$$\begin{array}{lll} \lambda_{i} & y_{i}, & \alpha, \mu \sim & \text{Gamma} \left(\alpha + y_{i}, \frac{\alpha}{\mu} + e_{i}\right) \\ \hline E\left[\lambda_{i} \mid y_{i}, \alpha, \mu\right] &= \frac{\alpha + y_{i}}{\frac{\alpha}{\mu} + e_{i}} \\ &= \frac{\alpha}{\mu} + e_{i} \\ &= \frac{\alpha + \mu e_{i}}{\alpha + \mu e_{i}} + \frac{\mu y_{i}}{\alpha + \mu e_{i}} \\ &= \frac{\alpha}{\alpha + \mu e_{i}} + \frac{\mu e_{i}}{\alpha + \mu e_{i}} \cdot \frac{y_{i}}{e_{i}} \\ &= \frac{\alpha}{\alpha + \mu e_{i}} + \frac{\mu e_{i}}{\alpha + \mu e_{i}} \cdot \frac{y_{i}}{e_{i}} \\ &= \frac{\alpha}{\alpha + \mu e_{i}} + \frac{\mu e_{i}}{\alpha + \mu e_{i}} \cdot \frac{y_{i}}{e_{i}} \\ &= \frac{\alpha}{\alpha + \mu e_{i}} + \frac{\mu e_{i}}{\alpha + \mu e_{i}} \cdot \frac{y_{i}}{e_{i}} \\ &= \frac{\alpha}{\alpha + \mu e_{i}} + \frac{\mu e_{i}}{\alpha + \mu e_{i}} \cdot \frac{y_{i}}{e_{i}} \\ &= \frac{\alpha}{\alpha + \mu e_{i}} + \frac{\mu e_{i}}{\alpha + \mu e_{i}} \cdot \frac{y_{i}}{e_{i}} \\ &= \frac{\alpha}{\alpha + \mu e_{i}} + \frac{\mu e_{i}}{\alpha + \mu e_{i}} \cdot \frac{y_{i}}{e_{i}} \\ &= \frac{\alpha}{\alpha + \mu e_{i}} + \frac{\mu e_{i}}{\alpha + \mu e_{i}} \cdot \frac{y_{i}}{e_{i}} \\ &= \frac{\alpha}{\alpha + \mu e_{i}} + \frac{\mu e_{i}}{\alpha + \mu e_{i}} \cdot \frac{y_{i}}{e_{i}} \\ &= \frac{\alpha}{\alpha + \mu e_{i}} + \frac{\mu e_{i}}{\alpha + \mu e_{i}} \cdot \frac{y_{i}}{e_{i}} \\ &= \frac{\alpha}{\alpha + \mu e_{i}} + \frac{\mu e_{i}}{\alpha + \mu e_{i}} \cdot \frac{y_{i}}{e_{i}} \\ &= \frac{\alpha}{\alpha + \mu e_{i}} + \frac{\mu e_{i}}{\alpha + \mu e_{i}} \cdot \frac{y_{i}}{e_{i}} \\ &= \frac{\alpha}{\alpha + \mu e_{i}} + \frac{\mu e_{i}}{\alpha + \mu e_{i}} \cdot \frac{y_{i}}{e_{i}} \\ &= \frac{\alpha}{\alpha + \mu e_{i}} + \frac{\mu e_{i}}{\alpha + \mu e_{i}} \cdot \frac{y_{i}}{e_{i}} \\ &= \frac{\alpha}{\alpha + \mu e_{i}} + \frac{\mu e_{i}}{\alpha + \mu e_{i}} \cdot \frac{y_{i}}{e_{i}} \\ &= \frac{\alpha}{\alpha + \mu e_{i}} + \frac{\mu e_{i}}{\alpha + \mu e_{i}} \cdot \frac{y_{i}}{e_{i}} \\ &= \frac{\alpha}{\alpha + \mu e_{i}} + \frac{\mu e_{i}}{\alpha + \mu e_{i}} \cdot \frac{y_{i}}{e_{i}} \\ &= \frac{\alpha}{\alpha + \mu e_{i}} + \frac{\mu e_{i}}{\alpha + \mu e_{i}} \cdot \frac{y_{i}}{e_{i}} \\ &= \frac{\alpha}{\alpha + \mu e_{i}} + \frac{\mu e_{i}}{\alpha + \mu e_{i}} \cdot \frac{y_{i}}{e_{i}} \\ &= \frac{\alpha}{\alpha + \mu e_{i}} + \frac{\mu e_{i}}{\alpha + \mu e_{i}} \cdot \frac{y_{i}}{e_{i}} \\ &= \frac{\alpha}{\alpha + \mu e_{i}} + \frac{\mu e_{i}}{\alpha + \mu e_{i}} \\ &= \frac{\alpha}{\alpha + \mu e_{i}} + \frac{\mu e_{i}}{\alpha + \mu e_{i}} \\ &= \frac{\alpha}{\alpha + \mu e_{i}} + \frac{\mu e_{i}}{\alpha + \mu e_{i}} \\ &= \frac{\alpha}{\alpha + \mu e_{i}} + \frac{\mu e_{i}}{\alpha + \mu e_{i}} \\ &= \frac{\alpha}{\alpha + \mu e_{i}} + \frac{\mu e_{i}}{\alpha + \mu e_{i}} \\ &= \frac{\alpha}{\alpha + \mu e_{i}} + \frac{\mu e_{i}}{\alpha + \mu e_{i}} \\ &= \frac{\alpha}{\alpha + \mu e_{i}} + \frac{\mu e_{i}}{\alpha + \mu e_{i}} \\ &= \frac{\alpha}{\alpha + \mu e_{i}} + \frac{\mu e_{i}}{\alpha + \mu e_{i}} \\ &= \frac{\alpha}{\alpha + \mu e_{i}} + \frac{\mu e_{i}}{\alpha + \mu e_{i}} \\ &= \frac{\alpha}{\alpha + \mu e_{i}} + \frac{\mu e_{i}}{\alpha + \mu e_{i}}$$