MATH 323 - Tutorial 5 Questions

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1. Political Party Preference. Suppose you are Vice President of Surveys and Sampling for the Generic McGill Political Science Club. There are $N = 100$ members in the club. Suppose that there are 3 main parties denoted Party A, B, and C respectfully and that in the Generic McGill Political Science club there are $n_A = 45$, $n_B = 30$, and $n_c = 25$ people who prefer Party A,B, and C respectfully. As Vice President of Surveys and Sampling, you are task will randomly surveying $R = 10$ members of the club.

a) What is the probability that exactly 4 of them prefer party A?

b) Write down a pmf for the number of students surveyed which prefer party A. What is this distribution called?

c) Now suppose, you want to take this survey national, sampling now 100 students from Generic Political Science Clubs all across the country. The problem is that you do not know how many total members there are of generic political science clubs in Canada, but you can reasonably assume that it is a large number. Suppose that $p_a = 0.5$, $p_b = 0.3$, and $p_c = 0.2$ are the probabilities that Generic Political Science members prefer the 3 parties nationally.

Justify an expression for the probability that at least 40 of the 100 randomly surveyed prefer Party B. What is the key assumption for this expression?

2. Consider a simple lottery, where two digits are drawn (with replacement) from the set $\{0, 1, 2, \ldots 9\}$. Players can buy tickets of two unordered numbers. If a player owns a winning ticket, they get the prize. This lottery occurs every week and the numbers are drawn identically and independently week to week.

Consider a player who plays the same strategy every week. They always buy all the tickets corresponding to the same digit twice. Suppose they play every week until they win. Let $Y :=$ "Number of weeks until the player wins" .

a) What values can Y take with positive probability, i.e the set $\{y : P(Y = y) > 0\}$ (the support)?

b) What is $P(Y \leq 2)$?

c) What is the p.m.f of Y? What kind of distribution does Y follow?

d) Come back to this question once we have covered expectations in a few weeks: What is $E[Y]$?

e) The player changes strategy. If the week is an odd week, they only buy the odd duplicates $\{(i,i) : i \in \{1,3,5,7,9\}\}$ and if the week is even then they only buy the even duplicates. What is the pmf of Y now? What kind of distribution is this?

f) Challenge Question: Suppose we don't know what week it is. Let the week be a random variable I, with support in $\{1,2,\ldots,n\}$, where n is a fixed end date that is known to be even. Let W_i be a random variable equal to 1 if player wins on the ith week and 0 otherwise. Let W_I be the random variable that depends on unknown week I. Suppose that during the unknown week we do know that one of the winning numbers is odd, but we don't know the other one. What is the probability that the player wins in that week?

3. Suppose that the number of goals scored by the home and away team in a hockey match are well-modelled by independent Poisson random variables, $Y_{home} \sim Poisson(\lambda_{home})$ and $Y_{away} \sim Poisson(\lambda_{away})$ respectively.

a) Find an expression for the probability that the home team scores strictly more than x goals in a game?

b) Challenge: Thinking of part a as a hint and making use of the independence assumption. Find an expression for the probability that the home team wins. Remember the home team wins if and only if they score strictly more goals than their opponent.