

# *Exoplanet Research*

*Empirical Predictions for the Period Distribution of Planets to be  
Discovered by TESS*

*Speaker: Xuan Ji*

## ■ *Introduction to exoplanets*

## ■ *Methods of detecting exoplanets*

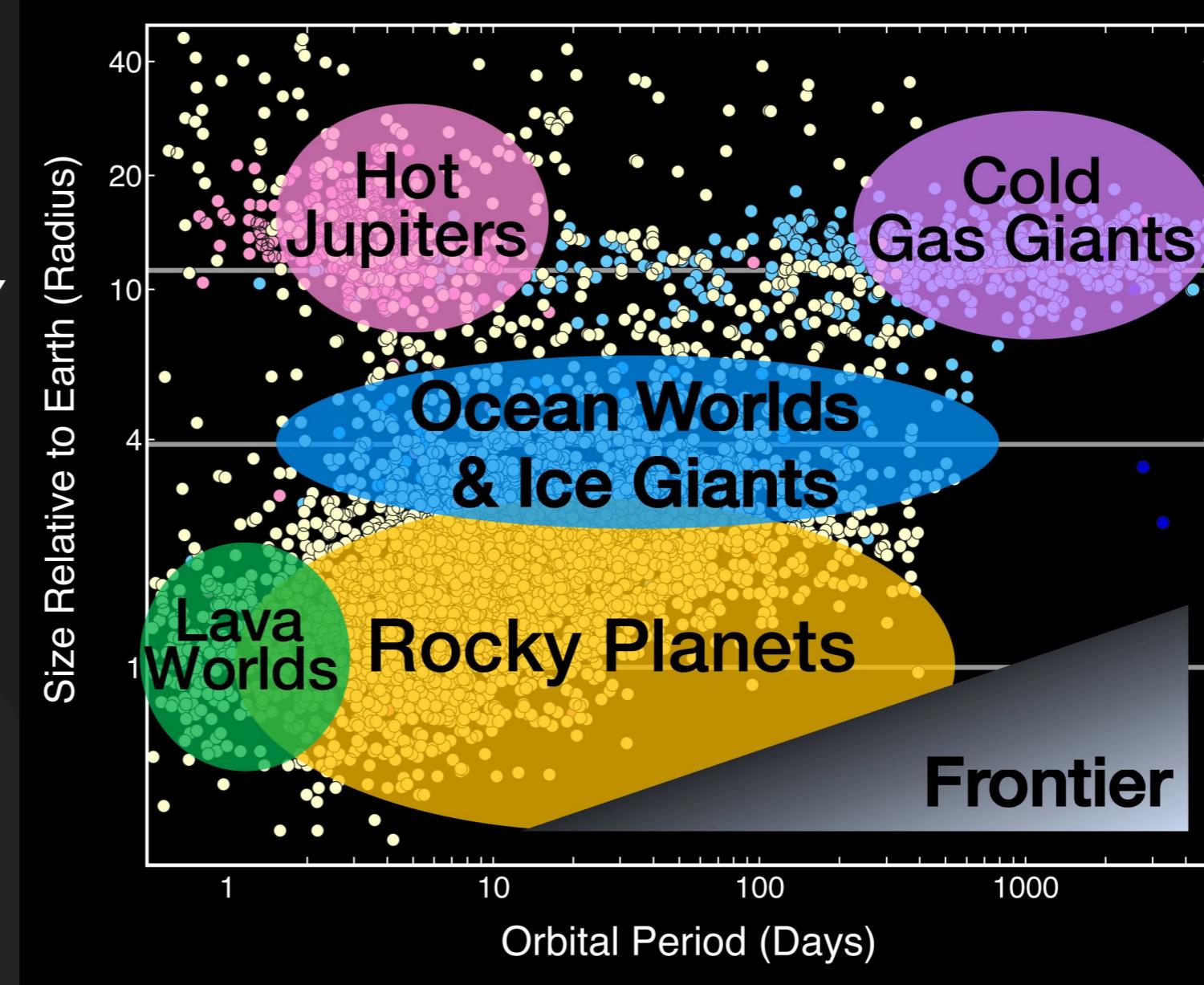
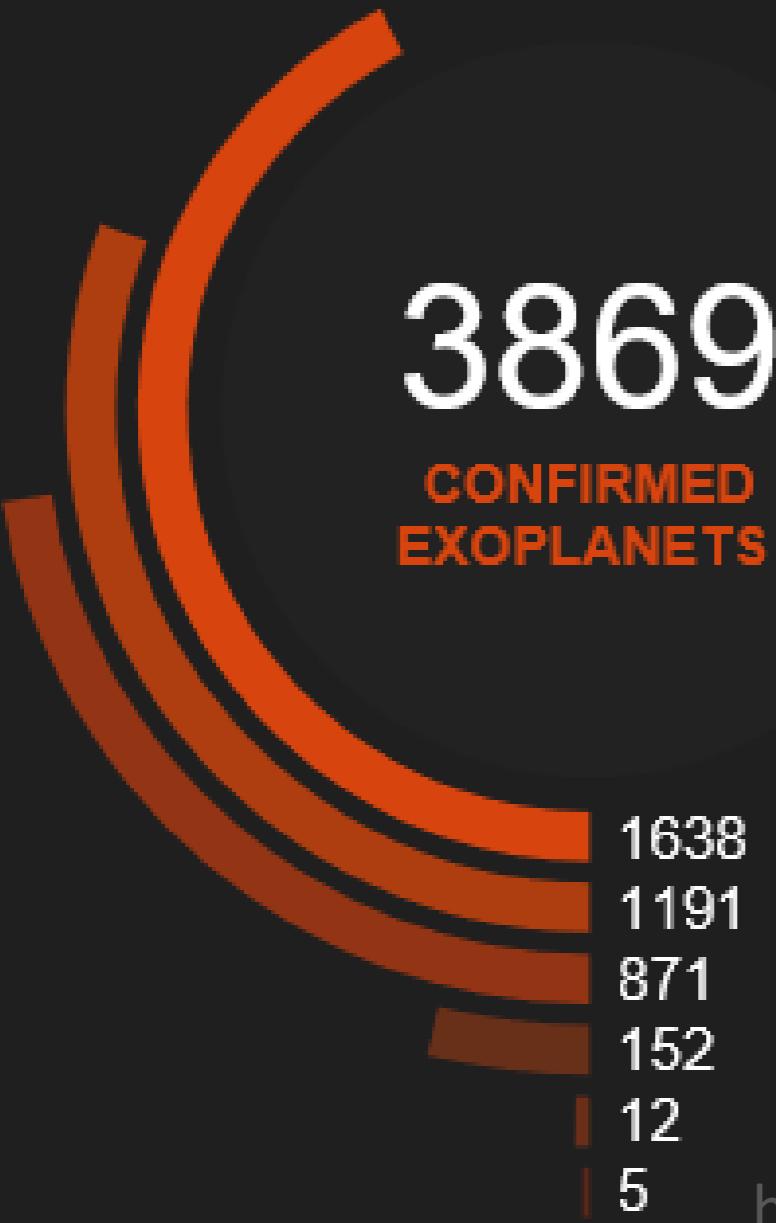
## ■ *My project*

✓ *Abstract*

✓ *Methodology*

✓ *Results and Summary*

# Exoplanets Demography

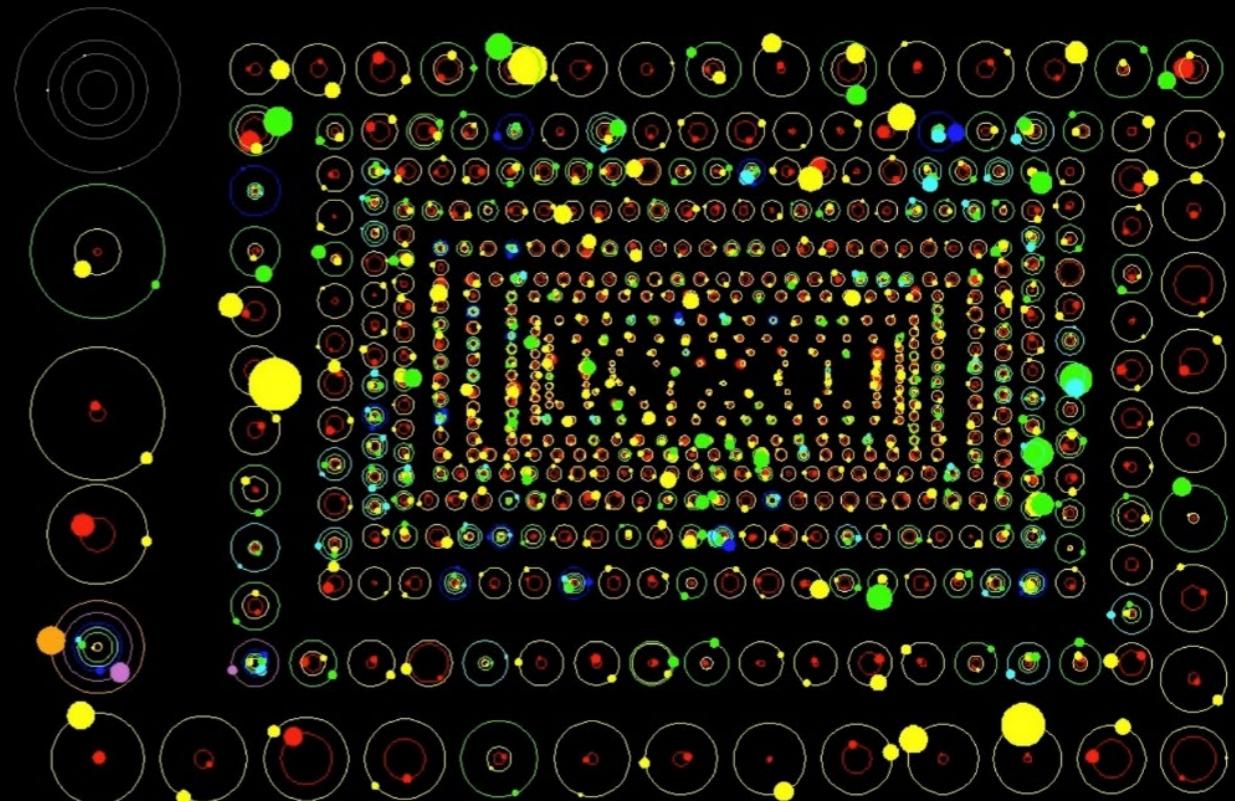
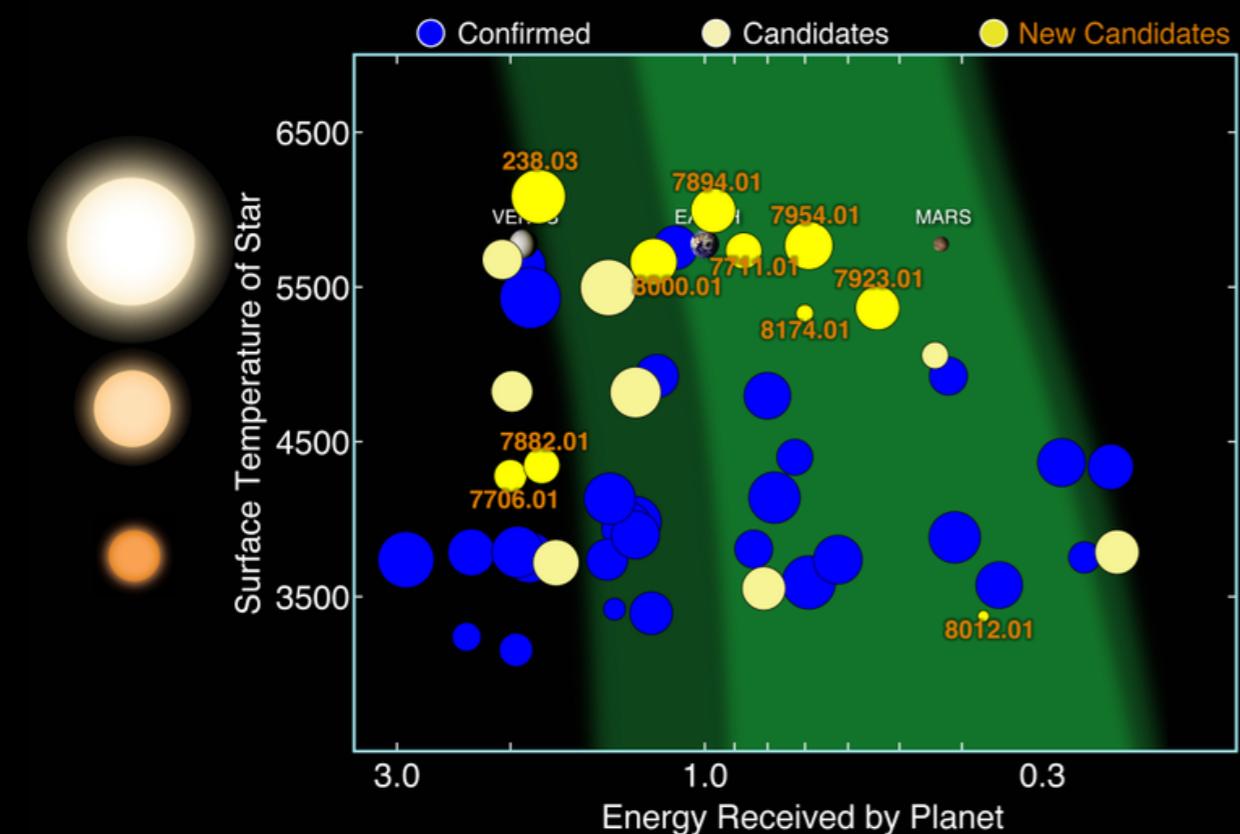


NASA/Ames Research Center/Natalie  
Batalha/Wendy Stenzel

# *What can we learn from exoplanets?*

- 1 ) *Life outside the Solar System*

- 2 ) *planetary formation and evolution*



NASA/Ames Research Center/Wendy Stenzel

NASA/Kepler/Dan Fabrycky



77.8% Transit



18.2% Radial Velocity



1.9% Microlensing



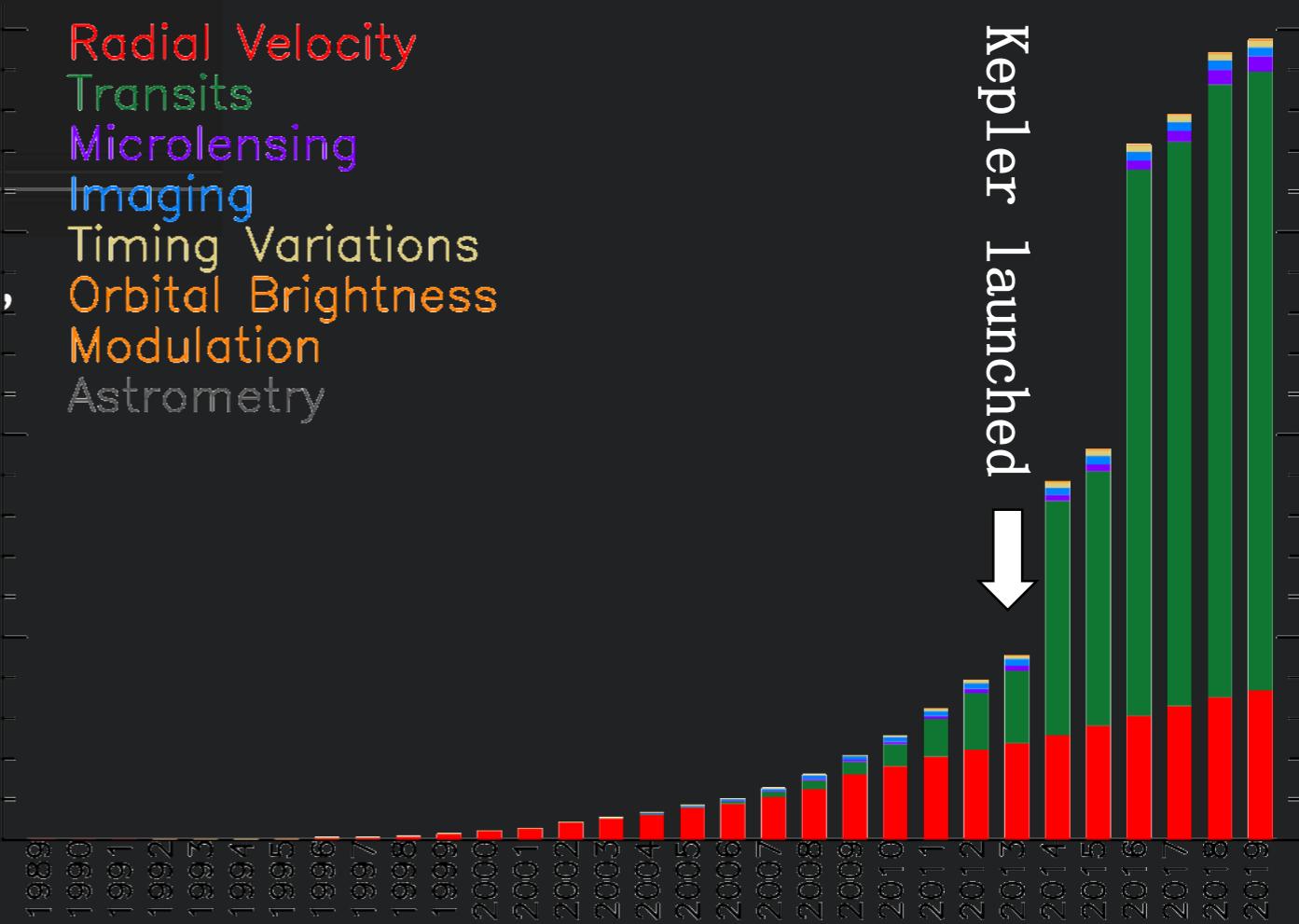
1.1% Imaging

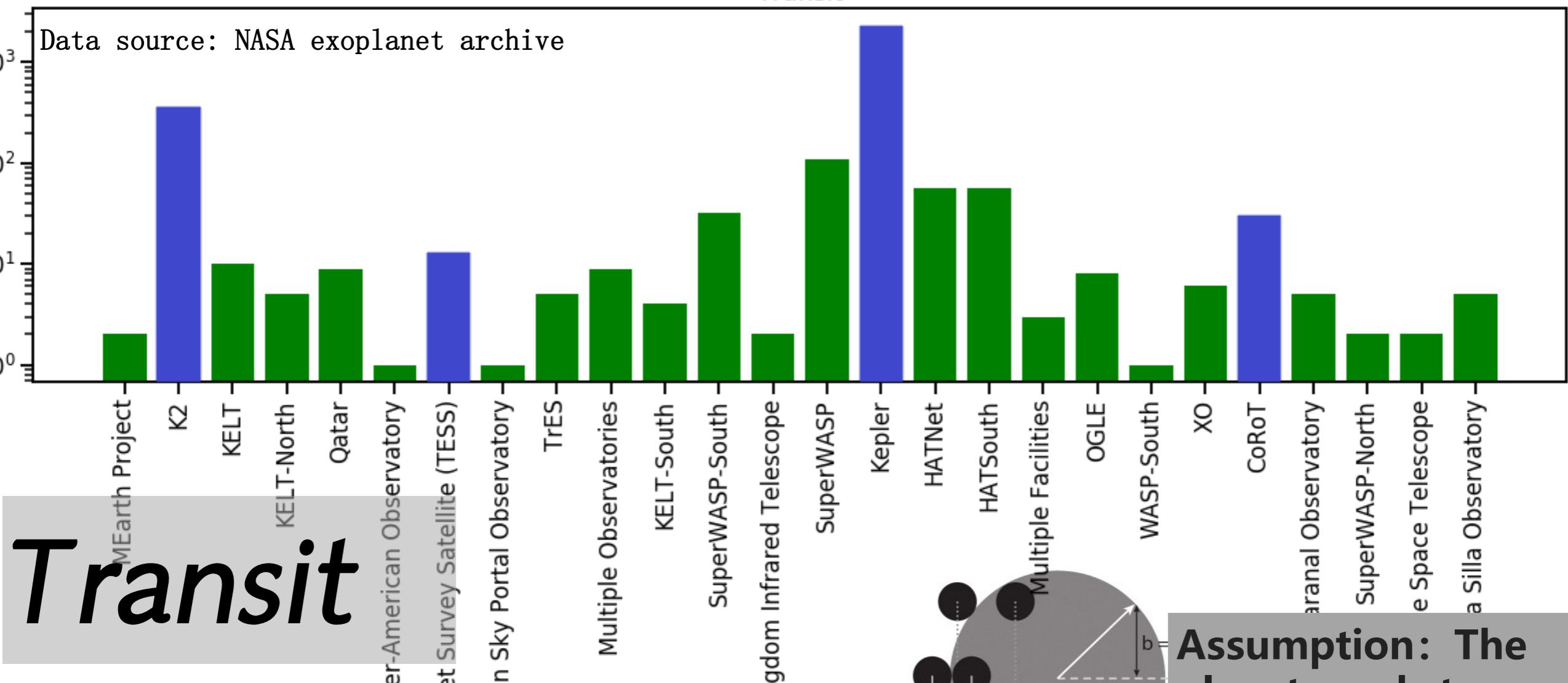
- 0.16% Orbital Brightness Modulation,
- 0.03% Astrometry
- 0.39% Transit Timing Variations,
- 0.23% Eclipse Timing Variations,
- 0.16% Pulsar Timing,
- 0.05% Pulsation Timing Variations,

Radial Velocity  
Transits  
Microlensing  
Imaging  
Timing Variations  
Orbital Brightness  
Modulation  
Astrometry

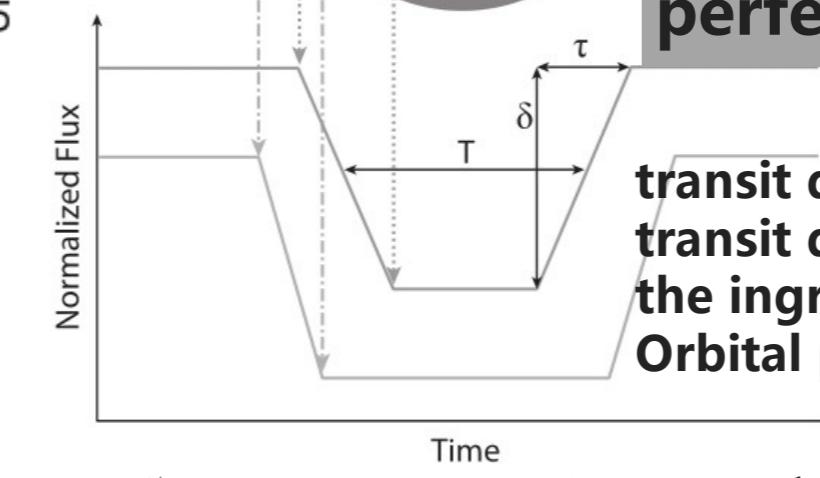
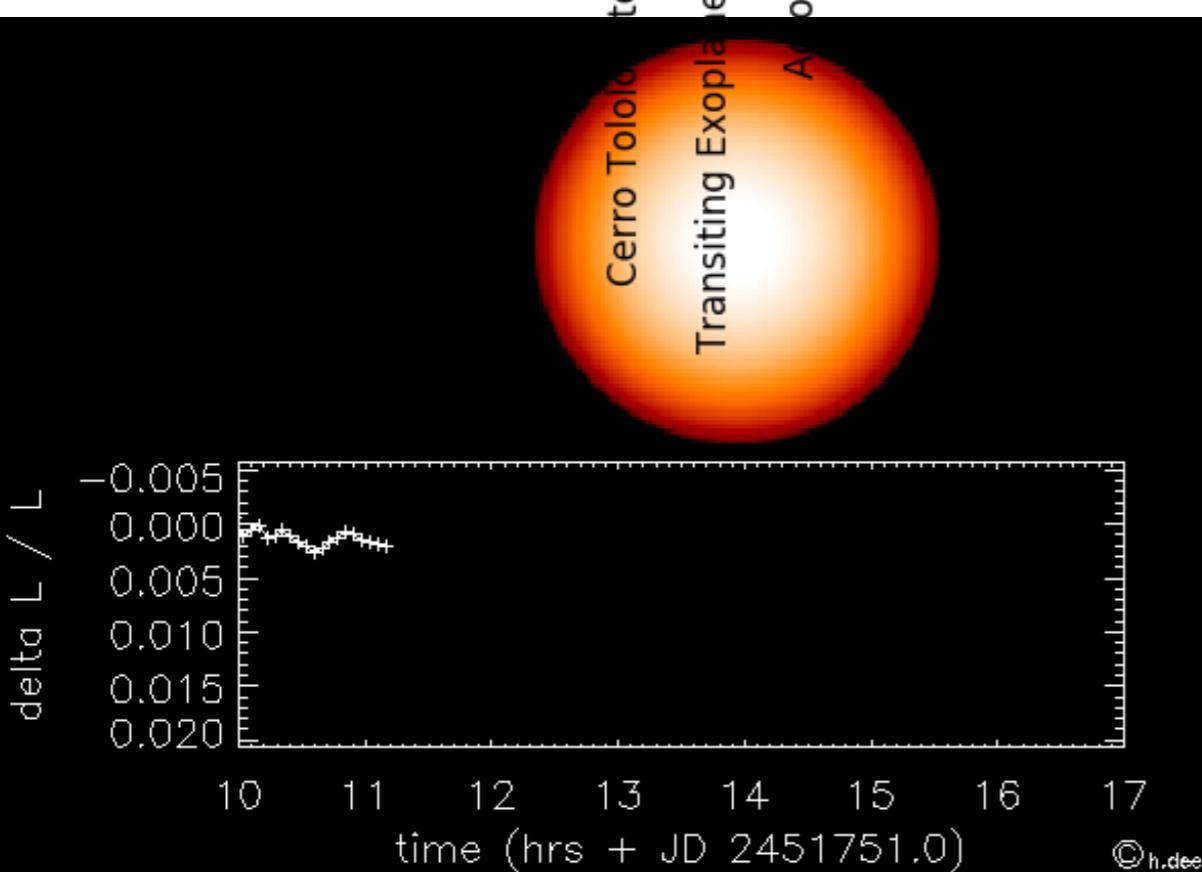


In 1995, Mayor and Queloz discovered the 1<sup>st</sup> exoplanet orbiting a sun-like star. Haute-Provence Observatory, France





# Transit



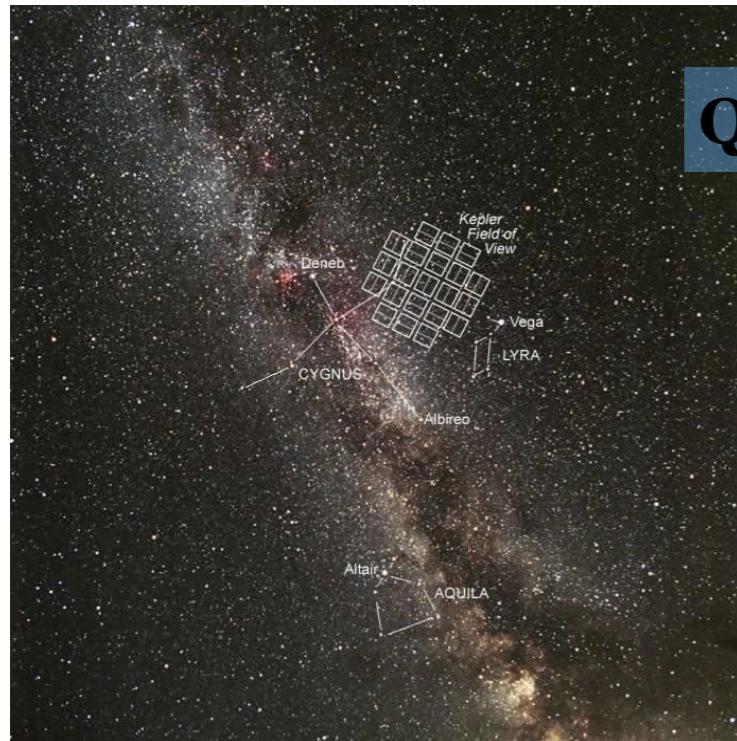
**Assumption:** The planets and stars are spherical; The orbit is perfect circle

transit depth ( $\delta$ )  
transit duration (T),  
the ingress/egress duration ( $\tau$ ),  
Orbital period(P)

Semi-major axis, stellar mass, stellar radius, planetary radius, eccentricity, inclination, ~~planetary mass~~

# Transit—Kepler

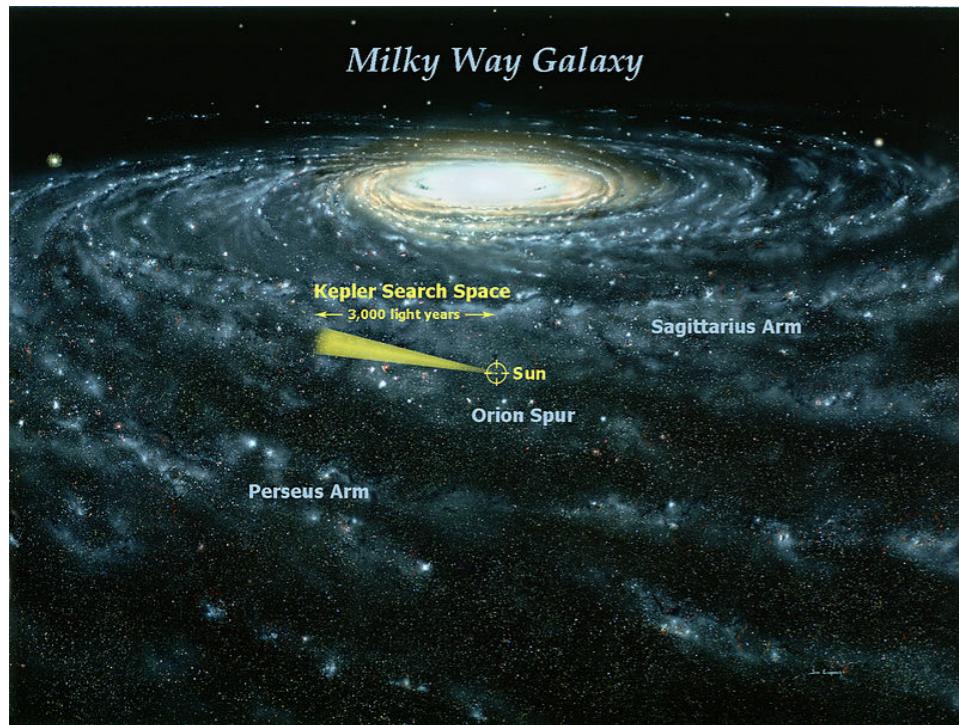
## Observing strategy:



### Quarterly rolls

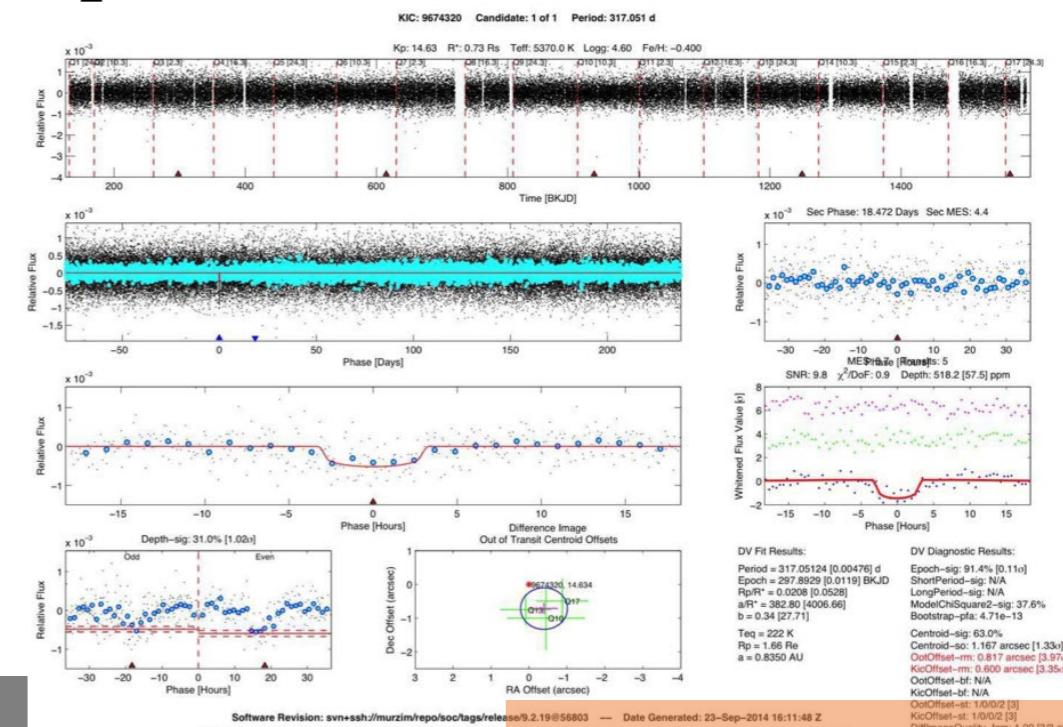
- 115 square degree
- 0.25% full sky
- 400 Kepler can cover the whole sky

[https://www.jpl.nasa.gov/news/press\\_kits/Kepler-presskit-2-19-smfile.pdf](https://www.jpl.nasa.gov/news/press_kits/Kepler-presskit-2-19-smfile.pdf)

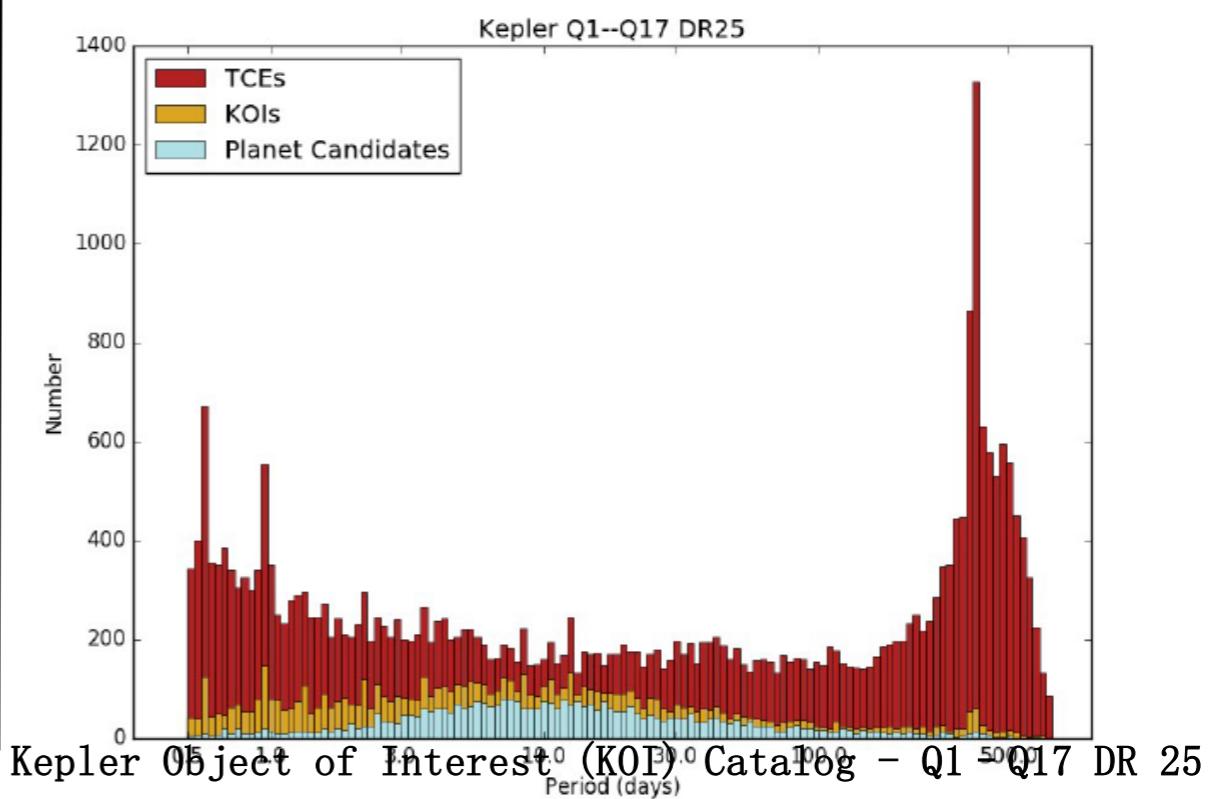


<http://kepler.nasa.gov/images/LombergA1600-full.jpeg>

## Pipeline:



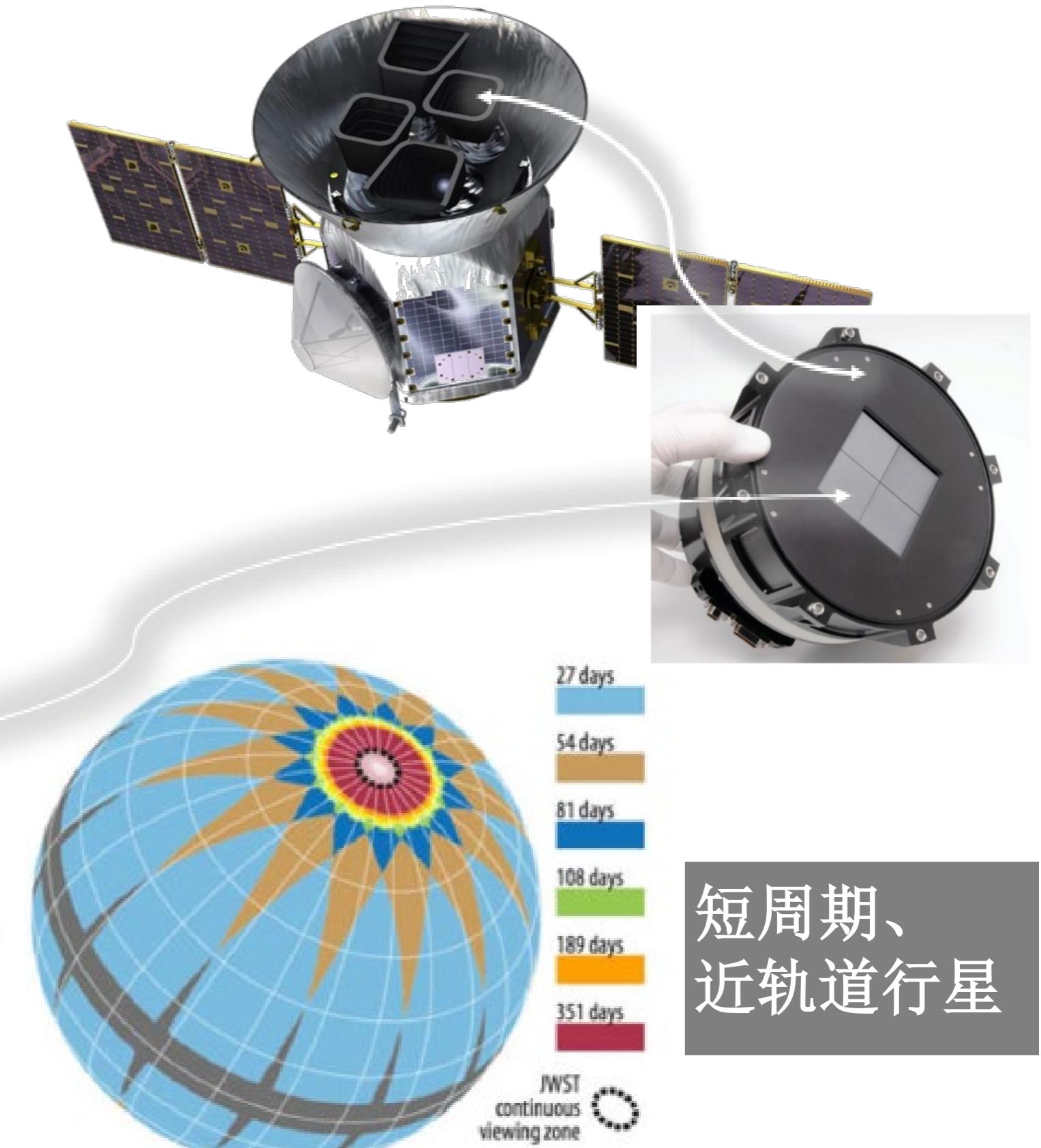
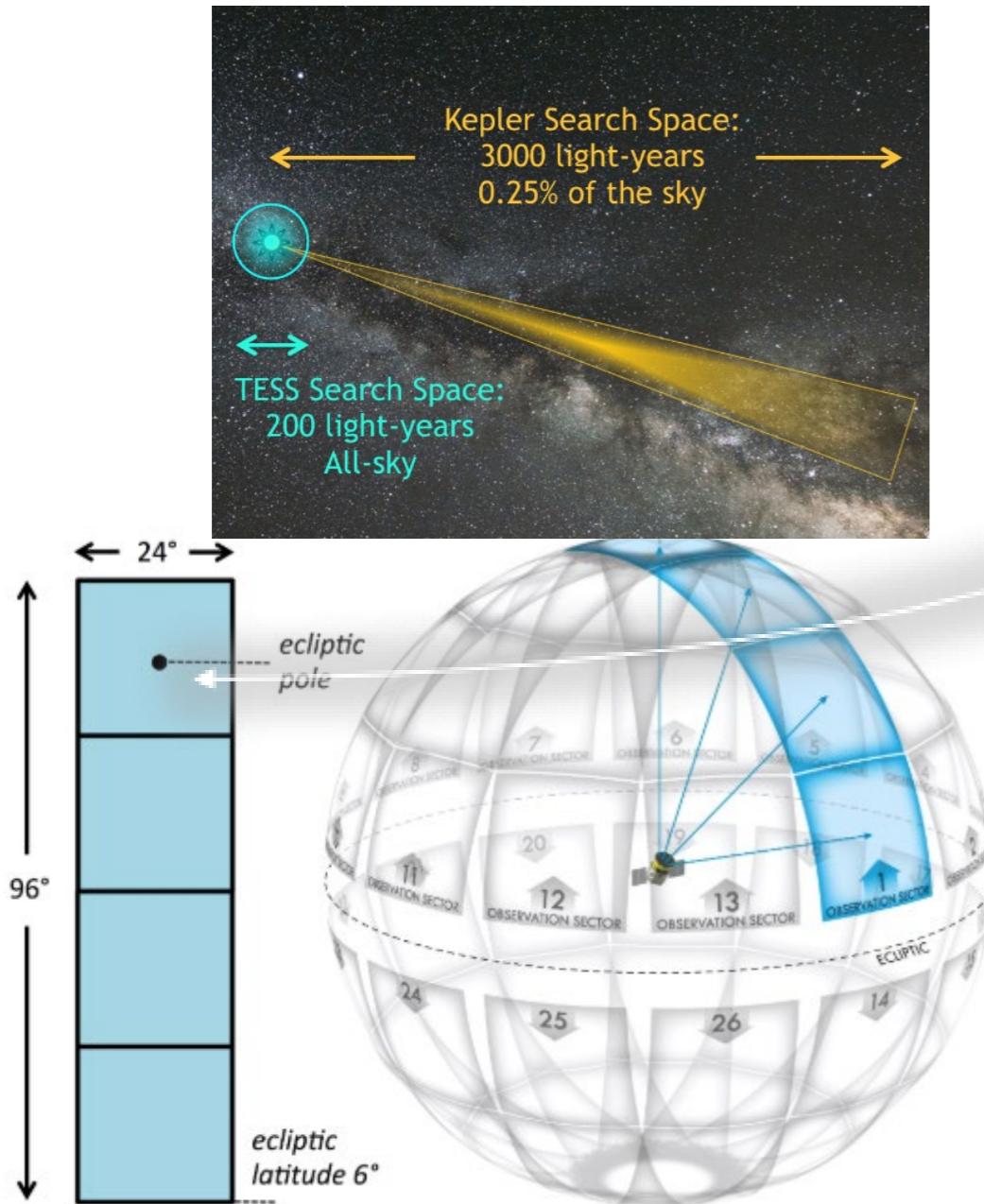
**≥3 transits!**  
Pixels → TCEs → KOIs  
→planet candidates  
→confirmed planets



# Transit—TESS

- \* 10 cm aperture
- \* Bandpass: 600 – 1100 nm
- \* 13.7–day elliptical orbit

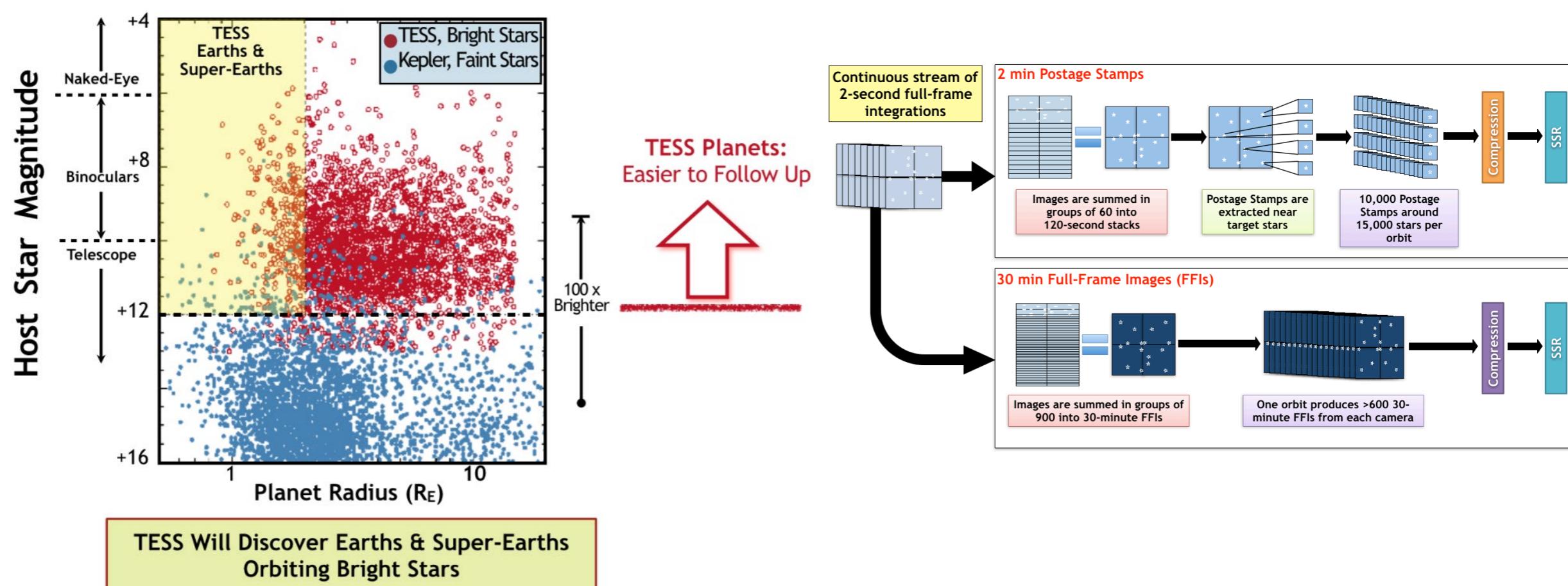
Launched April 18, 2018  
Started science operations July 25, 2018



# Transit—TESS

## Science Goal: Bright!

- \* Improve statistics for studies of the mass–radius relation of small planets as a function of distance from host stars.
- \* More temperate planets among which to select the best for atmospheric characterization with the JWST/ELTs
- \* .....



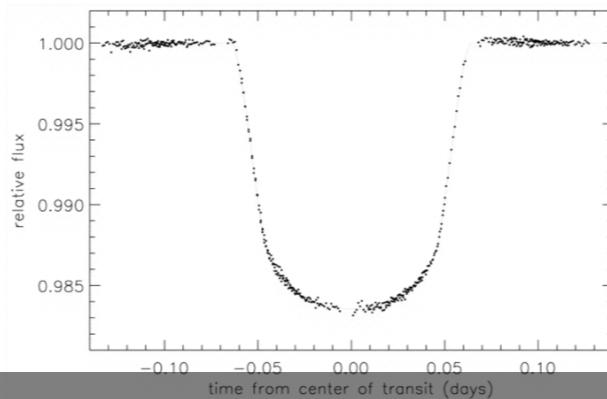
*Image Credit: MIT*

Villanueva, Dragomir & Gaudi (2019)

# Transit——TESS

## Single Transit

Stellar density +  
Eccentric



= Orbital Period

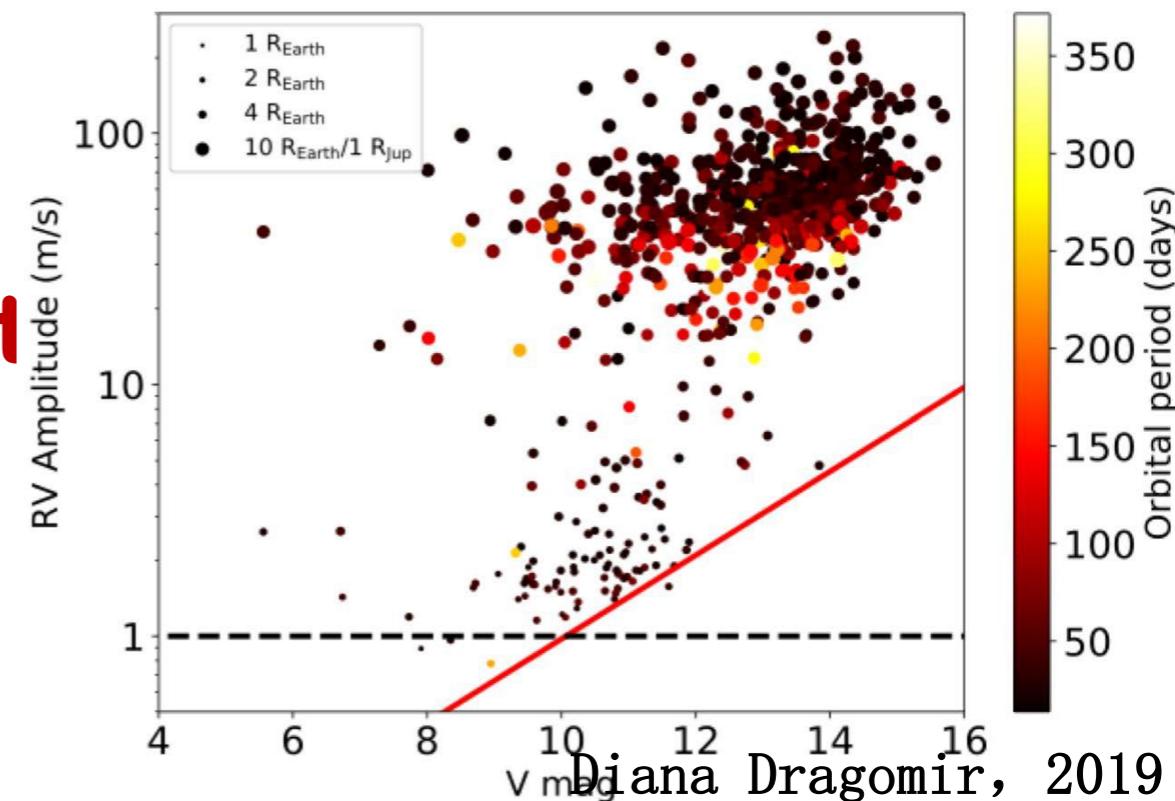
*Density:* Gaia (Radius) and spectroscopy/  
asteroseismology (spectroscopy).

*Eccentricity:* prior from known  
distribution).

Seager & Mallén-Ornelas (2003)  
Yee & Gaudi (2008)

## Radial Velocity Prospects for Single-Transit Planets

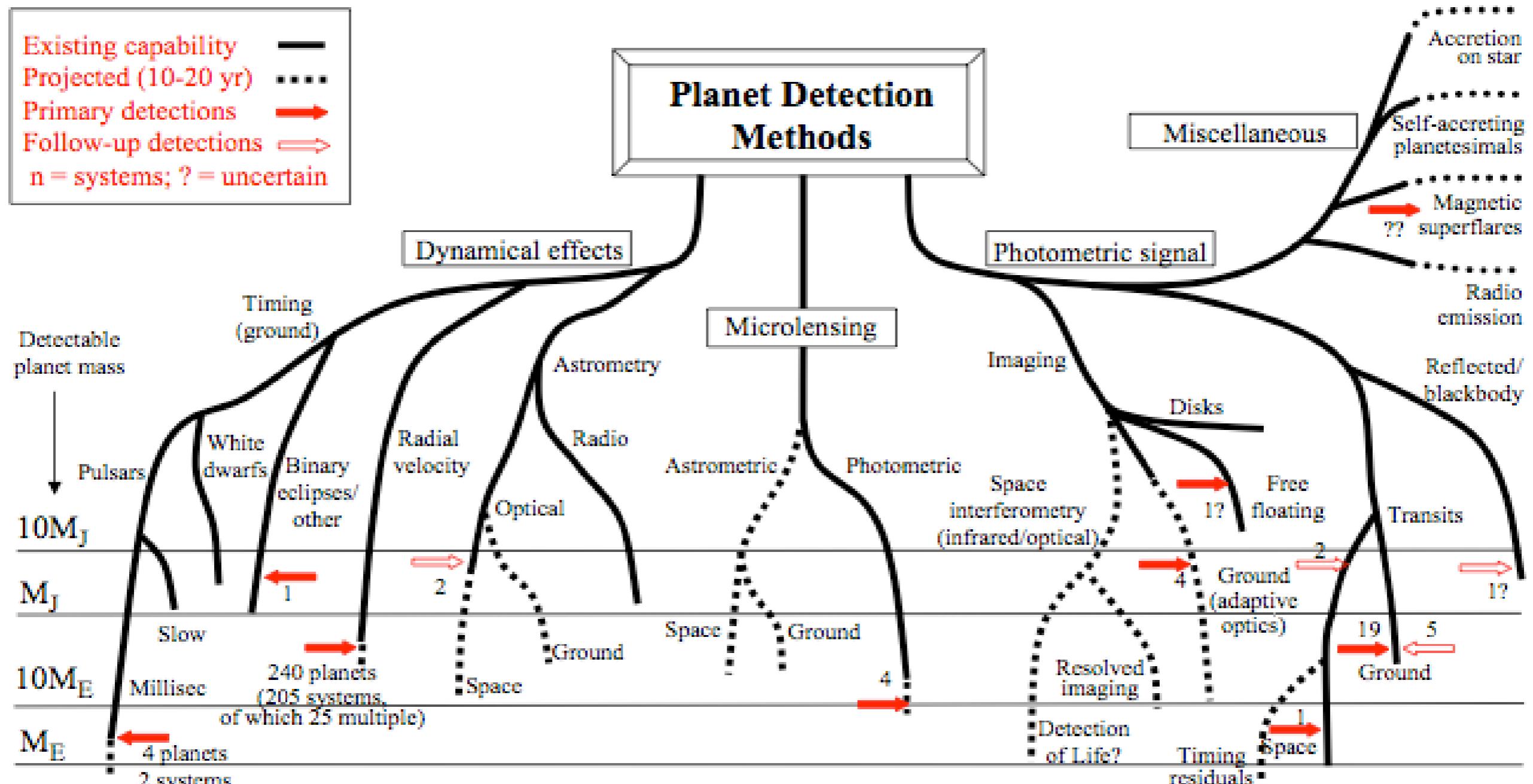
- \* If period constrained well enough
  - get photometry to catch next transit
- \* Not well enough
  - use RV measurements to improve constraint



# Detection Methods and Statistics

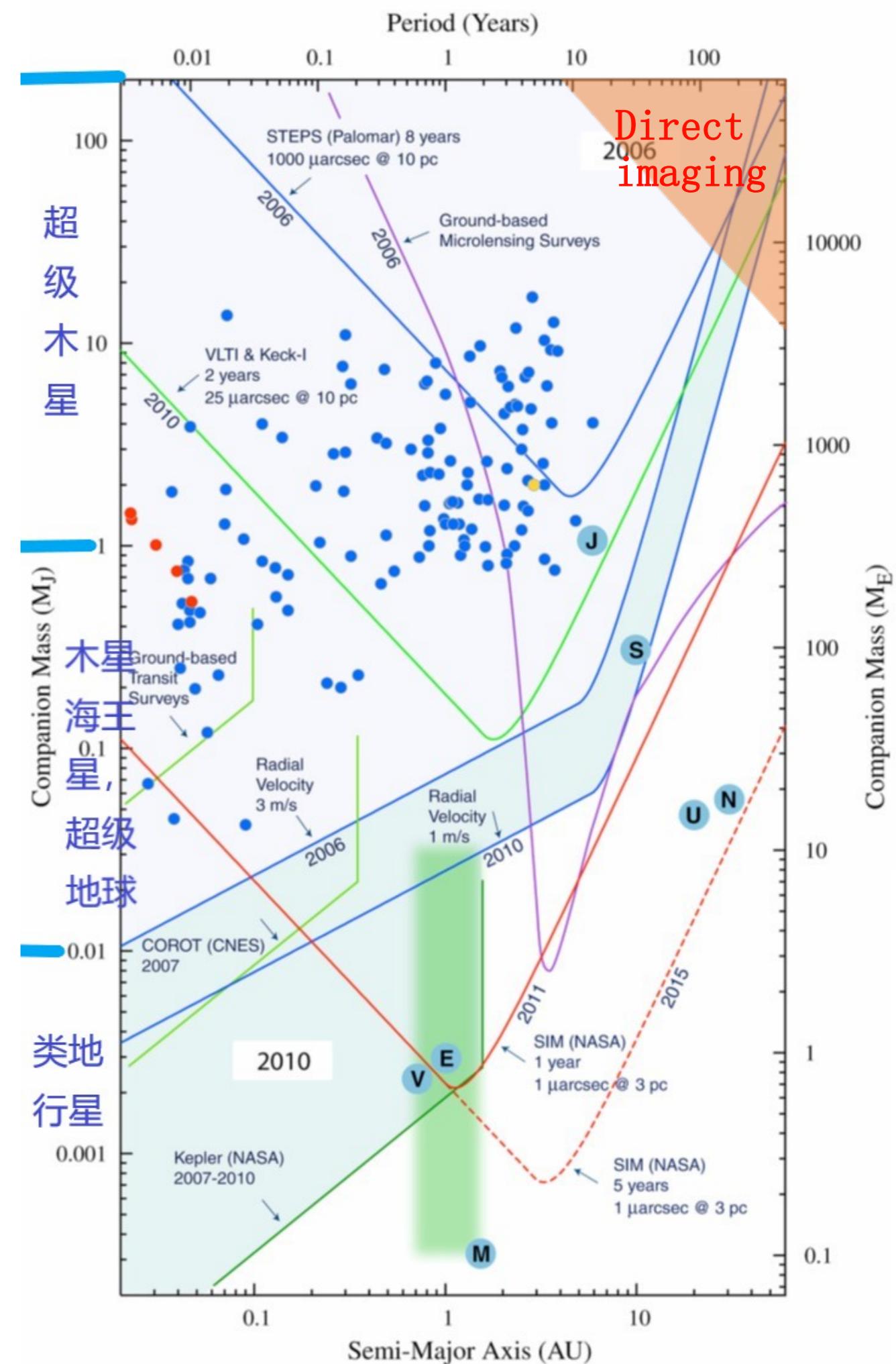
## Planet Detection Methods

Michael Perryman, Rep. Prog. Phys., 2000, 63, 1209 (updated 3 October 2007)



# Detection Method and Statistic

- \* habitable zone(green area)
- \* ~150 exoplanets detected in 2004
  - r.v. (blue)
  - transits (red)
  - microlensing (yellow)
  - pulsar timing (purple)
  - Imaging(magenta)



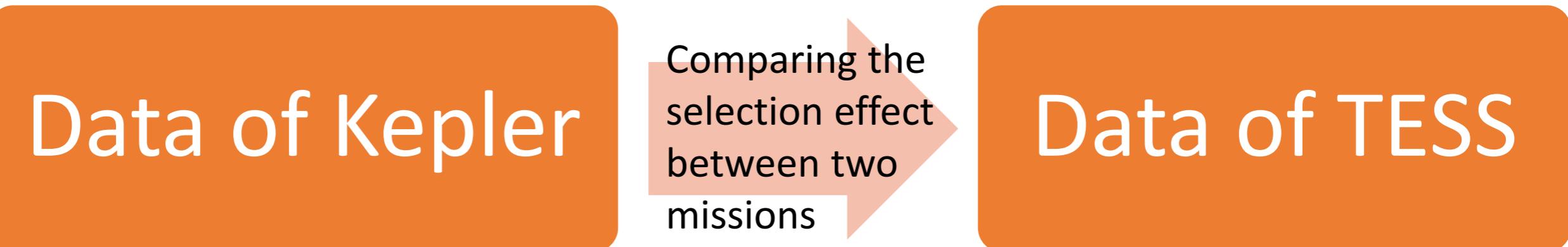
# My Project

- *Empirical Predictions for the Period Distribution of Planets to be Discovered by TESS*
- 

*Previous work :*



*My work :*



# Methodology

The probability that the orbital period of a detected exoplanet is  $P$  days :

$$\text{Prob}(P | \text{TESS}) = \frac{\text{Prob}(\text{TESS} | P) \times \text{Prob}(P)}{\text{Prob}(\text{TESS})}$$

The probability that an exoplanet of which orbital period is  $P$  days is detected by TESS

The occurrence rate of an exoplanet of which orbital period is  $P$  days

constant

$$\text{Prob}(\text{TESS} | P) = \text{Prob}(\text{Tr} | P) \times \text{Prob}(N\text{Tr}(\tau_1) | \text{Tr}, P) \times \text{Prob}(\text{SNR}_T | N\text{Tr}(\tau_1), \text{Tr}, P)$$

geometric probability of detecting a transit around a star for a fixed period

the probability of observing the transit(s) more than  $N$  times during the finite observing baseline of observations for TESS for a fixed period, given that the transit is detected

the probability that the signal-to-noise ratio (SNR) of the exoplanet is higher than the threshold given that it transits at least  $N$  times over the course of the observations.

# Methodology

TESS : CTL  
 (dwarfs and subgiants )  
**Input Catalogue**

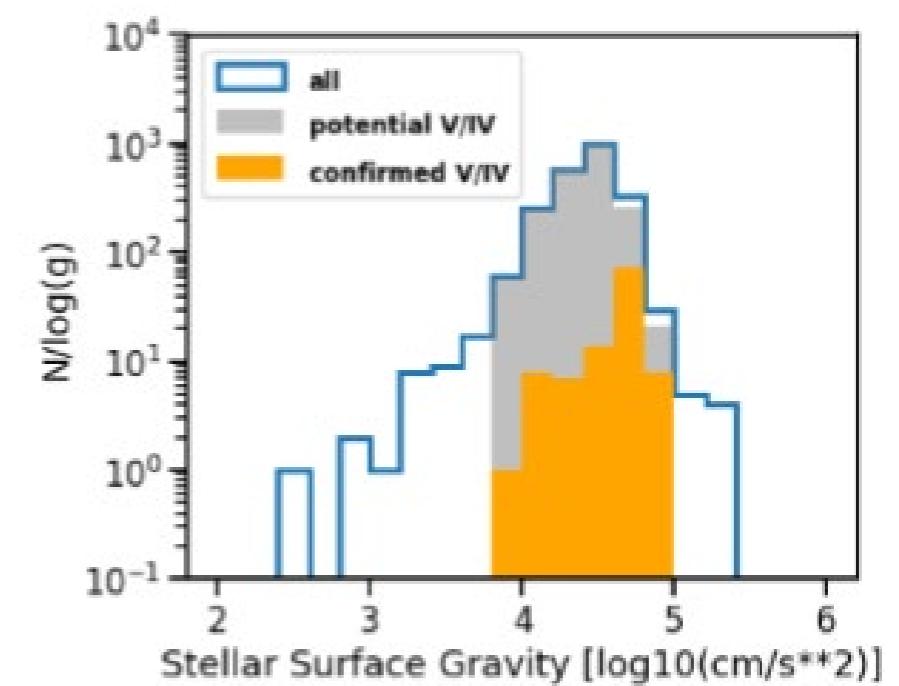
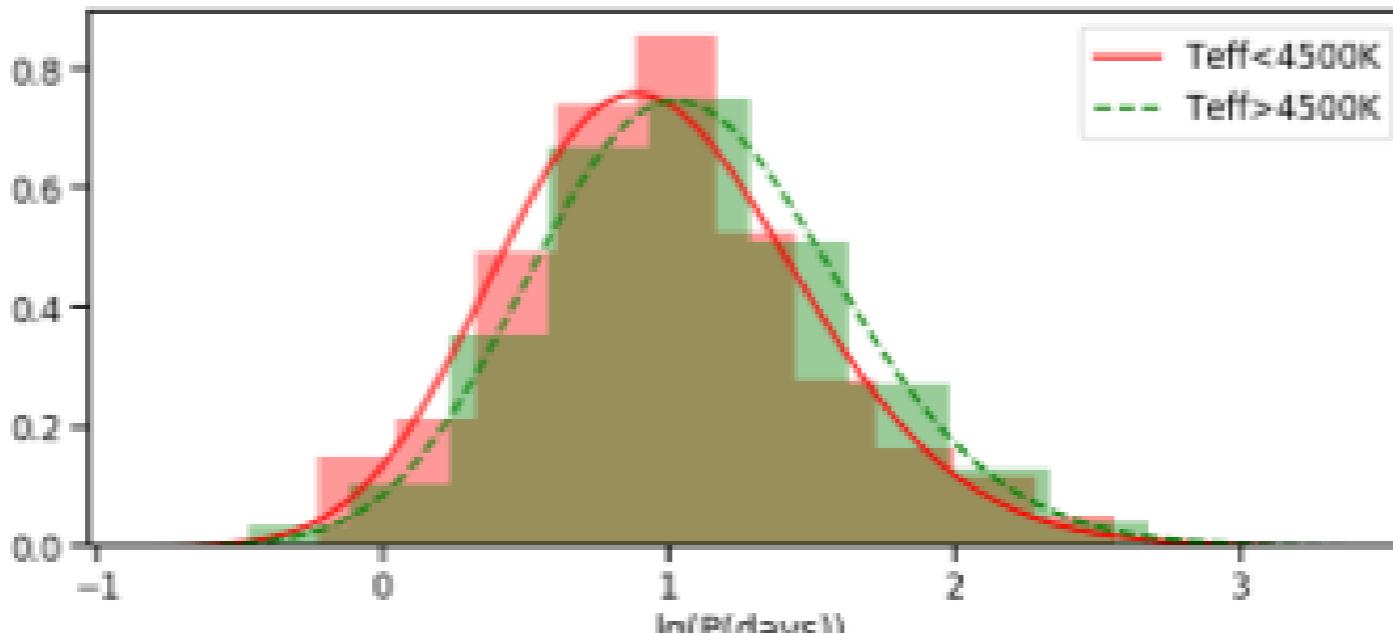
$$\text{Prob}(P|\text{TESS})$$

$$= \frac{\text{Prob}(tr|P) \cdot \text{Prob}(Ntrs_T|P, tr) \cdot \text{Prob}(\text{SNR}_T > \text{SNR}_{T\min}|P, tr, Ntrs_T) \cdot \text{Prob}(P)}{\text{Prob}(\text{TESS})}$$

Likewise, repeat the above analysis but for Kepler:

$$\text{Prob}(P|\text{Kepler})$$

$$= \frac{\text{Prob}(tr|P) \cdot \text{Prob}(Ntrs_K|P, tr) \cdot \text{Prob}(\text{SNR}_K > \text{SNR}_{K\min}|P, tr, Ntrs_K) \cdot \text{Prob}(P)}{\text{Prob}(\text{Kepler})}$$



# *Methodology*

# 1. Prob (tr | P)

$$\text{Prob}(tr|P) = \int \frac{R_*}{a} f_{R_*, a|P}(R_*, a) dR_* da = \int \left(\frac{4\pi^2}{G}\right)^{\frac{1}{3}} R_* M_*^{-\frac{1}{3}} P^{-\frac{2}{3}} f_{R_*, M_*, |P}(R_*, M_*) dR_* dM_*$$

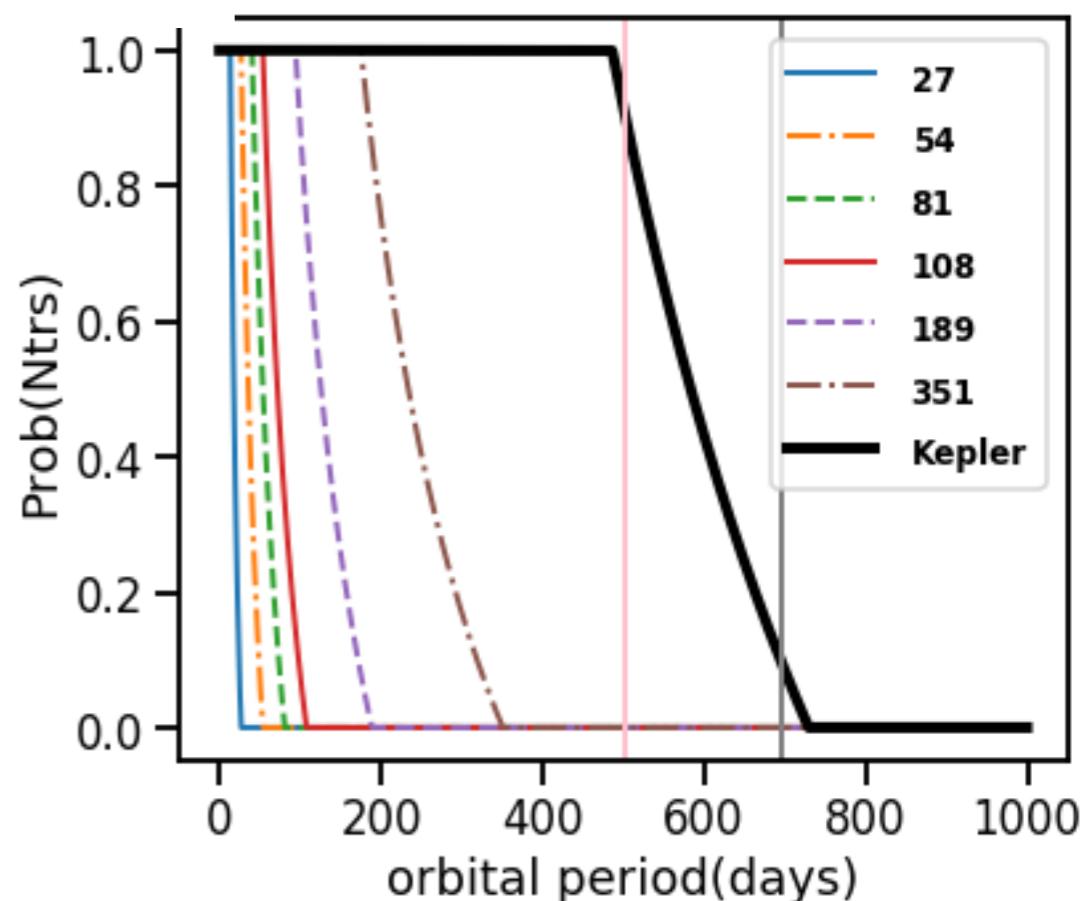
$$2. \text{ } Prob(2Tr(\tau_1)|Tr, P)$$

$$\text{Prob}(Ntrs_T | P, tr) = \begin{cases} 0, & t \leq (N-1)P \\ \frac{t - (N-1)P}{P}, & (N-1)P < t < NP \\ 1, & t \geq NP \end{cases}$$

Prob<sub>*i*</sub>(*P*|TESS)

$$= c_i \text{Prob}_i(P|Kepler) \frac{\text{Prob}_i(Ntrs_T|P,tr) \cdot \text{Prob}_i(\text{SNR}_T > \text{SNRT}_{min}|P,tr,Ntrs_T)}{\text{Prob}_i(Ntrs_K|P,tr) \cdot \text{Prob}_i(\text{SNR}_K > \text{SNRK}_{min}|P,tr,Ntrs_K)}$$

The gray vertical line is 694.76 days beyond which the probability that it can be detected by Kepler is less than 10% and the pink vertical line is 503.10 days within which the probability is higher than 90%



# Methodology

## 3. $\text{Prob}(\text{SNR}_T | 2\text{Tr}(\tau_1), \text{Tr}, P)$

$$\text{SNR} = R_p^2 P^{-\frac{1}{3}} \left( \frac{4\pi^2}{GM_*} \right)^{\frac{1}{6}} \sqrt{\frac{At_m}{4R_* r^2} \int_{\lambda_1}^{\lambda_2} \tau \pi B(\lambda, T_*) \left( \frac{\lambda}{hc} \right) d\lambda},$$

$$\text{SNR} = f(\mathbb{P})g(\mathbb{S})h(\mathbb{M}, T_*)$$

$$f(\mathbb{P}) = R_p^2 P^{-\frac{1}{3}}$$

$$g(\mathbb{S}) = \left( \frac{4\pi^2}{GM_*} \right)^{\frac{1}{6}} \sqrt{\frac{1}{4R_*}}$$

$$h(\mathbb{M}, T_*) = \sqrt{\frac{At_m}{r^2} \int_{\lambda_1}^{\lambda_2} \tau \pi B(\lambda, T_*) \left( \frac{\lambda}{hc} \right) d\lambda}$$

$$\text{SNR}_K = f(\mathbb{P})g(\mathbb{S})h(\mathbb{M}_K, T_*);$$

$$\text{SNR}_T = f(\mathbb{P})g(\mathbb{S})h(\mathbb{M}_T, T_*)$$

$$\text{SNR}_T = \text{SNR}_K \frac{h(\mathbb{M}_T, T_*)}{h(\mathbb{M}_K, T_*)}$$

$$= k(\mathbb{M}_T, \mathbb{M}_K, T_*) \cdot \text{SNR}_K$$

$T_*$	k (27 days)	k (54 days)	k (81 days)	k (108 days)	k (189 days)	k (351 days)
< 4500K	3974.41	0.0327	0.0463	0.0567	0.0655	0.0866
> 4500K	5653.53	0.0621	0.0878	0.1075	0.1242	0.1643

# Methodology

## 3. $\text{Prob}(\text{SNR}_T | 2\text{Tr}(\tau_1), \text{Tr}, P)$

$$f_{\text{SNRT}i}(\text{SNR}_T | P, \text{tr}) = f_{\text{SNRT}i}(k \cdot \text{SNR}_K | P, \text{tr}) = f_{\text{SNRK}i}(\text{SNR}_K | P, \text{tr})$$

$$f_{\text{SNRT}i}(\text{SNR} | P, \text{tr}) = f_{\text{SNRK}i}(\text{SNR}/k | P, \text{tr})$$

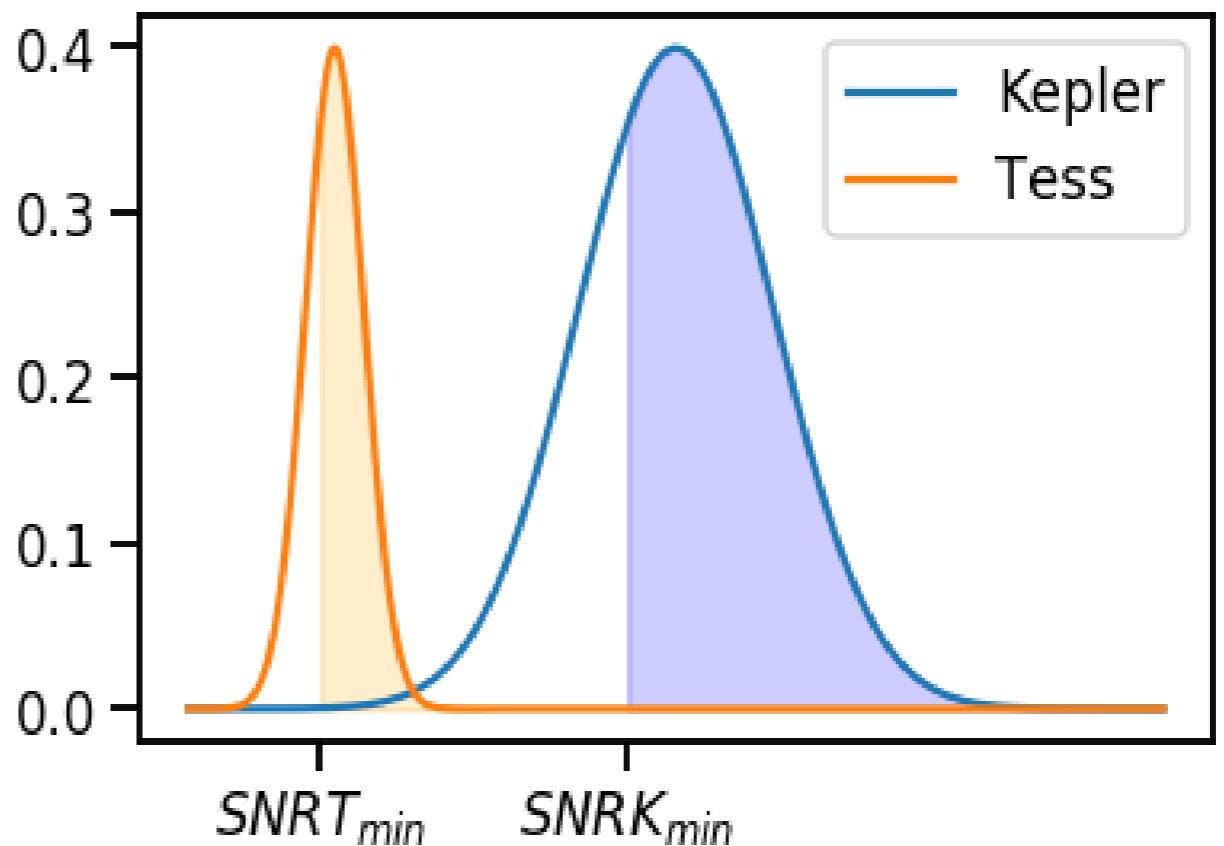
$$\text{Prob}_i(\text{SNR}_T > \text{SNRT}_{\min} | P, \text{tr})$$

$$= \int_{\text{SNRT}_{\min}}^{\infty} f_{\text{SNRT}i}(\text{SNR}' | P, \text{tr}) d\text{SNR}'$$

$$= \int_{\text{SNRT}_{\min}}^{\infty} f_{\text{SNRK}i}\left(\frac{\text{SNR}'}{k} | P, \text{tr}\right) d\text{SNR}'$$

$$= \int_{\frac{\text{SNRT}_{\min}}{k}}^{\infty} k \cdot f_{\text{SNRK}i}(\text{SNR}'' | P, \text{tr}) d\text{SNR}''$$

$$= k \cdot \text{Prob}_i\left(\text{SNR}_K > \frac{\text{SNRT}_{\min}}{k} | P, \text{tr}\right)$$



$$\frac{\text{Prob}_i(\text{SNR}_T > \text{SNRT}_{\min} | P, \text{tr})}{\text{Prob}_i(\text{SNR}_K > \text{SNRK}_{\min} | P, \text{tr})} = k \cdot \frac{\text{Prob}_i\left(\text{SNR}_K > \frac{\text{SNRT}_{\min}}{k} | P, \text{tr}\right)}{\text{Prob}_i(\text{SNR}_K > \text{SNRK}_{\min} | P, \text{tr})}$$



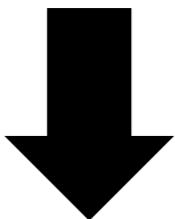
# Methodology

## 3. $\text{Prob}(\text{SNR}_T | 2\text{Tr}(\tau_1), \text{Tr}, P)$

$$\text{Prob}(TESS | P) = \text{Prob}(\text{Tr} | P) \times \text{Prob}(N\text{Tr}(\tau_1) | \text{Tr}, P) \times \text{Prob}(\text{SNR}_T | N\text{Tr}(\tau_1) | \text{Tr}, P)$$

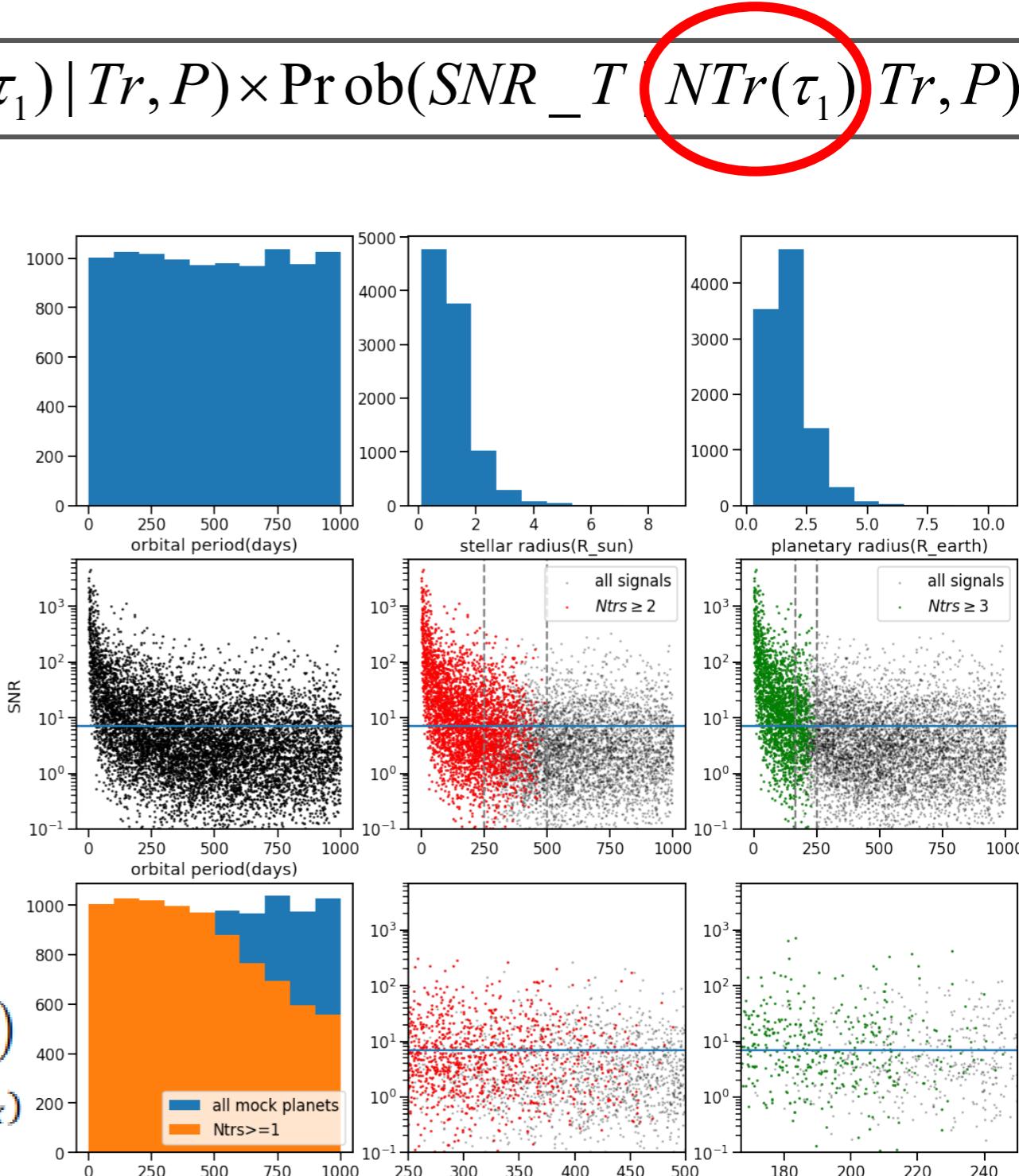
3 scenarios:

$$\begin{aligned} & \text{Prob}(Ntr_{ST} | P, tr) \\ &= \begin{cases} 0, & t \leq (N-1)P \\ \frac{t - N \cdot P}{P}, & (N-1)P < t < NP \\ 1, & t \geq N \cdot P \end{cases} \end{aligned}$$



$$\text{Prob}_i \left( \text{SNR}_K > \frac{\text{SNRT}_{min}}{k} | P, tr \right) \rightarrow \text{Prob}_i \left( \text{SNR}_K > \frac{\text{SNRT}_{min}}{k} | P, tr, 3trs_K \right)$$

$$\text{Prob}_i \left( \text{SNR}_K > \text{SNRK}_{min} | P, tr \right) \rightarrow \text{Prob}_i \left( \text{SNR}_K > \text{SNRK}_{min} | P, tr, 3trs_K \right)$$



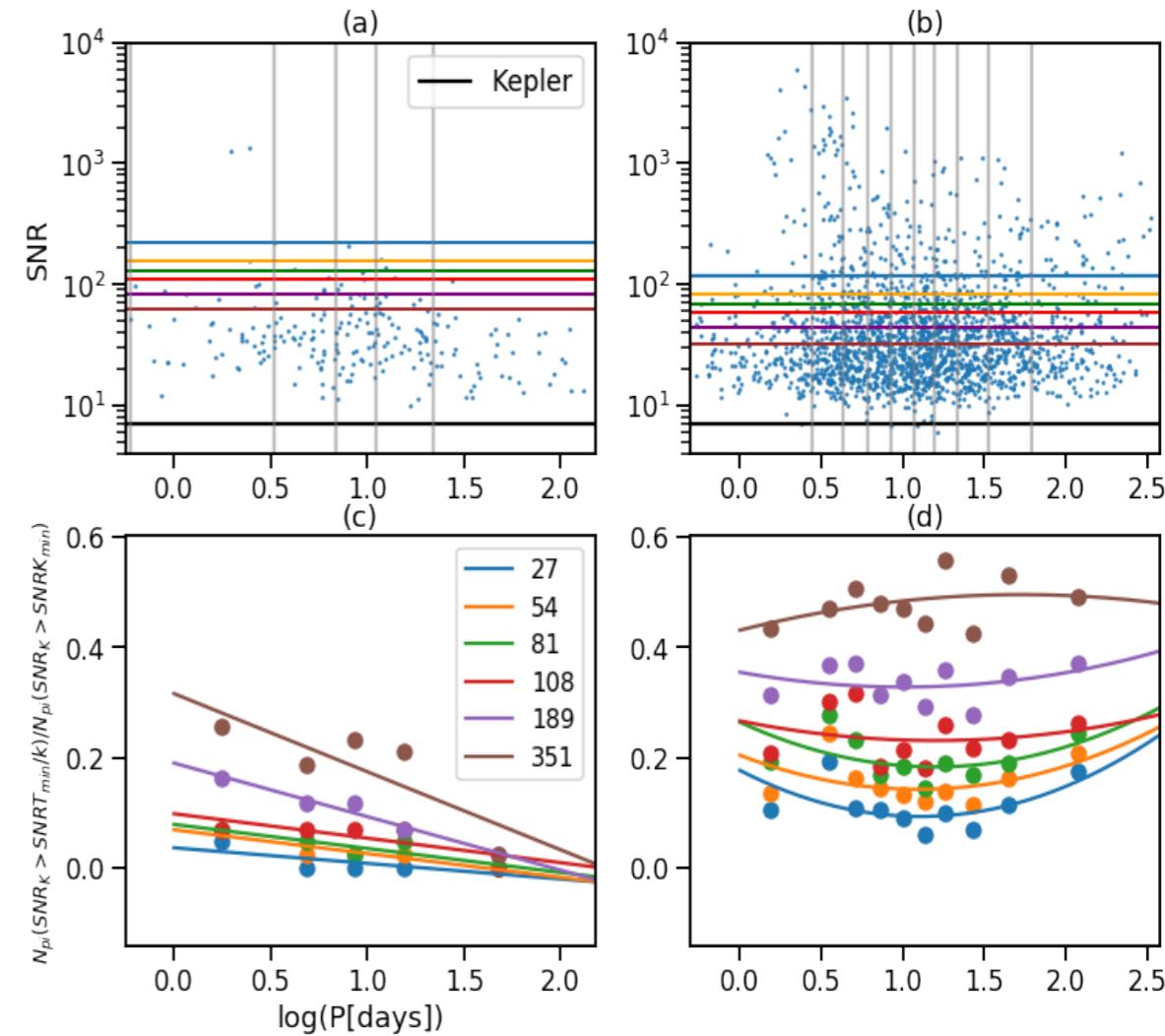
# Methodology

↓

$$\frac{\text{Prob}_i\left(\text{SNR}_K > \frac{\text{SNRT}_{min}}{k} | P, tr, 3trs_K\right)}{\text{Prob}_i(\text{SNR}_K > \text{SNRK}_{min} | P, tr, 3trs_K)} = \frac{N_{Pi}\left(\text{SNR}_K > \frac{\text{SNRT}_{min}}{k}\right) / N_{Pi}}{N_{Pi}(\text{SNR}_K > \text{SNRK}_{min}) / N_{Pi}}$$

$$= \frac{N_{Pi}\left(\text{SNR}_K > \frac{\text{SNRT}_{min}}{k}\right)}{N_{Pi}(\text{SNR}_K > \text{SNRK}_{min})}$$

↓

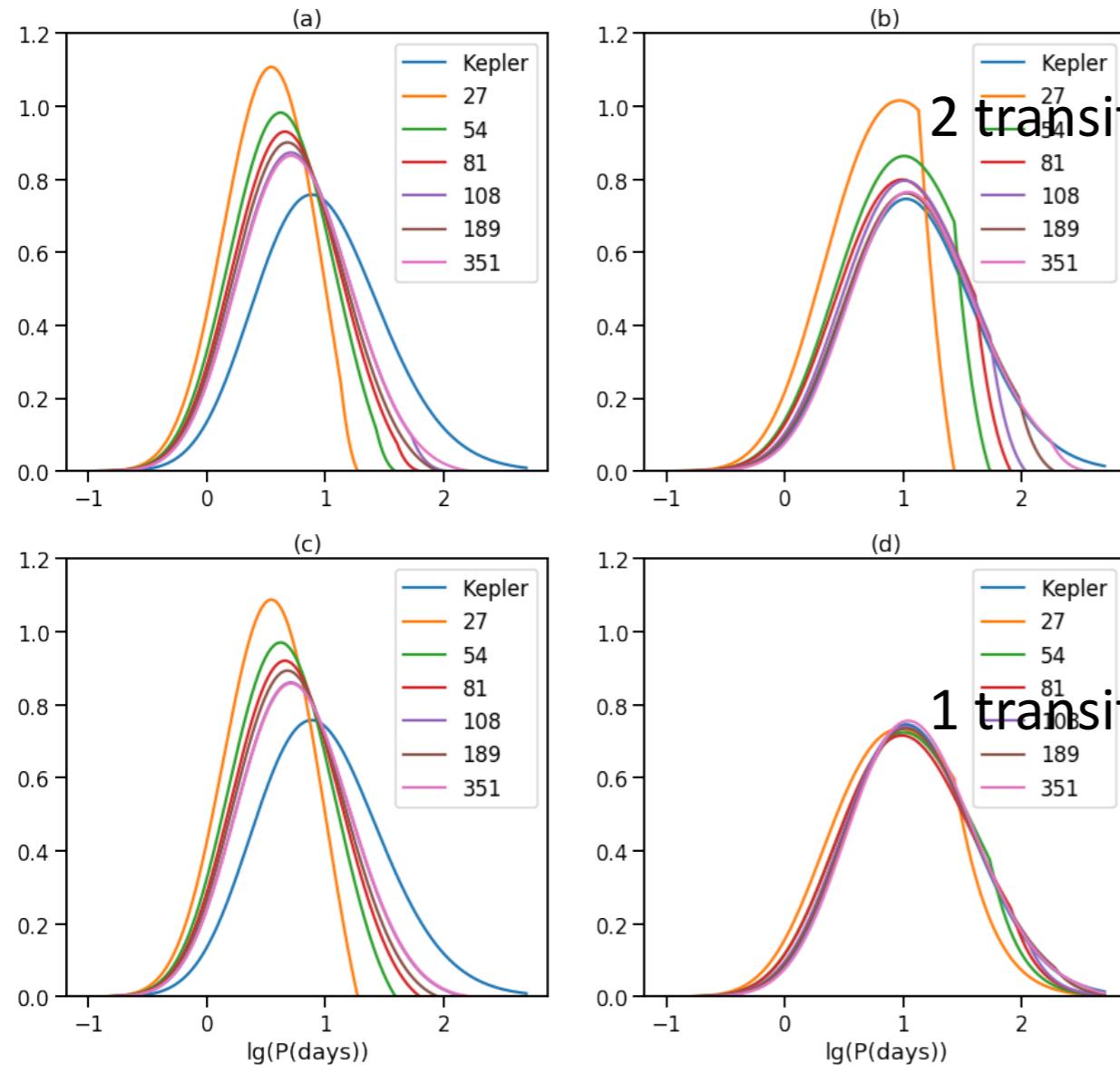


$\text{Prob}_i(P|TESS)$

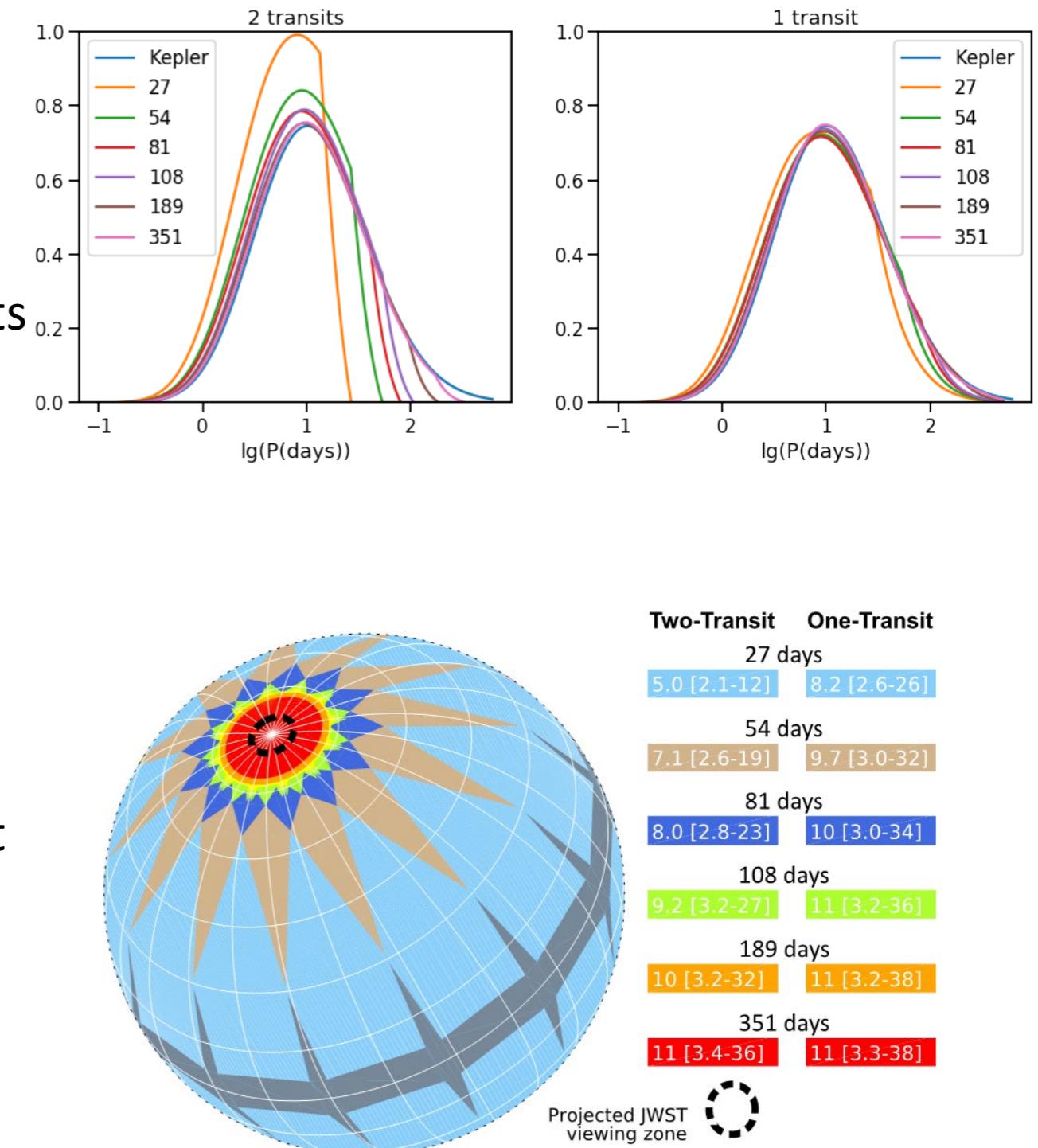
$$= c_i \text{Prob}_i(P|Kepler) \frac{\text{Prob}_i(Ntrs_T|P, tr) \cdot \text{Prob}_i(\text{SNR}_T > \text{SNRT}_{min} | P, tr, Ntrs_T)}{\text{Prob}_i(Ntrs_K|P, tr) \cdot \text{Prob}_i(\text{SNR}_K > \text{SNRK}_{min} | P, tr, Ntrs_K)}$$

# Results

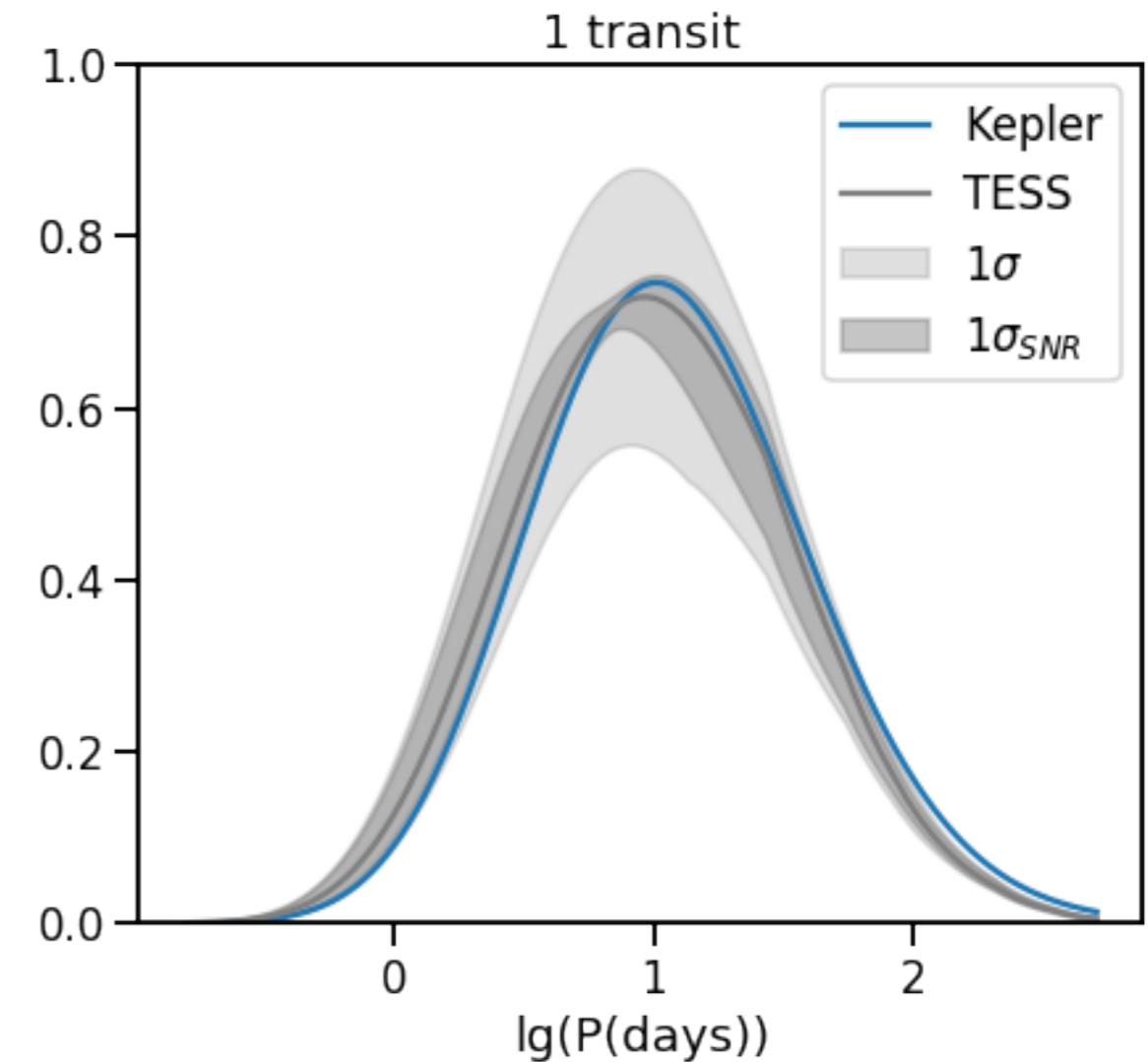
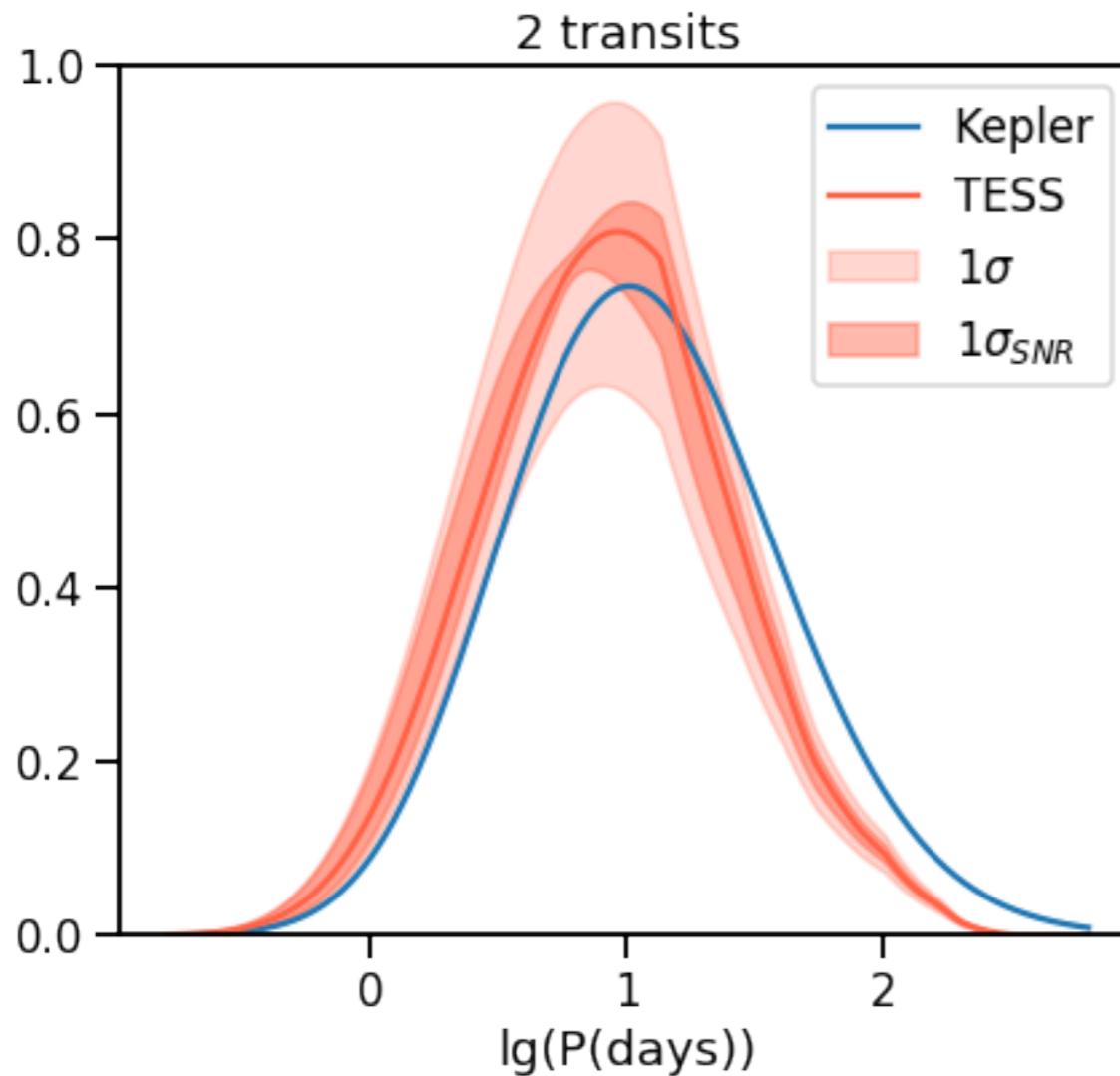
## 1. Results of two subsamples for different observation baseline



## 2. Results of different observation baseline



# Results

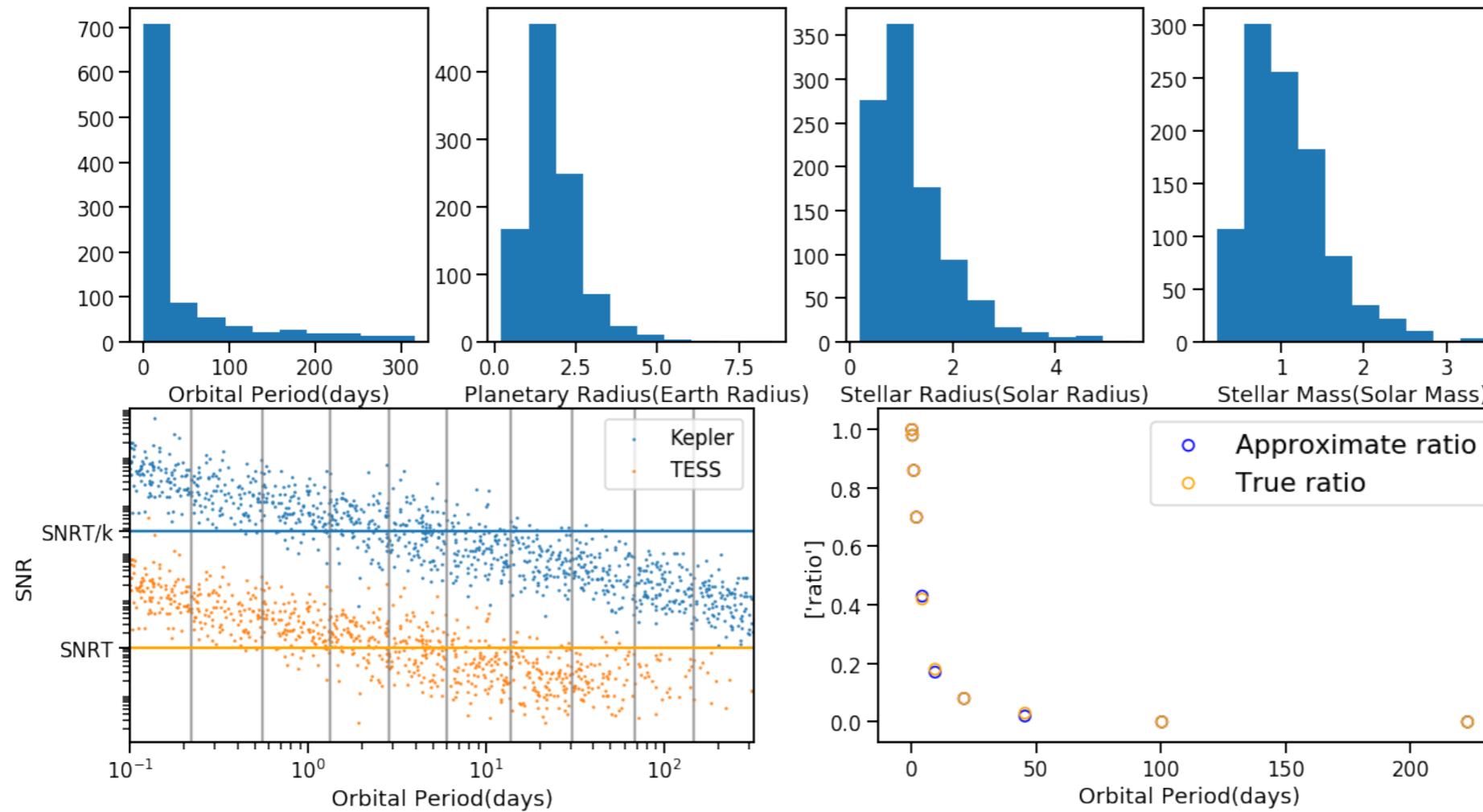


Duration of Observation (days)	TESS	Kepler
MP(days) -2 transits	8.47	11.89
$1\sigma$ (days) -2 transits	2.75-26.12	3.45-41.04
MP(days) -1 transit	10.09	-
$1\sigma$ (days) -1 transit	2.99-34.08	-

# Uncertainty

- Uncertainty of approximating  $N_{trs}$

$$\text{SNR} = R_p^2 \left( \frac{4\pi^2 P}{GM_*} \right)^{\frac{1}{6}} \sqrt{\frac{N_{trs} A}{4R_* r^2} \int_{\lambda_1}^{\lambda_2} \tau \pi B(\lambda, T_*) \left( \frac{\lambda}{hc} \right) d\lambda}$$



# Uncertainty

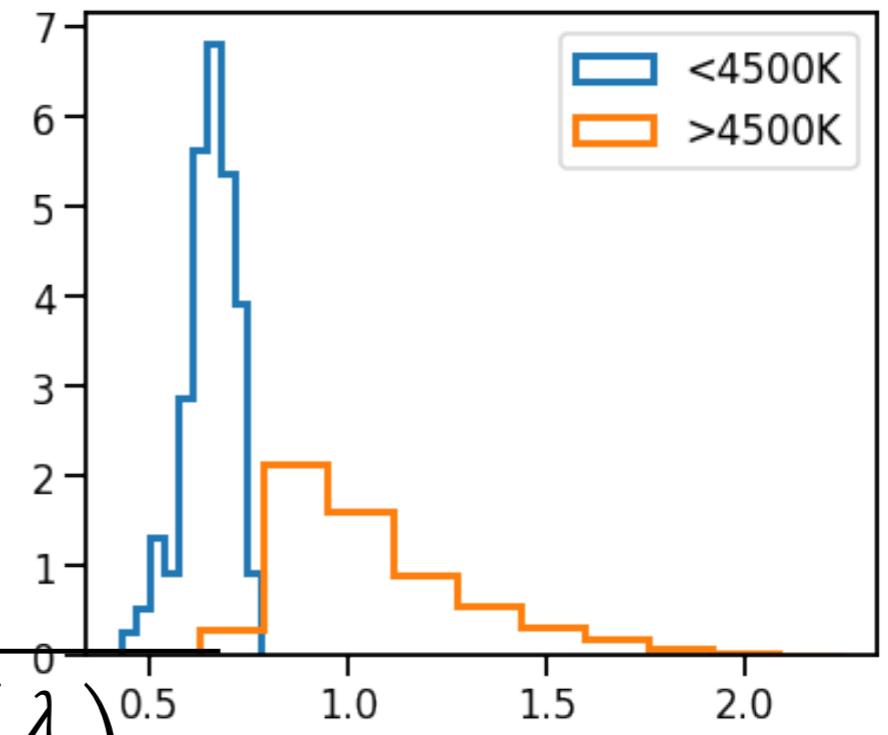
- Uncertainty of Stellar Parameters

$$\begin{aligned} \text{Prob}_i(P|TESS) \\ = c_i \text{Prob}_i(P|Kepler) \cdot \frac{\text{Prob}(tr|P)}{\text{Prob}(tr|P)} \cdot \frac{\text{Prob}_i(Ntrs_T|P, tr)}{\text{Prob}_i(Ntrs_K|P, tr)} \cdot \\ \cdot \frac{\text{Prob}_i(\text{SNR}_T > \text{SNR}_{Tmin}|P, tr)}{\text{Prob}_i(\text{SNR}_K > \text{SNR}_{Kmin}|P, tr)} \end{aligned}$$

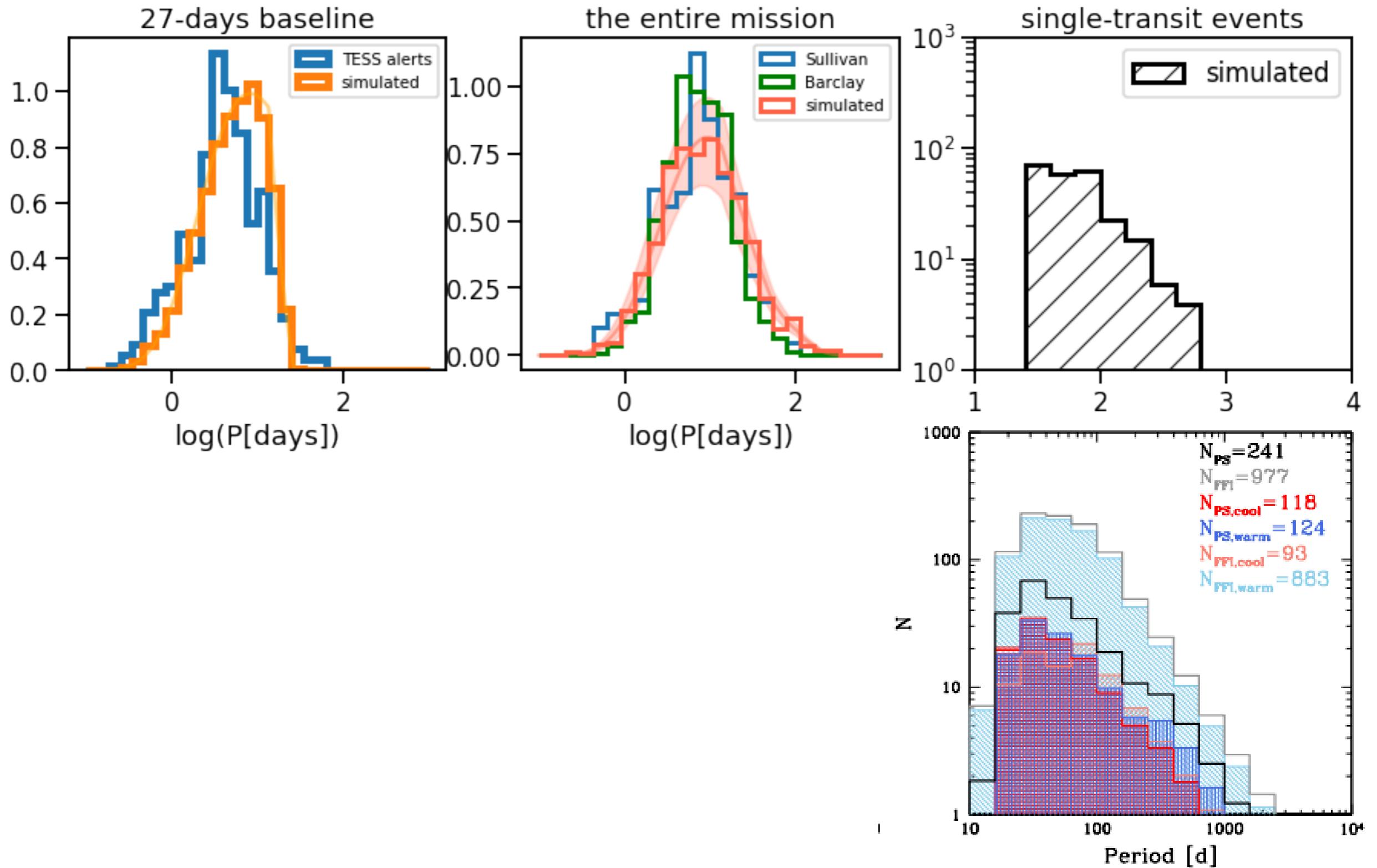
- Uncertainty of SNR model

$$\text{SNR} = R_p^2 P^{-\frac{1}{3}} \left( \frac{4\pi^2}{GM_*} \right)^{\frac{1}{6}} \sqrt{\frac{At_m}{4R_*r^2} \int_{\lambda_1}^{\lambda_2} \tau \pi B(\lambda, T_*) \left( \frac{\lambda}{hc} \right) d\lambda}, \quad R_*/(M_*)^{1/3}$$

$$h(M, T_*) = \sqrt{\frac{At_m}{r^2} \int_{\lambda_1}^{\lambda_2} \tau \pi B(\lambda, T_*) \left( \frac{\lambda}{hc} \right) d\lambda} \quad 1000 \text{ times}$$



# Comparison



# BRIGHT FUTURE!

