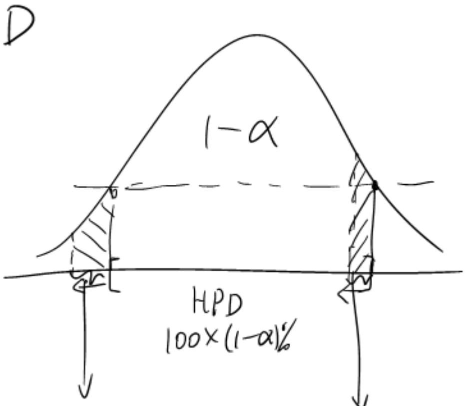
HPD



length > length

HPD has the shortest length among all C.I. with coverage $100\times(1-\alpha)\%$.

Poisson model

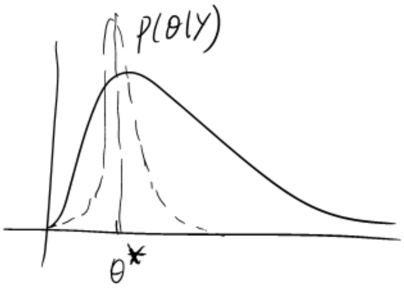
posterior : Gamma $(a_0+\Sigma y_i,\ b_0+n)$

$$E(\theta|Y)=\frac{a_0+\Sigma y_i}{b_0+n} \qquad Var(\theta|Y)=\frac{a_0+\Sigma y_i}{(b_0+n)^2}$$

$\rightsquigarrow \theta^*$

If $y_1,\cdots,y_n|\theta^* \sim Poisson(\theta^*)$ "truth"

by law of large numbers $\frac{\Sigma y_i}{n} \Rightarrow E(Y)=\theta^*$ $\underset{Var(Y)}{\parallel}$



If n is big, $Var(\theta|Y)\approx\frac{\Sigma y_i}{n^2}\approx\frac{\theta^*}{n}\ \left(\approx\frac{Var(Y)}{n}\right)$

central limit theorem

$$\sqrt{n}(\bar{X}-\mu)\to N(0,\sigma^2) \qquad X_1,\cdots,X_n\overset{iid}{\sim}F$$

$$\bar{X}\overset{approx}{\sim}N\left(\mu,\frac{\sigma^2}{n}\right) \qquad EX_i=\mu,\ Var(X_i)=\sigma^2$$

Gamma densities

$$\frac{b^a}{\Gamma(a)}x^{a-1}e^{-bx}$$

$P(\tilde{Y}=\tilde{y}|Y)$

$$=\int_0^\infty \frac{b^a}{\Gamma(a)\tilde{y}!}\cdot\underline{\theta^{\tilde{y}+a-1}e^{-(b+1)\theta}}\,d\theta$$

$$=\frac{b^a}{\Gamma(a)\tilde{y}!}\frac{1}{\boxed{}}\underbrace{\int_0^\infty \boxed{\frac{(b+1)^{\tilde{y}+a}}{\Gamma(\tilde{y}+a)}}\theta^{\tilde{y}+a-1}e^{-(b+1)\theta}\,d\theta}_{=1}$$

$p(y_1, \cdots, y_n|\theta)$, $\quad p(\theta)$

$p(\theta|y_1, \cdots, y_n)$ posterior of θ

uncertainty $\downarrow$ as $n \to \infty$

$$p(\tilde{y}|\theta) = \frac{\theta^{\tilde{y}}}{\tilde{y}!} e^{-\theta}$$

$Var(\tilde{y}|\theta) = \theta$

$Var(\tilde{y}|\theta^*) = \theta^*$ "truth"

uncertainty (remains the same)

$$p(\tilde{y}|y_1, \cdots, y_n)$$

$$Var(\tilde{y}|y_1, \cdots, y_n) = Var(\theta|y)\,(b_0 + n + 1)$$

$$= \frac{(a_0 + \Sigma y_i)}{(b_0 + n\ \ )^2} \cdot (b_0 + n + 1)$$

$$\overset{n \text{ is big}}{\approx} \frac{\Sigma y_i}{n} \approx \theta^* = Var(\tilde{y}|\theta^*)$$

tails of densities

$f(x) = \frac{c}{(1+x)^2}$, $x > 0$ heavy-tailed

$g(x) = e^{-x}$, $x > 0$

$h(x) = C \cdot e^{-x^2}$, $x > 0$ light-tailed

prior 1

$$\frac{p(y|\theta) \cdot p_1(\theta)}{\underbrace{\int p(y|\theta')\, p_1(\theta')\, d\theta'}_{m_1(y)}} = p_1(\theta|y) \quad \Longrightarrow \quad \underline{p(y|\theta)\, p_1(\theta)} = m_1(y)\, p_1(\theta|y)$$