

FAST IMPLEMENTATION OF ELLIPTIC CURVE DIFFIE-HELLMAN

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ABSTRACT

Describe in concise words what you do, why you do it (not necessarily in this order), and the main result. The abstract has to be self-contained and readable for a person in the general area. You should write the abstract last.

1. INTRODUCTION

Cryptography is a complex and widespread computational problem in modern computer science research and applications. To ensure the safety of the exchanging of information, encryption provides the core foundation of a secure information system. Among different cryptographic problems, key exchange is an essential topic for two distant users to be able to share a secret without meeting physically. Internet technology companies have been exploiting ways to have efficient yet high performance algorithms to fulfill the needs of exchanging keys in this age where network security has been concerned more than ever. Various key exchange tools have been generated. Elliptic Curve Cryptography (ECC) has shown its advantages over traditional RSA (Rivest, Shamir and Adleman). By using less key sizes, ECC can apply a more efficient method without reducing the secure level of the encryption, whereas RSA requires the involvement of larger primer number to reach the same goal [1].

As an important application domain of ECC, Elliptic Curve Diffie-Hellman (ECDH) key exchange has attracted researcher's attention due to its safety and efficiency. A fast implementation of ECDH is highly in demand, as the number of Internet users increases drastically with the growing needs for information security and the speed of computation. ECDH is widely used and there are some potential to be improved in its computational performance. The challenge lies in the complex computational models and its natural requires for large integers. In this paper, we propose a fast implementation of high performance of ECDH key exchange. By comparing with popular cryptography library we proved the feasibility of our improvement of our optimization methods.

Brown provides the backbone of this project by providing a general standard of Elliptic Curve Cryptography[2] and specifying a well-established deployment requirements. Malik's work regarding a particular implementation in a FPGA card [1] shows the possibility of performance oriented optimization. The use of projective coordinates, as the most efficient optimization methods during our implementation, was proven by Blake et al.[3] as an effective method for expensive field inversions. In this paper we combine multiply approaches of optimizations and achieved satisfactory performance improvement and execution speedup.

We give a thorough description of background in section 2, proposed optimization methods in section 3 and show our experimental results in section 4. In Section 5, we discuss and analyze our approach.

2. BACKGROUND: ELLIPTIC CURVE DIFFIE-HELLMAN

The purpose of this section is to familiarize the reader with the mathematical foundations of the Diffie-Hellman key exchange and which algorithms we used to achieve this task. This chapter follows a bottom-up approach.

Finite Field arithmetic [2, p. 3-4]. For elliptic curve cryptography one is mainly interested in the prime finite field \mathbb{F}_p and the characteristic 2 finite field \mathbb{F}_{2^m} . In this paper only elliptic curves over \mathbb{F}_p are discussed.

Montgomery multiplication.

The Montgomery modular multiplication algorithm allows to perform fast modular multiplication on large integers. In particular, the algorithm can be adapted for multi-precision arithmetic. Given two integer x and y and the value of the finite field \mathbb{F}_p , the algorithm computes $xy \bmod p$. In order to perform the modular multiplication, x and y are converted to Montgomery form: xR and yR where R is larger than p and co-prime with p . R is usually chosen as b^t where b matches the size of a processor's word, 2^{64} in our case. The algorithm performs a chain of multiplication mod b and division by b , which are simply mask and shift operations when b is in the appropriate form. The algorithm for the multi-precision representation is given below [4, p. 602]

INPUT:

$p = p_{n-1} \dots p_0)_b$
 $x = (x_{n-1} \dots x_0)_b$
 $y = (y_{n-1} \dots y_0)_b$

OUTPUT:

$xyR^{-1} \bmod m$

```

1.  $A \leftarrow 0$  ( $A = (a_n \dots a_0)_b$ )
2. For  $i$  from 0 to  $(n-1)$  do:
    2.1  $u_i \leftarrow (a_0 + x_i y_0) p' \bmod b$ 
    2.2  $A \leftarrow (A + x_i y + u_i p) / b$ 
3. If  $A \geq p$  then  $A \leftarrow A - p$ 
4. Return  $A$ 

```

Listing 1. Multiprecision Montgomery Modular Multiplication

The challenging part of the reduction modulo p resides in the high amount of division operations to be performed. The algorithm takes advantage of the underlying architecture by performing operations on machine words. The modulo operation in itself is only executed at the end, where A is certain to be between 0 and $2p - 1$. In the early phase of the project, we used a base $b = 2$ for R . We later changed the base to match the machine word and our block representation of the multi-precision integers.

Shifting using vector instructions.

The shift operation is used extensively in both our division and Montgomery multiplication algorithm. In the first implementation, we relied on multi-precision division which executes many shift-by-1-bit operations. Although the idea of using vector instruction can sound appealing, we did not see a major speed up when vectorizing the shift operation. The reason for this is that the shift needs to be performed across the vector and therefore also between two vector. This significantly slows the operation since the bits need to be moved between vectors but also across inner boundaries or vectors.

Elliptic Curve over \mathbb{F}_p [2, p. 6-7]. An elliptic curve is defined by the equation

$$y^2 \equiv x^3 + ax + b \pmod{p} \quad (1)$$

where p is an odd prime number and $a, b \in \mathbb{F}_p$ are the parameters of the curve. Furthermore a, b need to satisfy $4a^3 + 27b^2 \not\equiv 0$. The elliptic curve $E(\mathbb{F}_p)$ consists of the Points $P = (x, y)$ $x, y \in \mathbb{F}_p$ satisfying (1). Additionally we introduce \mathcal{O} the so called point at infinity. The addition of points (affine coordinates) is defined as follows [2]

1. $\mathcal{O} + \mathcal{O} = \mathcal{O}$
2. $(x, y) + \mathcal{O} = \mathcal{O} + (x, y) = (x, y) \quad \forall (x, y) \in E(\mathbb{F}_p)$

3. $(x, y) + (x, -y) = \mathcal{O} \quad \forall (x, y) \in E(\mathbb{F}_p)$
4. Assume $x_1 \neq x_2$. $(x_1, y_1) + (x_2, y_2) = (x_3, y_3)$ is defined as $x_3 \equiv \lambda^2 - x_1 - x_2 \pmod{p}$, $y_3 \equiv \lambda(x_1 - x_3) - y_1 \pmod{p}$ and $\lambda \equiv \frac{y_2 - y_1}{x_2 - x_1} \pmod{p}$
5. $(x_1, y_1) + (x_1, y_1) = (x_3, y_3)$ is defined as $x_3 \equiv \lambda^2 - 2x_1 \pmod{p}$, $\lambda(x_1 - x_3) - y_1 \pmod{p}$ and $\lambda \equiv \frac{3x_1^2 + a}{2y_1} \pmod{p}$

Rule 4. describes how to add two different points whereas rule 5. explains how to double a point. The set of points satisfying (1) and \mathcal{O} together with this addition form a commutative group.

Jacobian Coordinates [3, p. 59 - 60]. "In cases where field inversions are significantly more expensive than multiplications, it is efficient to implement projective coordinates" [3] In Jacobian coordinates a point is represented as a triplet (x, y, z) satisfying (2).

$$y^2 \equiv x^3 + axz^4 + bz^6 \pmod{p} \quad (2)$$

In this project Jacobian coordinates were implemented. For the definition of the rules see [3, p. 59-60]

Double-and-add method. In order to implement the Diffie-Hellman key exchange we need to calculate dP fast.

Input $P \in E(\mathbb{F}_p)$, $d \in \mathbb{N}$

Output: $d \cdot P \in E(\mathbb{F}_p)$

```

 $N \leftarrow P$ 
 $Q \leftarrow \mathcal{O}$ 
for  $i$  from 0 to  $m$  do
    if  $d_i = 1$  then
         $Q \leftarrow \text{point\_add}(Q, N)$ 
     $N \leftarrow \text{point\_double}(N)$ 
return  $Q$ 

```

Listing 2. double-and-add method

where $d = d_0 + d_1 2 + \dots + d_m 2^m$ $d_i \in \{0, 1\}$

Diffie Hellman key exchange [5, p. 170-171]. Alice and Bob want to establish a secret over a public channel. We assume that the elliptic curve parameter: a prime p , $a, b \in \mathbb{F}_p$ a point G with high order are publicly known.

1. Alice chooses a secret integer u , computes $G_u = uG$, and sends G_u to Bob.
2. Bob chooses a secret integer v , computes $G_v = vG$, and sends G_v to Alice.
3. Alice computes $uG_v = uvG$
4. Bob computes $vG_u = vuG$.

One can verify that $uG_v = uvG = vuG = vG_u$. An eavesdropper knows G, G_u, G_v and his goal is to calculate

uvG . This is known as the Diffie-Hellman Problem and is assumed to be a hard problem.

Complexity.

Cost measure.

3. YOUR PROPOSED METHOD

Baseline - Implementation without memory optimization
Memory optimization - Implementation with memory optimization
Comparison with OpenSSL
Jacobian coordinates - Algorithmic changes, jacobian coordinates
Final - Final performance optimization

Fig. 1. approach

Baseline. Some information about the first baseline

Memory optimization.

First comparison with OpenSSL. Realization that the chosen algorithm is not optimal.

Jacobian coordinates. Introduction of the Jacobian coordinates Precomputation of the Points

Final. Final optimizations like function stitching and inline assembly

Now comes the “beef” of the paper, where you explain what you did. Again, organize it in paragraphs with titles. As in every section you start with a very brief overview of the section.

For this class, explain all the optimizations you performed. This mean, you first very briefly explain the baseline implementation, then go through locality and other optimizations, and finally SSE (every project will be slightly different of course). Show or mention relevant analysis or assumptions. A few examples: 1) Profiling may lead you to optimize one part first; 2) bandwidth plus data transfer analysis may show that it is memory bound; 3) it may be too hard to implement the algorithm in full generality: make assumptions and state them (e.g., we assume n is divisible by 4; or, we consider only one type of input image); 4) explain how certain data accesses have poor locality. Generally, any type of analysis adds value to your work.

As important as the final results is to show that you took a structured, organized approach to the optimization and that you explain why you did what you did.

Mention and cite any external resources including library or other code.

Good visuals or even brief code snippets to illustrate what you did are good. Pasting large amounts of code to fill the space is not good.

4. EXPERIMENTAL RESULTS

To evaluate the optimization result, we test the implementation in a 64 bit Arch Linux machine with Intel CPU of Skylake i7-6600U CPU @ 3GHz. The compiler and arithmetic environment are as following:

- GCC 6.1.1 compiler
- 64 bit multiplication (mul, mulx): 1 op/cycle
- 64 bit addition/subtraction (add, sub): 4 op/cycle
- 64 bit addition with carry (adc, adcx, adox): 1 op/cycle
- Carry addition only: peak performance of 2 ops/cycle 6 Gops/s on 1 core
- Compile flag: -O3 -mavx2 -mbmi2 -madx

Based on the recommended Elliptic curve parameters from the guidance standard [2], we have chosen 5 predefined curves with various key size : 192, 224, 256, 384, 521 bits. All versions of implementation has been validated by using Google Unit test, by checking the key exchange result. The benchmark is done by calculating the CPU cycles, in order to testify the real performance by eliminating the influence of the machine. In the end we compare our implementation with OpenSSL, the well known open source library to check our implementation effect. The OpenSSL ECDH mechanism is deployed and the library has been imported as a package of GCC.

Speedup plot compared to Baseline

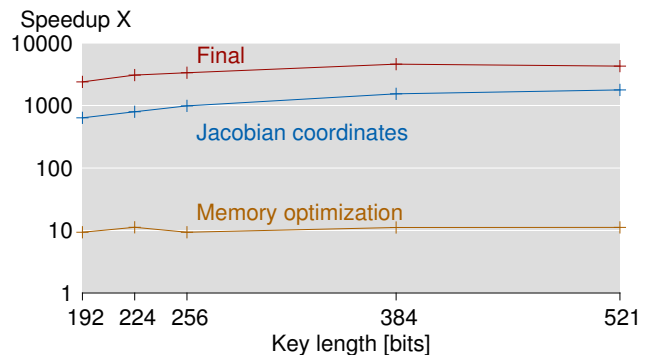


Fig. 2. Speedup plot compared to Baseline

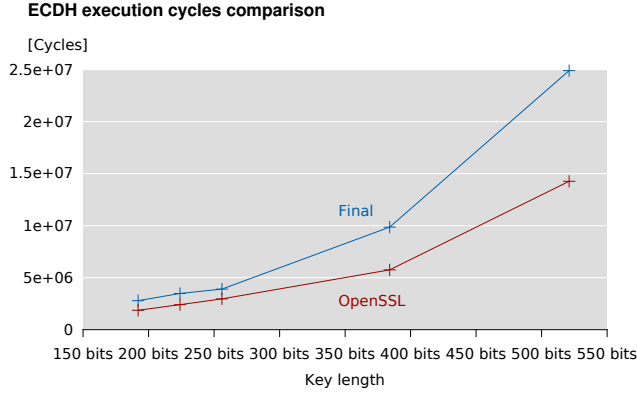


Fig. 3. ECDH execution cycles comparison

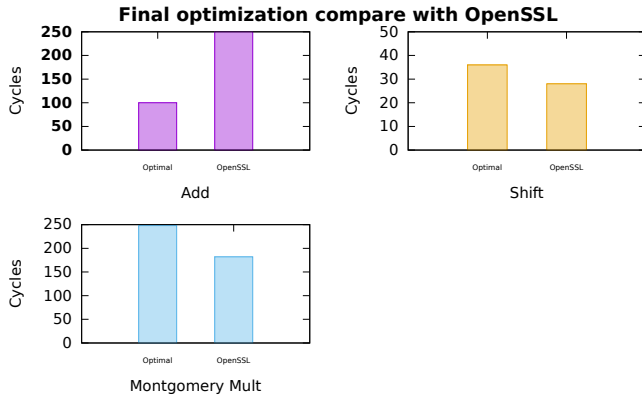


Fig. 4. Final optimization compared with OpenSSL

In the speedup comparison(Fig. 2) three crucial numbers are shown. It can be clearly seen that the Final implementation can execute the key exchange process 5000 times faster than the baseline in almost all the key lengths. In all three cases the speedup increases as the key size grows. It can be explained that the program is not fully functioning when the key size is relatively small, considering that the big integer operations(64 bits base) will take full operations even with smaller input size. The first milestone of memory optimization gives us about 10 times speedup, whereas Jacobian coordinates can run 1000 times faster. The speedup result is satisfactory and reasonable as the baseline implementation contains costly and expensive memory allocations.

Next we run the same operation with OpenSSL library and compare their computation cycles with us. It is no surprise that with the increasing of the key length the required cycles also increase rapidly(Fig. 3). Our required cycles remain the same level with OpenSSL with key length smaller than 256 bits, yet fall behind OpenSSL with larger key length.

Individual operations of integers are the main compo-

nent of our algorithm. To see how good is our implementation we chose integer addition, shift and Montgomery multiplication to compare with OpenSSL(Fig. 4). This comparison is done with fixed key length of 192 bits. In the comparison of integer addition, our final implementation uses less cycles than OpenSSL big integer addition thus being faster than OpenSSL. Yet in both shift and Montgomery multiplication our implementation is more expensive and slower than OpenSSL.

Performance plot Part 1

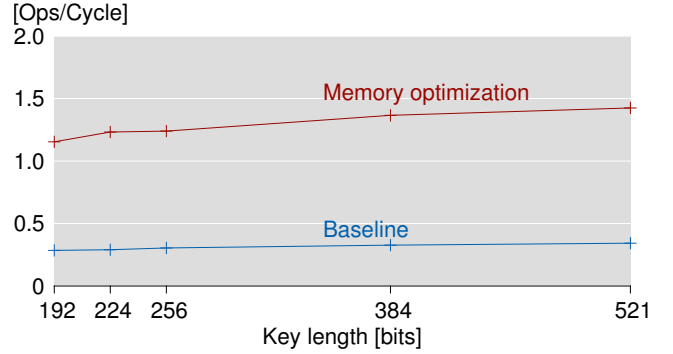


Fig. 5. Performance plot of baseline and memory optimization

Performance plot Part 2

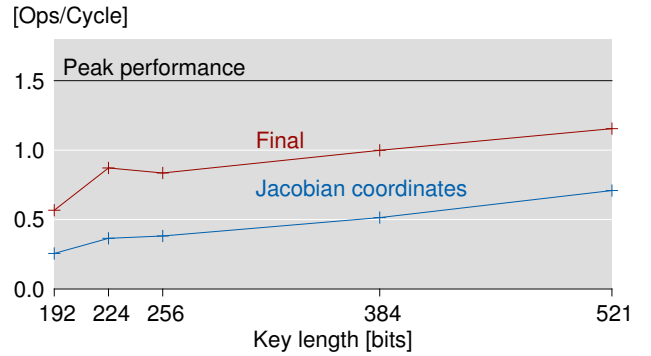


Fig. 6. Performance plot of Final optimization and Jacobian coordinates

Performance plots(Fig. 5 and Fig. 6) are consisted with the division of operation counts and cycles. It reflects if the implementation can reach up the machines' computation ability. Normal operations counts would stay constant as the optimization goes on. Yet in our case since the integer operations does not only include the normal integer operations, but also include the index counts. Thus we have to compromise by using code generated operation counts. In the following performance plot the operation counts changes, but it does reflect the computational efficiency of the code

in certain level. In Fig. 5 the first comparison is done by showing the effect of memory optimization. It is clear that with huge amount of memory allocation, the baseline performance is really limited. In Fig. 6 we begin to see the real performance boost that comes from later optimization. The peak performance here is calculated by considering the ratio of additions and multiplications. With 1 add and 1 mult per cycle and two times add versus one mult, we can draw the line of peak performance at 1.5 ops/cycle. As we can see the final optimization are close to the peak performance with large key size, proving the optimization work successful.

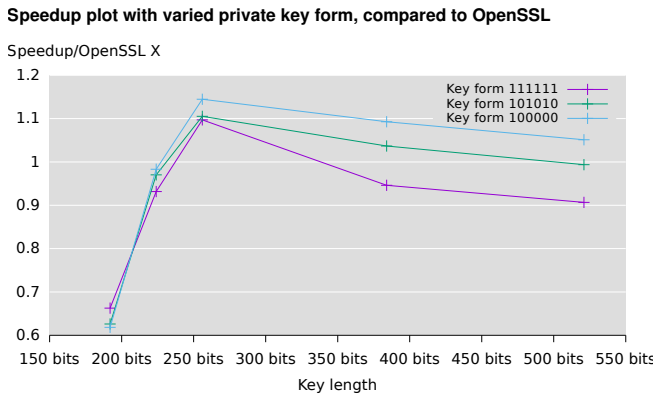


Fig. 7. Speedup plot with varied private key form, compared to OpenSSL

Fig. 5 gives us another perspective of how different key form would influence the speedup. In the simplified private key form that consist of only zeros and ones, our speed of processing EDCH come closely to OpenSSL. The key form that consist only ones has the worst performance since it requires redundant integer operations that are unavoidable.

5. CONCLUSIONS

Here you need to briefly summarize what you did and why this is important. *Do not take the abstract* and put it in the past tense. Remember, now the reader has (hopefully) read the paper, so it is a very different situation from the abstract. Try to highlight important results and say the things you really want to get across (e.g., the results show that we are within 2x of the optimal performance ... Even though we only considered the DFT, our optimization techniques should be also applicable) You can also formulate next steps if you want. Be brief.

6. REFERENCES

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nal processor," in *Advanced Communication Technology (ICACT), 2010 The 12th International Conference on*. IEEE, 2010, vol. 2, pp. 1464–1468.

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