MATH 323 - DunderMifflin Golden Tickets

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Email tyrel.stokes@mail.mcgill.ca for any questions about these problems. A selection of these problems will be covered in Tuesdays tutorial which takes place from 3-4pm. (See mycourses → content → Tyrel's Tutorial → Zoom link for office hourse and tutorial for the link)

1. a) Let $Y_1 \sim Geometric(p)$. Find the Moment Generating function of Y_1.

b) Suppose that $Y_1, Y_2, \ldots, Y_k \sim Geometric(p)$ are identically and independently distributed geometric variables with parameter p. Show that X, $X = \sum_{i=1}^{k} Y_i$ has a negative binomial distribution.

Use the fact that if $X \sim NegBin(r, p)$ is a negative binomial, the it has the following moment generating function

$$M_X(t) = \left(\frac{pe^t}{(1-(1-p)e^t} \right)^r, t < -\ln(1-p)$$

.

In this parametrization of the negative binomial, X models the number of trials until the rth success, so that $P(X = x) = \binom{x-1}{r-1} p^r (1-p)^{x-r}, x \in r, r+1, r+2, \ldots$.

(If you want more MGF practice, derive the MFG above yourself. I will include a solution to get this result when I release the solutions)

c) Michael Scott from the Dundermifflin paper company has organized a promotional sale. He has hidden 200 golden tickets in different boxes of paper in the warehouse. A Dundermifflin customer who finds a golden ticket is entitled to a 0.5% discount on all purchases for next year for each ticket they find. Darryl, the warehouse manager, is worried that Michael forgot to randomly distribute the tickets across the warehouse and that a single customer might end up with several or many of the tickets and the company will go bankrupt. Darryl sends Creed (the quality assurance representative) to perform a check to see if the tickets are well-dispersed. Creed starts randomly opening boxes in the southwest corner of the warehouse checking whether there is a golden ticket in it or not. He keeps track of how many tickets he finds, but forgets to remember which boxes he opens (and thus when he randomly picks a new box he may be opening a box he has already checked). Creed finds his 100th ticket on exactly the 800th try. If Michael had truly randomly dispersed the tickets, what is the probability of creed finding his 100th ticket on his 800th time. Suppose that there are 2,000 boxes in the warehouse. Do not find the numerical value, but an expression which evaluates correctly the desired probability.

d) If Michael had truly randomly distributed the golden tickets, what is the probability that we would expect to find Creed to have found his 100th in 800 or more attempts (using his less than perfect inspection method). Find an approximation to this probability using the information from the previous parts. Use a table or calculator to find the value of the approximation.

e) Supposing again that Michael truly randomly distributed the golden tickets, what is the expected number of boxes for Creed to open to find 100 golden tickets? Using what you have learned in class bound the probability of Creed finding 100 golden tickets within or equal to 2 standard deviations away from the mean.