$$p(\theta|y) = \frac{p(y|\theta)\,p(\theta)}{\int p(y|\theta')\,p(\theta')\,d\theta'} \longrightarrow \text{a density function}$$

$$\int p(\theta|y)\,d\theta = \int \frac{p(y|\theta)\,p(\theta)}{\int p(y|\theta')\,p(\theta')\,d\theta'}\,d\theta = \frac{\int p(y|\theta)\,p(\theta)\,d\theta}{\int p(y|\theta')\,p(\theta')\,d\theta'} = 1$$

$$p(y) = \int p(y|\theta')\,p(\theta')\,d\theta'$$

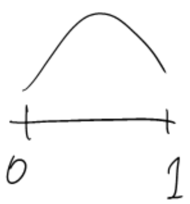
$$\int p(y) = \iint p(y|\theta')\,p(\theta')\,d\theta'\,dy \overset{\text{Fubini}}{=} \int \Big[\underbrace{\int p(y|\theta')\,dy}_{=1}\Big]\,p(\theta')\,d\theta' = \int p(\theta')\,d\theta' = 1$$

$p(y)$ is a density.

" $\propto$ " drop all multiplicative constants

$$g(x) = \frac{f(x)}{100} \propto f(x) \quad ✓$$

$$g(x) = f(x) + 100 \quad \not\propto \quad f(x)$$



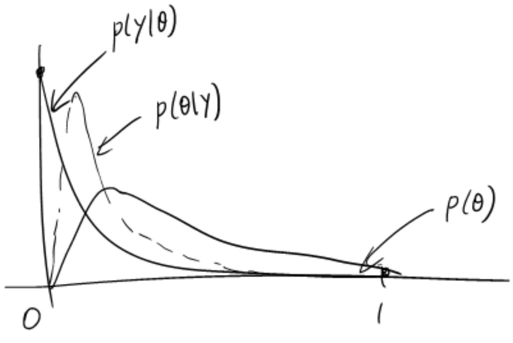
$\theta^{a-1}(1-\theta)^{b-1} \rightarrow$ must be beta distribution

$$p(\theta|y) \propto \theta^{2-1}(1-\theta)^{40-1} \Rightarrow \theta|y \sim \text{Beta}(2, 40)$$

$$p(\theta|y) = \frac{\theta^{2-1}(1-\theta)^{40-1}}{B(2, 40)}$$

likelihood function is a function of θ. (sometimes written as $L(\theta|y)$, $L(\theta;y)$)

$\underline{p(y|\theta)}$

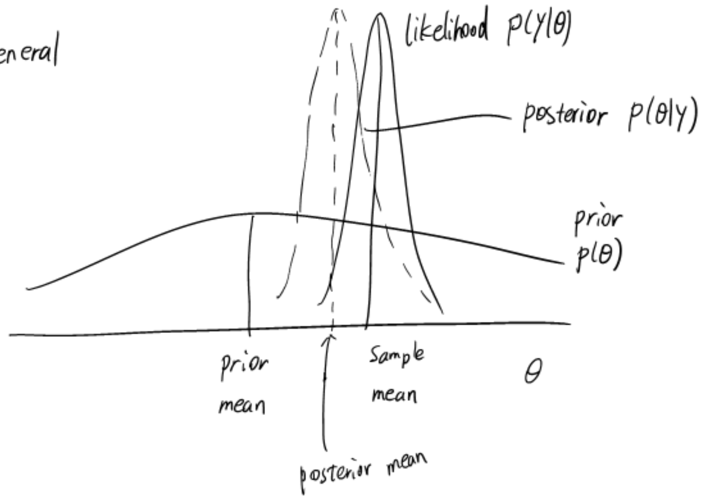


$$p(y|\theta) = (1-\theta)^{20}$$

$$p(\theta) = \frac{\theta^{2-1}(1-\theta)^{20-1}}{B(2,20)}$$

$$p(\theta|y) = \frac{\theta^{2-1}(1-\theta)^{40-1}}{B(2, 40)}$$

In general



$$\theta|y \sim \text{Beta}\left(w\theta_0+y,\ \underline{(1-\theta_0)w +}\ n-y\right) \qquad a = w\theta_0 \quad b = (1-\theta_0)w$$

$$P(\theta<0.10|y) = \int_0^{0.1} \frac{\theta^{w\theta_0+y-1}(1-\theta)^{(1-\theta_0)w+n-y-1}}{B(w\theta_0,\ (1-\theta_0)w+n-y)}\, d\theta \rightsquigarrow \text{a function of } (w, \theta_0)$$