Advanced Macroeconomics
Problemset Class 1

Due Date: 25 January 2021 8.00 am GMT
Maarten De Ridder

Question 1: Recursive Formulation and Optimal Allocations

Consider the following RBC model. There is a representative household that solves the following maximization problem:

$$\begin{aligned} & \max_{C_s, L_s, K_{s+1}} \mathrm{E}_t \sum_{s=0}^{\infty} \beta^s \left(\ln C_{t+s} - \frac{L_{t+s}^{1+1/\eta}}{1+1/\eta} \right) \\ \text{such that } K_{t+1} + C_t & = (1 + r_t - \delta) K_t + w_t L_t \end{aligned}$$

Workers consume an amount C_t of the consumption good and supply capital K_t and labor L_t to firms on competitive markets, in exchange for rental rate r_t wage w_t. There is a continuum of price-taking firms that produce that produce using a Cobb-Douglas technology :

$$Y_t = Z_t K_t^{\alpha} L_t^{1-\alpha}$$

where Z_t follows a stationary AR(1) process.

(a) Solve the firm's problem to find factor prices w_t and r_t. Interpret your results.

(b) Set up the recursive formulation for the household's optimization problem. What are the state variables? What are the controls? Using the recursive formulation, find the first order conditions and the envelope condition for the household's optimization problem. Provide an intuitive interpretation of the results.

(c) Define the competitive equilibrium.

(d) Now set up the problem from the perspective of a benevolent social planner. Take the first order conditions and compare the equilibrium to the decentralized case in part (d). Explain your results.

Question 2: Analytical RBC Model

Consider the following simplified RBC model. The continuum of infinitely lived representative households has measure 1 and each household is endowed with a single unit of labor per period which is supplied competitively. They maximize the present value of utility subject to the usual no-ponzi and flow budget constraints. Households have log-utility over consumption and no disutility of labor. The Cobb-Douglas production function is $Y_t = Z_t K_t^{\alpha} L_t^{1-\alpha}$ where productivity Z_t is stochastic. Capital fully depreciates each period and investments take 1 year to build, such that $K_{t+1} = I_t$. Households own capital and rent it to firms on competitive markets.

(a) Find the Euler equation for consumption and the price-taking firm's profit maximizing capital and labor choices. Explain your results.

(b) Use guess-and-verify to solve for investment and consumption as functions of output with parameters α and β. Explain your results.

(c) Find steady state output using your results in (b). Explain your results.

(d) Assume that productivity is stationary and subject to white noise ε_t such that: $Z_t = Z\exp(\varepsilon_t)$. Say the economy is initially in the steady state. Find the **impulse response functions** for $\varepsilon_t = 1$. That is, find the expected path of output, consumption and investments in log-deviations from the steady state using $\mathrm{E}_t(\varepsilon_s = 0)$ for all $s > t$. Explain your results. What's the source of propagation in the model?

(e) How do your results in (d) change if productivity was a random walk without drift, such that $Z_t = Z_{t-1}\exp(\varepsilon_t)$ with $Z_0 > 0$?

Question 3: Output Gaps and the HP Filter

The HP filter solves for the sequence of potential GDP estimates that minimizes the sum of the squared cyclical variations and changes in the growth rate of potential output, weighed by a smoothing parameter λ:

$$\{y_t^g\} = \underset{\{y_t^g\}}{\arg\min} \sum_{t=1}^{T} \left(y_t - y_t^g\right)^2 + \lambda \sum_{t=2}^{T-1} (\Delta y_{t+1}^g - \Delta y_t^g)^2 \tag{1}$$

(a) Write equation (1) in matrix notation, in terms of a vector of potential output y^g, data y, smoothing parameter λ and a $T-2$ by T matrix K:

$$K = \begin{bmatrix} 1 & -2 & 1 & 0 & 0 & .. & .. & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 & .. & .. & 0 & 0 & 0 \\ .. & 0 & 1 & -2 & 1 & .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. & .. & .. & .. & .. & .. \\ .. & .. & .. & .. & .. & .. & 1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 & .. & .. & .. & 1 & -2 & 1 \end{bmatrix}$$

(b) Show that the solution to the minimization problem is $Ay^g = y$, where $A = (I_T + \lambda K'K)$

(c) Using your favorite coding software (I will use Matlab in class), write a function that takes as inputs a column vector for log GDP y_t and a scalar for the smoothing parameter λ, which returns potential output y_t^g and output gap y_t^c using the condition provided in (b).

(d) Download the files 'germany.csv' and 'greece.csv' from Moodle. The files contain 3 columns, where the first is the level of GDP, the second is the quarter and the third is the year of the data. Using your code, obtain the HP-filter estimates for Germany when $\lambda = 1600$ and when $\lambda = 10$. Explain and interpret the difference. Now compare the estimated output gaps for Germany and Greece using $\lambda = 1600$. How do their output gaps compare around the Great Recession?