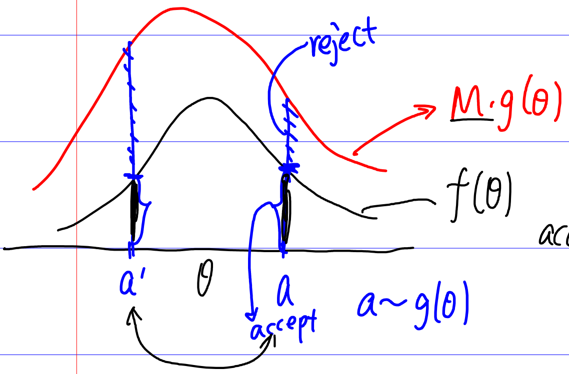

rejection - acceptance

Draw $U \sim unif(0,1)$

accept a

if $U < \frac{\text{lower section}}{\text{entire section}}$

$= \frac{f(a)}{Mg(a)}$

reject a

if $U \geq \frac{f(a)}{Mg(a)}$ (wasted)

Ex.

$f(\theta)$ is Cauchy

$f(\theta) = \frac{1}{\pi(1+\theta^2)}$

$g(\theta) = \frac{1}{\sqrt{2\pi}} e^{-\frac{\theta^2}{2}}$ as proposal

We must have $f(\theta) \leq M g(\theta)$ for some $M < \infty$

for all $\theta \in \mathbb{R}$

We can always choose $\boxed{M = \sup_\theta \frac{f(\theta)}{g(\theta)}}$

$$\frac{f(\theta)}{g(\theta)} = \frac{\frac{1}{\pi(1+\theta^2)}}{\frac{1}{\sqrt{2\pi}} e^{-\theta^2/2}} = \frac{\sqrt{2\pi}}{\pi} \frac{e^{\theta^2/2}}{1+\theta^2} \overset{\times}{\underset{?}{\leq}} M$$

as $\theta \to \infty$, $\frac{e^{\theta^2/2}}{1+\theta^2} \boxed{\to \infty}$ problem $\Rightarrow \sup_\theta \frac{f(\theta)}{g(\theta)}$ is not fini

$\Rightarrow g(\theta)$ can't be a normal density

if $g(\theta)$ is a proposal density for rejection sampling from $f(\theta)$ (which is Cauchy)

acceptance : $U \leq \frac{f(\theta)}{Mg(\theta)}$

$$\Leftrightarrow \log U \leq \underbrace{\log f(\theta) - \log g(\theta)}_{\text{diff()}} - \log M$$

Importance Sampling

$$E[\varphi(\theta)|y] = \frac{\int \varphi(\theta)\, w(\theta)\, g(\theta) d\theta}{\underbrace{\int w(\theta) g(\theta) d\theta}}$$

g : proposal density

$$w(\theta) = \frac{f(\theta)}{g(\theta)} \star$$

posterior $p(\theta|y)$

Weight (computable)

✓ $\int w(\theta) g(\theta) d\theta = E_g[w(\theta)]$

$\int \varphi(\theta) w(\theta) g(\theta) d\theta = E_g[\varphi(\theta) w(\theta)]$

If $\theta_1, \dots, \theta_S \overset{iid}{\sim} g(\theta)$

by LLN, $\int w(\theta) g(\theta) d\theta = E_g[w(\theta)] \approx \frac{1}{S}\sum_{s=1}^{S} w(\theta_s)$ ✓

$\int \varphi(\theta) w(\theta) g(\theta) d\theta = E_g[\varphi(\theta) w(\theta)] \approx \frac{1}{S}\sum_{s=1}^{S} \varphi(\theta_s) w(\theta_s)$ ✓

$$\Rightarrow E[\varphi(\theta)|y] \approx \frac{\cancel{\frac{1}{S}} \sum_{s=1}^{S} \varphi(\theta_s) \boxed{w(\theta_s)}}{\boxed{\cancel{\frac{1}{S}} \sum_{s=1}^{S} w(\theta_s)}}$$

$$= \sum_{s=1}^{S} \varphi(\theta_s) \cdot W(\theta_s)$$

$$W(\theta_s) = \frac{w(\theta_s)}{\sum_{s=1}^{S} w(\theta_s)}$$

Tutorial 6

Prob 1. $y \sim \text{Binomial}(n, \theta)$

MLE : $\hat{\theta} = \frac{y}{n}$, $Var(\hat{\theta}) = \frac{\theta(1-\theta)}{\underbrace{n}}$

posterior of normal approx $\approx N\left(\frac{y}{n}, \frac{\frac{y}{n}(1-\frac{y}{n})}{n}\right)$

poisson model

laplace approx 2 : $\dfrac{\left(1+\frac{1}{a_0+t-1}\right)^{a_0+t-\frac{1}{2}}}{\underbrace{e}} \cdot \dfrac{a_0+t}{\underbrace{b_0+n}} \to$ real posterior mean

$\left(1+\frac{1}{x}\right)^x$

$\to 1$ if $t \to \infty$ $\quad (t = \sum y_i)$