Math 323 Tutorial 1 Questions

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1. Let A, B, C be arbitrary sets such that $A, B, C \subseteq \Omega$. Using the sets themselves and the operations $\cap, \cup, \setminus$ and set complement write the following events:

i) Precisely 2 events occur
ii) At least 1 event occurs
iii) No more than 2 events occur
iv) Only A occurs

2. Let A, B be arbitrary sets such that $A, B \subseteq \Omega$. Are the following relations true or false, justify your answer with a brief proof:

a) $(A \cup B) = (A \cap B^c) \cup B$
b) $(A^c \cap B) \cup (A \cap B^c) = (A \cup B) \cap (A \cap B)^c$

3. Suppose we are testing a patient for a disease. Each patient either has the disease, denoted D, or they do not, denoted D^c. Suppose we administer a blood test. The blood test will either come back positive, denoted B, or negative, B^c.

a) Use the sets D, D^c, B, B^c to describe the event that someone was misclassified by the Blood Test. First consider the two ways that a person could be misclassified and then use set notation to write this down as an event.

b) Suppose in addition to the blood test (B), the patient consulted a Physician and took an x-ray. Let the event P denote the physician *believes* that the patient has the disease and the event X denote that the x-ray came back positive. Suppose the hospital uses the following classification rule to decide overall if they believe the patient has the disease or not. In order to be classified as having the disease, the Physician and at least one of the tests needs to come back positive. Write the event that the patient was correctly classified

(NB: There is a difference between a patient actually having the disease D and a test indicating that the patient has it or not (P, X, and/or B). The difference is a test can be wrong. D represents the truth which we cannot observe, but that we can still describe. This is a very foundational concept, in statistics generally. Many of the questions you get asked later in this will require us to reason about the probabilities of the truth given (uncertain) evidence.)

4. Show that for arbitrary $A, B \subseteq \Omega$ that:

a) $P(A \cap B) \geq P(A) + P(B) - 1$
b) $P(A \cap B) \leq min(P(A), P(B))$

(Ask yourself when the bound in a) is a useful one, that is when it tells you something that you don't already know from the basic rules and axioms of probability. What does this look like (draw it if it helps you)? Sometimes in math we use the word non-trivial to describe a useful bound, you may see this in futures questions. A trivial probablity bound would be a bound from below less than or equal to 0 since we know that all probablities must be greater than or equal to 0 from Kolmogorov's axioms already. Similarly, a trivial bound from above would be greater than or equal to 1.)