ST444: Exercise Sheet 1

Part I: Python

1. Recall that the relative error is the absolute error divided by the magnitude of the exact value. Give an example (in Python) to show that the sum of three floating-point numbers can have a very large *relative* error.

2. Read in a string name, e.g. "Richard", then print a greeting of the form "Hello Richard!".

Part II: Introduction to Computational Data Science

1. Show that the largest integer that can be stored in m bits (of `unsigned` type) is $2^m - 1$.

2. Study the IEEE standard for both `float` type and `double` type.

   See for instance *https://en.wikipedia.org/wiki/Double-precision_floating-point_format*

3. Let "$\tilde{}$" denote the result of floating-point conversion. Prove or disprove that

$$(\tilde{x}\tilde{+}\tilde{y})\tilde{+}\tilde{z} = \tilde{x}\tilde{+}(\tilde{y}\tilde{+}\tilde{z}).$$

4. ($\star$) Let $y = 1 + x$ for a positive number x. If y is stored as the `double float` value $\tilde{y}$ according to the IEEE standard, under what conditions will $\tilde{y} = 1$?

5. Recall the numerical differentiation example (i.e. Example 4) from Lecture 1. Try out different values of `delta` and see at which values the procedure breaks down. Could this cut-off value be (roughly) explained without running the code?

6. Recall the linear regression example (i.e. Example 5) from Lecture 1. Try shifting `x` by different values and see at which values the procedure breaks down.

7. ($\star$) Recall the matrix multiplication example (i.e. Example 6) from Lecture 1. Let n be an positive integer, and let $a_1, \ldots, a_{n+1}$ be $n + 1$ positive integers. Suppose that you are given n matrices, where the i-th matrix $\mathbf{B}_i$ is $a_i \times a_{i+1}$. How to determine the optimal parenthesization of a product of these n matrices, $\mathbf{B}_1\mathbf{B}_2 \cdots \mathbf{B}_n$?

   (Hint: use dynamic programming; to be discussed in more detail later in the course; for those who are interested, search for "matrix chain multiplication")