Elliptic Curve Cryptography

Fast implentation of the Diffie-Hellman key exchange

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Elliptic Curve

Algorithm

▶ An elliptic curve $E(\mathbb{F}_p)$ consists of the set of the points P(x,y), $x, y \in \mathbb{F}_p$ satisfying

$$y^2 \equiv x^3 + ax + b \pmod{p}$$

Possible to define an addition rule to add points on E



Diffie Hellman key exchange

Public parameters

Algorithm

$$y^2 \equiv x^3 + ax + b \pmod{p}$$

 $a, b \in \mathbb{F}_p$, a prime p and a base point G are known

Private computations

Alice Bob

Compute P = uGCompute Q = vG

Public exchange of values

Alice \xrightarrow{P} Bob Bob

Further private computations

Alice Bob

Compute uQCompute *vP*

The shared secret is uQ = u(vG) = v(uG) = vP

Adaption of Table 2.2 J. Hoffstein et al., An Introduction to Mathematical Cryptography

Double-and-add-Method

- ▶ Input $P \in E(F_p)$, $d \in \mathbb{N}$
- ▶ Output: $d \cdot P \in E(F_p)$

Algorithm

```
N < -P
Q < - \mathcal{O}
for i from 0 to m do
  if d_i = 1 then
     Q \leftarrow point_add(Q, N)
N \leftarrow point_double(N)
return Q
```

where
$$d = d_0 + d_1 2 + ... + d_m 2^m$$
 $d_i \in \{0, 1\}$

https://en.wikipedia.org/wiki/Elliptic_curve_point_multiplication#Double-and-add



Implementation

Bigint

```
typedef uint64_t block;
2
  typedef struct
3
       uint64_t significant_blocks;
       block blocks[BIGINT_BLOCKS_COUNT];
6
7
     __BigInt;
```

Corresponding operations (Addition, Multiplication, Division, ...)

- Elliptic Curve and ECDH
 - ► Elliptic Curve definition and key exchange mechanism
 - 5 predefined curves: 192, 224, 256, 384, 521 bits



Cost Anaylsis

- Index integer operation matters
- Cost measure
 - $ightharpoonup C = C_{add} + C_{mult} + C_{shift}$
 - Code generated operations counts

Optimizations

Stages

Baseline - Implementation without memory optimization

Memory optimization - Implementation with memory optimization

Comparison with OpenSSL

Algorithm

Performance

Algorithm

Performance

Final - Final performance optimization



Optimizations

Overview

Baseline

- Improved memory allocation
- ► Change the base from 8 bit to 64 bit for the big integers

Memory optimization

- Change the base from 2 bit to 64 bit for Montogomery
- ▶ Precomputation of $2^k \cdot G$ where $k \in \{1, ..., \log_2(p)\}$ and $G \in E$
- Introduction of Jacobian coordinates
- ▶ Vectorize shifting using AVX2

Jacobian coordinates

- function stiching
- Inlining functions
- Unrolling



Intel ADX

C Code

```
low_m1 = _mulx_u64(a->blocks[i], b, &hi_m1);
add.carry_m1 = _addcarryx_u64(add_carry_m1, carry_m1, low_m1, &temp_m1);
add_carry_1 = _addcarryx_u64(add_carry_1, res->blocks[i], temp_m1, tmp->blocks[i]);
carry_m1 = hi_m1;
```

Created Assembly code

x86 icc 13.0.1 -m64 -march=CORE-AVX2 -O3

http://gcc.godbolt.org/



Intel ADX

C Code

```
low_m1 = _mulx_u64(a->blocks[i], b, &hi_m1);
add_carry_m1 = _addcarryx_u64(add_carry_m1, carry_m1, low_m1, &temp_m1);
add_carry_1 = _addcarryx_u64(add_carry_1, res->blocks[i], temp_m1, tmp->blocks[i]);
carry_m1 = hi_m1;
```

Created Assembly code

x86 gcc 5.3 -m64 -march=haswell -O3

```
mulx
            48(% rsi), %r9, %r10
            %r9. %r11
   adda
            %r10, %r9
   movq
            %bpl
   setc
          $-1. %bl
   addb
6
           48(% rdi), %r11
   adca
7
            %r11, 2288(%rax)
   movq
    setc
            %b1
            -1. \%bpl
   addh
```

http://gcc.godbolt.org/



ADX vs AVX2

- Bottleneck operation: BigInt block multiplication
- Unavoidable dependecies in carry chain -> vectorization by processing 4 multiplications in parallel

Approach Lower bound		Bottleneck	
ADX	8 cycles/iteration	ADX throughput	
AVX2 (base 32)	10 cycles/iteration	Emulation of carry	
AVX2 (base 64)	24 cycles/iteration	flag	

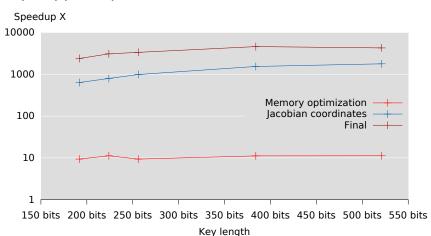
- Further AVX2 downsides
 - higher mul latencies
 - unfriendly data layout
 - multiplications not always independent



Experiment result

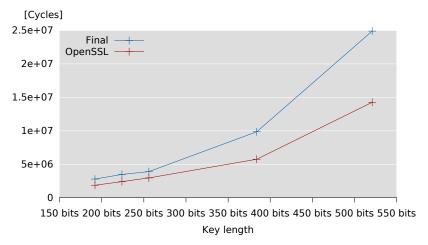
- Environment
 - Platform:Arch Linux 64 bit, GCC 6.1.1 compiler
 - Skylake i7-6600U CPU @ 3GHz
 - ▶ 64 bit multiplication (mul, mulx): 1 op/cycle
 - ▶ 64 bit addition/subtraction (add, sub): 4 op/cycle
 - ▶ 64 bit addition with carry (adc, adcx, adox): 1 op/cycle
 - Carry addition only: peak performance of 2 ops/cycle 6 Gops/s on 1 core
 - Compile flag: -O3 -mavx2 -mbmi2 -madx



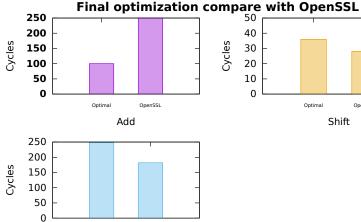


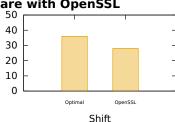


ECDH execution cycles comparison







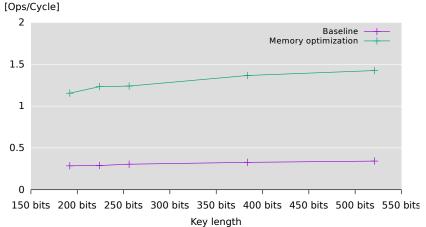


Optimal

Montgomery Mult

OpenSSL

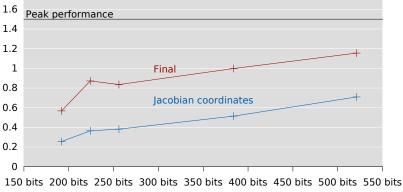






Performance plot Part 2

[Ops/Cycle]



Key length



Performance plot operation counts

Key size	Baseline	Memory optimization	Precomputation	Jacobian coordinates	Final
192	2117237004	912515528	16277780	2977446	1585686
224	3666675252	1384425854	25816754	5796852	3035341
256	4901227919	2122930314	35137355	6206895	3257705
384	18873323391	7047549105	109331889	19264827	9848295
521	48749705798	18063182851	282776551	56794800	28765815



Speedup plot with varied private key form, compared to OpenSSL

Speedup/OpenSSL X

