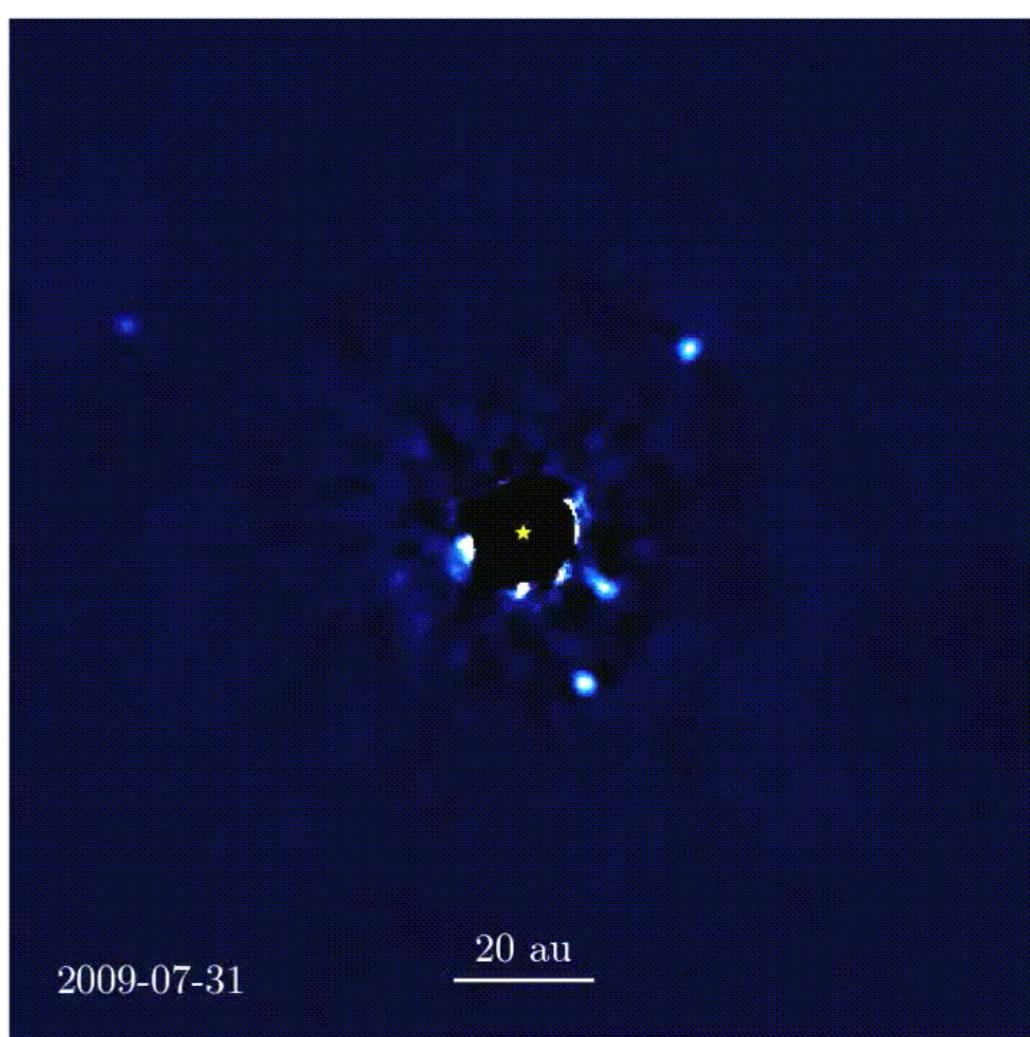
\* 遮挡恒星，日冕仪



2M1207b , 2004年第一个成像法发现的系外行星



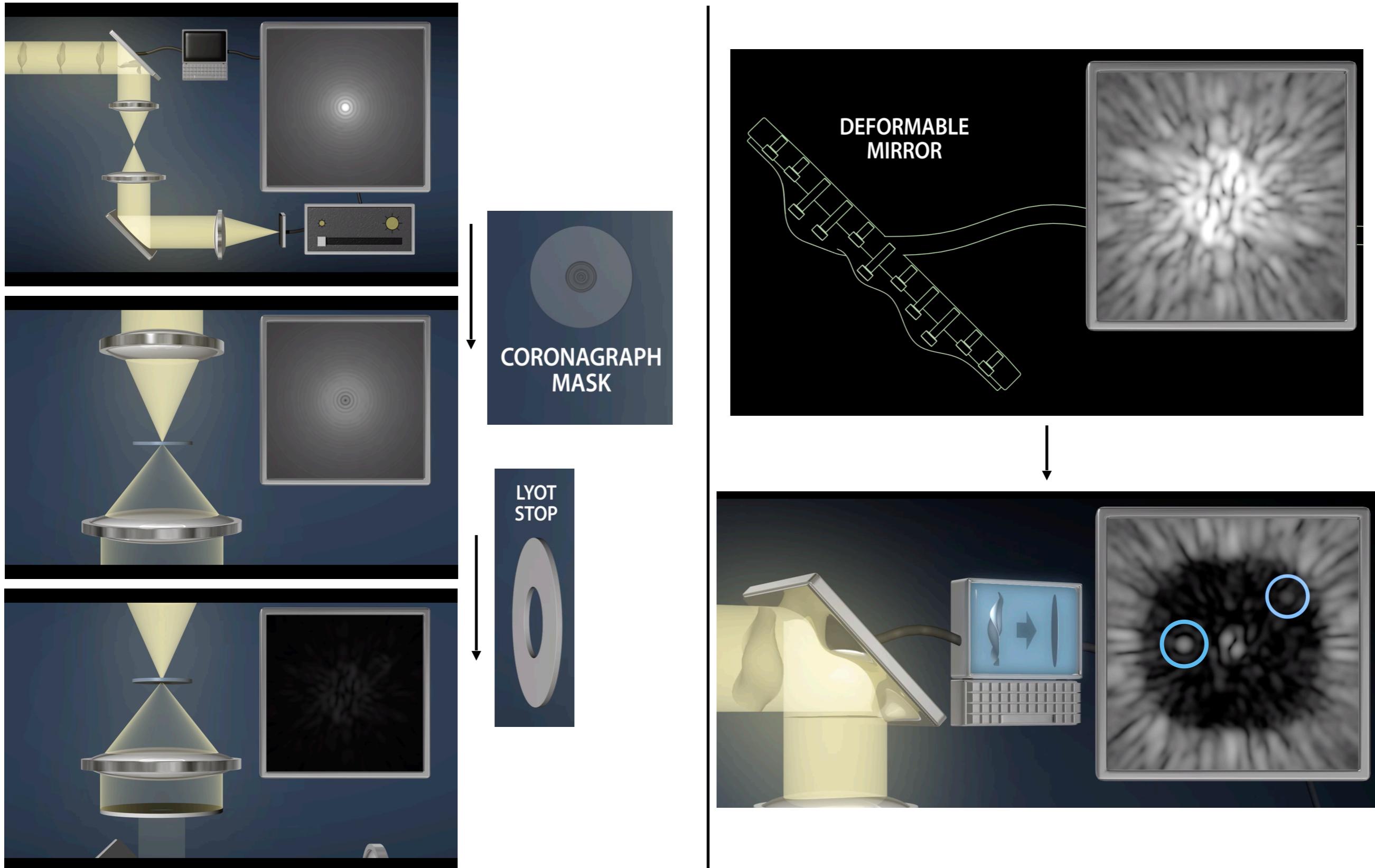
Fomalhautb, HST



HR8799, Keck 近红外

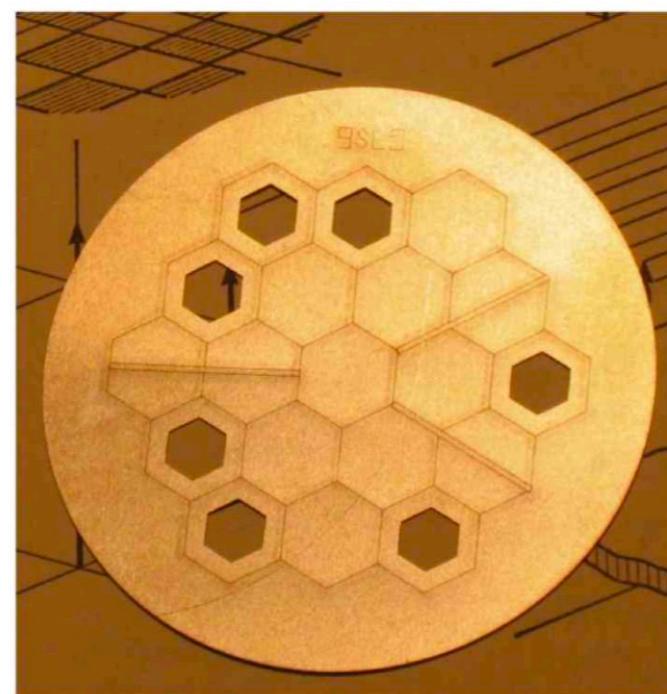
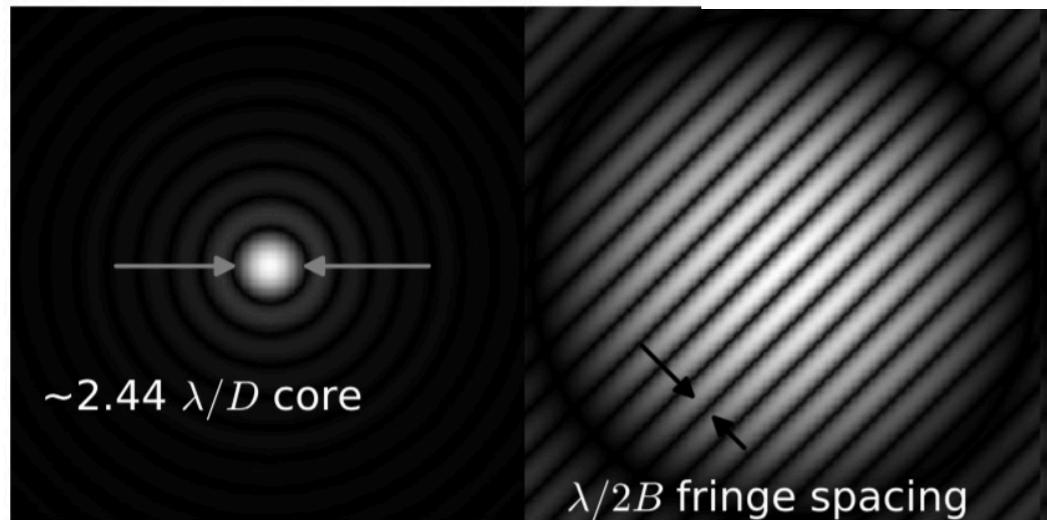
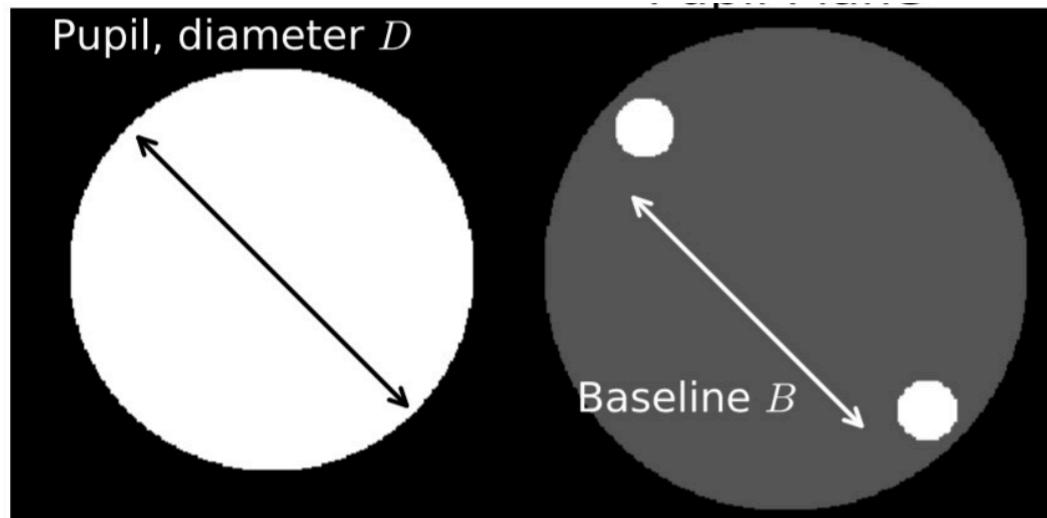
# Imaging

## \* 下一代空间望远镜 WFIRST 搭载的日冕仪

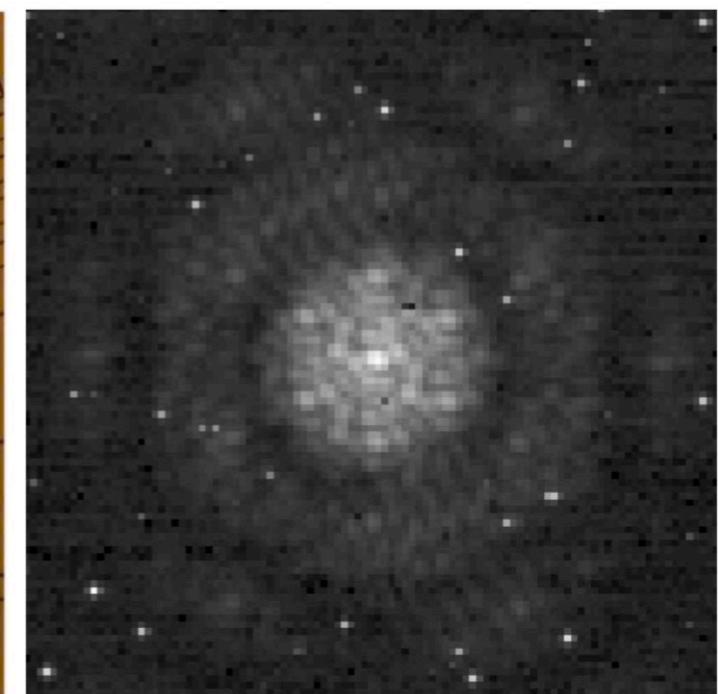


# Imaging

- \* 难点2：分辨率
  - \* 衍射极限 – 空间X, 地面 + 自适应光学
  - \* 干涉 – Aperture Masking Interferometry



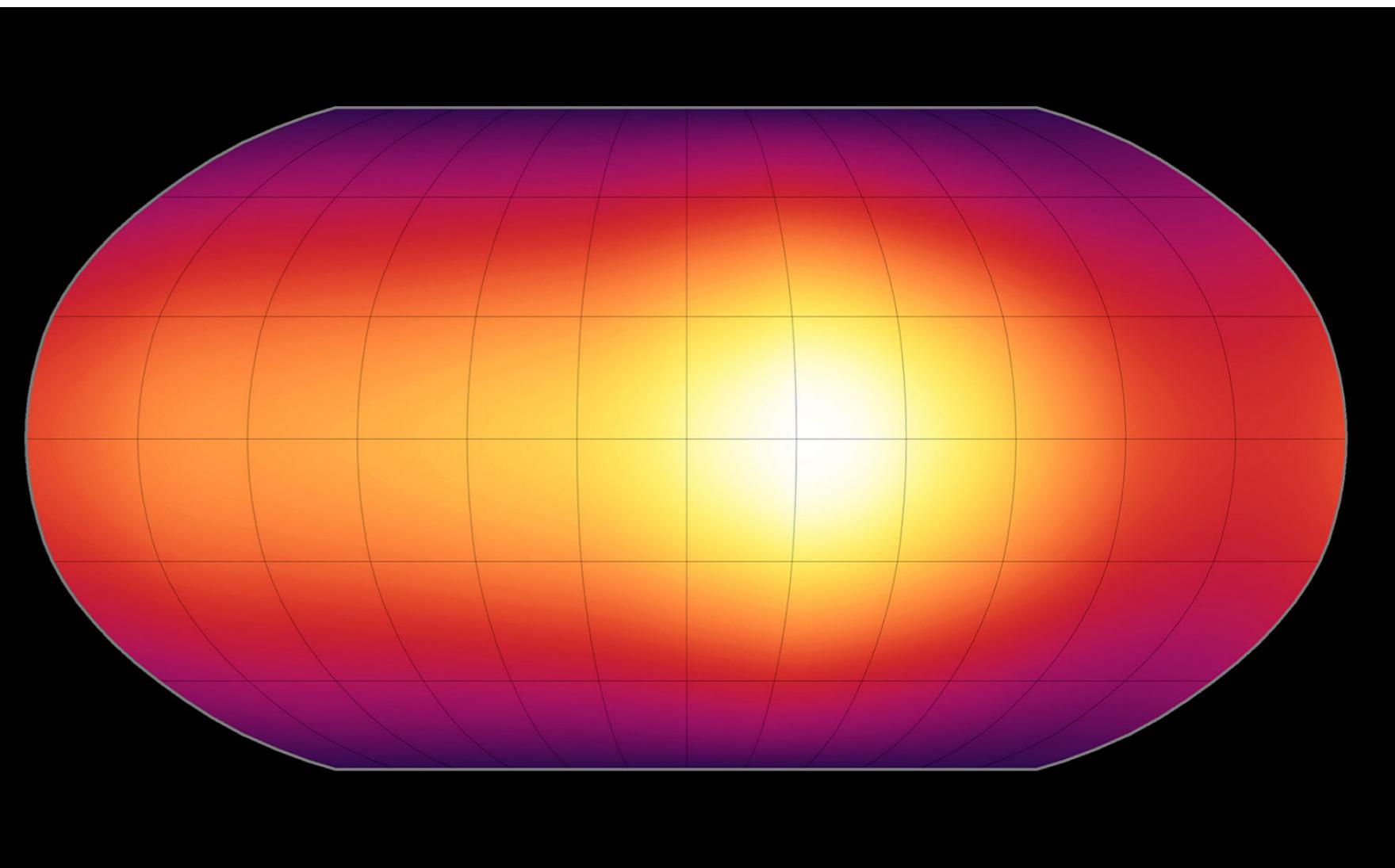
JWST mask



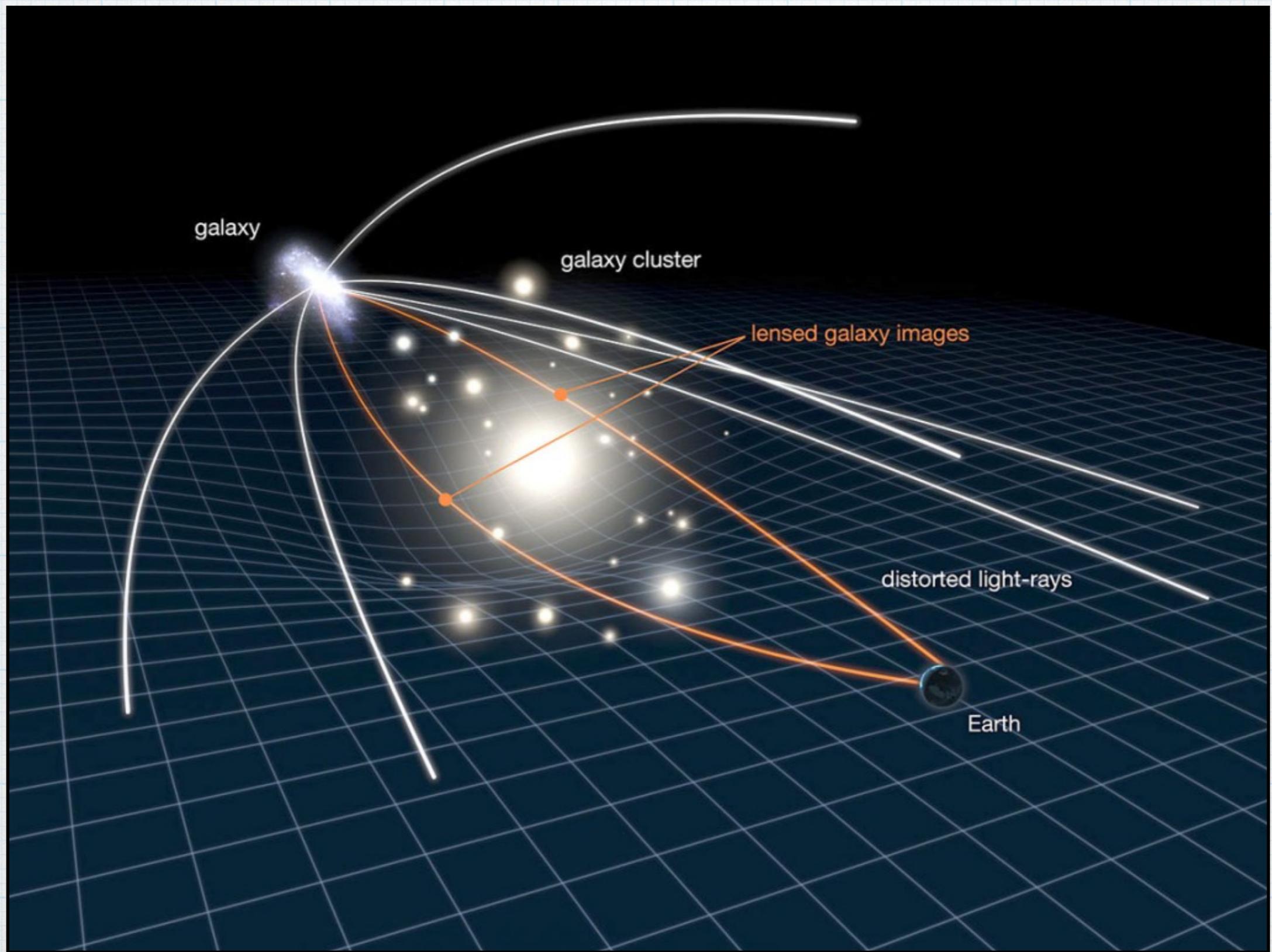
# Imaging

| 望远镜                     | 数量  |
|-------------------------|-----|
| Imaging 总计              | 45  |
| Palomar Observatory     | 12  |
| Gemini Observatory      | 8   |
| W. M. Keck Observatory  | 8   |
| Hubble Space Telescope  | 3   |
| Spitzer Space Telescope | 3   |
| Subaru Telescope        | 3   |
| ...                     | ... |

- \* Spitzer 通过行星的红外光 – 测温度
- \* HD 189733b
  - \* 潮汐锁定
  - \* 黑暗/阳光 - 650°C/930°C
  - \* 强风 - 9600 km/h



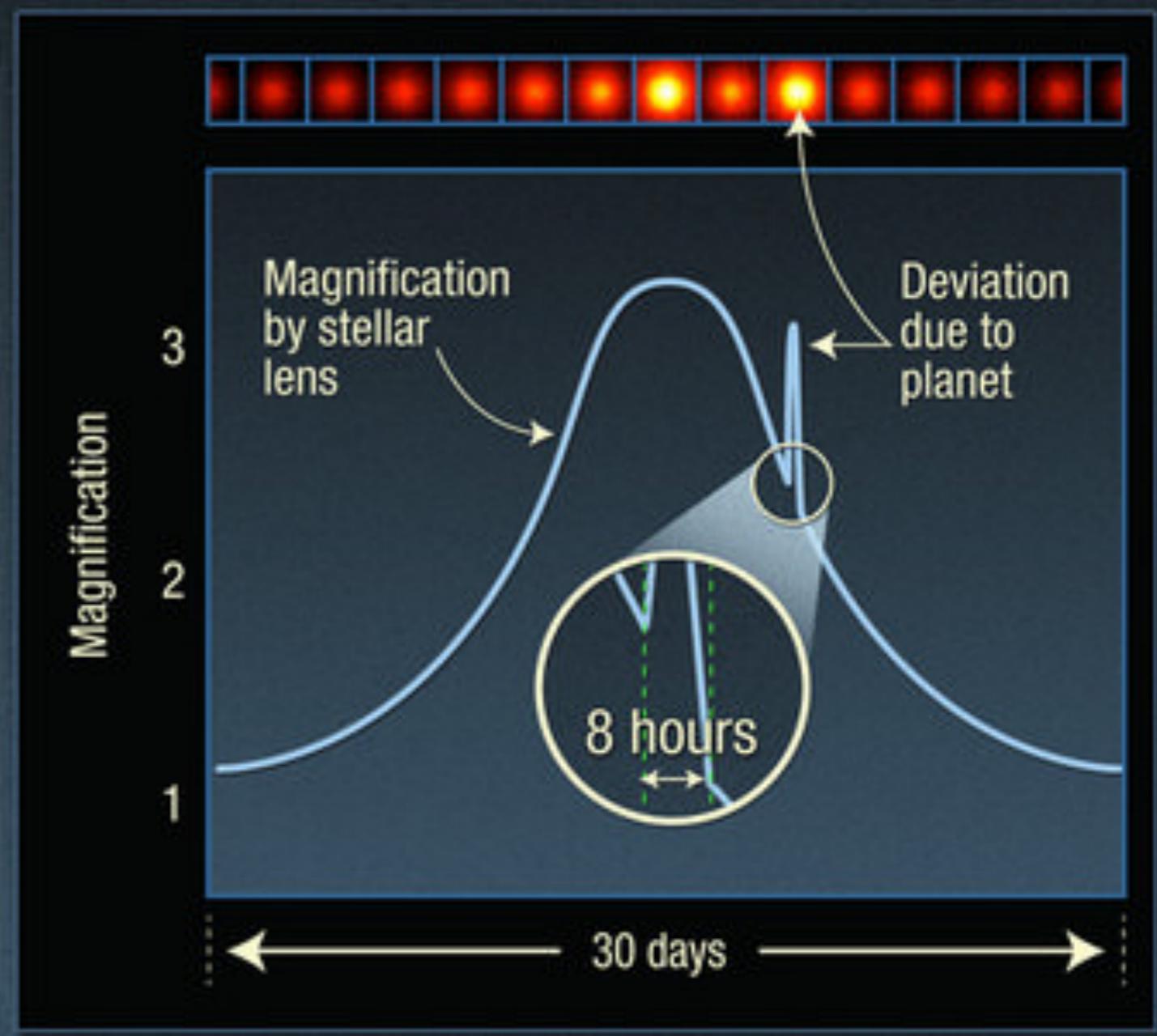
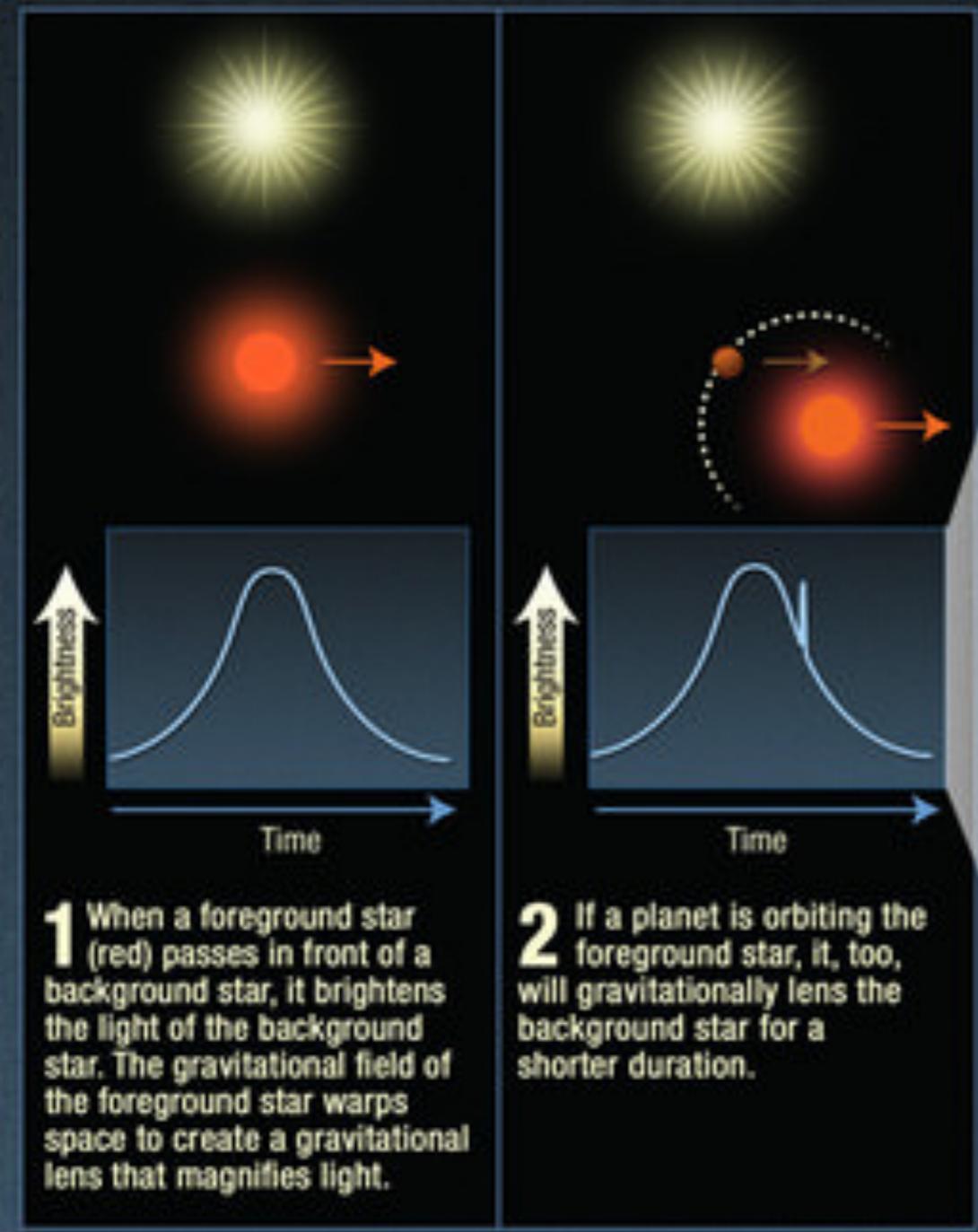
# Microlensing



# Microlensing

## \* 原理

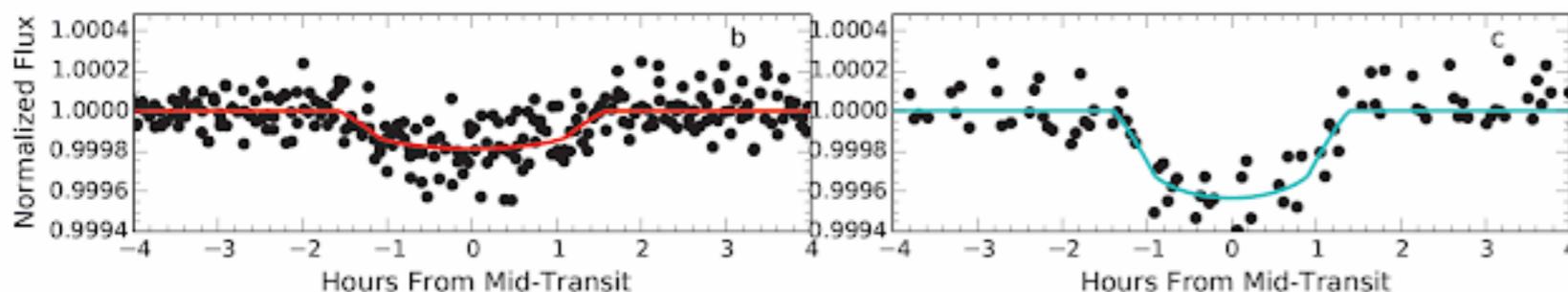
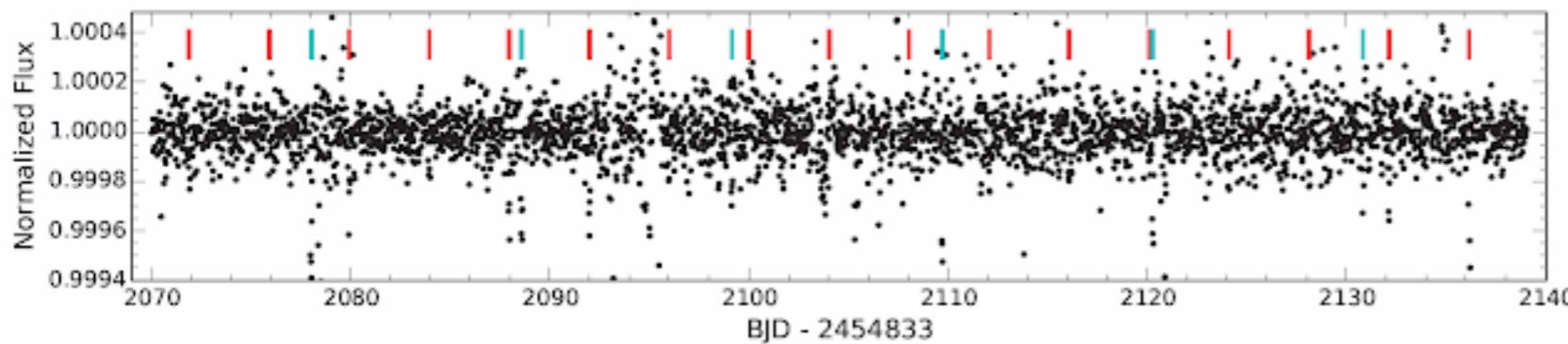
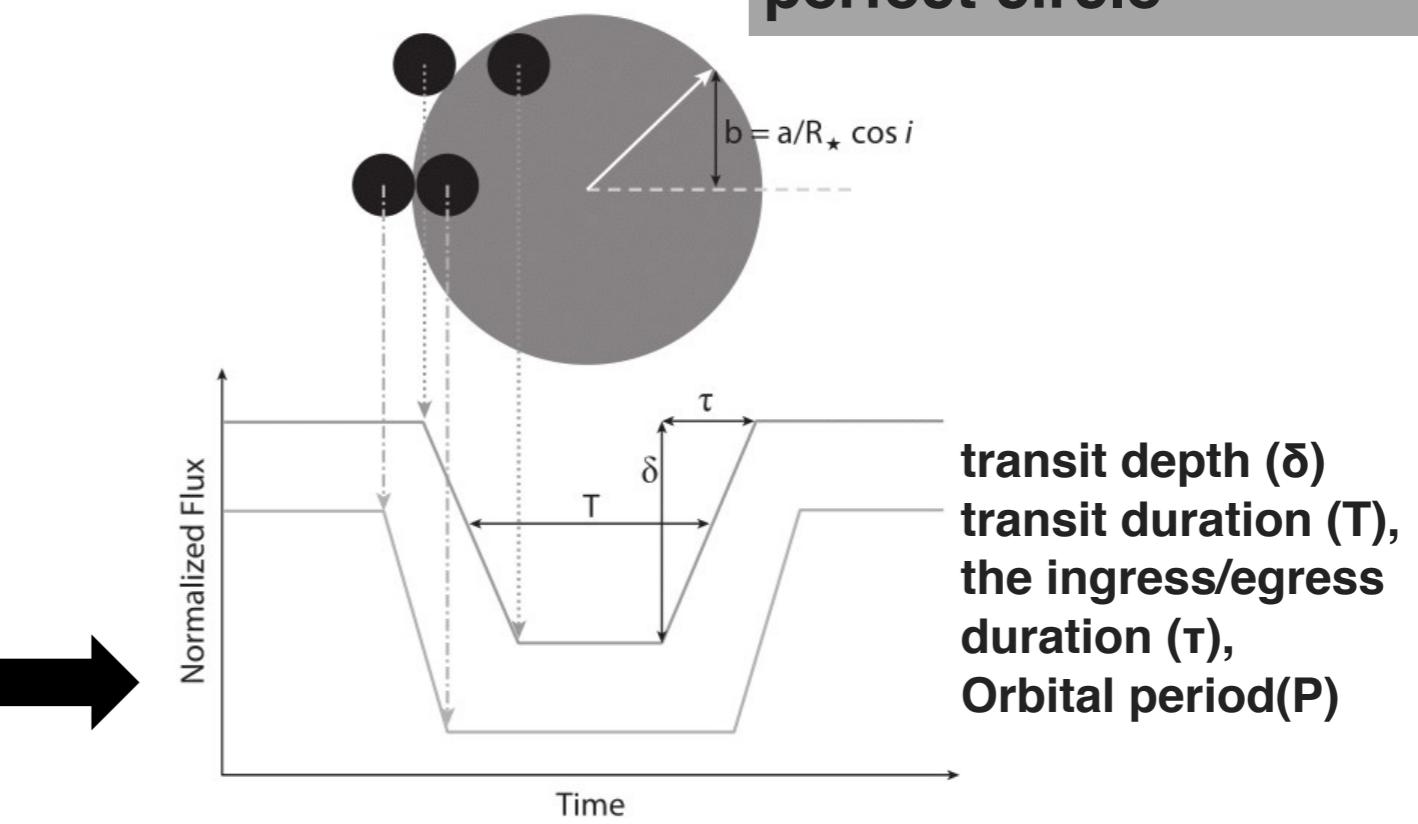
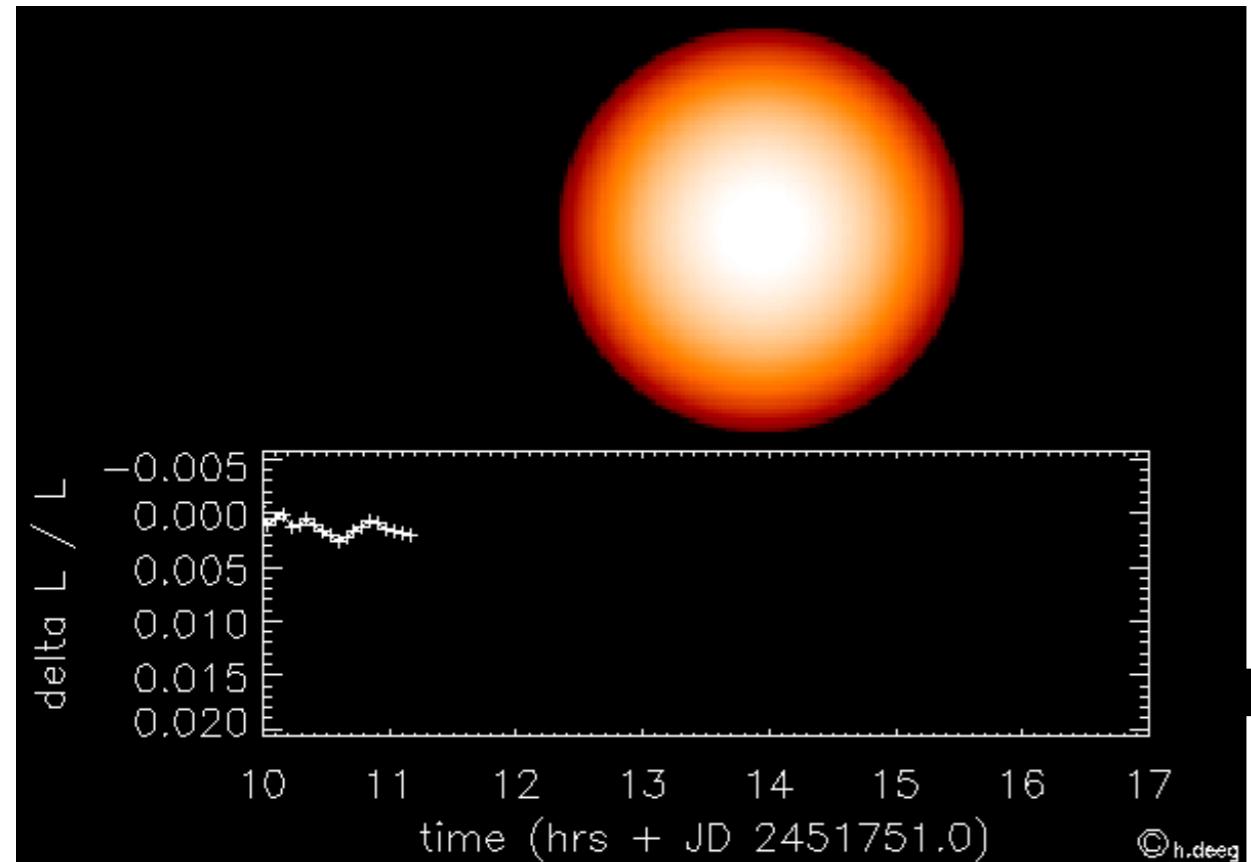
### Extrasolar planet detected by gravitational microlensing



A sketch of a microlensing signature with a planet in the lens system. Image Credit: NASA / ESA / K. Sahu / STScI

# Transit

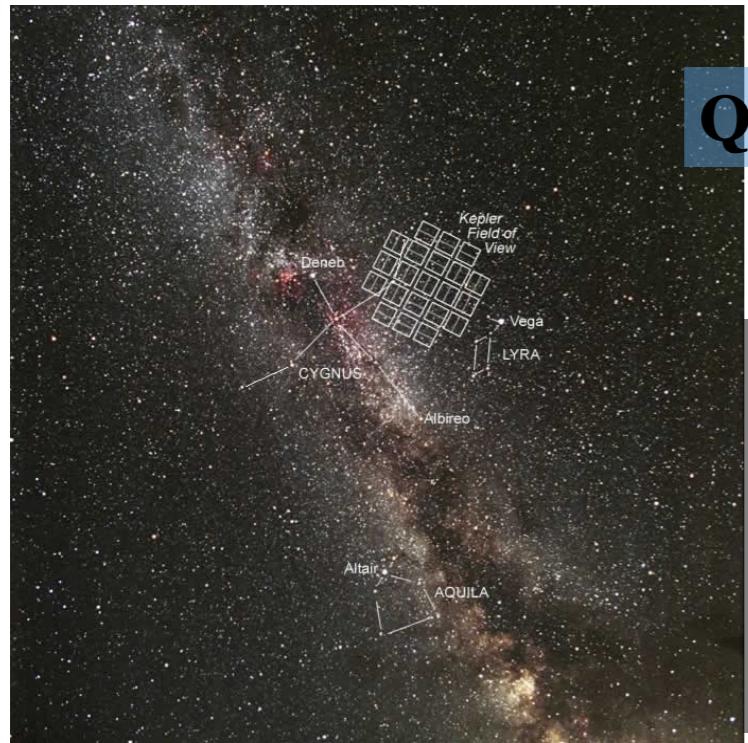
**Assumption:** The planets and stars are spherical; The orbit is perfect circle



Semi-major axis, stellar mass, stellar radius, planetary radius, eccentricity, inclination, ~~planetary mass~~

# Transit—Kepler Pipeline:

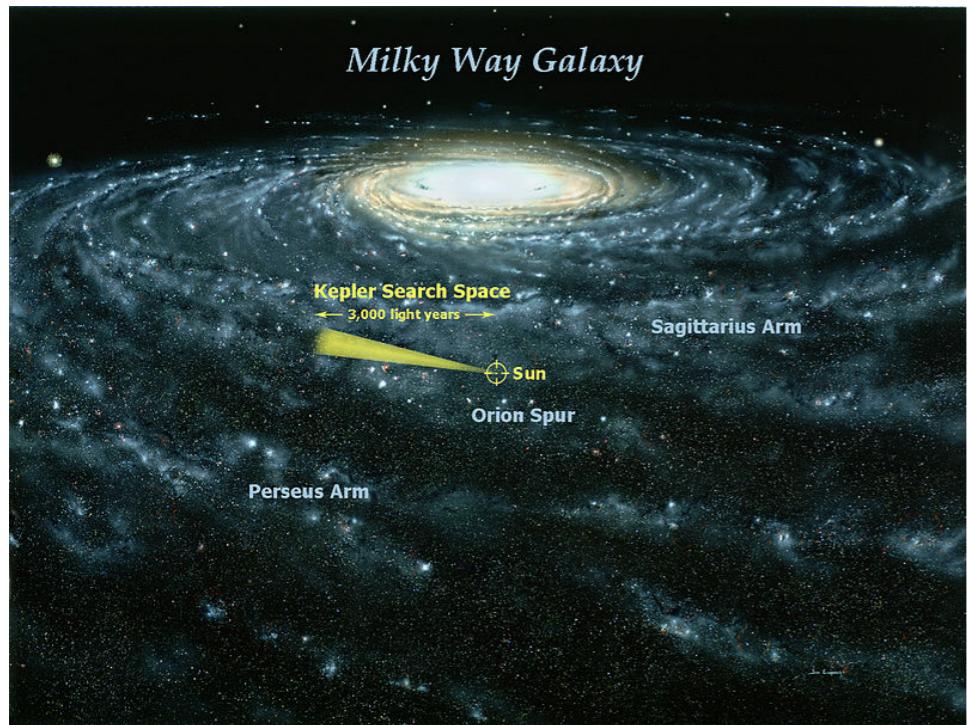
## Observing strategy:



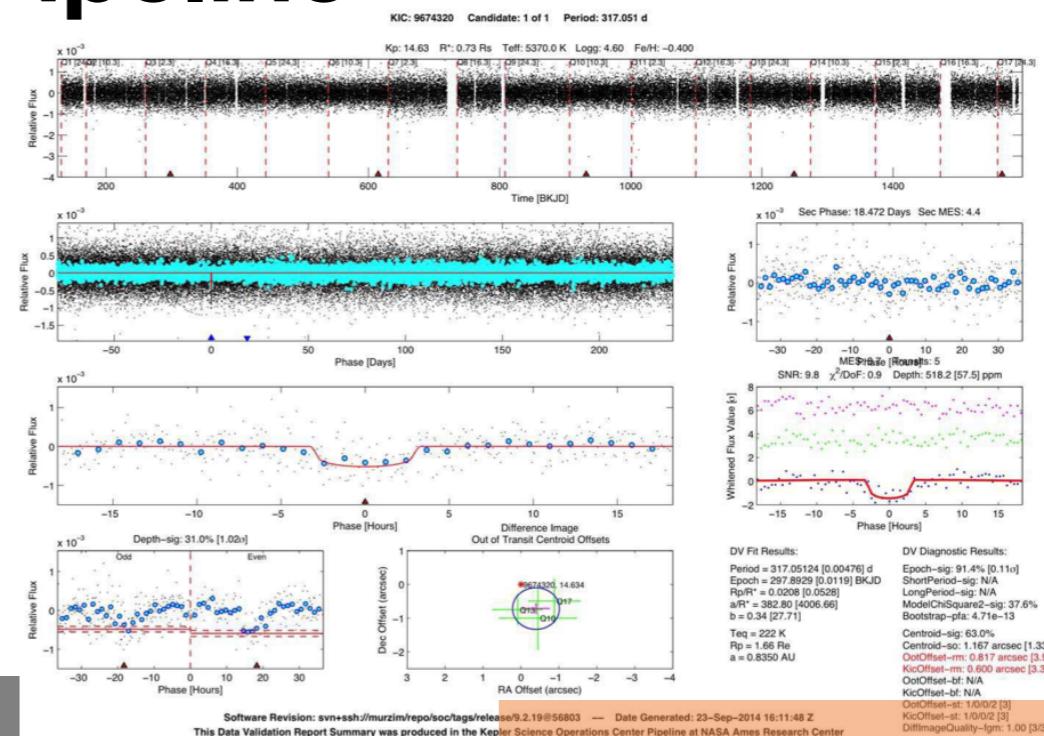
Quarterly rolls

- 115 square degree
- 0.25% full sky
- 400 Kepler can cover the whole sky

[https://www.jpl.nasa.gov/news/press\\_kits/Kepler-presskit-2-19-smfile.pdf](https://www.jpl.nasa.gov/news/press_kits/Kepler-presskit-2-19-smfile.pdf)

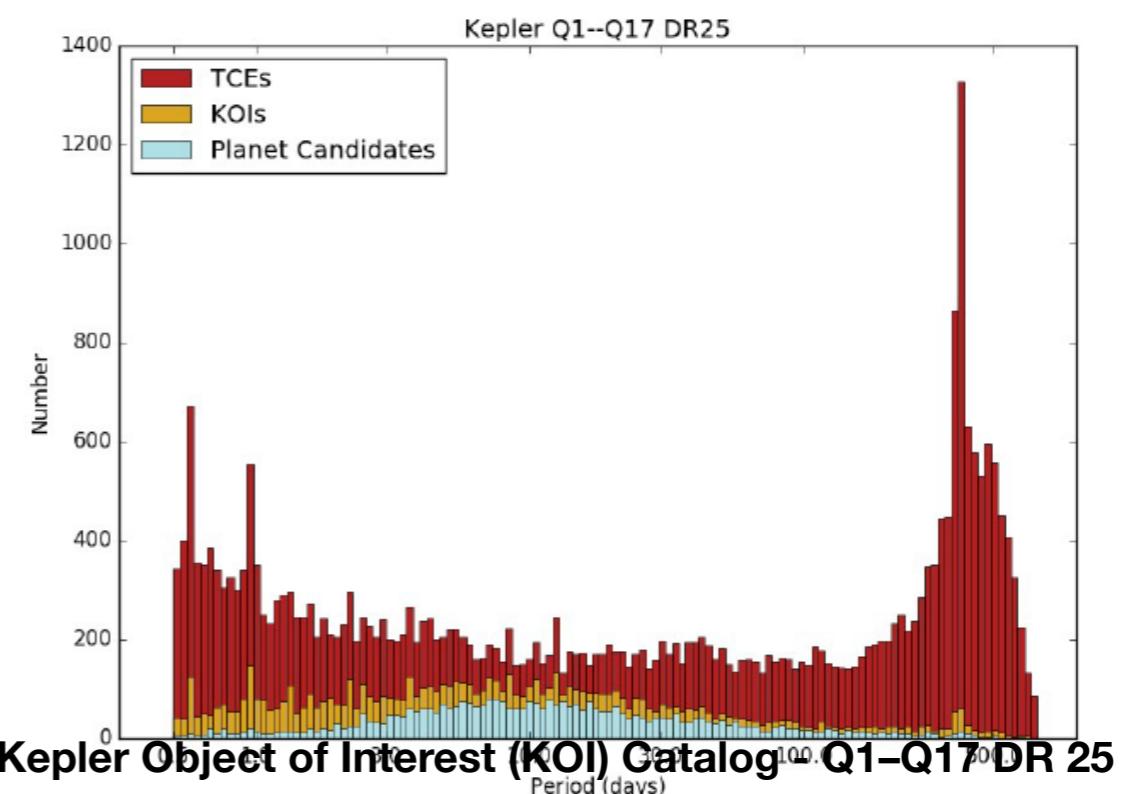


<http://kepler.nasa.gov/images/LombergA1600-full.jpeg>



DV Fit Results:  
Period = 317.05104 [0.00476] d  
Epoch = 237.05104 [0.01119] BKJD  
SemiMajorAxis = 1.167 AU  
LongPeriod = N/A  
Rp/R\* = 0.0206 [0.0528]  
a/R\* = 382.80 [4006.66]  
b = 0.34 [27.71]  
Teq = 222 K  
Rp = 1.66 R\*  
a = 0.8350 AU  
Centroid-sig: 63.0%  
Centroid-so: 1.167 arcsec [1.33a]  
OutOffset-mm: 0.817 arcsec [3.35a]  
KicOffset-mm: 0.800 arcsec [3.35a]  
OutOffset-bf: N/A  
KicOffset-bf: N/A  
OutOffset-st: 1/0/0/2 [3]  
KicOffset-st: 1/0/0/2 [3]  
DiffImage-Quality-fgm: 1.00 [3/3 of 5]

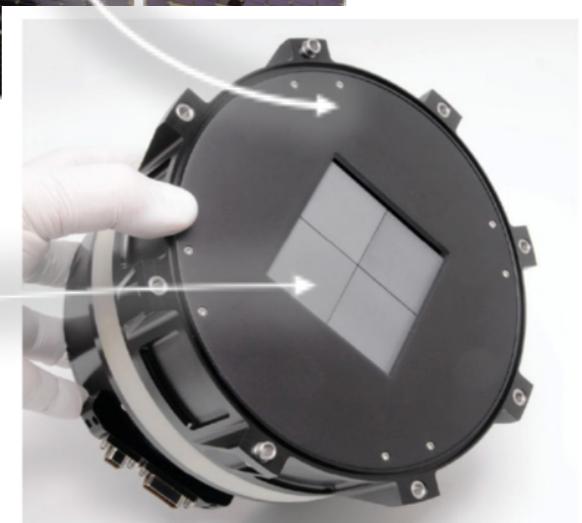
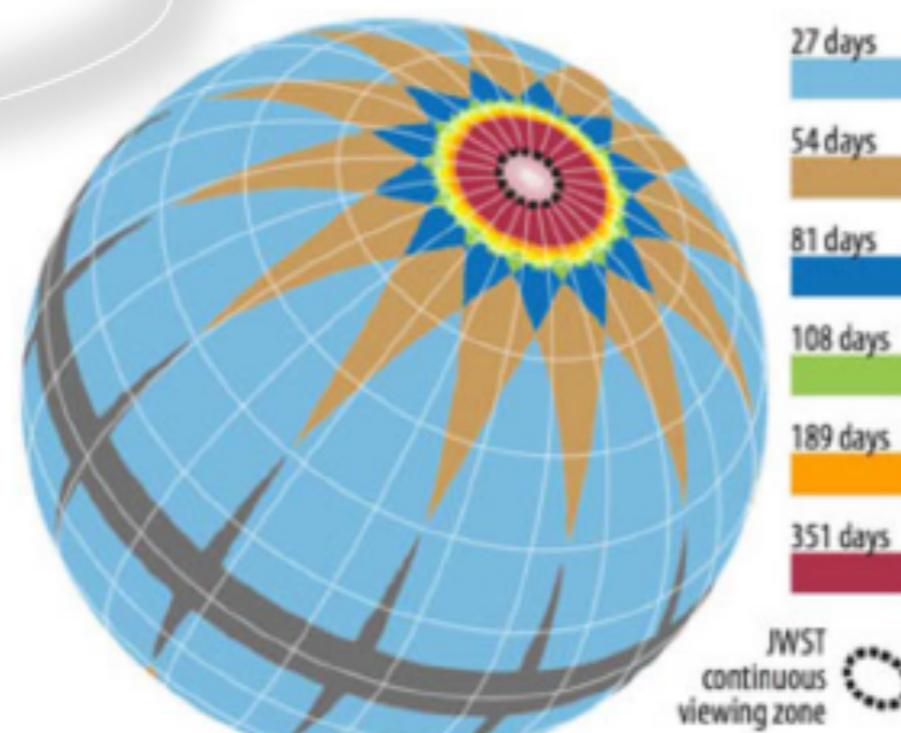
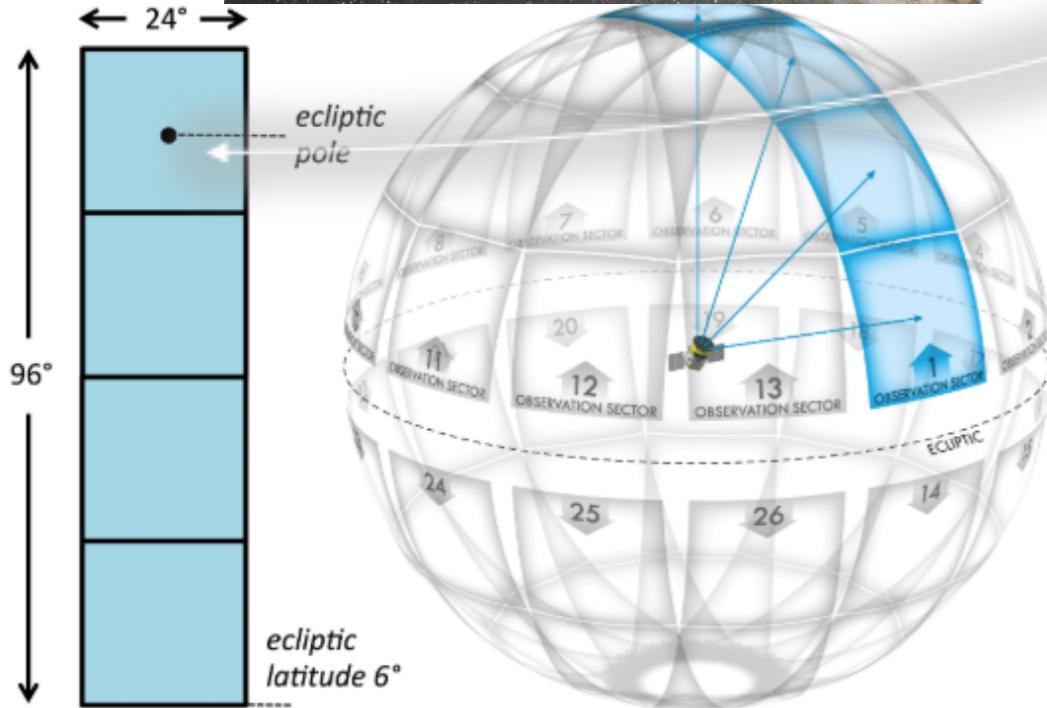
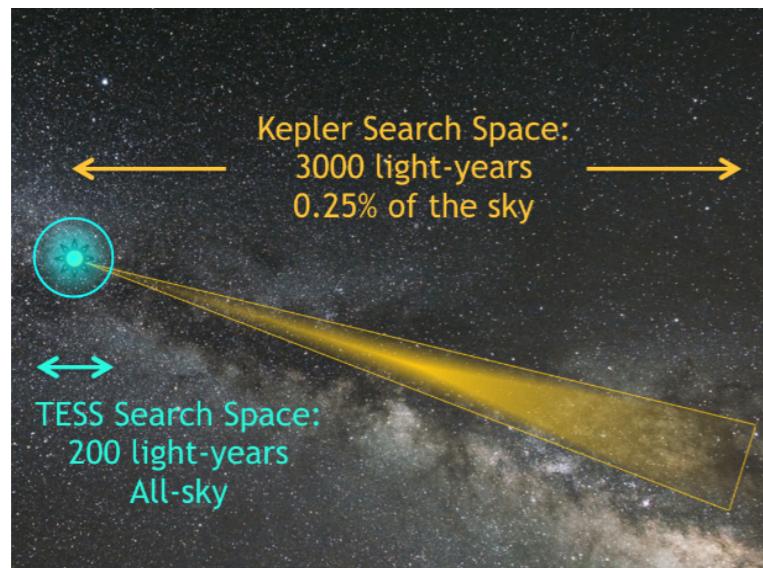
**≥3 transits!**  
**Pixels → TCEs → KOIs**  
**→planet candidates**  
**→confirmed planets**



# Transit—TESS

Launched April 18, 2018  
Started science operations July 25, 2018

- \* 10 cm aperture
- \* Bandpass: 600 - 1100 nm
- \* 13.7-day elliptical orbit



短周期、  
近轨道行星

# Transit—TESS

## Science Goal: Bright!

- \* Improve statistics for studies of the mass-radius relation of small planets as a function of distance from host stars.
- \* More temperate planets among which to select the best for atmospheric characterization with the JWST/ELTs
- \* .....

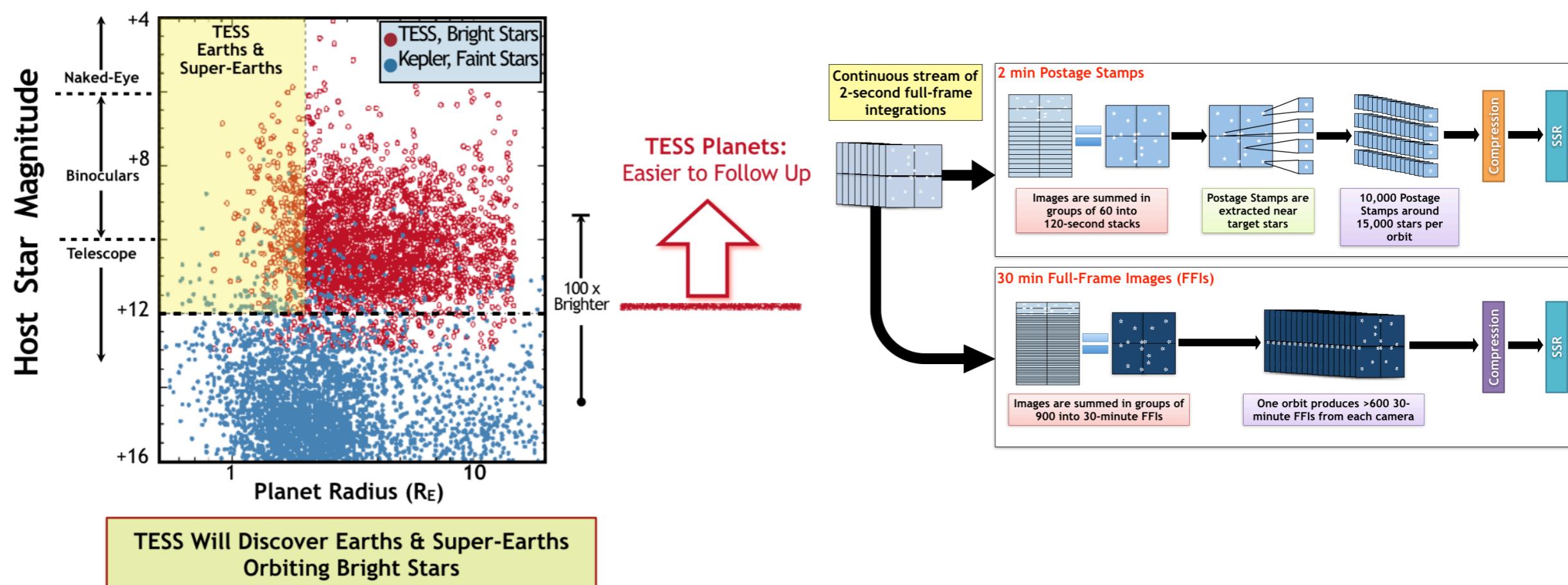


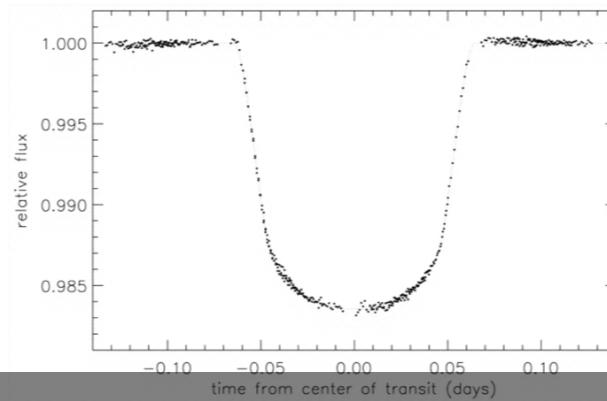
Image Credit: MIT

Villanueva, Dragomir & Gaudi (2019)

# Transit—TESS

## Single Transit

Stellar density +  
Eccentric



= Orbital Period

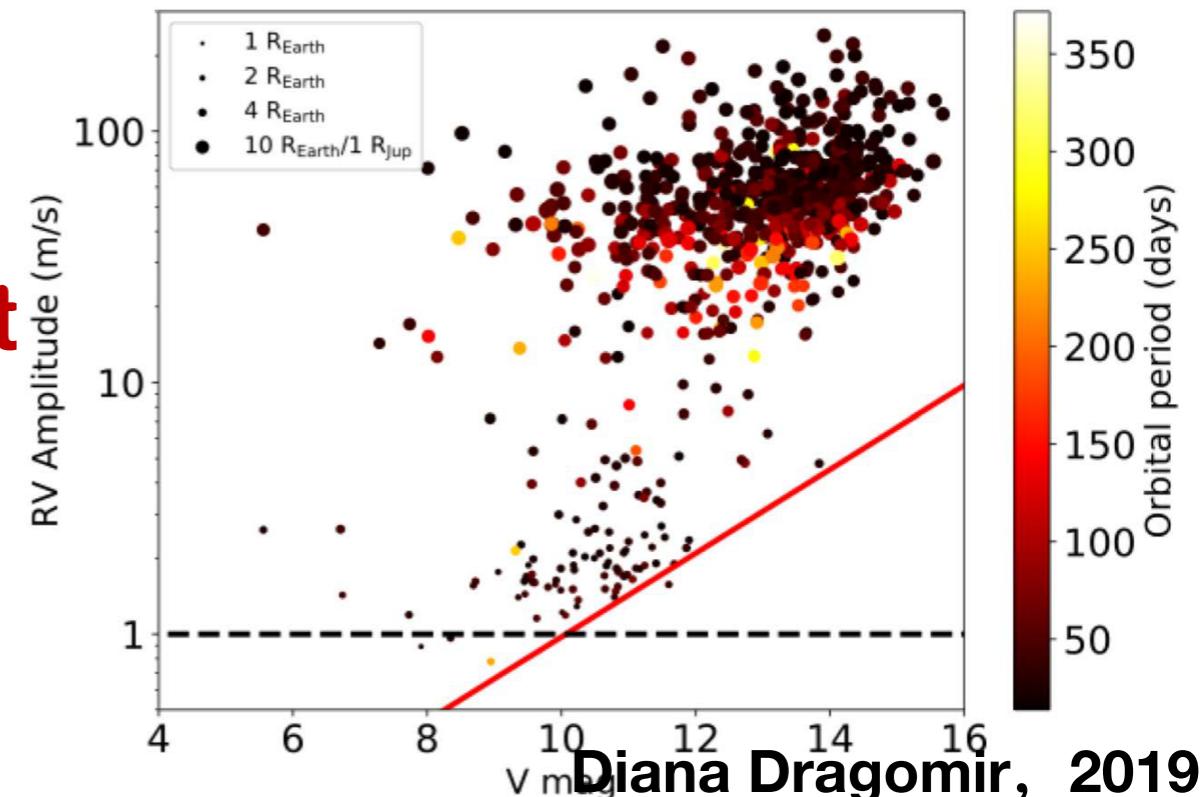
*Density:* Gaia (Radius) and spectroscopy/  
asteroseismology (spectroscopy).

*Eccentricity:* prior from known  
distribution).

Seager & Mallén-Ornelas (2003)  
Yee & Gaudi (2008)

**Radial Velocity Prospects  
for Single-Transit Planets**

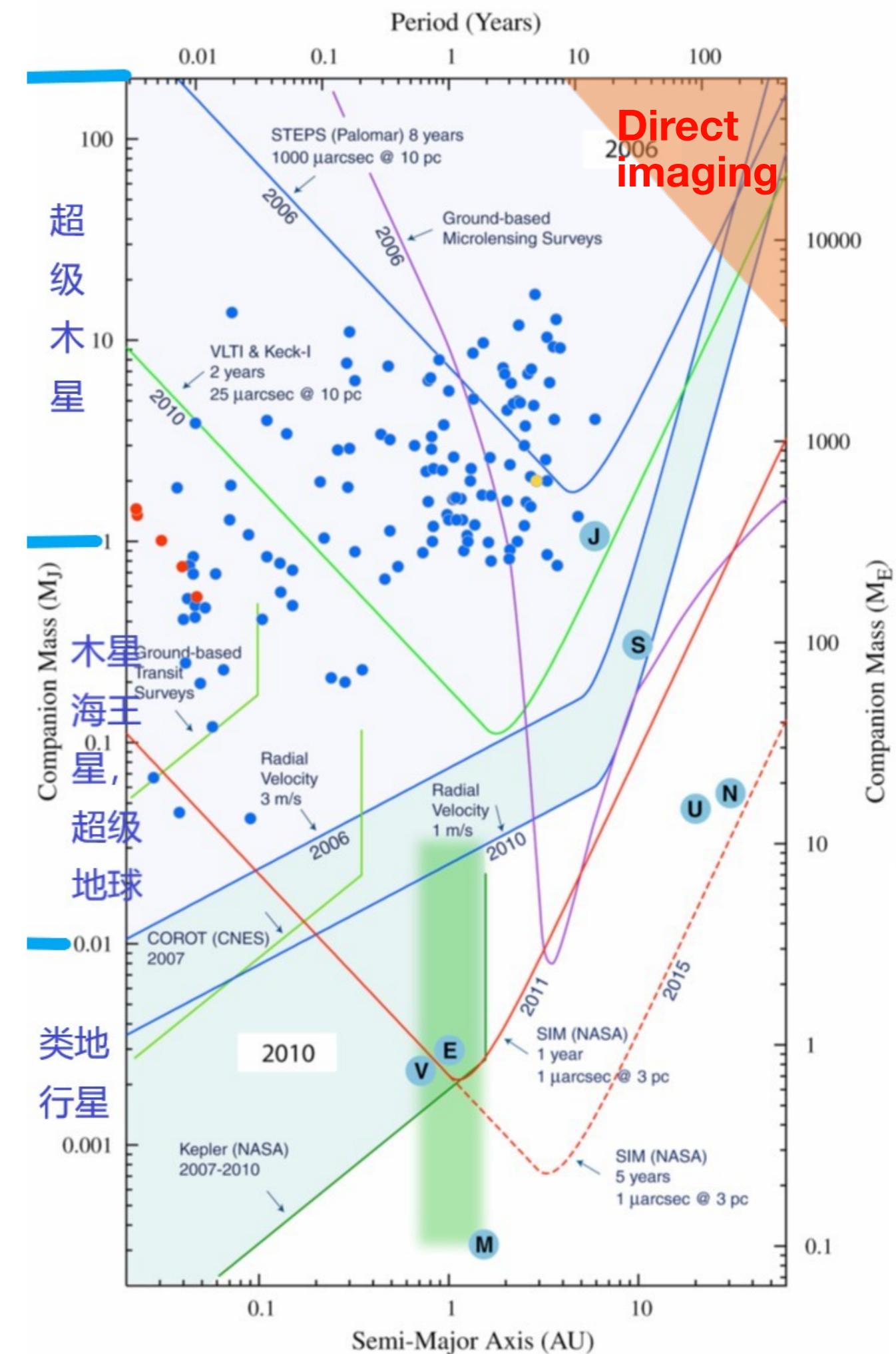
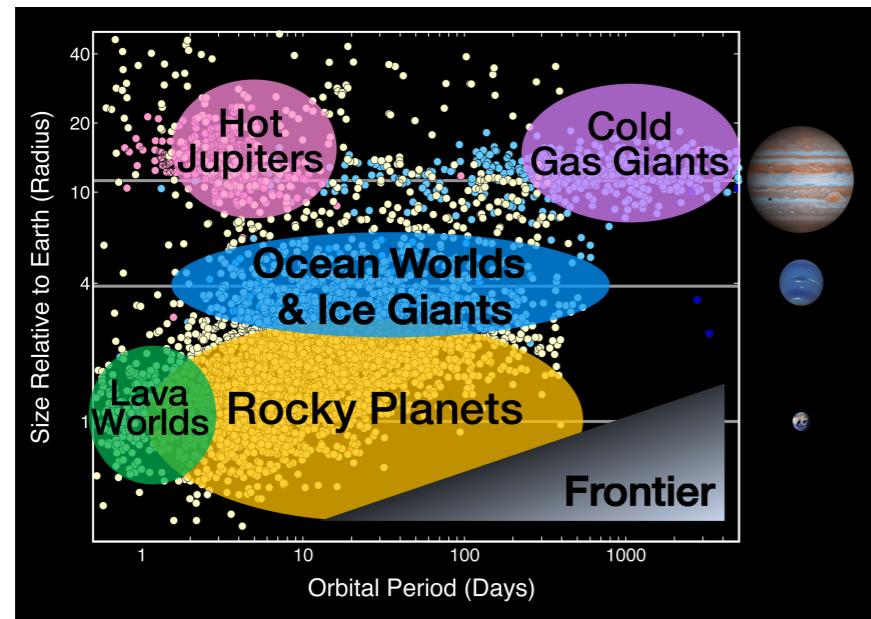
- \* If period constrained well enough
  - get photometry to catch next transit
- \* Not well enough
  - use RV measurements to improve constraint



Diana Dragomir, 2019

# Detection Method and Statistic

- \* habitable zone(green area)
- \* ~150 exoplanets detected in 2004
  - r.v. (blue)
  - transits (red)
  - microlensing (yellow)
  - pulsar timing (purple)
  - Imaging(magenta)



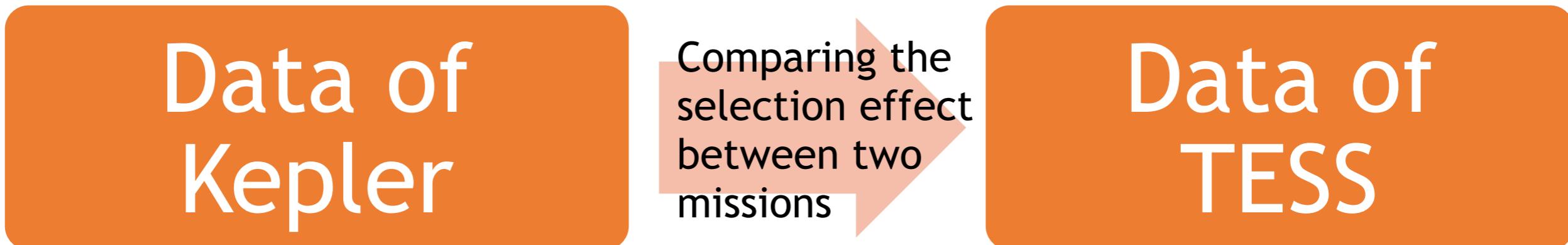
# Our Project

- Empirical Predictions for the Period Distribution of Planets to be Discovered by TESS
- 

Previous work :



My work :



# Methodology

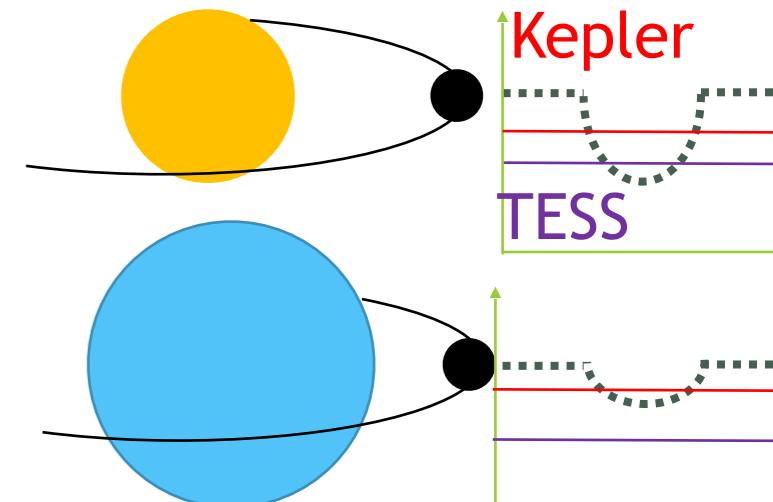
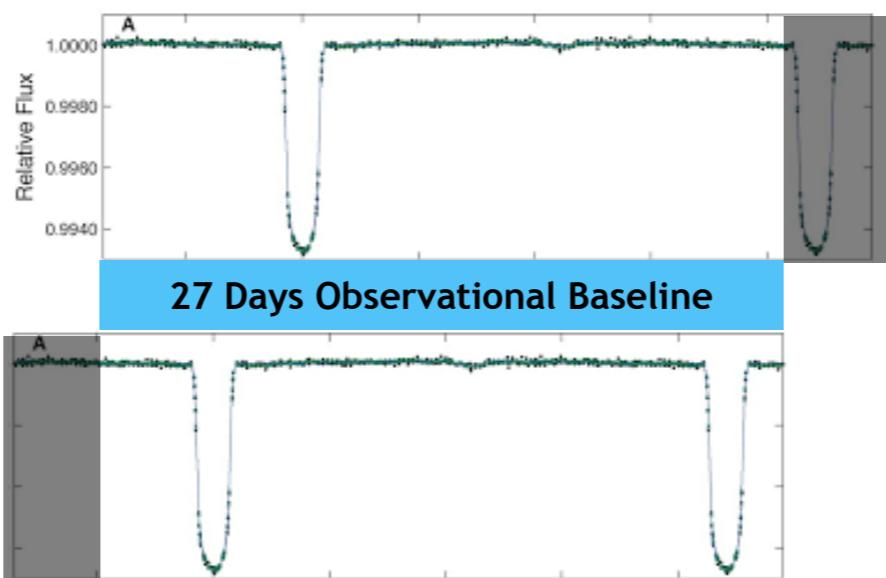
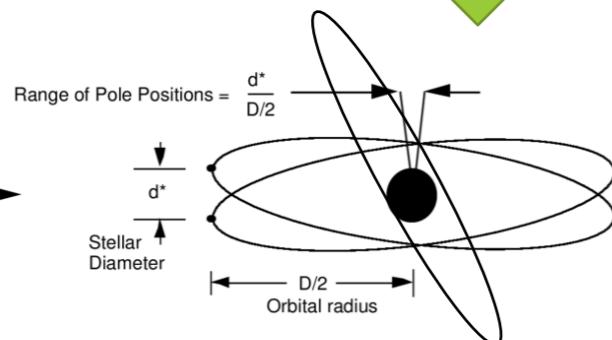
The probability that an exoplanet of which orbital period is P days is detected by TESS

The occurrence rate of an exoplanet of which orbital period is P days

$$Prob(P, R_p | TESS) = \frac{Prob(TESS | P, R_p) * Prob(P, R_p)}{Prob(TESS)}$$

constant

$$Prob(TESS | P, R_p) = Prob(Tr | P, R_p) \times Prob(NTr(\tau_1) | Tr, P, R_p) \times Prob(SNR\_T | NTr(\tau_1), Tr, P, R_p)$$



# Methodology

TESS : CTL  
(dwarfs and subgiants )  
**Input Catalogue**

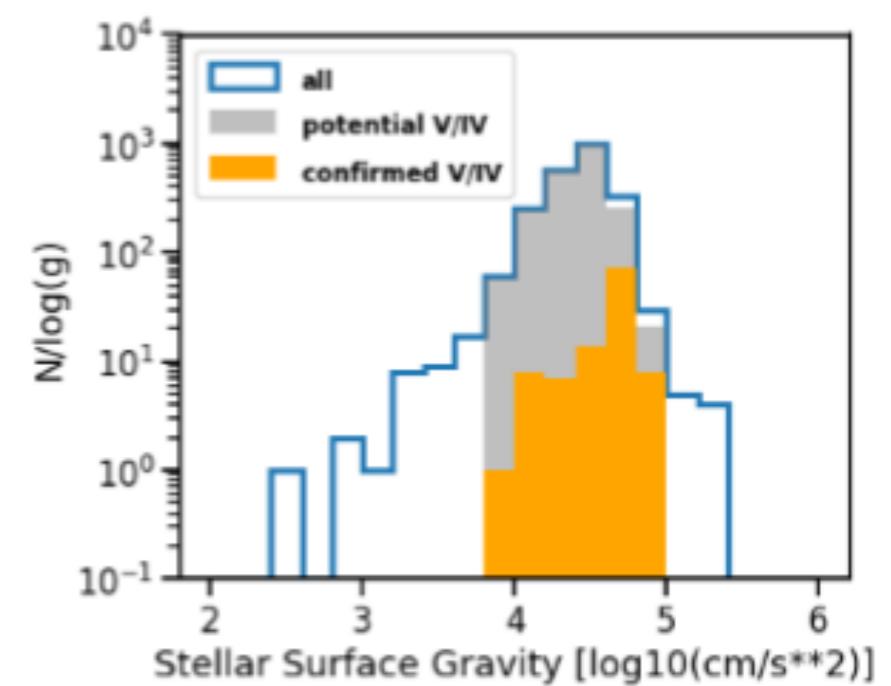
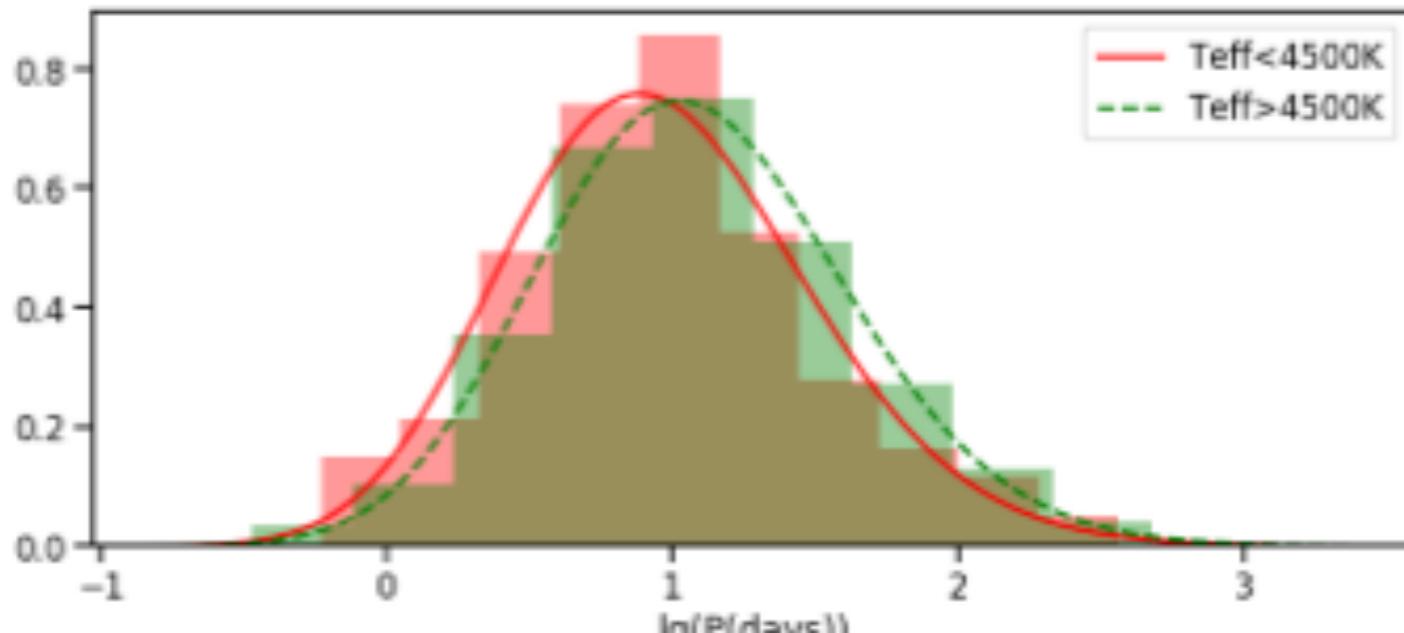
$$\text{Prob}(P|\text{TESS})$$

$$= \frac{\text{Prob}(tr|P) \cdot \text{Prob}(Ntrs_T|P, tr) \cdot \text{Prob}(\text{SNR}_T > \text{SNR}_{T\min}|P, tr, Ntrs_T) \cdot \text{Prob}(P)}{\text{Prob}(\text{TESS})}$$

Likewise, repeat the above analysis but for Kepler:

$$\text{Prob}(P|\text{Kepler})$$

$$= \frac{\text{Prob}(tr|P) \cdot \text{Prob}(Ntrs_K|P, tr) \cdot \text{Prob}(\text{SNR}_K > \text{SNR}_{K\min}|P, tr, Ntrs_K) \cdot \text{Prob}(P)}{\text{Prob}(\text{Kepler})}$$



# Methodology

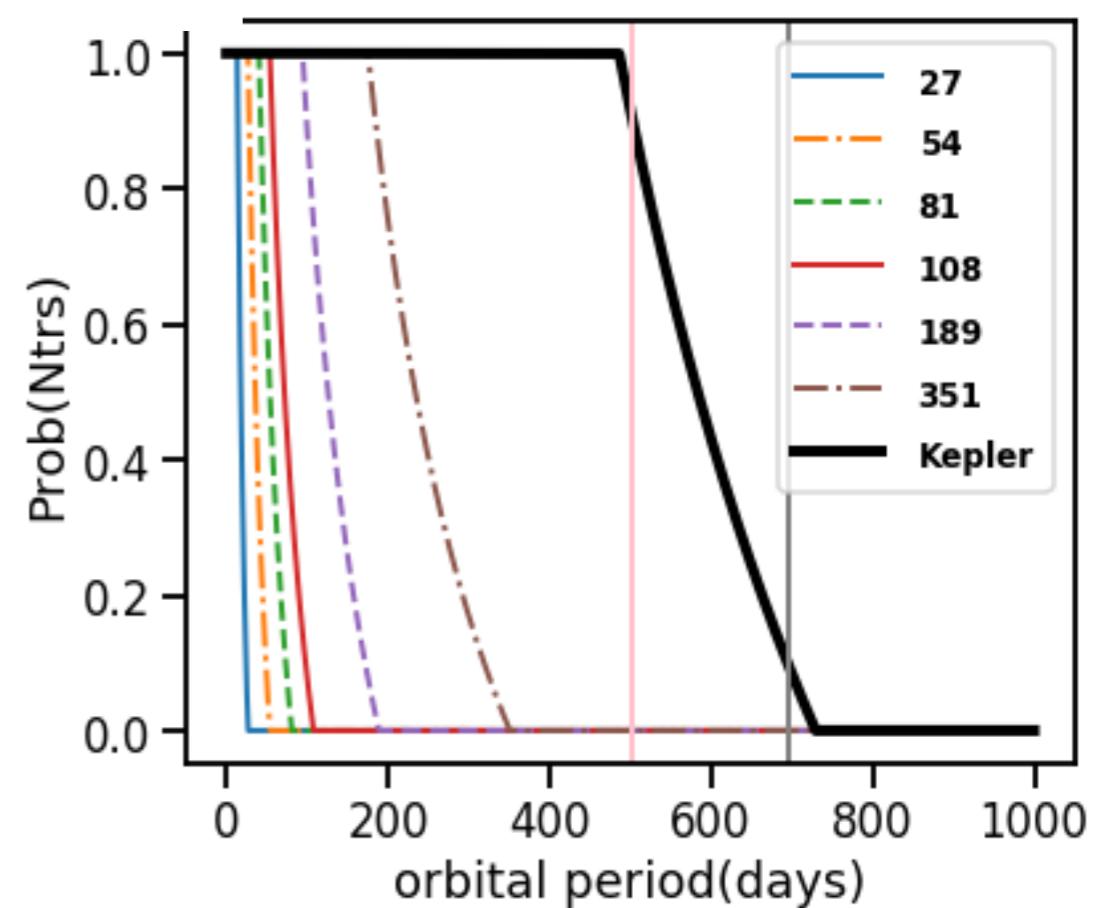
## 1. Prob ( $tr \mid P$ )

$$\text{Prob}(tr|P) = \int \frac{R_*}{a} f_{R_*, a|P}(R_*, a) dR_* da = \int \left( \frac{4\pi^2}{G} \right)^{\frac{1}{3}} R_* M_*^{-\frac{1}{3}} P^{-\frac{2}{3}} f_{R_*, M_*|P}(R_*, M_*) dR_* dM_*$$

The gray vertical line is 694.76 days beyond which the probability that it can be detected by Kepler is less than 10% and the pink vertical line is 503.10 days within which the probability is higher than 90%

$$2. \text{ } Prob(2Tr(\tau_1) | Tr, P)$$

$$\text{Prob}(Ntrs_T | P, tr) = \begin{cases} 0, & t \leq (N-1)P \\ \frac{t-(N-1)P}{P}, & (N-1)P < t < NP \\ 1, & t \geq NP \end{cases}$$



$$\text{Prob}_i(P|TESS) = c_i \text{Prob}_i(P|Kepler) \frac{\text{Prob}_i(Ntrs_T|P,tr) \cdot \text{Prob}_i(\text{SNR}_T > \text{SNRT}_{min} | P,tr,Ntrs_T)}{\text{Prob}_i(Ntrs_K|P,tr) \cdot \text{Prob}_i(\text{SNR}_K > \text{SNRK}_{min} | P,tr,Ntrs_K)}$$

# Methodology

3.  $\text{Prob}(\text{SNR}_T | 2Tr(\tau_1), Tr, P)$

$$\text{SNR} = R_p^2 P^{-\frac{1}{3}} \left( \frac{4\pi^2}{GM_*} \right)^{\frac{1}{6}} \sqrt{\frac{At_m}{4R_* r^2} \int_{\lambda_1}^{\lambda_2} \tau \pi B(\lambda, T_*) \left( \frac{\lambda}{hc} \right) d\lambda},$$

$$\text{SNR} = f(P)g(S)h(M, T_*)$$

$$f(P) = R_p^2 P^{-\frac{1}{3}}$$

$$g(S) = \left( \frac{4\pi^2}{GM_*} \right)^{\frac{1}{6}} \sqrt{\frac{1}{4R_*}}$$

$$h(M, T_*) = \sqrt{\frac{At_m}{r^2} \int_{\lambda_1}^{\lambda_2} \tau \pi B(\lambda, T_*) \left( \frac{\lambda}{hc} \right) d\lambda}$$

$$\text{SNR}_K = f(P)g(S)h(M_K, T_*);$$

$$\text{SNR}_T = f(P)g(S)h(M_T, T_*)$$

$$\text{SNR}_T = \text{SNR}_K \frac{h(M_T, T_*)}{h(M_K, T_*)} = k(M_T, M_K, T_*) \bullet \text{SNR}_K$$

| SubSample | k (27 days) | k (54 days) | k (81 days) | k (108 days) | k (189 days) | k (351 days) |
|-----------|-------------|-------------|-------------|--------------|--------------|--------------|
| < 4500K   | 3974.41     | 0.0327      | 0.0463      | 0.0567       | 0.0655       | 0.0866       |
| > 4500K   | 5653.53     | 0.0621      | 0.0878      | 0.1075       | 0.1242       | 0.1643       |

# Methodology

## 3. Prob( $SNR_T | 2Tr(\tau_1), Tr, P$ )

$$f_{SNRTi}(SNR_T | P, tr) = f_{SNRTi}(k \cdot SNR_K | P, tr) = f_{SNRKi}(SNR_K | P, tr)$$

$$f_{SNRTi}(SNR | P, tr) = f_{SNRKi}(SNR/k | P, tr)$$

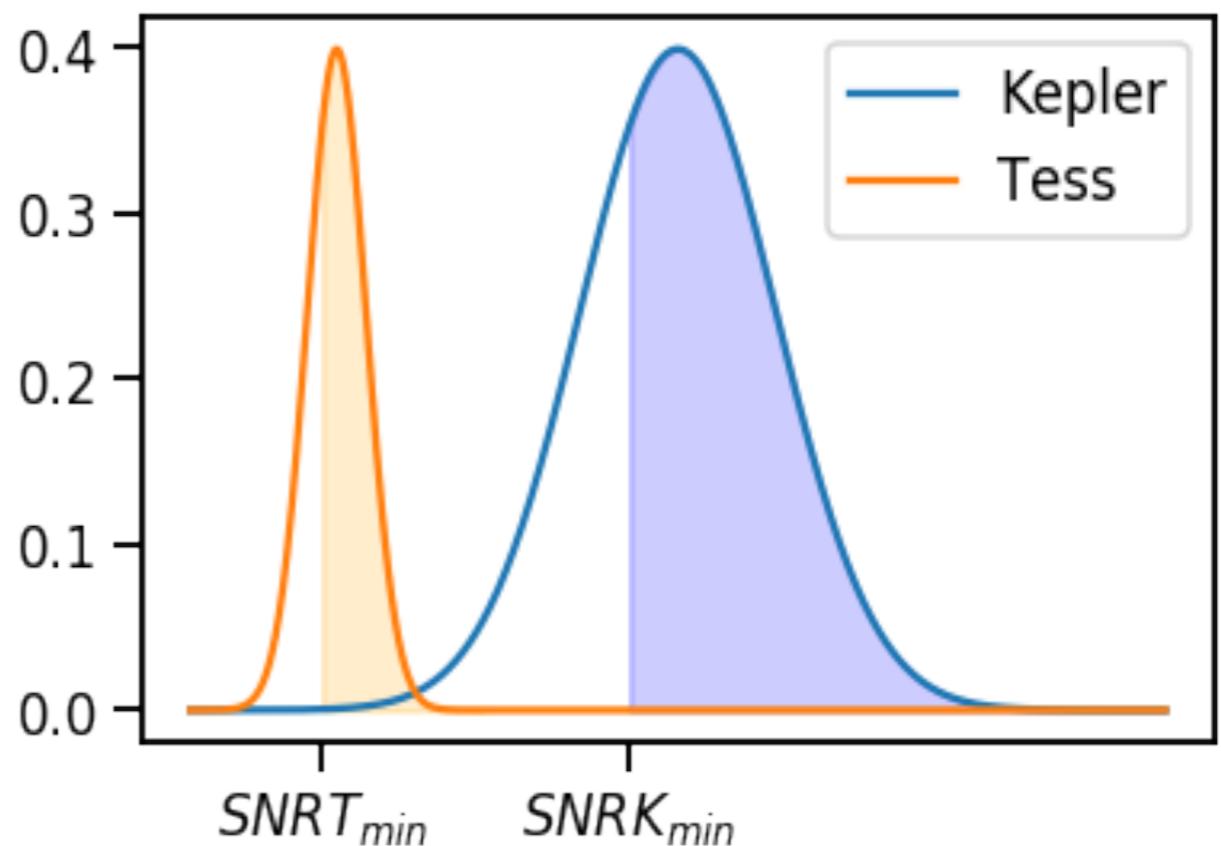
$$\text{Prob}_i(SNR_T > SNRT_{min} | P, tr)$$

$$= \int_{SNRT_{min}}^{\infty} f_{SNRTi}(SNR' | P, tr) dSNR'$$

$$= \int_{SNRT_{min}}^{\infty} f_{SNRKi}\left(\frac{SNR'}{k} | P, tr\right) dSNR'$$

$$= \int_{\frac{SNRT_{min}}{k}}^{\infty} k \cdot f_{SNRKi}(SNR'' | P, tr) dSNR''$$

$$= k \cdot \text{Prob}_i\left(SNR_K > \frac{SNRT_{min}}{k} | P, tr\right)$$



$$\frac{\text{Prob}_i(SNR_T > SNRT_{min} | P, tr)}{\text{Prob}_i(SNR_K > SNRK_{min} | P, tr)} = k \cdot \frac{\text{Prob}_i\left(SNR_K > \frac{SNRT_{min}}{k} | P, tr\right)}{\text{Prob}_i(SNR_K > SNRK_{min} | P, tr)}$$



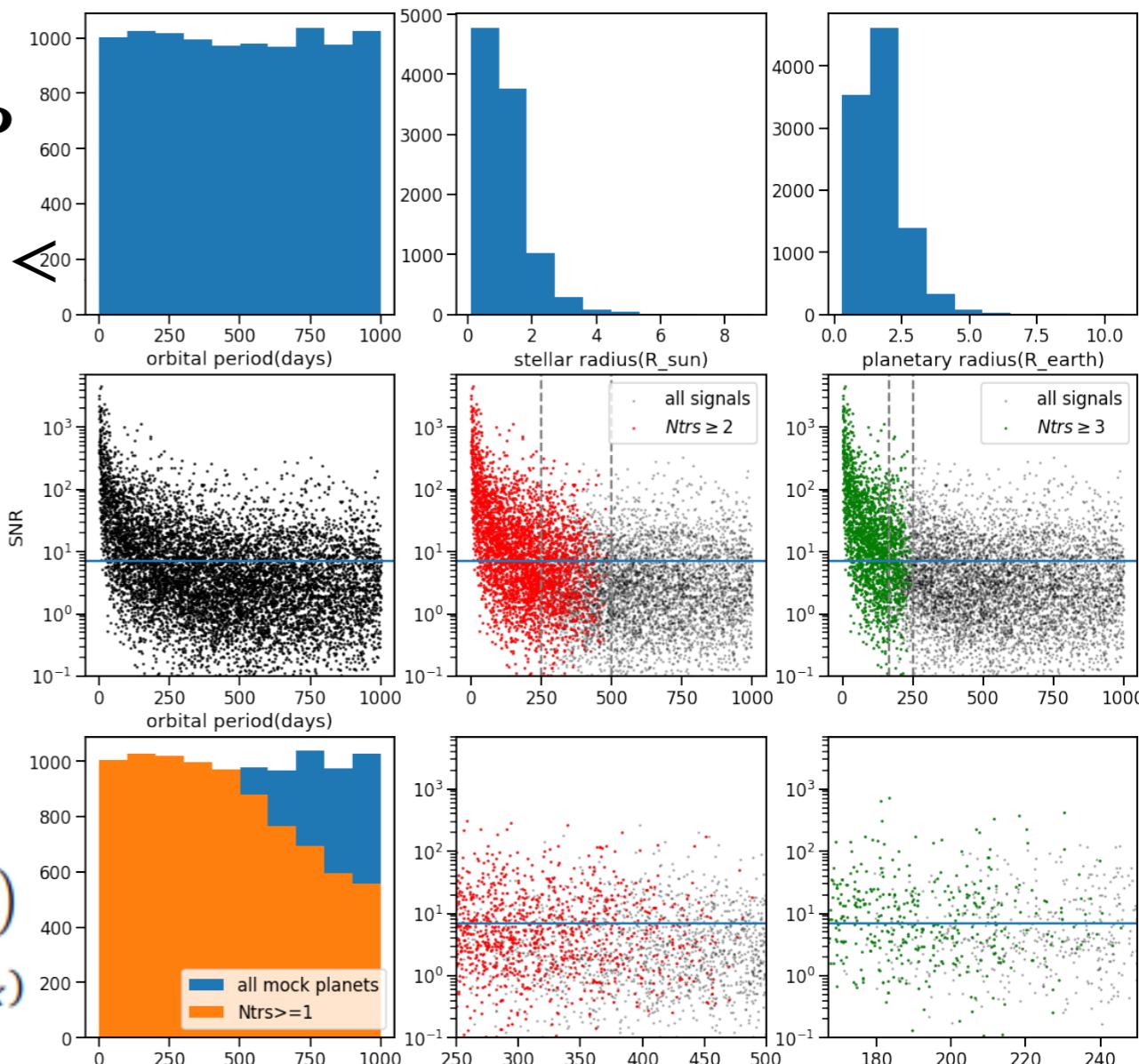
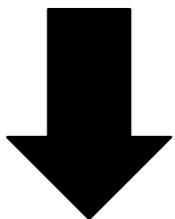
# Methodology

$$3. \text{ Prob}(SNR_T | 2Tr(\tau_1), Tr, P)$$

$$\text{Prob}(TESS | P) = \text{Prob}(Tr | P) \times \text{Prob}(NTr(\tau_1) | Tr, P) \times \text{Prob}(SNR_T | NTr(\tau_1) | Tr, P)$$

3 scenarios:

$$(Ntrs_T | P, tr) = \begin{cases} 0, & t \leq (N-1)P \\ \frac{t - N \cdot P}{P}, & (N-1)P < t < N \cdot P \\ 1, & t \geq N \cdot P \end{cases}$$



$$\text{Prob}_i \left( \text{SNR}_K > \frac{\text{SNRT}_{\min}}{k} | P, tr \right) \rightarrow \text{Prob}_i \left( \text{SNR}_K > \frac{\text{SNRT}_{\min}}{k} | P, tr, 3trs_K \right)$$

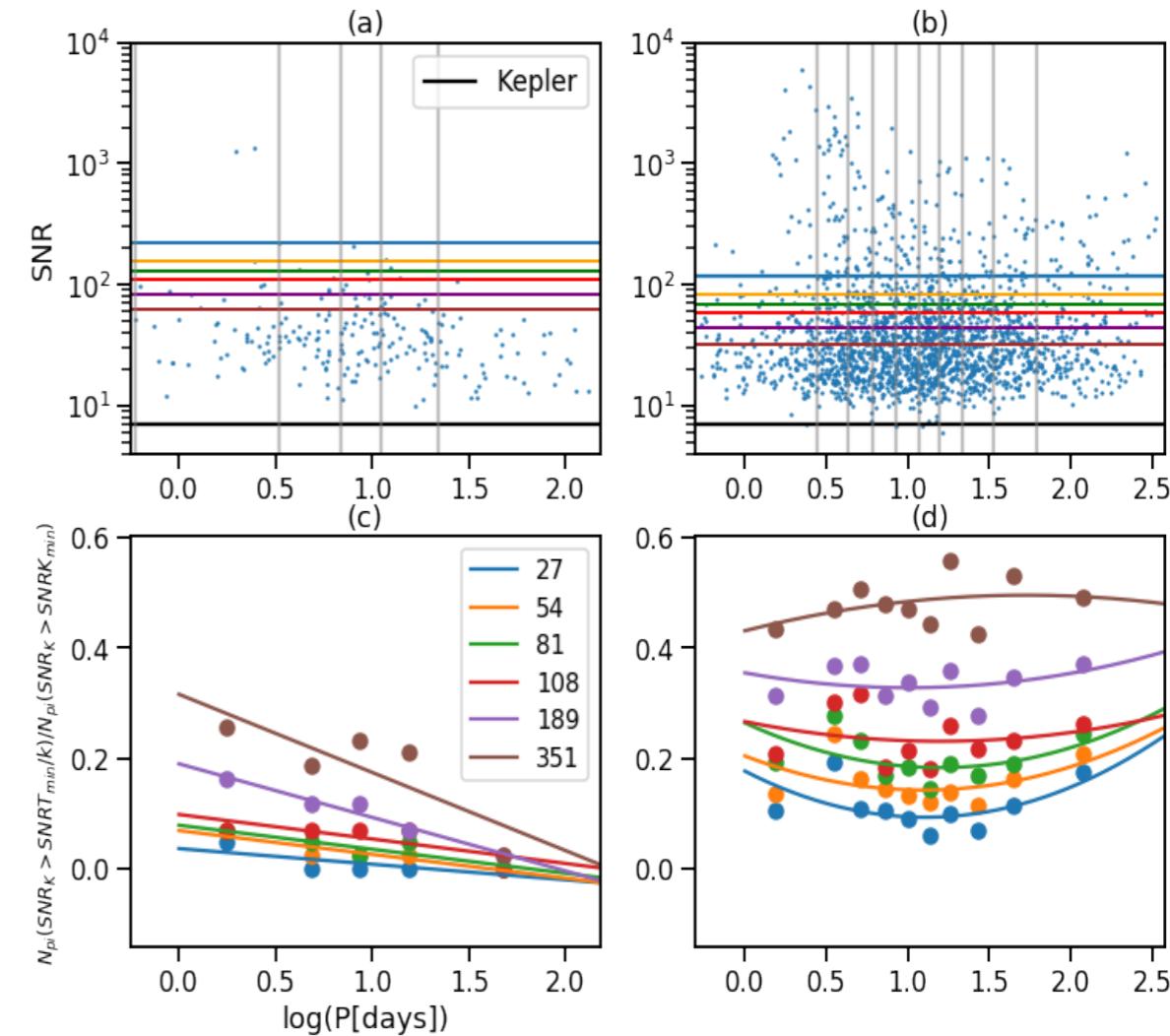
$$\text{Prob}_i \left( \text{SNR}_K > \text{SNRK}_{\min} | P, tr \right) \rightarrow \text{Prob}_i \left( \text{SNR}_K > \text{SNRK}_{\min} | P, tr, 3trs_K \right)$$

# Methodology



$$\frac{\text{Prob}_i\left(\text{SNR}_K > \frac{\text{SNRT}_{min}}{k} \mid P, tr, 3trs_K\right)}{\text{Prob}_i(\text{SNR}_K > \text{SNRK}_{min} \mid P, tr, 3trs_K)} = \frac{N_{Pi}\left(\text{SNR}_K > \frac{\text{SNRT}_{min}}{k}\right) / N_{Pi}}{N_{Pi}(\text{SNR}_K > \text{SNRK}_{min}) / N_{Pi}}$$

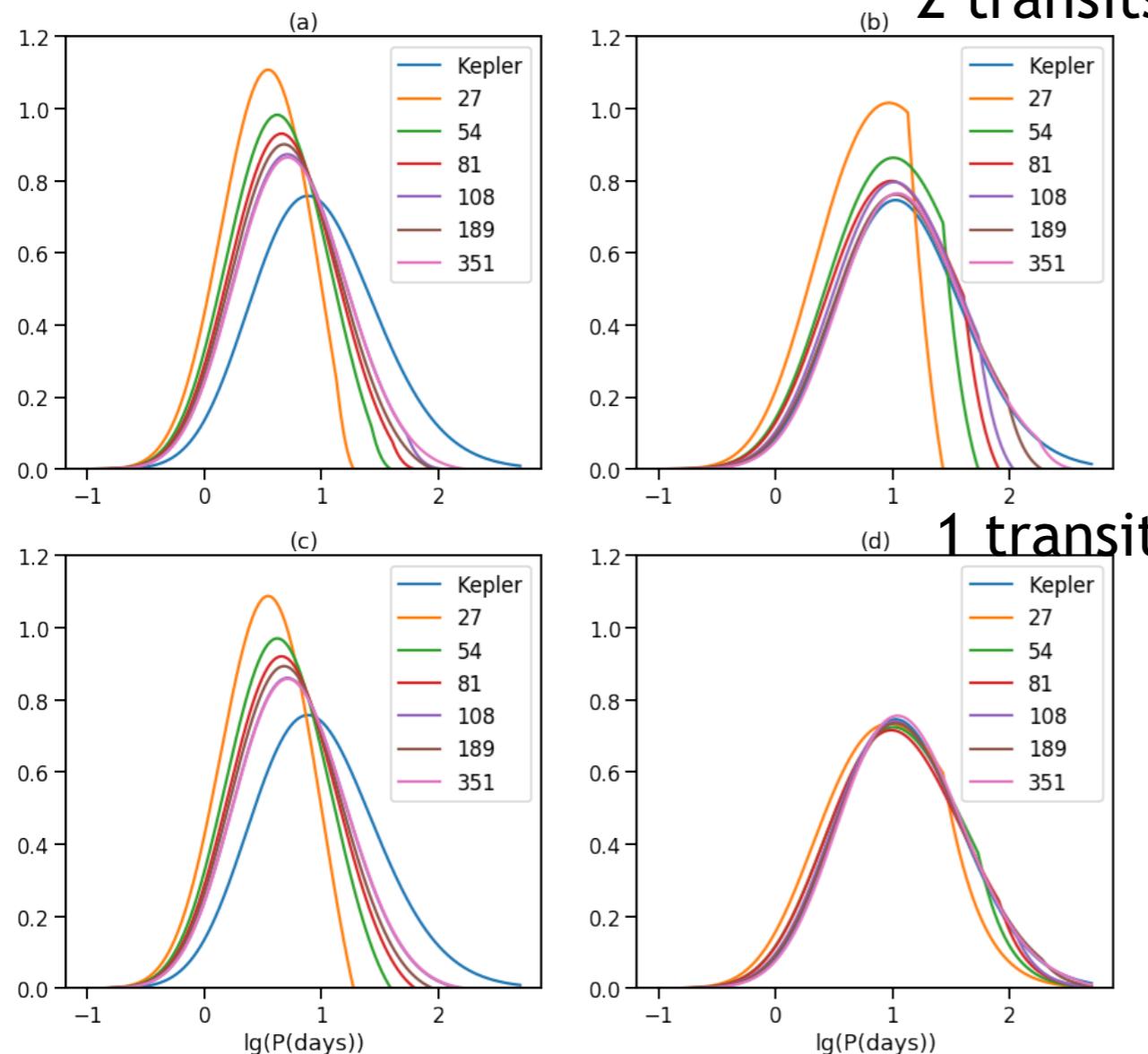
$$= \frac{N_{Pi}\left(\text{SNR}_K > \frac{\text{SNRT}_{min}}{k}\right)}{N_{Pi}(\text{SNR}_K > \text{SNRK}_{min})}$$



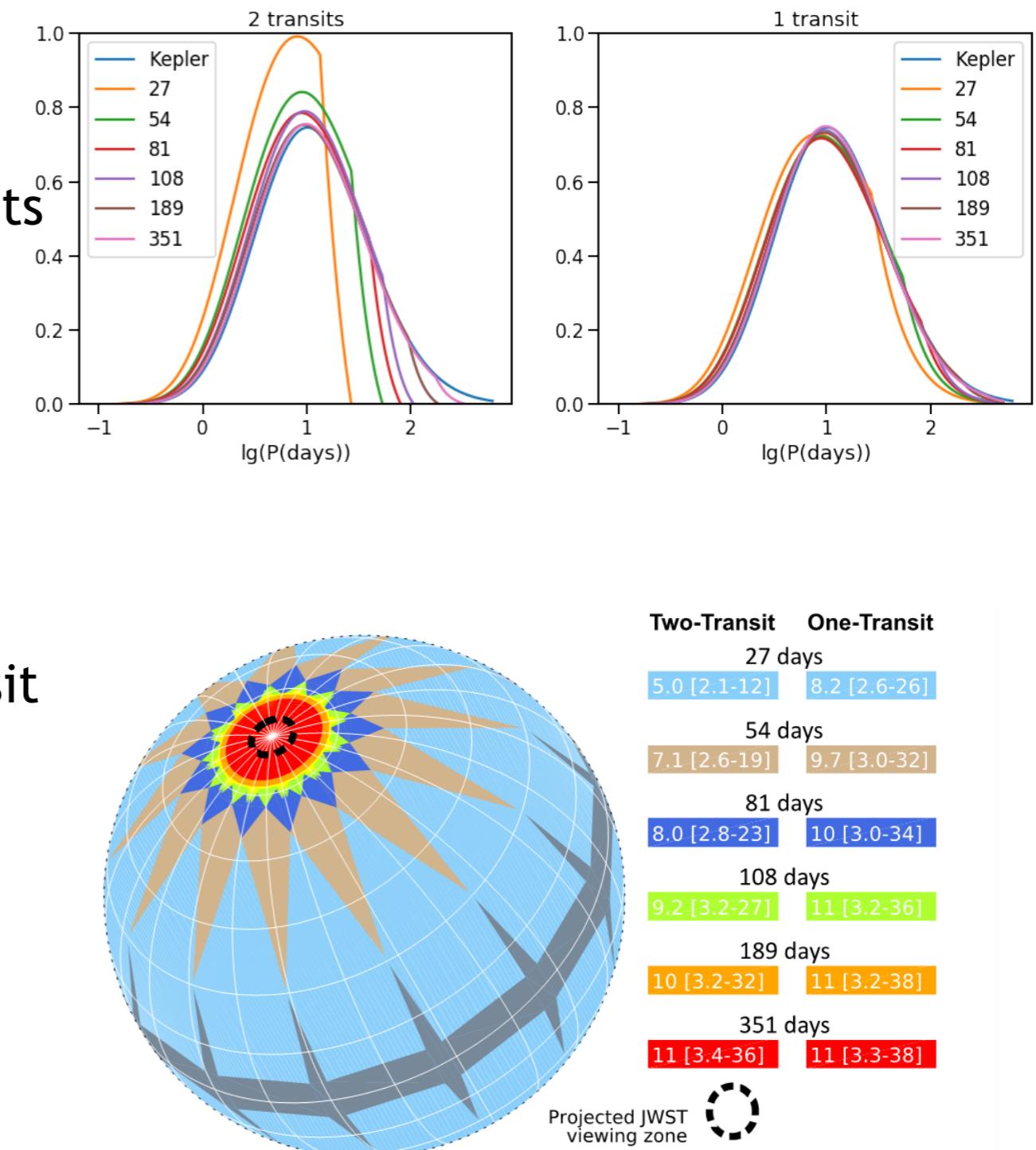
$$\text{Prob}_i(P \mid TESS) = c_i \text{Prob}_i(P \mid Kepler) \frac{\text{Prob}_i(Ntrs_T \mid P, tr) \cdot \text{Prob}_i(\text{SNR}_T > \text{SNRT}_{min} \mid P, tr, Ntrs_T)}{\text{Prob}_i(Ntrs_K \mid P, tr) \cdot \text{Prob}_i(\text{SNR}_K > \text{SNRK}_{min} \mid P, tr, Ntrs_K)}$$

# Results

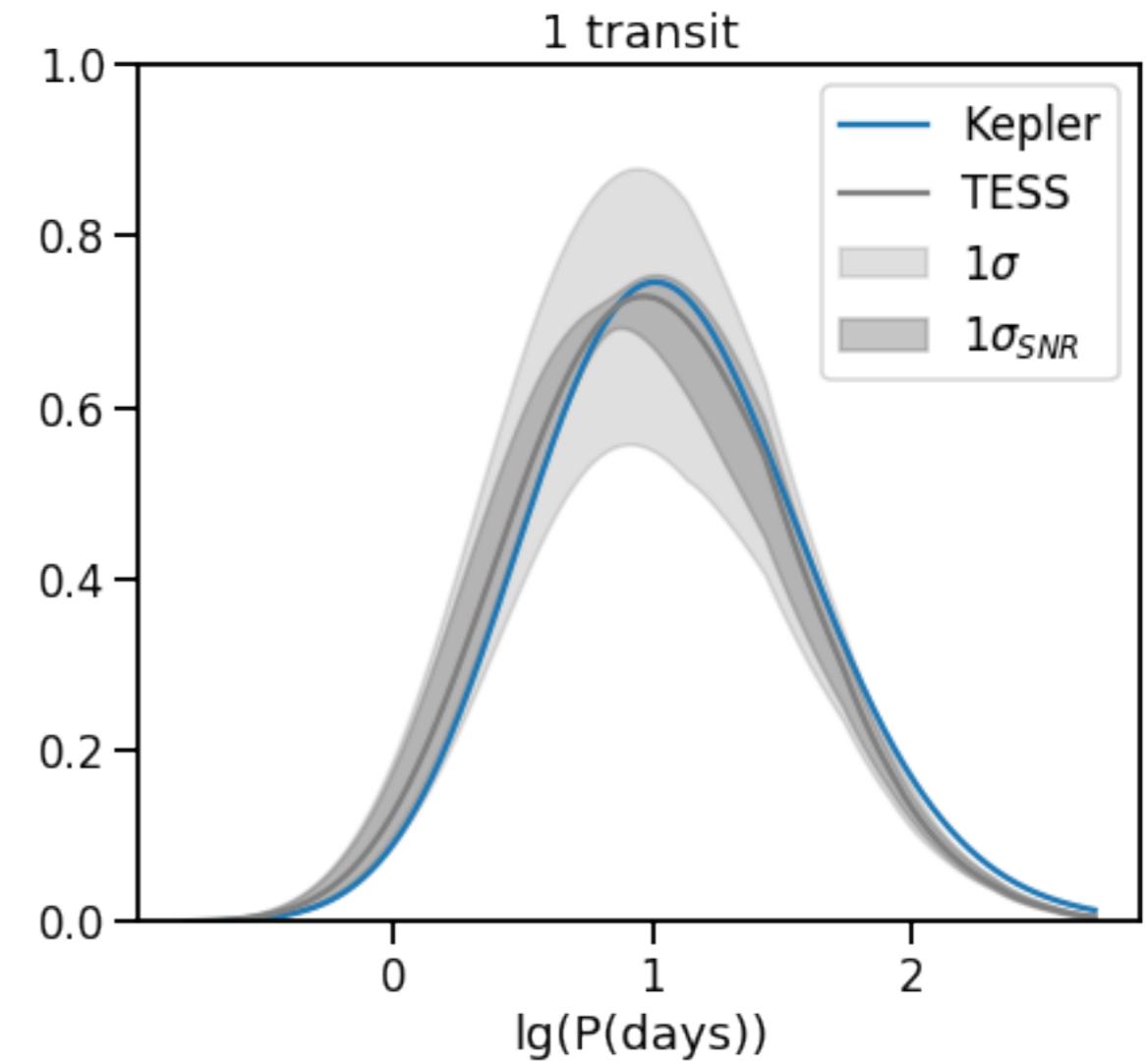
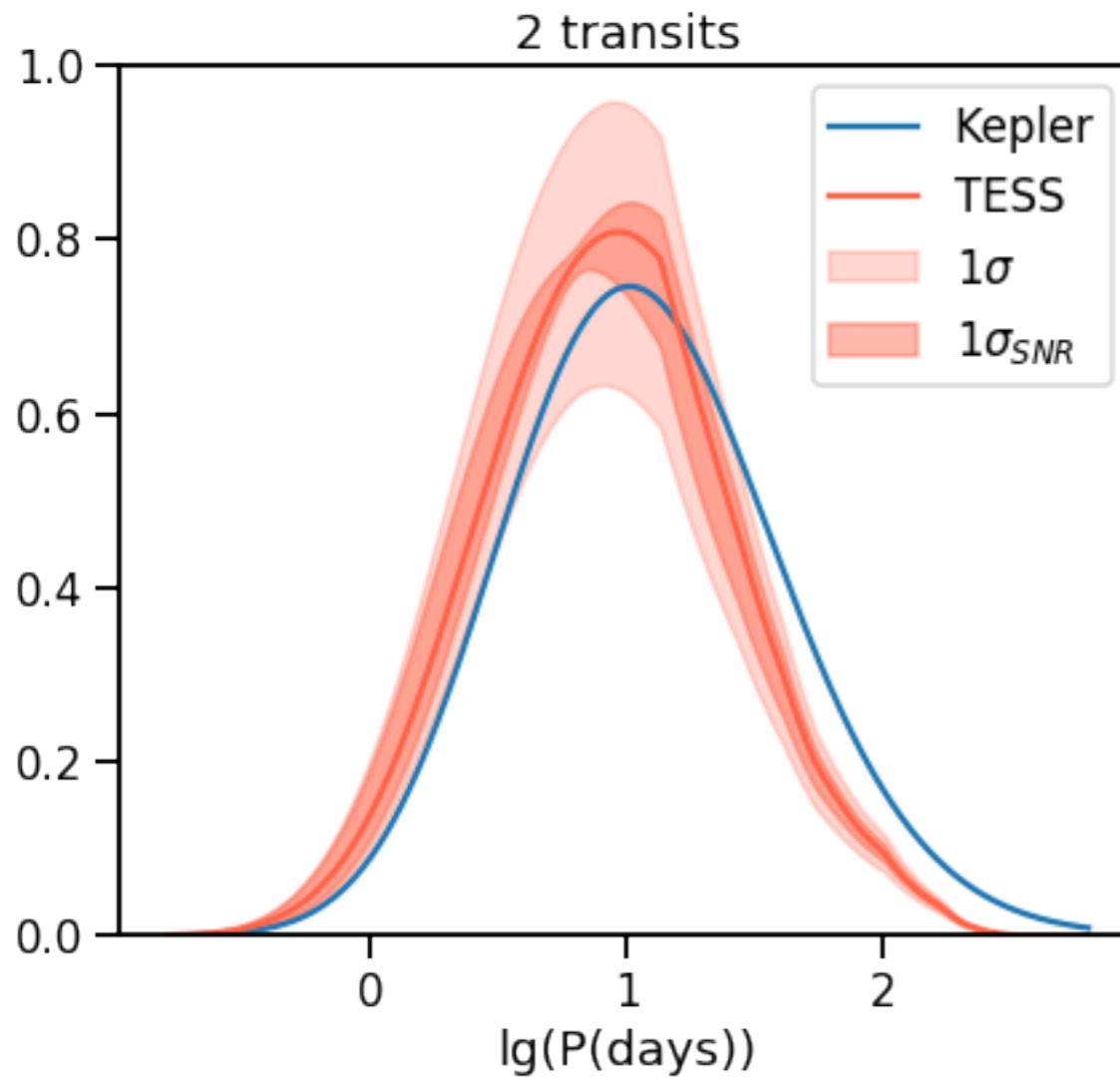
## 1. Results of two substitution samples for different observation baseline



## 2. Results of different observation baseline



# Results

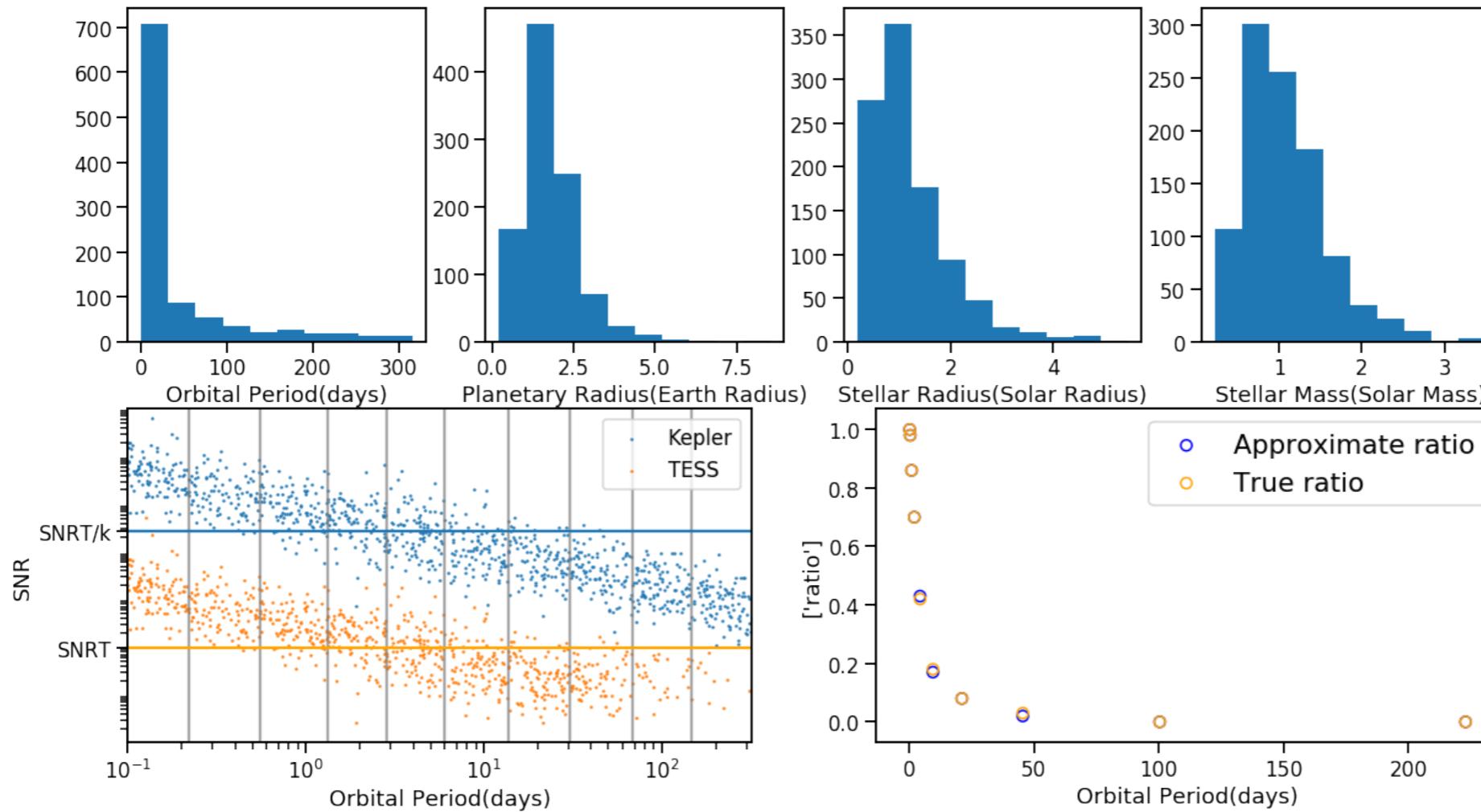


| Duration of Observation (days) | TESS       | Kepler     |
|--------------------------------|------------|------------|
| MP(days) - 2 transits          | 8.47       | 11.89      |
| 1sigma(days) - 2 transits      | 2.75-26.12 | 3.45-41.04 |
| MP(days) - 1 transit           | 10.09      | -          |
| 1sigma(days) - 1 transit       | 2.99-34.08 | -          |

# Uncertainty

- Uncertainty of approximating  $N_{trs}$

$$\text{SNR} = R_p^2 \left( \frac{4\pi^2 P}{GM_*} \right)^{\frac{1}{6}} \sqrt{\frac{N_{trs} A}{4R_* r^2} \int_{\lambda_1}^{\lambda_2} \tau \pi B(\lambda, T_*) \left( \frac{\lambda}{hc} \right) d\lambda}$$



# Uncertainty

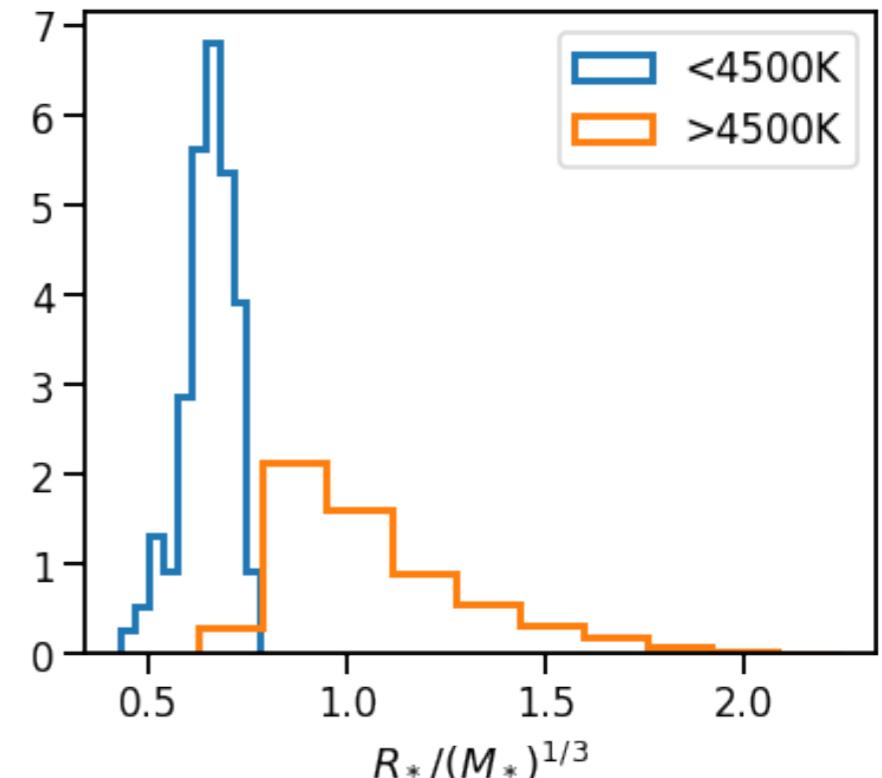
## Uncertainty of Stellar Parameters

$$\text{Prob}_i(P|TESS) = c_i \text{Prob}_i(P|Kepler) \cdot \frac{\text{Prob}(tr|P)}{\text{Prob}(tr|P)} \cdot \frac{\text{Prob}_i(Ntrs_T|P,tr)}{\text{Prob}_i(Ntrs_K|P,tr)} \cdot \dots \frac{\text{Prob}_i(\text{SNR}_T > \text{SNR}_{T_{min}}|P,tr)}{\text{Prob}_i(\text{SNR}_K > \text{SNR}_{K_{min}}|P,tr)}$$

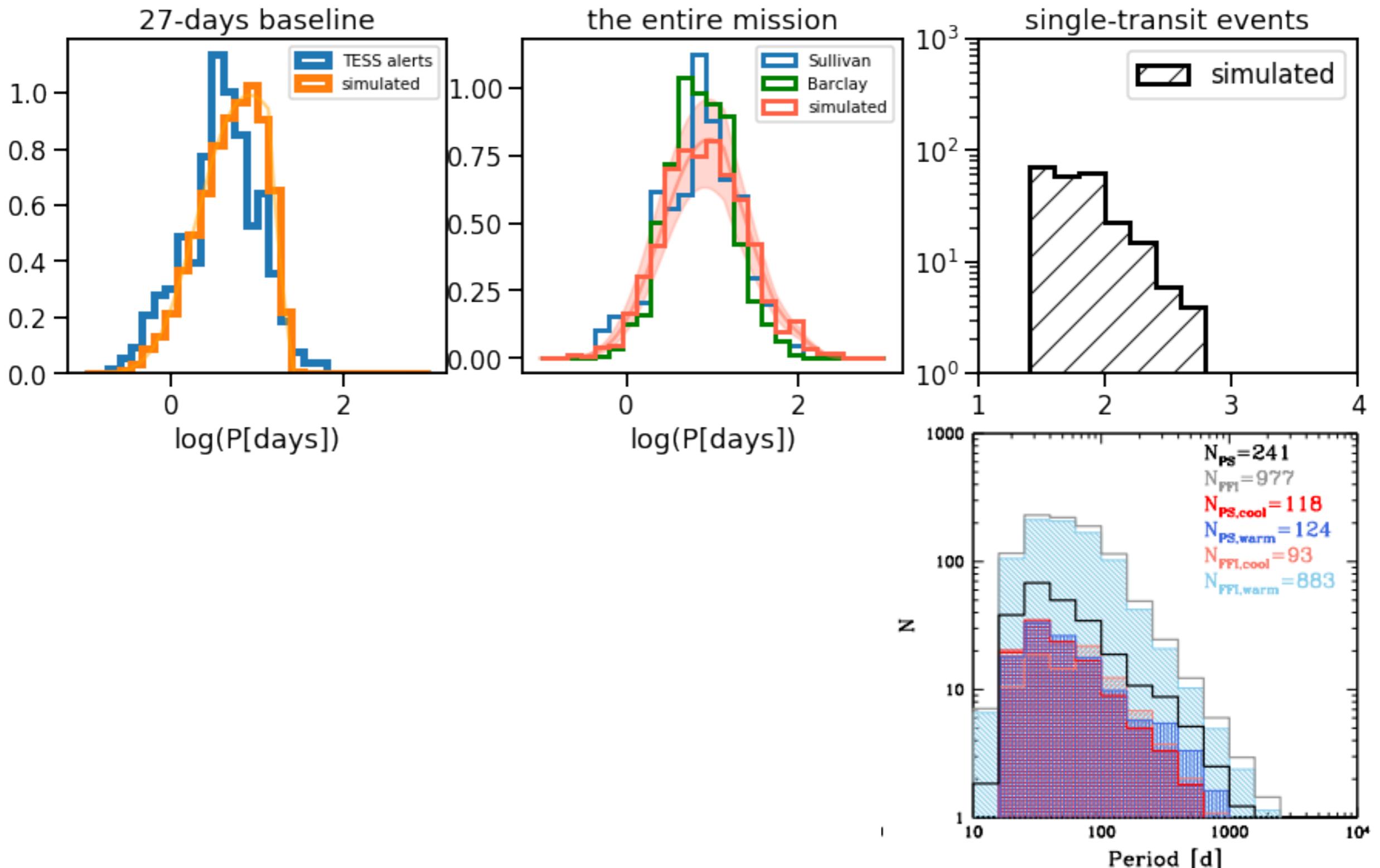
- Uncertainty of SNR model

$$\text{SNR} = R_p^2 P^{-\frac{1}{3}} \left( \frac{4\pi^2}{GM_*} \right)^{\frac{1}{6}} \sqrt{\frac{At_m}{4R_*r^2} \int_{\lambda_1}^{\lambda_2} \tau \pi B(\lambda, T_*) \left( \frac{\lambda}{hc} \right) d\lambda},$$

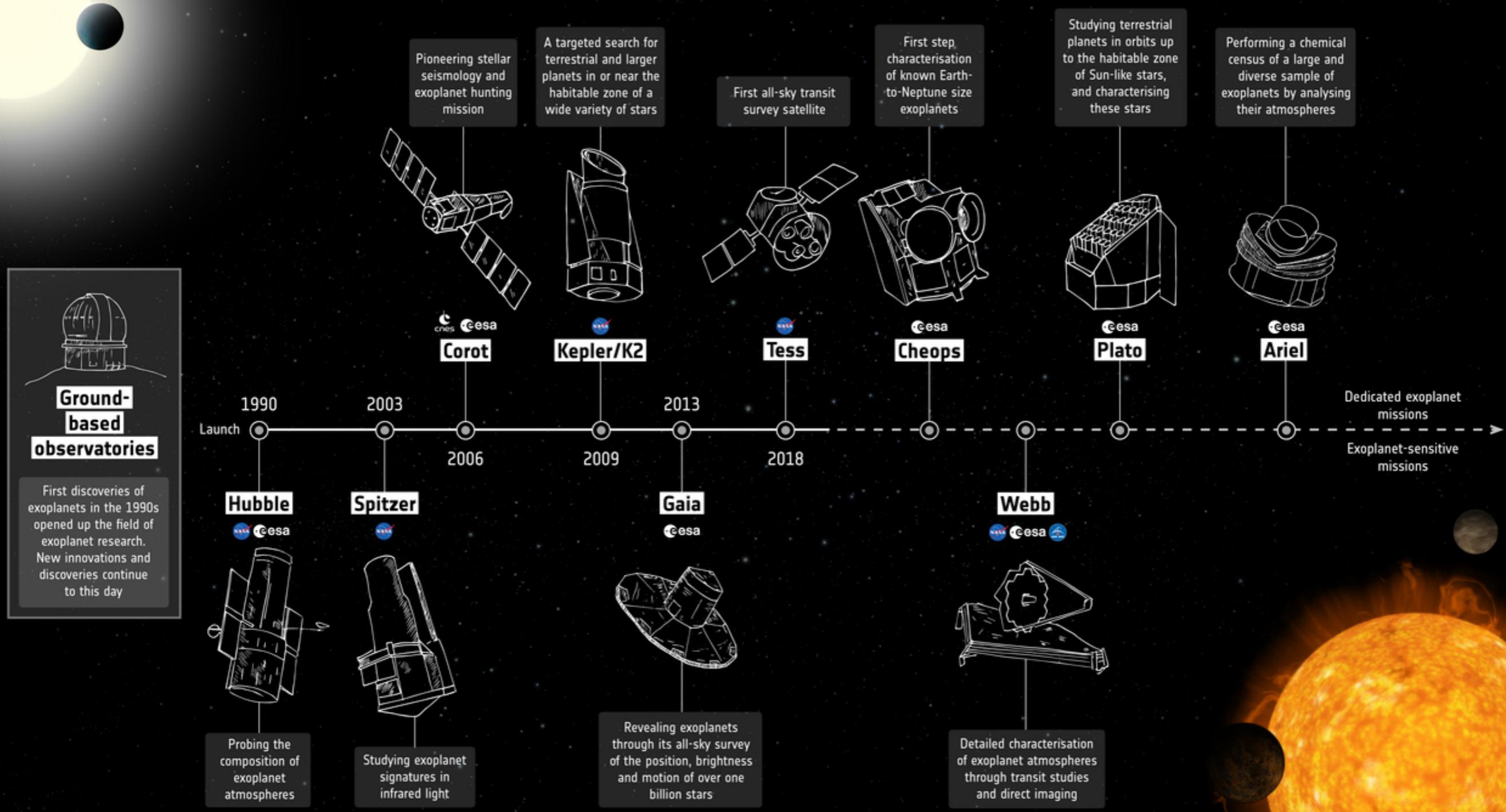
$$h(M, T_*) = \sqrt{\frac{At_m}{r^2} \int_{\lambda_1}^{\lambda_2} \tau \pi B(\lambda, T_*) \left( \frac{\lambda}{hc} \right) d\lambda} \quad 1000 \text{ times}$$



# Comparison



# Bright Future!



Any Questions?