MATH 323 - Tutorial 4 Questions

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1. Stonks!

Consider two different stocks, X_1 and X_2. Suppose that there are three different states of the world, say for example there is a state where the economy crashes (S_b), another where status quo is maintained (S_m), and the state of the world where the economy booms S_g. The stocks will have different values depending on the state of the world. You can assume that the value for both stocks will be lowest in S_b, at least as great as the bad state in S_m, and at least as great as that in the boom economy S_b. It is known that $P(S_b) = \frac{1}{2}$, $P(S_m) = \frac{1}{6}$, $P(S_g) = \frac{1}{3}$.

Suppose we know the CDF of X_1 is:

$$P(X_1 \leq x_1) = \begin{cases} = 0, x_1 < 0 \\ = \frac{1}{2}, 0 \leq x_1 < 4 \\ = \frac{2}{3}, 4 \leq x_1 < 7 \\ = 1, x \geq 7 \end{cases}$$

For X_2 we know that if the economy is bad then it will be worth 1 and it will be worth 5 if the economy either stays the same or booms.

a) This is not a real probability question, but imagine you were forced to buy one of the two stocks (without being able to predict the state of the economy, i.e only knowing the marginal probabilities above). Which would you prefer?

b) Find the PMF of X_1 and the CDF of X_2 (draw it).

We will return to this question in a few weeks and explore more of the Stocks properties.

2. When coded messages are sent, there are sometimes errors in transmission. In particular, Morse code uses dots and dashes, which are know to be sent in the proportions $3dots : 4dashes$. Suppose there is interference in the signal and potentially human error in properly distinguishing a dot from a dash, so that sometimes a dot which is sent is received as a dash and vice versa. Suppose that it is known that with probability $\frac{1}{4}$ a dot is mistakely encoded as a dash and with probability

$\frac{1}{3}$ a dash is encoded as a dot.

a) If a dash is recieved, what is the probability that a dash was actually sent?

b) Now consider the random variable X, where X := "The number of dots recieved" is the sample space generated from the experiment of sending and receiving n independent messages.

i) What is the support of X?
ii) what is the pmf of X written in terms of $p_{dot_R} = P(DOT\ Received)$?

3. Consider a sequence of random variables $Y_1, Y_2, \ldots Y_n$ generated from the experiment of flipping a coin n times independently.

Let

$$Y_i = \begin{cases} 1 \text{ if the ith flip is heads} \\ 0 \text{ if the ith flip is tails} \end{cases}$$

Suppose that the probability of a heads is $P(Y_i = 1) = p_y, p_y \in (0,1), \forall i = 1, 2, \ldots, n$. These are bernouilli variables.

Now consider a random variable, which is a function of the sequence of variables.

Let $W_1 = min(Y_1, Y_2, \ldots, Y_n)$.

a) i) What is the support of the random variable W_1?
(You may not have seen this word support in class and that is okay. All it is the set of numbers that the random variable can take with positive probability, $\{x : P(X = x) > 0\}$ for some random variable X)
ii) Write down the PMF of W_1 in terms of p_y

b) Consider the random variable $W_2 = max(Y_1, Y_2, \ldots, Y_n)$
i) What is it's support
ii) Write down the PMF in terms of p_y

c) Assume now that $p_y = 0.5$ and that $n = 5$.
i) Find the expectations of W_1 and W_2 respectively.
ii) Challenge: Find the expectation: $E[|W_1 - W_2|]$? [There is a way to simplify this, do you see it?]