Elliptic Curve Cryptography

Fast implentation of the Diffie-Hellman key exchange

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Elliptic Curve

- An elliptic curve $E(\mathbb{F}_p)$ consists of the set of the points $P(x, y)$, $x, y \in \mathbb{F}_p$ satisfying

$$y^2 \equiv x^3 + ax + b \pmod{p}$$

- Possible to define an addition rule to add points on E

Diffie Hellman key exchange

Public parameters	
$y^2 \equiv x^3 + ax + b \pmod{p}$	
$a, b \in \mathbb{F}_p$, a prime p and a base point G are known	
Private computations	
Alice	**Bob**
Compute $P = uG$	Compute $Q = vG$
Public exchange of values	
Alice $\xrightarrow{\quad P \quad}$ **Bob**	**Alice** $\xleftarrow{\quad Q \quad}$ **Bob**
Further private computations	
Alice	**Bob**
Compute uQ	Compute vP
The shared secret is $uQ = u(vG) = v(uG) = vP$	

Adaption of Table 2.2 J. Hoffstein et al., *An Introduction to Mathematical Cryptography*

Double-and-add-Method

- Input $P \in E(F_p)$, $d \in \mathbb{N}$ Complexity of $\mathcal{O}(log_2(d) \cdot log_2^2(p))$
- Output: $d \cdot P \in E(F_p)$

```
N <- P
Q <- O
for i from 0 to m do
  if d_i = 1 then
    Q <- point_add(Q, N)
  N <- point_double(N)
return Q
```

where $d = d_0 + d_1 2 + ... + d_m 2^m \quad d_i \in \{0, 1\}$

https://en.wikipedia.org/wiki/Elliptic_curve_point_multiplication#Double-and-add

Implementation

- Bigint

```
typedef uint64_t block;

typedef struct
{
    uint64_t significant_blocks;
    block blocks[BIGINT_BLOCKS_COUNT];
} __BigInt;
```

Corresponding operations (Addition, Multiplication, Division, ...)

- Elliptic Curve and ECDH
  - Elliptic Curve definition and key exchange mechanism
  - 5 predefined curves : 192, 224, 256, 384, 521 bits

Cost Anaylsis

- Index integer operation matters
- Cost measure
  - $C = C_{\text{add}} + C_{\text{mult}} + C_{\text{shift}}$
  - Code generated operations counts

Optimizations

Stages

Baseline - Implementation without memory optimization

Performance

Memory optimization - Implementation with memory optimization

Comparison with OpenSSL

Algorithm

Jacobian coordinates - Algorithmic changes, jacobian coordinates

Performance

Final - Final performance optimization

Optimizations

Overview

- **Memory optimization**
  - Change the base from 2 bit to 64 bit for Montogomery
  - Precomputation of $2^k \cdot G$ where $k \in \{1, ..., \log_2(p)\}$ and $G \in E$
  - Vectorize shifting using AVX2
- **Jacobian coordinates**
- **Final optimization**
  - Function stiching
  - Inlining functions
  - Unrolling

Intel ADX

C Code

```
low_m1 = _mulx_u64(a->blocks[i], b, &hi_m1);
add_carry_m1 = _addcarryx_u64(add_carry_m1, carry_m1, low_m1, &temp_m1);
add_carry_1 = _addcarryx_u64(add_carry_1, res->blocks[i], temp_m1, tmp->blocks[i]);
carry_m1 = hi_m1;
```

Created Assembly code

x86 icc 13.0.1 -m64 -march=xCORE-AVX2 -O3

```
mov     rdx, QWORD PTR [48+rsi]
mulx    rdx, rbx, rax
adox    rbp, rdx
adcx    rbp, QWORD PTR [48+rdi]
mov     QWORD PTR [2288+r9], rbp
```

http://gcc.godbolt.org/

Intel ADX

C Code

```
low_m1 = _mulx_u64(a->blocks[i], b, &hi_m1);
add_carry_m1 = _addcarryx_u64(add_carry_m1, carry_m1, low_m1, &temp_m1);
add_carry_1 = _addcarryx_u64(add_carry_1, res->blocks[i], temp_m1, tmp->blocks[i]);
carry_m1 = hi_m1;
```

Created Assembly code

x86 gcc 5.3 -m64 -march=haswell -O3

```
mulx    48(%rsi), %r9, %r10
addq    %r9, %r11
movq    %r10, %r9
setc    %bpl
addb    $-1, %bl
adcq    48(%rdi), %r11
movq    %r11, 2288(%rax)
setc    %bl
addb    $-1, %bpl
```

http://gcc.godbolt.org/

ADX vs AVX2

- Bottleneck operation: BigInt - block multiplication
- Unavoidable dependecies in carry chain -> vectorization by processing 4 multiplications in parallel

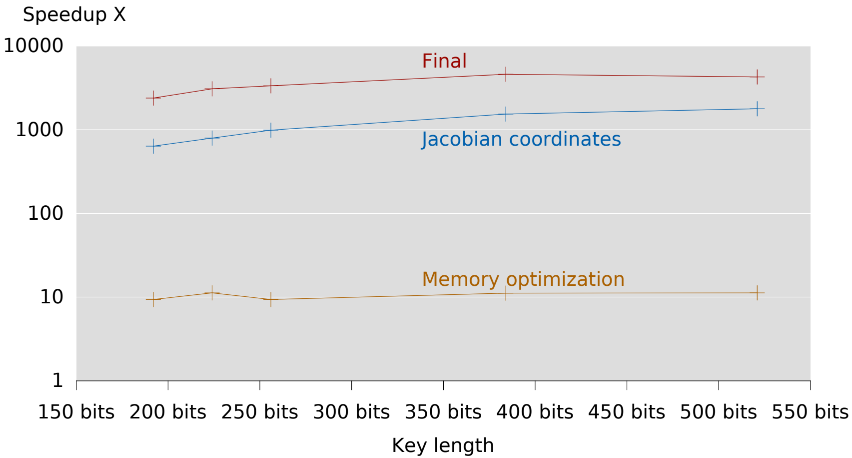
Approach	Lower bound	Bottleneck
ADX	8 cycles/iteration	ADX throughput
AVX2 (base 32)	10 cycles/iteration	Emulation
AVX2 (base 64)	24 cycles/iteration	of carry flag

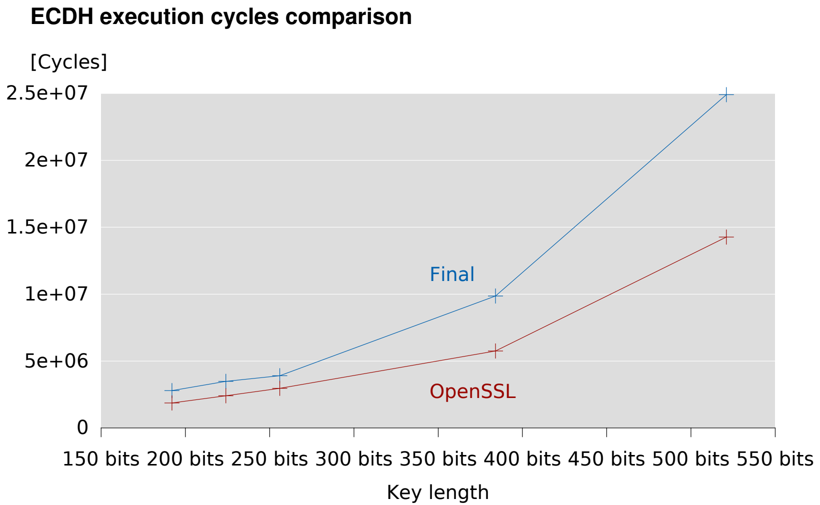
- Further AVX2 downsides
  - higher mul latencies
  - unfriendly data layout
  - multiplications not always independent

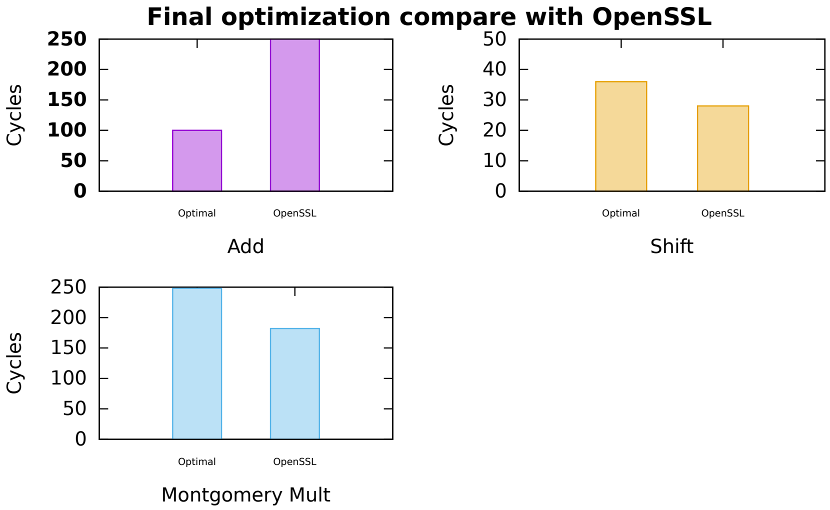
Experiment result

- Environment
  - Platform:Arch Linux 64 bit, GCC 6.1.1 compiler
  - Skylake i7-6600U CPU @ 3GHz
  - 64 bit multiplication (mul, mulx): 1 op/cycle
  - 64 bit addition/subtraction (add, sub): 4 op/cycle
  - 64 bit addition with carry (adc, adcx, adox): 1 op/cycle
  - Carry addition only: peak performance of 2 ops/cycle $\sim$ 6 Gops/s on 1 core
  - Compilation flags: -fomit-frame-pointer -O3 -mavx2 -mbmi2 -madx

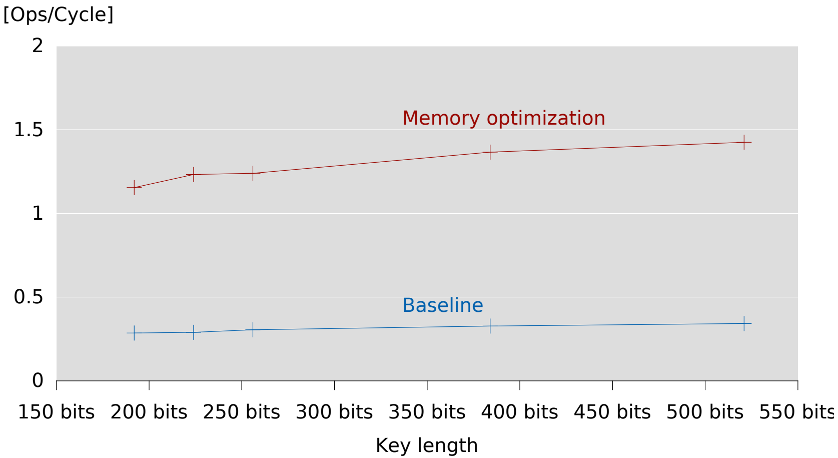
Speedup plot compared to Baseline




ECDH execution cycles comparison
[Cycles]
2.5e+07
2e+07
1.5e+07
1e+07
5e+06
0
Final
OpenSSL
150 bits 200 bits 250 bits 300 bits 350 bits 400 bits 450 bits 500 bits 550 bits
Key length

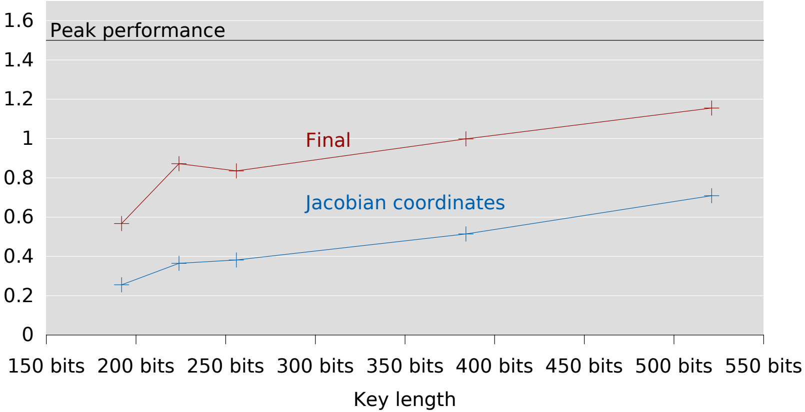

Final optimization compare with OpenSSL
Cycles
250
200
150
100
50
0
Optimal
OpenSSL
Add
Cycles
50
40
30
20
10
0
Optimal
OpenSSL
Shift
Cycles
250
200
150
100
50
0
Optimal
OpenSSL
Montgomery Mult

Performance plot Part 1



Performance plot Part 2

[Ops/Cycle]



Speedup plot with varied private key form, compared to OpenSSL

(u=1000000..., v=1000000... || u=1010101..., v=1010101... || u=1111111..., v=1111111...)