

$$g(\theta^* | \theta^{(t-1)}) = h(\theta^* - \theta^{(t-1)})$$

$$g(\theta_b | \theta_a) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(\theta_b - \theta_a)^2}{2\sigma^2}\right], \quad h(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left(-\frac{x^2}{2\sigma^2}\right)$$

if  $\theta$  is univariate  $\theta_b | \theta_a \sim N(\theta_a, \sigma^2)$

$$g(\theta_b | \theta_a) = g(\theta_a | \theta_b) \Rightarrow \theta_a | \theta_b \sim N(\theta_b, \sigma^2)$$

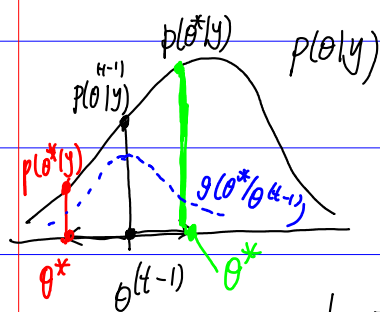
In MH algorithm, if  $g(\theta_b | \theta_a) = g(\theta_a | \theta_b)$  symmetry for any  $\theta_a, \theta_b$ ,

$$r = \frac{f(\theta^*) g(\theta^{(t-1)} | \theta^*)}{f(\theta^{(t-1)}) g(\theta^* | \theta^{(t-1)})} = \frac{f(\theta^*)}{f(\theta^{(t-1)})} \quad \begin{matrix} p(\theta | y) \\ \propto f(\theta) \end{matrix}$$

$$= \frac{p(\theta^* | y)}{p(\theta^{(t-1)} | y)}$$

$$p(\theta | y) = \frac{f(\theta)}{C}$$

MH algorithm: If  $r > 1$ , always accept  $\theta^*$ , set  $\theta^{(t)} = \theta^*$   
If  $r < 1$ , accept  $\theta^*$  with prob  $r$ .



$\theta^{(1)}, \theta^{(2)}, \theta^{(3)}, \dots$

In this posterior Markov chain sample, the relative frequencies of  $\theta$ 's should roughly match with the "height" of their posterior density values.