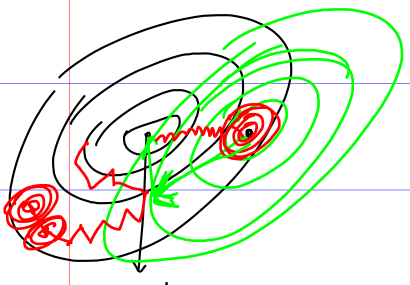
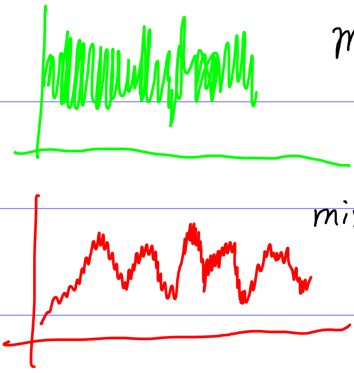


posterior mode



mix well ✓

mix poorly ✗

Markov chain mixes well $\approx$ draws don't have high serial dependence/correlation

mixing

t distribution:

$y_1, \ldots, y_n$

$$\left.\begin{aligned} y_i \mid \lambda_i, \mu, \sigma^2 &\sim N\left(\mu, \frac{\sigma^2}{\lambda_i}\right) \\ \lambda_i &\sim \text{Gamma}\left(\frac{\nu}{2}, \frac{\nu}{2}\right) \end{aligned}\right\} \Rightarrow y_i \mid \mu, \sigma^2 \sim t_\nu \text{ center } \mu \text{ scale } \sigma^2$$

$$f(y \mid \mu, \sigma^2) = \int_0^\infty f(y, \lambda \mid \mu, \sigma^2)\, d\lambda$$

$$= \int_0^\infty \underbrace{f(y \mid \boxed{\lambda}, \mu, \sigma^2)}_{\text{normal}} \underbrace{f(\boxed{\lambda} \mid \mu, \sigma^2)}_{\text{gamma}}\, d\lambda$$

$$\propto \int_0^\infty \frac{1}{\sqrt{2\pi \frac{\sigma^2}{\lambda}}} \exp\left\{-\frac{(y-\mu)^2}{2\sigma^2/\lambda}\right\}$$

$$\times \lambda^{\frac{\nu}{2}-1} e^{-\frac{\nu}{2}\lambda}\, d\lambda$$

$$\propto \int_0^\infty \boxed{?}\, \lambda^{\underbrace{\frac{\nu+1}{2}}_{\alpha}-1} \exp\left\{-\underbrace{\frac{(y-\mu)^2/\sigma^2+\nu}{2}}_{\beta}\lambda\right\} d\lambda$$

$$\propto \frac{\Gamma\left(\frac{\nu+1}{2}\right)}{\left[\frac{(y-\mu)^2}{2\sigma^2} + \frac{\nu}{2}\right]^{\frac{\nu+1}{2}}}$$

$$\propto \frac{1}{\left[1 + \frac{1}{\nu}\left(\frac{y-\mu}{\sigma}\right)^2\right]^{\frac{\nu+1}{2}}} \rightarrow t_\nu \text{ density with center } \mu \text{ and scale } \sigma^2$$