# 7

Traffic flow theory and modelling

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7.1 Introduction

When do traffic jams emerge? Can we predict, given certain demand levels, when queuing will occur, how long the queues will be, how they will propagate in space and time and how long it will take for the congestion to resolve? Why does an overloaded traffic network underperform? This chapter gives a basic introduction to traffic flow theory which can help to answer these kinds of questions.

We start this chapter by explaining how it connects with the other chapters in this book (see Figure 7.1). In the top left of the figure, the reader will recognize the conceptual model used in Chapters 2 to 6, in a highly simplified form, to explain transport and traffic volumes.

One of the results of the interplay between people's and shippers' needs and desires, the locations of activities and the transport resistance factors (Figure 7.1, top left) is a certain volume of road traffic (Figure 7.1, middle left). Road traffic, and this is where this chapter starts, can be described by using flow variables such as speed and density (Figure 7.1, middle right). The density of traffic is the number of vehicles that are present on a roadway per unit distance. Road traffic flows on certain road stretches during certain time periods can be either free or congested and/or the flows can be unreliable. In the last two cases the transport resistance on these road stretches will be relatively high, as explained in Chapter 6 using concepts such as value of time and value of reliability, among other things. Consequently, high transport resistance implies negative repercussions on road traffic volumes (see the arrow from flow variables to transport resistance, Figure 7.1, top).
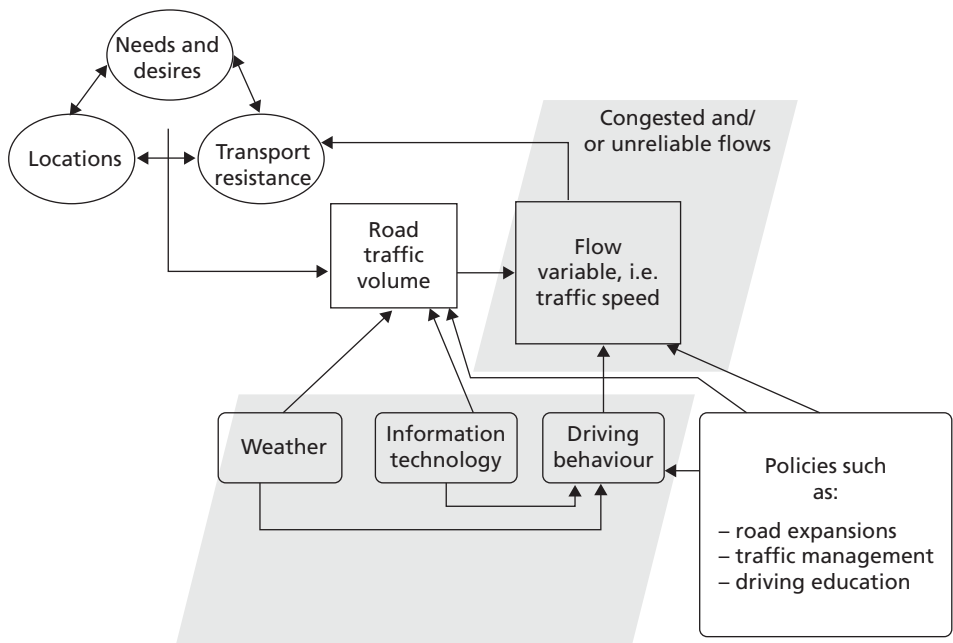
Figure 7.1 The connection between this chapter (grey area) and the simple conceptual framework (top left) as described in Chapter 2

To be clear, this chapter focuses on the road traffic flow variables and the interactions with aspects such as driving behaviour, weather, information technology and so forth (the grey areas in Figure 7.1). Thus traffic flow operations on a road facility are explained for a given traffic demand profile. Factors such as weather and information technology (e.g. navigation systems) can influence traffic flow characteristics through driving behaviour. Additionally, policies such as road expansions and traffic management measures can have an impact on traffic flow operations, either directly or indirectly, by influencing driving behaviour. Transport policies are discussed in Chapter 12.

Traffic flow theory entails the knowledge of the fundamental characteristics of traffic flows and the associated analytical methods. Examples of such characteristics are the road capacities, the relation between flow and density, and headway distributions. Examples of analytical methods are shockwave theory and microscopic simulation models.

Using the presented material, the reader will be able to interpret, analyse and – for simple situations – predict the main characteristics of traffic flows. For the greater part, the chapter considers traffic flow operations in simple infrastructure elements (uninterrupted traffic flow operations, simple discontinu-

ities), although an important side step is made to look at network dynamics. In doing so, the chapter takes both a microscopic and a macroscopic perspective. The microscopic perspective reflects the behaviour of individual drivers interacting with surrounding vehicles, while the macroscopic perspective considers the average state of traffic. We discuss empirical facts, and some well-known analytical tools, such as shockwave theory, kinematic wave models and microscopic simulation models.

Section 7.2 introduces the basic variables on the microscopic level (the vehicle level), and section 7.3 the macroscopic variables, that is, the flow level. Section 7.4 discusses flow characteristics. Then, in section 7.5, traffic flow dynamics and the (self-)organization of traffic are discussed. Section 7.6 presents several theories on multi-lane traffic (i.e. motorways). Section 7.7 discusses car-following models, that is, microscopic flow models, and section 7.8 discusses the macroscopic flow models. Section 7.9 adds the dynamics of networks to this. Finally, in section 7.10 the conclusions are presented.
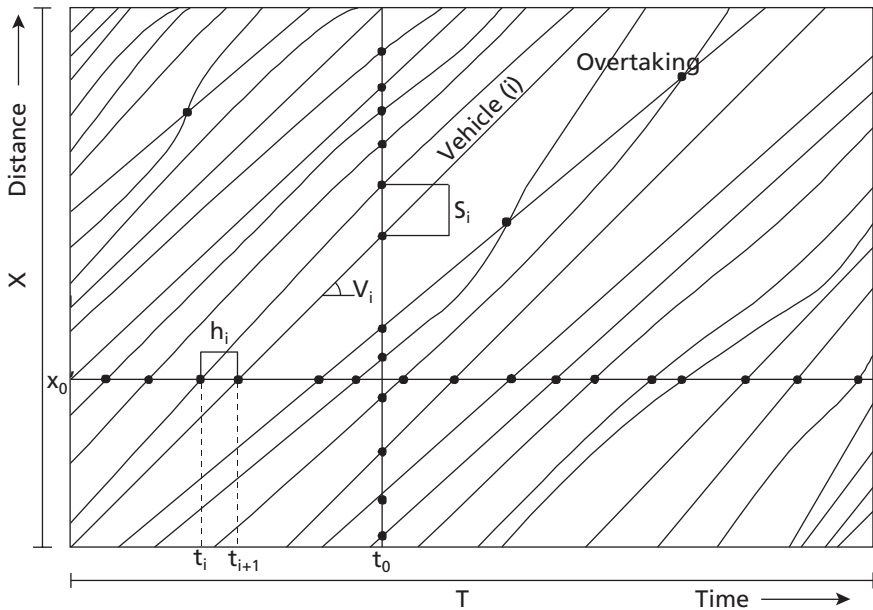
7.2 Vehicle trajectories and microscopic flow variables

The vehicle trajectory (often denoted as $x_i(t)$) of a vehicle (i) describes the position of the vehicle over time (t) along the roadway. The trajectory is the core variable in traffic flow theory which allows us to determine all relevant microscopic and macroscopic traffic flow quantities. Note that, for the sake of simplicity, the lateral component of the trajectory is not considered here.

To illustrate the versatility of trajectories, Figure 7.2 shows several vehicle trajectories. From the figure, it is easy to determine the distance headway S_i, and the time headway h_i, overtaking events (crossing trajectories), the speed $v_i = dx_i/dt$, the size of the acceleration (see top left where one vehicle accelerates to overtake another vehicle), the travel time TT_i and so forth.

However, although the situation is rapidly changing owing to so-called floating car data becoming more common, trajectory information is seldom available. Floating car data is information from mobile phones in vehicles that are being driven. In most cases, vehicle trajectory measurements only contain information about average characteristics of the traffic flow, provide only local information, or aggregate information in some other way (e.g. travel times from automatic vehicle identification or licence plate cameras).

Most commonly, traffic is measured by (inductive) loops measuring local (or time-mean) traffic flow quantities, such as (local) traffic flow q and local

Figure 7.2 Vehicle trajectories and key microscopic flow characteristics

mean speed *u*. First, we will discuss the main microscopic traffic flow variables in detail. This type of flow variable reflects the behaviour of individual drivers interacting with surrounding vehicles.

Gross and net headways

The (gross) time headway (h) is one of the most important microscopic flow variables. It describes the difference between passage times t_i at a cross-section x of the rear bumpers of two successive vehicles:

$$h_i(x) = t_i(x) - t_{i-1}(x) \tag{1}$$

The time headway, or simply headway, is directly determined by the behaviour of the driver, vehicle characteristics, flow conditions and so on. Its importance stems from the fact that the (minimal) headways directly determine the capacity of a road, a roundabout and so forth. Typically, these minimal headways are around 1.5 seconds in dry conditions. Time headways, combined with the speeds, lead to the distance headways (see 'Gross and net distance headways' below).

The net time headway or gap is defined by the difference in passage times between the rear bumper of the lead vehicle and the front bumper of the

following vehicle. This value is particularly important for driving behaviour analysis, for instance when analysing and modelling the amount of space drivers need to perform an overtaking manoeuvre (critical gap analysis).

Gross and net distance headways

We have seen in the preceding sub-section that time headways are *local* microscopic variables: they relate to the behaviour of an individual driver and are measured at a cross-section. On the contrary, distance headways (often denoted by the symbol *s*) are *instantaneous* (measured at one moment in time) microscopic variables, measuring the distance between the rear bumper of the leader and the rear bumper of the follower at time instant *t*:

$$s_i(t) = x_{i-1}(t) - x_i(t) \tag{2}$$

In congested conditions, distance headways are determined by the behaviour of drivers, which in turn depends on the traffic conditions, driver abilities, vehicle characteristics, weather conditions and so forth. In free flow with no interaction between the drivers, the headways are determined largely by the demand (that is, they are determined by the moments when drivers enter the freeway).

Net distance headways are defined, similarly to the net time headways, as the distance between the position of the rear bumper of the leader and the front bumper of the follower.

It should be clear that the time headways and the distance headways are strongly correlated. If v_{i-1} denotes the speed of the leading vehicle, it is easy to see that:

$$s_i = v_{i-1}h_i \tag{3}$$

7.3 Macroscopic flow variables

So far, we have mainly looked at microscopic traffic flow variables. Macroscopic flow variables, such as flow, density, speed and speed variance, reflect the average state of the traffic flow in contrast to the microscopic traffic flow variables, which focus on individual drivers. Let us take a closer look at the most important variables.

Traditional definitions of flow, density and speed

In general, the flow q (also referred to as intensity or volume) is tradition-ally defined by the 'average number of vehicles (n) that pass a cross-section during a unit of time (T)'. According to this definition, flow is a local variable (since it is defined at a cross-section). We have:

$$q = \frac{n}{T} = \frac{n}{\sum_{i=1}^{n} h_i} = \frac{1}{\bar{h}} \tag{4}$$

This expression shows that the flow can be computed easily by taking the number of vehicles n that have passed the measurement location during a period of length $\underline{T}$. The expression also shows how the flow q relates to the average headway $\bar{h}$, thereby relating the macroscopic flow variable to average microscopic behaviour (i.e. time headways).

In a similar way, the density k (or concentration) is defined by the 'number of vehicles per distance unit'. Density is, therefore, a so-called instantaneous variable (i.e. it is computed at a time instance), defined as follows:

$$k = \frac{m}{X} = \frac{m}{\sum_{i=1}^{m} s_i} = \frac{1}{\bar{s}} \tag{5}$$

This expression shows that the density can be computed by taking a snapshot of a roadway segment of length X, and counting the number of vehicles m that occupy the road at that time instant. The expression also shows how density relates to average microscopic behaviour (i.e. distance headways, s). Note that, contrary to the flow, which can generally be easily determined in practice by using cross-sectional measurement equipment (such as inductive loops), the density is not so easily determined, since it requires observations of the entire road at a time instant (e.g. via an aerial photograph).

Similarly to the definitions above, average speeds u can be computed in two ways: at a cross-section (local mean speed or time-mean speed u_L), or at a time instant (instantaneous mean speed or space-mean speed u_M). As will be shown in the following sub-section, the difference between these defi-nitions can be very large. Surprisingly, in practice the difference is seldom determined. For instance, the Dutch motorway monitoring systems collect time-mean speeds, while for most applications (e.g. average travel time) the space-mean speeds are more suitable.
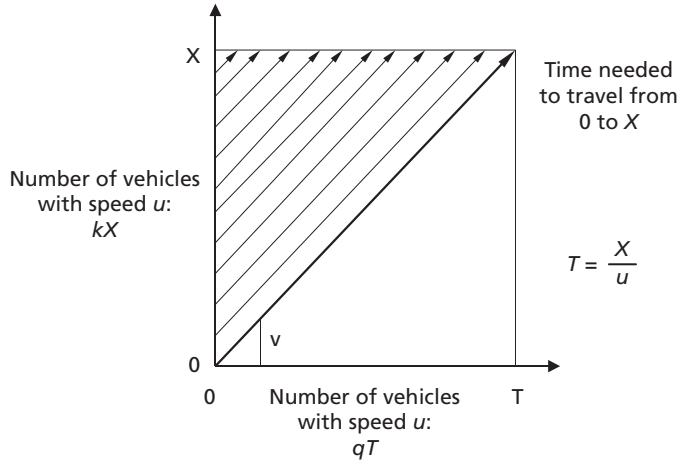
**Figure 7.3** Derivation of the continuity equation

Continuity equation

An important relation in traffic flow theory is the continuity equation: $q = ku$ (flow equals density times the speed). This equation is used to relate the instantaneous characteristic density to the local characteristic flow. The derivation of this equation is actually quite straightforward (Figure 7.3).

Consider a road of length X. All vehicles on this road drive at an equal speed u. Let us define the period T by $T = X/u$. Under this assumption, it is easy to see that the number of vehicles that are on the road at time $t = 0$ – which is equal to the density k times the length X of the roadway segment – is equal to the number of vehicles that will pass the exit at $x = X$ during period $[0,T]$, which is in turn equal to the flow q times the duration of the period T. That is:

$$kX = qT \Leftrightarrow q = k\frac{X}{T} = ku \tag{6}$$

Clearly, the continuity equation holds when the speeds are constant. The question is whether the equation $q = ku$ can also be applied when the speeds are not constant (e.g. u represents an average speed) and, if so, which average speed (time-mean or space-mean speed) is to be used. It turns out that $q = ku$ can indeed be applied, but only if $u = u_M$, that is, if we take the space-mean speed.

Intuitively, one can understand this as follows (mathematical proof can be found in May, 1990). A detector lies at location x_{det}. Now we reconstruct which vehicles will pass in the time of one aggregation period. For this,
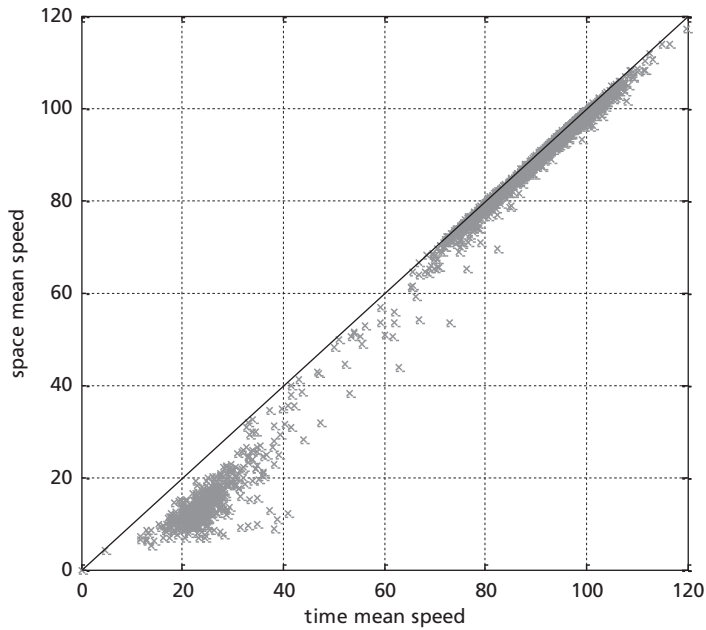
Figure 7.4 Differences between time-mean speed and space-mean speed for the A9 motorway

the vehicle must be closer to the detector than the distance it travels in the aggregation time t_{agg}:

$$x_{det} - x_j \leq t_{agg} v_i \qquad (7)$$

In this formula, x is the position on the road. For faster vehicles, this distance is larger. Therefore, if one takes the local arithmetic mean, one overestimates the influence of the faster vehicles. If the influence of the faster vehicles on speeds is overestimated, the average speed u is overestimated (compared to the space-mean speed u_m).

The discussion above might be conceived as academic. However, if we look at empirical data, then the differences between the time-mean speeds and space-mean speeds become apparent. Figure 7.4 shows an example where the time-mean speed and space-mean speed have been computed from motorway individual vehicle data collected on the A9 motorway near Amsterdam, the Netherlands. Figure 7.4 clearly shows that the differences between the speeds can be as high as 100 per cent. Also note that the space-mean speeds are always lower than the time-mean speeds. Since, in most countries where inductive loops are used to monitor traffic flow operations, arithmetic mean speeds are computed and stored, average speeds are generally overestimated,

affecting travel time estimations. Furthermore, since $q = ku$ can only be used for space-mean speeds, we cannot determine the density k from the local speed and flow measurements, complicating the use of the collected data, for example for traffic information and traffic management purposes.

Generalized traffic flow variables

Alternative measurement methods, such as automatic vehicle identification (AVI), radar and floating car measurements, provide new ways to determine the flow variables described above. One of the benefits of these new methods is that they provide information about the temporal and spatial aspects of traffic flow. For instance, using video we can observe the density in a region directly, rather than by determining the density from local observations.

For the relation between instantaneous and local variables, the work of Edie (1965) is very relevant. Edie (1965) introduces generalized definitions of flow, density and speed. These apply to regions in time and space, and will turn out to be increasingly important with the advent of new measurement techniques.

Consider a rectangular region in time and space with dimensions T and X respectively (see Figure 7.5). Let d_i denote the total distance travelled by vehicle i during period T and let r_i denote the total time spent in region X. Let us define the total distance travelled by all vehicles by:
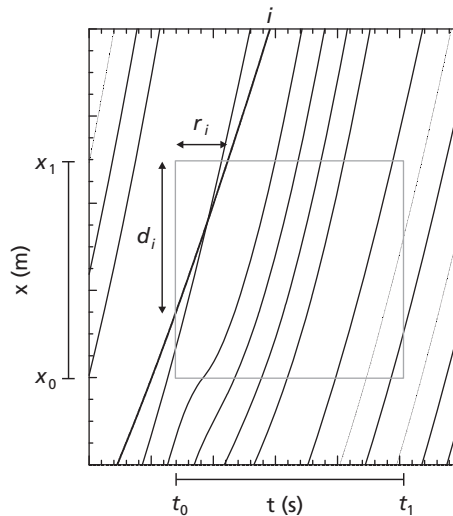
Figure 7.5 Generalization according to Edie (1965)

$$P = \sum_i d_i \qquad (8)$$

Based on this quantity P, which is referred to as the *performance*, Edie defined the generalized flow as follows:

$$q = \frac{P}{XT} \qquad (9)$$

Note that we can rewrite this equation as follows:

$$q = \frac{\sum_i d_i / X}{T} \qquad (10)$$

Let us now define the total travel time R as follows:

$$R = \sum_i r_i \qquad (11)$$

Edie defines the generalized density by:

$$k = \frac{R}{XT} = \frac{\sum_i r_i / T}{X} \qquad (12)$$

For the generalized speed, the following intuitive definition is used:

$$u = \frac{q}{k} = \frac{P}{R} = \frac{\text{total distance travelled}}{\text{total time spent}} \qquad (13)$$

These definitions can be used for any regions in space–time, even non-rectangular ones.

7.4 Microscopic and macroscopic flow characteristics

The preceding sections introduced the different microscopic and macroscopic variables. This section shows the most common flow characteristics, entailing both relations between the flow variables, or typical distribution, and so on. These flow characteristics, in a sense, drive the traffic flow dynamics that will be discussed below. As well as providing a short description of the characteristics and their definition, this section will discuss empirical examples as well as key issues in identifying these parameters.

Headway distributions

If we were to collect headways at a specific location x, then we would observe that these headways are not constant but rather follow some probability distribution function. This is also the case when the flow is stationary during the data collection period. The causes are manifold: there are large differences in driving behaviour between different drivers and differences in the vehicle characteristics, but there is also variation within the behaviour of one driver. A direct and important consequence of this is that the capacity of the road, which is by and large determined by the driving behaviour, is not constant either, but a stochastic variable.

The headway distribution can be described by a probability density function $f(h)$. In the literature, many different kinds of distribution functions have been proposed, with varying success. It can be shown that if the flows are small – there are few vehicle interactions – the exponential distribution will be an adequate model. When the flows become larger, there are more interactions amongst the vehicles, and other distributions are more suitable. A good candidate in many situations is the log-normal distribution; we refer to Cowan (1975) for more details. In Hoogendoorn (2005), an overview is given of estimation techniques for the log-normal distributions in specific situations.

The main problem with these relatively simple models is that they are only able to represent available measurements but cannot be extrapolated to other situations. If, for instance, we are interested in a headway distribution for another flow level than the one observed, we need to collect new data and re-estimate the model.

To overcome this, so-called composite headway models have been proposed. The main characteristic of these models is that they distinguish between vehicles that are flowing freely and those that are constrained by the vehicle in front. Buckley (1968) was one of the first proposing these models, assuming that the headways of the free driving vehicles are exponentially distributed. He showed that the probability density function $f(h)$ of the observed headways h can be described by the following function:

$$f(h) = \phi g(h) + (1 - \phi) w(h) \tag{14}$$

In this equation, g describes the probability density function of the headways of vehicles that are following (also referred to as the distribution of the empty zones), while w denotes the probability density function of those vehicles
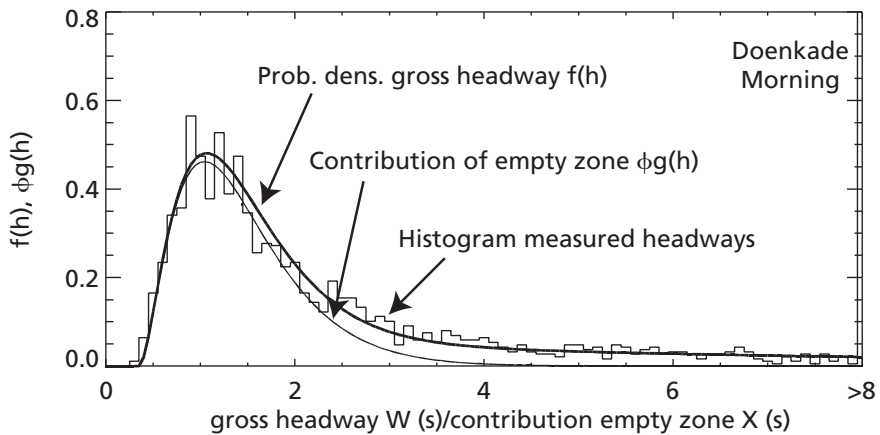
Note: W denotes the gross headway, which is composed of the empty zone X and the free headway W-X.

Source: Hoogendoorn and Botma (1997).

Figure 7.6 Composite headway probability density function (p.d.f.) applied to data on Doenkade site

that are driving freely. For the latter, an exponential distribution is assumed, and φ denotes the fraction of vehicles which are following.

There are different ways to estimate these probability density functions from available headway observations. Wasielewski (1974), later improved by Hoogendoorn (2005), proposed an approach in which one does not need to choose a prior form of the constrained headway distribution. To illustrate this, Figure 7.6 shows an example of the application of this estimation method on a two-lane motorway in the Netherlands in the morning (the location is the Doenkade).

This example nicely illustrates how the approach can be applied for estimating capacities, even if no capacity observations are available. We find the maximum flow (or capacity flow, C) when all drivers are following (as opposed to driving freely). We directly observe from the Buckley model (Buckley, 1968; see above) that the observed headways in that case follow g. The number of vehicles per hour equals 3600 seconds per hour divided by the average headway (H, following distribution g) in seconds (or the expectation value thereof, indicated by E). We therefore get:

$$C = 3600/E(H) \text{ where } H \sim g \tag{15}$$

In other words, the capacity flow equals one over the mean (minimum) headway value, on the condition that all vehicles are following. Using this approach, we can find estimates for the capacity even if there are no direct capacity observations available. For the example above, we can compute the mean empty zone value by looking at the p.d.f. $g(h)$, which turns out to be equal to 1.69. Based on this value, we find a capacity estimate of 3600/1.69 = 2134 vehicles per hour.

Desired speed distributions

Generally, the free speed or desired speed of a driver–vehicle combination (hereafter, simply called vehicle or driver) is defined by the speed driven when other road users do not influence the driver. Knowledge of free speeds on a road under given conditions are relevant for a number of reasons. For instance, the concept of free speed is an important element in many traffic flow models. As an illustration, the free speed distribution is an important input for many microscopic simulation models. Insights into free speeds and their distributions are also important from the viewpoint of road design and for determining suitable traffic rules for a certain facility. For instance, elements of the network should be designed so that drivers using the facility can traverse the road safely and comfortably. It is also of interest to see how desired speed distributions change under varying road, weather and ambient conditions and how these distributions vary for different types of travellers. So speed distribution is also an important characteristic amongst drivers for design issues.

The free speed will be influenced by the characteristics of the vehicle, the driver, the road and (road) conditions such as weather and traffic rules (speed limits). Botma (1999) describes how individual drivers choose their free speed, discussing a behavioural model relating the free speed of a driver to a number of counteracting mental stresses a driver is subjected to. A similar model can be found in Jepsen (1998). However, these models have not been successful in their practical application. The problem of determining free speed distributions from available data is not trivial. In Botma (1999), an overview of alternative free speed estimation approaches is presented. Botma (1999) concluded that all the methods he reviewed have severe disadvantages, which is the reason why another estimation approach is proposed. This approach is based on the concept of censored observations (Nelson, 1982) using a parametric estimation approach to estimate the parameters of the free speed distribution. Speed observations are marked as either censored (constrained) or uncensored (free flowing) using subjective criteria (headway and

relative speed). Hoogendoorn (2005) presents a new approach to esti-
mating the distribution of free speeds based on the method of censored
observations.

Gap acceptance and critical gaps

Gap acceptance is a process that occurs in different traffic situations, such
as crossing a road, entering a roundabout or performing an overtaking
manoeuvre on a bi-directional road. The minimum gap that a driver will
accept is generally called the critical gap. Mathematical representations
of the gap acceptance process are an important part of traffic simulation
models, for instance.

In general terms the gap acceptance process can be described as follows:
traffic participants who want to make a manoeuvre estimate the space they
need and estimate the available space. Based on the comparison between
required and available space, they decide to start the manoeuvre or to post-
pone it. The term 'space' is deliberately somewhat vague; it can be expressed
either in time or in distance. The required space is dependent on characteris-
tics of the traffic participant, the vehicle and the road. The available space is
dependent on the characteristics of, for instance, the on-coming vehicles and
the vehicle to be overtaken (the passive vehicle). Traffic participants have
to perceive all these characteristics, process them and come to a decision.
Humans differ highly in perception capabilities; for example, the ability to
estimate distances can vary substantially between persons, and they differ in
the acceptance of risk. The total acceptance process is dependent on many
factors, of which only a subset is observable. This has led to the introduction
of stochastic models.

Many different methods to estimate the distribution of critical gaps by
observing the gap acceptance process in reality can be found in the literature
(Brilon et al., 1999). Let us consider the problem of estimating the critical
gap distribution. Suppose, as an example, a driver successively rejects gaps
of 3, 9, 12 and 7 *s* and accepts a gap of 19 *s*. The only thing one can conclude
from these observations is that this driver has a critical gap between 12 and
19 *s*. Stated in other words, the critical gap cannot be observed directly. The
observations are, thus, censored. Note that it can also be concluded that only
the maximum of the rejected gaps is informative for the critical gap (assum-
ing that the driver's behaviour is consistent); the smaller gaps are rejected by
definition.

Capacity and capacity estimation

Capacity is usually defined as follows: 'The maximum hourly rate at which people or vehicles can reasonably be expected to traverse a point or uniform section of a lane or roadway during a given time period (usually 15 minutes) under prevailing roadway, traffic and control conditions.'

Maximum flows (maximum free flows of queue discharge rates) are not constant values and vary under the influence of several factors. Factors influencing the capacity are, among other things, the composition of the vehicle fleet, the composition of traffic with respect to trip purpose, weather, road, ambient conditions and so on. These factors affect the behaviour of driver–vehicle combinations and thus the maximum number of vehicles that can pass a cross-section during a given time period. Some of these factors can be observed and their effect can be quantified. Some factors, however, cannot be observed directly. Furthermore, differences exist between drivers, implying that some drivers will need a larger minimum time headway than other drivers, even if drivers belong to the same class of users. As a result, the minimum headways will not be constant values but follow a distribution function (see the discussion on headway distribution modelling in 'Headway distributions' above). Observed maximum flows thus appear to follow a distribution. The shape of this distribution depends, among other things, on the capacity definition and measurement method or period. In most cases, a normal distribution can be used to describe the capacity.

Several researchers have pointed out the existence of two different maximum flow rates, namely pre-queue and queue discharge. Each of these has its own maximum flow distribution. We define the pre-queue maximum flow as the maximum flow rate observed at the downstream location just before the onset of congestion (a queue or traffic jam) upstream. These maximum flows are characterized by the absence of queues or congestion upstream of the bottleneck, high speeds and instability leading to congestion onset within a short period and maximum flows showing a large variance. The queue discharge flow is the maximum flow rate observed at the downstream location as long as congestion exists. These maximum flow rates are characterized by the presence of a queue upstream of the bottleneck, lower speeds and densities, and a constant outflow with a small variance which can be sustained for a long period, but with lower flow rates than in the pre-queue flow state. Both capacities can only be measured downstream of the bottleneck location. Average capacity drop changes are in the range of −1 to −15 per cent, but −30 per cent changes are also reported (see section 7.5 for more on capacity drop changes).

There are many approaches that can be applied to compute the capacity of a specific piece of infrastructure. The suitability of the approach depends on a number of factors, such as:

1. type of infrastructure (e.g. motorway without on- or off-ramps, on-ramp, roundabout, unsignalized intersection, etc.);
2. type of data (individual vehicle data, aggregate data) and time aggregation;
3. location of data collection (upstream of, in or downstream of the bottleneck);
4. traffic conditions for which data are available (congestion, no congestion).

We refer to Minderhoud et al. (1996) for a critical review of approaches that are available to estimate road capacity.
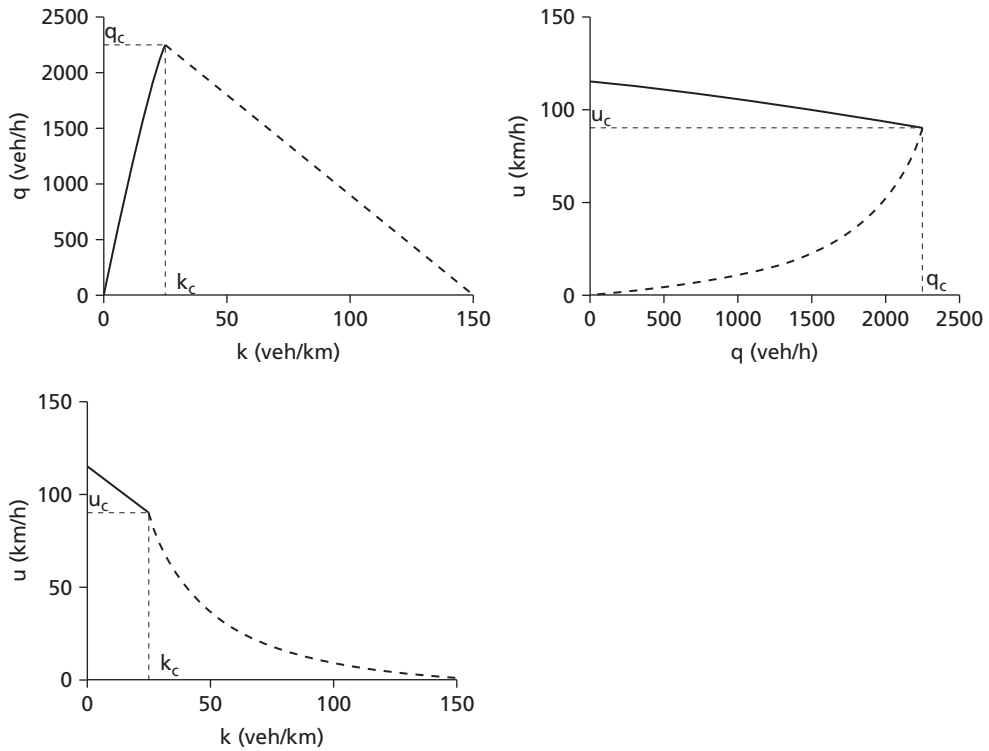
Fundamental diagrams

The fundamental diagram describes a statistical relation between the macroscopic traffic flow variables of flow, density and speed. There are different ways to represent this relation, but the most often used is the relation $q = Q(k)$ between the flow and the density. Using the continuity equation, the other relations $u = U(k)$ and $u = U(q)$ can be easily derived.

To understand the origin of the fundamental diagram, we can interpret the relation from a driving behaviour perspective. To this end, recall that the flow and the density relate to the (average) time headway and distance headway according to Equations (5) and (6) respectively. Based on this, we can clearly see which premise underlies the existence of the fundamental diagram: *under similar traffic conditions, drivers will behave in a similar way.* That is, when traffic operates at a certain speed u, then it is plausible that (on average) drivers will maintain (on average) the same distance headway $s = 1/k$. This behaviour – and therewith the relation between speed and density – is obviously dependent on factors like weather, road characteristics, composition of traffic, traffic regulations and so forth.

Figure 7.7 shows typical examples of the relation between flow, density and speed. The figure shows the most important points in the fundamental diagram, which are the roadway capacity C, the critical density k_c and the critical speed u_c (the density and speed occurring at capacity operations), the jam density k_{jam} (density occurring at zero speed) and the free speed u_0. In

Figure 7.7 Example of the fundamental diagram

the figure, we clearly see the difference between the free conditions $(k < k_c)$ and the congested conditions $(k > k_c)$.

It is tempting to infer causality from the fundamental diagram: it is often stated that the relation $u = U(k)$ describes the fact that, with increasing density (e.g. reduced spacing between vehicles), the speed is reducing. It is, however, more the other way around. If we take a driving behaviour perspective, then it seems more reasonable to assume that, with reduced speed of the leader, drivers need smaller distance headways to drive safely and comfortably.

Fundamental diagrams are often determined from real-life traffic data. This is usually done by assuming that stationary periods can be identified during data measurements. To obtain meaningful fundamental diagrams, the data collection must be performed at the correct location during a selected time period.

7.5   Traffic flow dynamics and self-organization

So far, we have discussed the main microscopic and macroscopic characteristics of traffic flow. In doing so, we have focused on static characteristics of traffic flow. However, there are different characteristics, which are dynamic in nature or, rather, have to do with the dynamic properties of traffic flow.

Capacity drop

The first phenomenon that we discuss is the so-called capacity drop. The capacity drop describes the fact that, once congestion has formed, drivers are not maintaining a headway that is as close as it was before the speed breakdown. Therefore the road capacity is lower. This effect is considerable, and values of a reduction up to 30 per cent are quoted (Hall and Agyemang-Duah, 1991; Cassidy and Bertini, 1999; Chung et al., 2007). The effect of the capacity drop is illustrated in Figure 7.8.
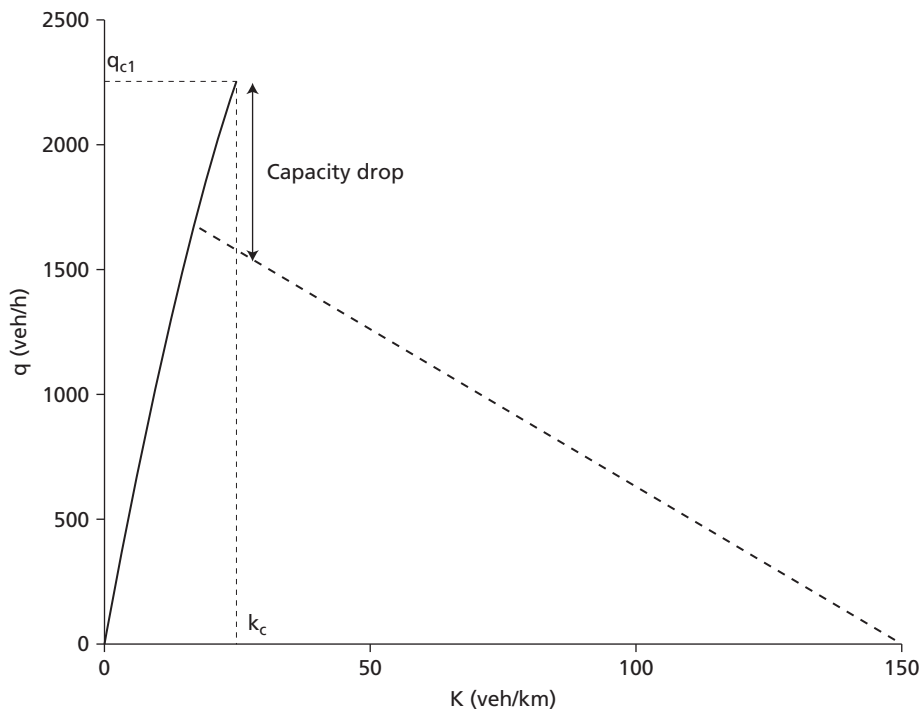
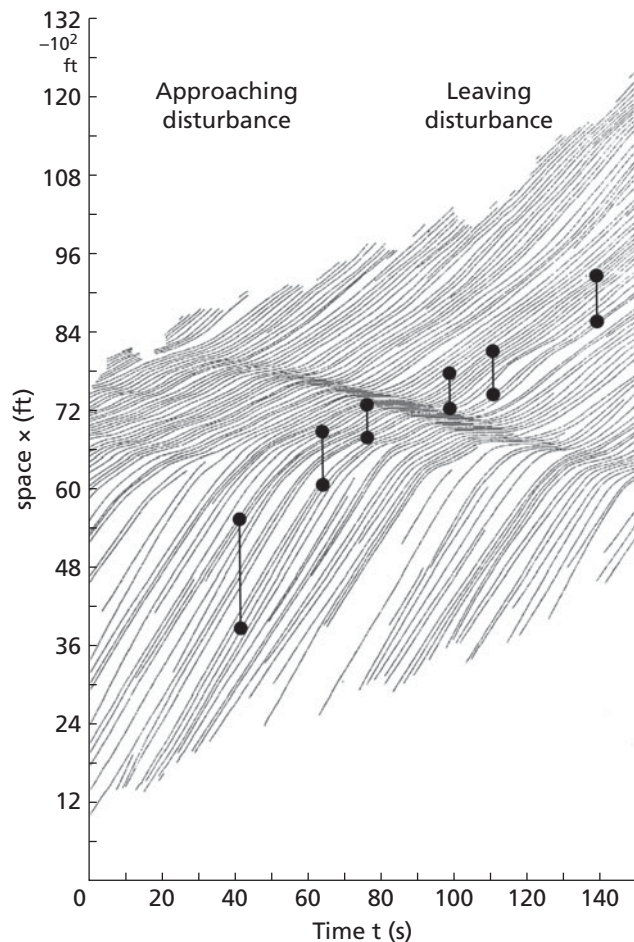**Figure 7.8** The capacity drop in the flow-density diagram

Traffic hysteresis

The different microscopic processes that constitute the characteristics of a traffic flow take time: a driver needs time to accelerate when the vehicle in front drives away when the traffic signal turns green. When traffic conditions in a certain location change, for instance when the head of a queue moves upstream, it will generally take time for the flow to adapt to these changing conditions.

Generally, however, we may assume that given that the conditions remain unchanged for a sufficient period of time – say, five minutes – traffic conditions will converge to an average state. This state is often referred to as the equilibrium traffic state. When considering a traffic flow, this equilibrium state is generally expressed in terms of the fundamental diagram. That is, when considering traffic flow under stationary conditions, the flow operations can – on average – be described by some relation between speed, density and flow. This is why the speed–density relation is often referred to as the equilibrium speed.

From real-life observations of traffic flow, it can be observed that many of the data points collected are not on the fundamental diagram. While some of these points can be explained by stochastic fluctuations (e.g. vehicles have different sizes, drivers have different desired speeds and following distances), some can be structural, and stem from the dynamic properties of traffic flow. That is, they reflect so-called transient states, that is, changes from congestion to free flow (acceleration phase) or from free flow to congestion (deceleration phase) in traffic flow. It turns out that generally these changes in the traffic state are not on the fundamental diagram. In other words, if we consider the average behaviour of drivers (assuming stationary traffic conditions), observed mean speeds will generally not be equal to the 'equilibrium' speed. The term 'equilibrium' reflects the fact that the observed speeds in time will converge to the equilibrium speed, assuming that the average conditions remain the same. That is, the average speed does not adapt instantaneously to the average or equilibrium speed.

This introduces traffic hysteresis, which means that for the same distance headway drivers choose a different speed during acceleration from that chosen during deceleration. Figure 7.9 shows the first empirical observation thereof by Treiterer and Myers (1974). The figure shows the time it takes for a platoon to pass a point along the roadway. The longer the arrow is, the longer that time is and hence the lower the flow (vehicles/hour). The arrow is long at the beginning, since some drivers are not car-following yet.

Figure 7.9 Vehicle trajectories collected from an airborne platform clearly showing differences in average platoon length before and after the disturbance

At the second arrow, all vehicles are car-following and the flow is high (short arrow). In the disturbance, the flow is very low and we find a long arrow. After the disturbance, the flow increases but the headways are longer than before the vehicles entered the disturbance. Note also that, in exiting the traffic jam, all vehicles will be in car-following mode.

Three-phase traffic flows, phase transitions and self-organization

Amongst the many issues raised by Kerner (2004) is the fact that there are three phases (free flow, synchronized flows and jams) rather than two (free

Flow rate, q

Synchronized
flow

$q_{max}^{(free)}$

$q_{max}^{(syn)}$

F

$V_{min}^{(syn)}$

0 $\rho_{max}^{(free)}$ Density, ρ
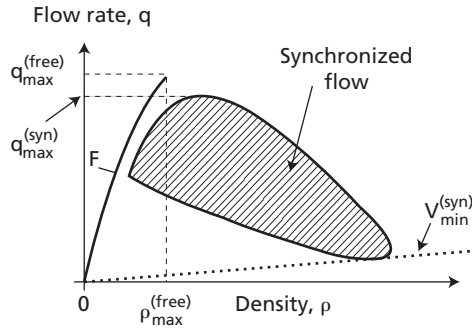
Figure 7.10 The fundamental diagram according to Kerner (2004)

flow and congestion). As in other theories, the first phase is the free flow phase. The second phase, in three-phase traffic flow theory, is the synchronized flow. In this phase, the speed between the lanes is more or less the same, rather than in free flow where the overtaking lane has a higher speed than the slow lane. Furthermore, Kerner claims that there are different equilibrium spacings (densities) for the same flow value, such that the phase should not be described by a line but by an area in the flow-density plane (Figure 7.10). The third phase, the wide moving jam, is identified by a (near) standstill of the vehicles. Owing to the very low flow, the queue will grow at the tail. At the same time, vehicles at the head of the queue can accelerate. This means that the queue moves in the opposite direction to the traffic; the wave speed is approximately 18 km/h propagating backwards from the driving direction.

As well as the distinguishing of the three phases, Kerner discusses transitions between the different traffic phases. Some of these transitions are induced ('forced'). An example is a phase transition induced by a bottleneck, such as an on-ramp. In this situation, the simple fact that traffic demand is at some point in time larger than the rest capacity (being the motorway capacity minus the inflow from the on-ramp) causes a transition from the free flow phase to the synchronized flow phase. Note that these kinds of phase transitions can be described by basic flow theories and models (shockwave theory, kinematic wave models) adequately. As an additional remark, note that these transitions are, although induced, still random events, since both the free flow capacity and the supply are random variables.

However, not all phase transitions are induced (directly); some are caused by intrinsic ('spontaneous') properties of traffic flow. An example is the spontaneous transition from synchronized flow to jammed flow (referred to by Kerner as wide moving jams). Owing to the unstable nature of specific
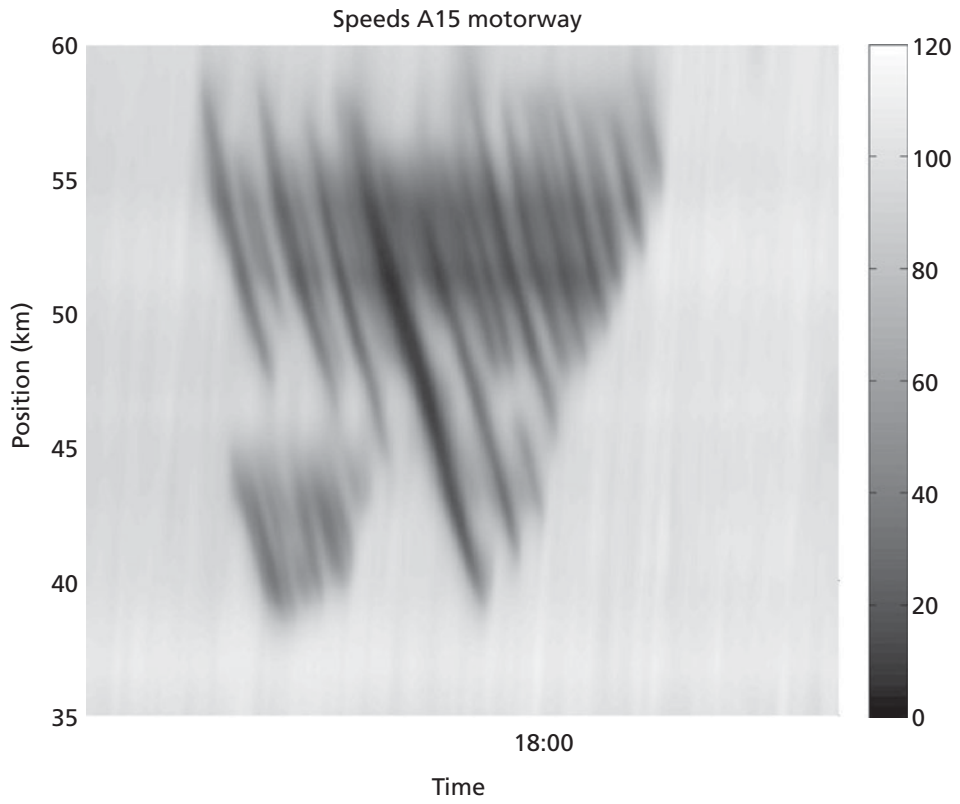
Figure 7.11 Typical traffic pattern on the A15 motorway in the Netherlands. A bottleneck can be determined at 55 km, and one can find wide moving jams propagating backwards at approximately 18 km/h

synchronized flow regimes, small disturbances in the congested flow will grow over time. For instance, a small localized high density cluster caused by a vehicle braking a bit too hard because the driver was temporarily distracted may grow because of vehicles moving at the back of the localized cluster subsequently needing to brake as well (and often doing so because of the finite reaction times of drivers). As a result, this upstream moving disturbance will gain in amplitude and will, in the end, become a wide moving jam.

This phenomenon is quite common in day-to-day motorway traffic operations. Figure 7.11 shows an example of the A15 motorway in the Netherlands. The picture clearly shows the frequent occurrence of these spontaneous transitions from synchronized to jammed flow, resulting in numerous upstream moving, wide moving jams. Note that, as wide moving jams have an outflow rate which is about 30 per cent lower than the free flow capacity,

these jams are actually quite undesirable from a traffic efficiency perspective. Furthermore, they imply additional braking and acceleration, yielding increased fuel consumption and emission levels.

7.6   Multi-lane traffic flow facilities

Up to now, the chapter has considered each lane of the freeway to be equal. However, there are considerable differences between them. This section introduces just the basic concept. For a deeper insight, we refer to the literature mentioned in this chapter. For the sake of simplicity, we here assume driving on the right. For countries where a left-hand driving rule applies, like Japan, the United Kingdom or Australia, the lanes are exactly opposite. Daganzo (2002a, 2002b) poses a theory classifying traffic as slugs, defined by a low desired speed, and rabbits, defined by a high free flow speed. He states that, as soon as the speed in the right lane goes under a certain threshold, rabbits will move to faster lanes to the left. Furthermore, the theory states that, even if the density in the right lane is lower than in the left lane, the rabbits will not change to the right lane as long as the speeds in the left lane are higher. This traffic state, with two different speeds, is called a *two pipe regime*, since traffic is flowing as it were in two different, unrelated pipes. In this state, there is no equal density in both lanes. Only when the density in the left lane increases so much that the speed decreases to a value lower than the speed in the right lane will the rabbits move towards the right lane. Then the rabbits will redistribute themselves in such a way that the traffic in both lanes flows at the same speed. This is called a *one pipe regime*.

Note that the speeds in different lanes at the same densities can be different, owing to these effects or, basically, owing to the driver population in that lane. This leads to different fundamental diagrams in the left and right lanes. Usually, the free flow speed in the left lane is higher than in the right lane, owing to the higher fraction of rabbits in that lane. Kerner (2004) poses a similar theory on multi-lane traffic flow facilities.

7.7   Traffic flow models

Traffic flow models can be used to simulate traffic, for instance to evaluate ex ante the use of a new part of the infrastructure. The models can be helpful tools in answering the questions posed in the introduction to this chapter, such as: When do traffic jams emerge? How will they propagate in space and time? And how long does it takes for the congestion to resolve? Additionally, the models can be used to improve road safety.

Table 7.1 Overview of traffic flow model classification

Representation	Behavioural rules	
	Microscopic	Macroscopic
Vehicle based	Microscopic flow models	Particle models
Flow based	Gas-kinetic models	Macroscopic models

Traffic flow models may be categorized using various dimensions (deterministic or stochastic, continuous or discrete, analytical or simulation, and so forth). The most common classification is the distinction between microscopic and macroscopic traffic flow modelling approaches. However, this distinction is not unambiguous, owing to the existence of hybrid models. This is why models are categorized here based on the following aspects:

1. **representation** of the traffic flow in terms of flows (macroscopic), groups of drivers (macroscopic) or individual drivers (microscopic);
2. **underlying behavioural theory**, which can be based on characteristics of the flow (macroscopic) or individual drivers (microscopic behaviour).

The remainder of this section uses this classification to discuss some important flow models. Table 7.1 depicts an overview of these models.

The observed behaviour of drivers, that is, headways, driving speeds and driving lane, is influenced by different factors, which can be related to the driver–vehicle combination (vehicle characteristics, driver experience, age, gender and so forth), the traffic conditions (average speeds, densities), infrastructure conditions (road conditions) and external situational influences (weather, driving regulations). Over the years, different theories have been proposed to (dynamically) relate the observed driving behaviour to the parameters describing these conditions.

In the process, different driver sub-tasks are often distinguished. Table 7.2 provides a rough but useful classification of these tasks (Minderhoud, 1999). In general, two types of driver tasks are distinguished: longitudinal tasks (acceleration, maintaining speed, maintaining distance relative to the leading vehicle) and lateral tasks (lane changing, overtaking). In particular the longitudinal and (to a lesser extent) the lateral interaction sub-tasks have received quite a lot of attention in traffic flow theory research.

Table 7.2 Driving sub-tasks overview

	Longitudinal	Lateral
Infrastructure	Free speed	Course keeping
Interaction	Car-following	Mandatory and discretionary lane changing

<div align="right">Source: Minderhoud (1999).</div>

A microscopic model provides a description of the movements of individual vehicles that are considered to be a result of the characteristics of drivers and vehicles, the interactions between driver–vehicle elements, the interactions between driver–vehicle elements and the road characteristics, external conditions and the traffic regulations and control. Most microscopic simulation models assume that a driver will only respond to the one vehicle that is driving in the same lane directly in front of him (the leader).

When the number of driver–vehicle units on the road is very small, the driver can freely choose his speed given his preferences and abilities, the roadway conditions, curvature, prevailing speed limits and so forth. In any case, there will be little reason for the driver to adapt his speed to the other road users. The target speed of the driver is the so-called free speed. In real life, the free speed will vary from one driver to another, but the free speed of a single driver will also change over time. Most microscopic models assume however that the free speeds have a constant value that is driver-specific. When traffic conditions deteriorate, drivers will no longer be able to choose the speed freely, since they will not always be able to overtake or pass a slower vehicle. The driver will need to adapt his speed to the prevailing traffic conditions, that is, the driver is following. In the rest of this section, we will discuss some of these car-following models. Models for the lateral tasks, such as deciding to perform a lane change and gap acceptance, will not be discussed in this section in detail. Ahmed et al. (1996) provide a concise framework of lane changing modelling.

Safe-distance models

The first car-following models were developed by Pipes (1953) and were based on the assumption that drivers maintain a safe distance. A good rule for following vehicle $i-1$ at a safe distance s_i is to allow at least the length S_0 of a car between vehicle i and a part which is linear with the speed v_i at which i is travelling:

$$s_i = S(v_i) = S_0 + T_r v_i \tag{16}$$

Here, S_0 is the effective length of a stopped vehicle (including additional distance in front), and T_r denotes a parameter (comparable to the reaction time). A similar approach was proposed by Forbes et al. (1958). Both Pipes's and Forbes's theories were compared to field measurements. It was concluded that, according to Pipes's theory, the minimum headways are slightly less at low and high velocities than observed in empirical data. However, considering the models' simplicity, the way they were in line with real-life observations was amazing (see Pignataro, 1973).

Stimulus response models

However, safe-distance models do not seem to capture much of the phenomena observed in real-life traffic flows, such as hysteresis, traffic instabilities and so on. Stimulus response models are dynamic models that describe more realistically the reaction of drivers to things like changes in distance, speeds and so on relative to the vehicle in front, by considering a finite reaction time, for example. These models are applicable to relatively busy traffic flows where the overtaking possibilities are small and drivers are obliged to follow the vehicle in front of them. Drivers do not want the gap in front of them to become too large so that other drivers can enter it. At the same time, the drivers will generally be inclined to keep a safe distance.

Stimulus response models assume that drivers control their acceleration (a). The well-known model of Chandler et al. (1958) is based on the intuitive hypothesis that a driver's acceleration is proportional to the relative speed $v_{i-1} - v_i$:

$$a_i(t) = \frac{d}{dt} v_i(t) = \alpha(v_{i-1}(t - T_r) - v_i(t - T_r)) \tag{17}$$

where T_r again denotes the overall reaction time, and α denotes the sensitivity. Based on field experiments, conducted to quantify the parameter values for the reaction time T_r and the sensitivity α, it was concluded that α depended on the distance between the vehicles: when the vehicles were close together, the sensitivity was high, and vice versa.

Stimulus response models have been applied mainly to single lane traffic (e.g. tunnels; see Newell, 1961) and traffic stability analysis (Herman, 1959; May, 1990). It should be noted that no generally applicable set of parameter estimates has been found so far, that is, estimates are site-specific. An

overview of parameter estimates can be found in Brackstone and McDonald (1999).

Psycho-spacing models

The two car-following models discussed so far have a mechanistic character. The only human element is the presence of a finite reaction time T_r. However, in reality a driver is not able to:

1. observe a stimulus lower than a given value (perception threshold);
2. evaluate a situation and determine the required response precisely, for instance because of observation errors resulting from radial motion observation;
3. manipulate the acceleration and brake pedals precisely.

Furthermore, owing to the need to distribute his attention between different tasks, a driver will generally not be permanently occupied with the car-following task. This type of consideration has inspired a different class of car-following models, namely the psycho-spacing models. Michaels (1963) provided the basis for the first psycho-spacing, based on theories borrowed from perceptual psychology (see Leutzbach and Wiedemann, 1986).

The so-called action point models (an important psycho-spacing model) form the basis for a large number of contemporary microscopic traffic flow models. Brackstone and McDonald (1999) conclude that it is hard to come to a definitive conclusion on the validity of these models, mainly because the calibration of its elements has not been successful.

7.8   Macroscopic traffic flow models

In the previous section we have discussed different microscopic traffic flow modelling approaches. In this section, we will discuss the main approaches that have been proposed in the literature taking a macroscopic perspective.

Deterministic and stochastic queuing theory

The most straightforward approach to model traffic dynamics is probably the use of queuing theory. In queuing theory we keep track of the number of vehicles in a queue (n). A queue starts whenever the flow to a bottleneck is larger than the bottleneck capacity, where the cars form a virtual queue. The outflow of the queue is given by the infrastructure (it is the outflow capacity
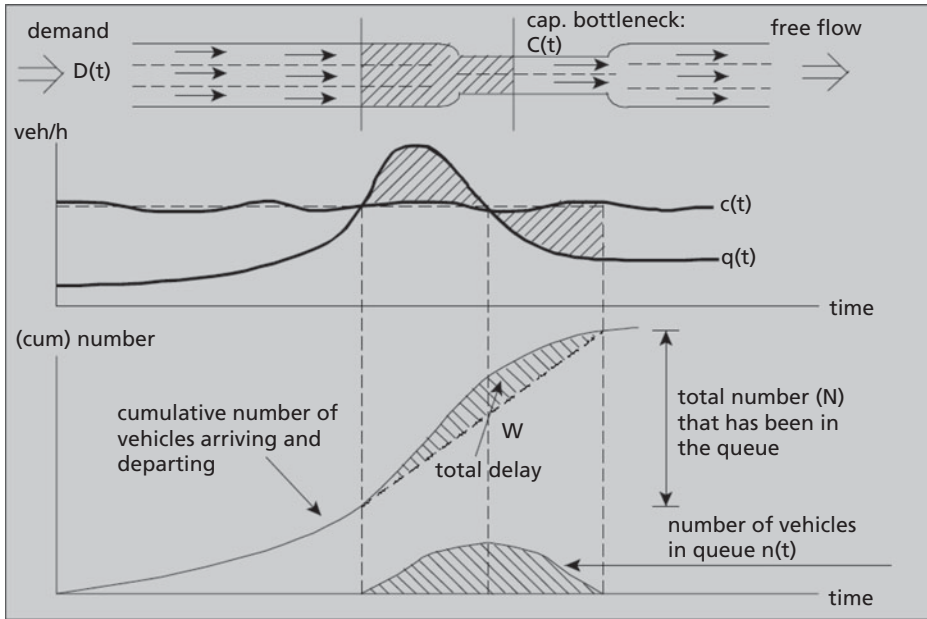
Figure 7.12 Functioning of queuing theory

of the bottleneck, given by C), whereas the inflow is the flow towards the bottleneck (q) as given by the traffic model. In an equation, this is written as:

$$dn = q(t)\,dt - C(t)\,dt \qquad (18)$$

The number of vehicles in the queue (n; dn stands for the change in the number of vehicles in the queue) will evolve in this way until the queue has completely disappeared. Note that both the inflow and the capacity are time dependent in the description. For the inflow, this is due to the random distribution pattern of the arrival of the vehicles. Vehicles can arrive in platoons or there can be large gaps in between two vehicles. The capacity is also fluctuating. On the one hand, there are vehicle-to-vehicle fluctuations. For instance, some drivers have a shorter reaction time, hence a shorter headway leading to a higher capacity. On the other hand, on a larger scale, the capacities will also depend on road or weather conditions (e.g. wet roads, night-time).

Figure 7.12 shows how the number of vehicles in the queue, n, fluctuates with time for a given inflow and outflow curve.

The disadvantage of the queuing theory is that the queues have no spatial dimension, and they do not have a proper length either (they do not occupy

space). Other models, which overcome these problems, are discussed below.

Shockwave theory

Queuing theory provides some of the simplest models that can be used to model traffic flow conditions. However, the spatial dimension of traffic congestion in particular is not well described or – in the case of vertical queuing models – not described at all. Shockwave theory is able to describe the spatio-temporal properties of queues more accurately. This sub-section briefly introduces shockwave theory.

A shockwave describes the boundary between two traffic states that are characterized by different densities, speeds and/or flow rates. Shockwave theory describes the dynamics of shockwaves, in other words how the boundary between two traffic states moves in time and space.

Suppose that we have two traffic states: states 1 and 2. Let S denote the wave that separates these states. The speed of this shockwave S can be computed by:

$$\omega_{12} = \frac{q_2 - q_1}{k_2 - k_1} \tag{19}$$

In other words, the speed of the shockwave equals the jump in the flow over the wave divided by the jump in the density. This yields a nice graphical interpretation (Figure 7.13): if we consider the line that connects the two traffic states 1 and 2 in the fundamental diagram, then the slope of this line is exactly the same as the speed of the shock in the time–space plane.

Shockwave theory provides a simple means to predict traffic conditions in time and space. These predictions are largely in line with what can be observed in practice, but they have their limitations:

1. Traffic driving away from congestion does not accelerate smoothly towards the free speed but continues driving at the critical speed.
2. Transition from one state to the other always occurs in jumps, not taking into account the bound acceleration characteristics of real traffic.
3. There is no consideration of hysteresis.
4. There are no spontaneous transitions from one state to the other.
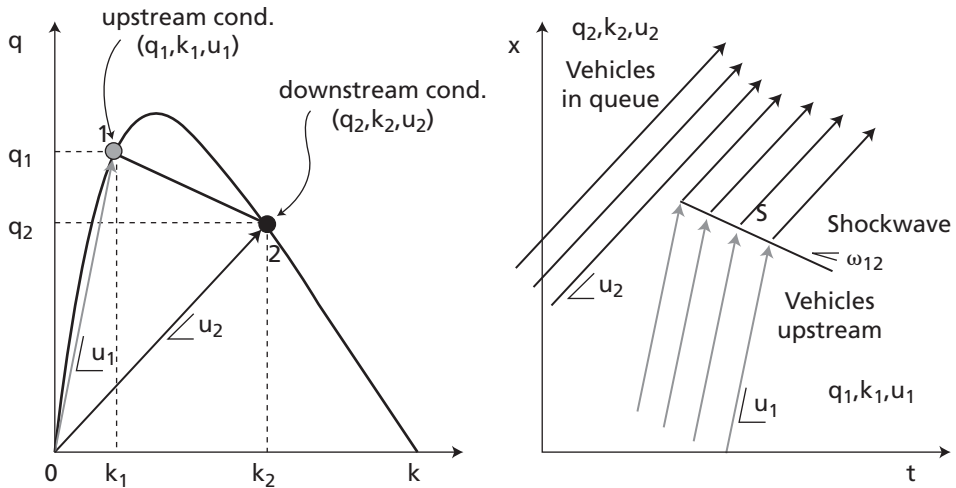5. Location of congestion occurrence is not in line with reality.

Figure 7.13 Graphical interpretation of shockwave speed

As a result, more advanced approaches have been proposed. Let us now consider the most important ones.

Continuum traffic flow models

Continuum traffic flow deals with traffic flow in terms of aggregate variables, such as flow, densities and mean speeds. Usually, the models are derived from the analogy between vehicular flow and the flow of continuous media (e.g. fluids or gases), complemented by specific relations describing the average macroscopic properties of traffic flow (e.g. the relation between density and speed). Continuum flow models generally have a limited number of equations that are relatively easy to handle.

Most continuum models describe the dynamics of density $k = k(x,t)$, mean instantaneous speed $u = u(x,t)$ and the flow $q = q(x,t)$. The density $k(x,t)$ describes the expected number of vehicles per unit length at instant t. The flow $q(x,t)$ equals the expected number of vehicles flowing past cross-section x during the time unit. The speed $u(x,t)$ equals the mean speed of the vehicle defined according to $q = ku$. Some macroscopic traffic flow models also contain partial differential equations of the speed variance $q = q(x,t)$, or the traffic pressure $P = P(x,t) = rq$. For an overview of continuum flow models, we refer to Hoogendoorn and Bovy (2001).

7.9   Network dynamics

In the preceding sections, we have presented some of the main traffic flow characteristics. Using the microscopic and macroscopic models discussed, flow operations on simple infrastructure elements can be explained and predicted. Predicting flow operations in a network is, obviously, more involved, since it also requires predicting the route traffic demand profiles, which in turn means modelling route choice, departure time choice, mode choice and so on.

Interestingly, it turns out that the overall dynamics of a traffic network can be described using a remarkably simple relation, referred to as the *macroscopic* or *network fundamental diagram (NFD)*. This diagram relates the vehicle accumulation – or average vehicle density – to the network performance. The network performance is defined by the flow, weighted by the number of lanes, and the length of the roadway segment for which the measured flow is representative.

This relation, which will be discussed in the following sections, shows one of the most important properties of network traffic operations, namely that its performance decreases when the number of vehicles becomes larger. In other words, when it is very busy in the network, performance goes down and fewer vehicles are able to complete their trip per unit of time. As a consequence, problems become even bigger.

Macroscopic fundamental diagram

Vehicular traffic network dynamics are atypical. Contrary to many other networks, network production (average rate at which travellers complete their trip) deteriorates once the number of vehicles in the network has surpassed the critical accumulation. Pioneering work by Daganzo and Geroliminis (2008) shows the existence of the NFD, clearly revealing this fundamental property. Figure 7.14 shows an example of the NFD. Knowledge of this fundamental property and its underlying mechanisms is pivotal in the design of effective traffic management.

Developing a macroscopic description of traffic flow is not a new idea. Thomson (1967) found the relationship between average speed and flow using data collected from central London streets. Wardrop (1968) stated that this relation between average speed and flow decreased monotonically, and Zahavi (1972) enriched Wardrop's theory by analysing real traffic data collected from various cities in the United Kingdom and United States.
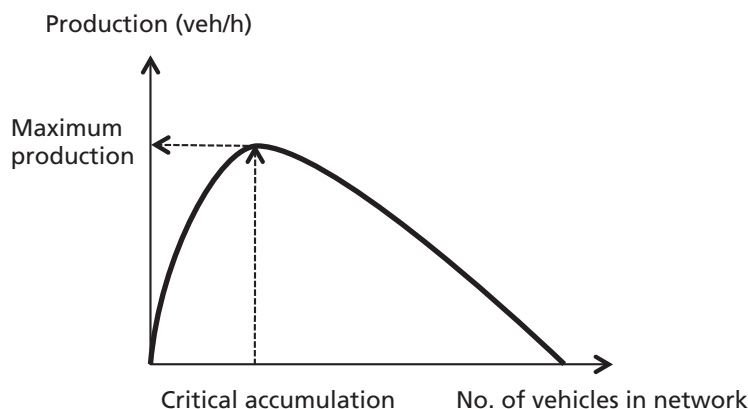
Figure 7.14 Example of the network fundamental diagram

Geroliminis and Daganzo (2008) have proven that NFDs exist in small networks, revealing the relation between the outflow and accumulation in the network. The accumulation is the number of vehicles in the network. The outflow is also called trip completion rate, reflecting the rate at which travellers reach their destinations. Similarly to a conventional link fundamental diagram, relating the local flow and density, three states are demonstrated on an NFD. When only a few vehicles use the network, the network is in the free flow condition and the outflow is low. With an increase in the number of vehicles, the outflow rises to the maximum. Like the critical density in a link fundamental diagram, the value of the corresponding accumulation when maximum outflow is reached is also an important parameter, called 'sweet spot'.

As the number of vehicles further increases, travellers will experience delay. If vehicles continue to enter the network, it will result in a congested state where vehicles block each other and the outflow declines (congested conditions). Furthermore, macroscopic feedback control strategies were introduced with the aim of keeping accumulation at a level at which outflow is maximized for areas with a high density of destination.

Causes of network degeneration

The two main causes of the production deterioration of overloaded networks are spill-back of queues possibly resulting in grid-lock effects, and the capacity drop. Spill-back occurs because of the simple fact that queues occupy space: a queue occurring at a bottleneck may propagate so far upstream that it will affect traffic flows that do not have to pass the bottleneck, for example

when the queue passes a fork or an intersection upstream of the active bottle-neck. As a result, congestion will propagate over other links of the network, potentially causing grid-lock phenomena. The capacity drop describes the fact that the free flow freeway capacity is considerably larger than the queue discharge rate.

7.10 Conclusions

The most important conclusions of this chapter are as follows:

1. Traffic flow theory and modelling are important in order to design comfortable and safe roads, to solve road congestion problems and to design adequate traffic management measures, amongst other things.
2. Traffic flow theory entails knowledge of the fundamental characteristics of traffic flows.
3. In traffic flow theory a basic distinction is made between microscopic and macroscopic traffic flow variables. Microscopic traffic flow variables focus on individual drivers. Macroscopic traffic flow variables reflect the average state of the traffic flow.
4. The fundamental diagram in traffic flow theory describes a statistical relation between the macroscopic flow variables of flow, density and speed. The basic premise underlying the fundamental diagram is that under similar traffic conditions drivers will behave in a similar way.
5. Traffic flow models can be used to simulate traffic, for instance to evaluate ex ante the use of a new part of the infrastructure. Models can be categorized based on, firstly, representation of the traffic flow in terms of flows (macroscopic), groups of drivers (macroscopic) or individual drivers (microscopic) and, secondly, underlying behavioural theory, which can be based on characteristics of the flow (macroscopic) or individual drivers (microscopic behaviour).
6. The overall dynamics of a traffic network can be described using a remarkably simple relation, referred to as the macroscopic or network fundamental diagram (NFD). This relation shows one of the most important properties of network traffic operations, namely that their performance decreases when the number of vehicles becomes greater.

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