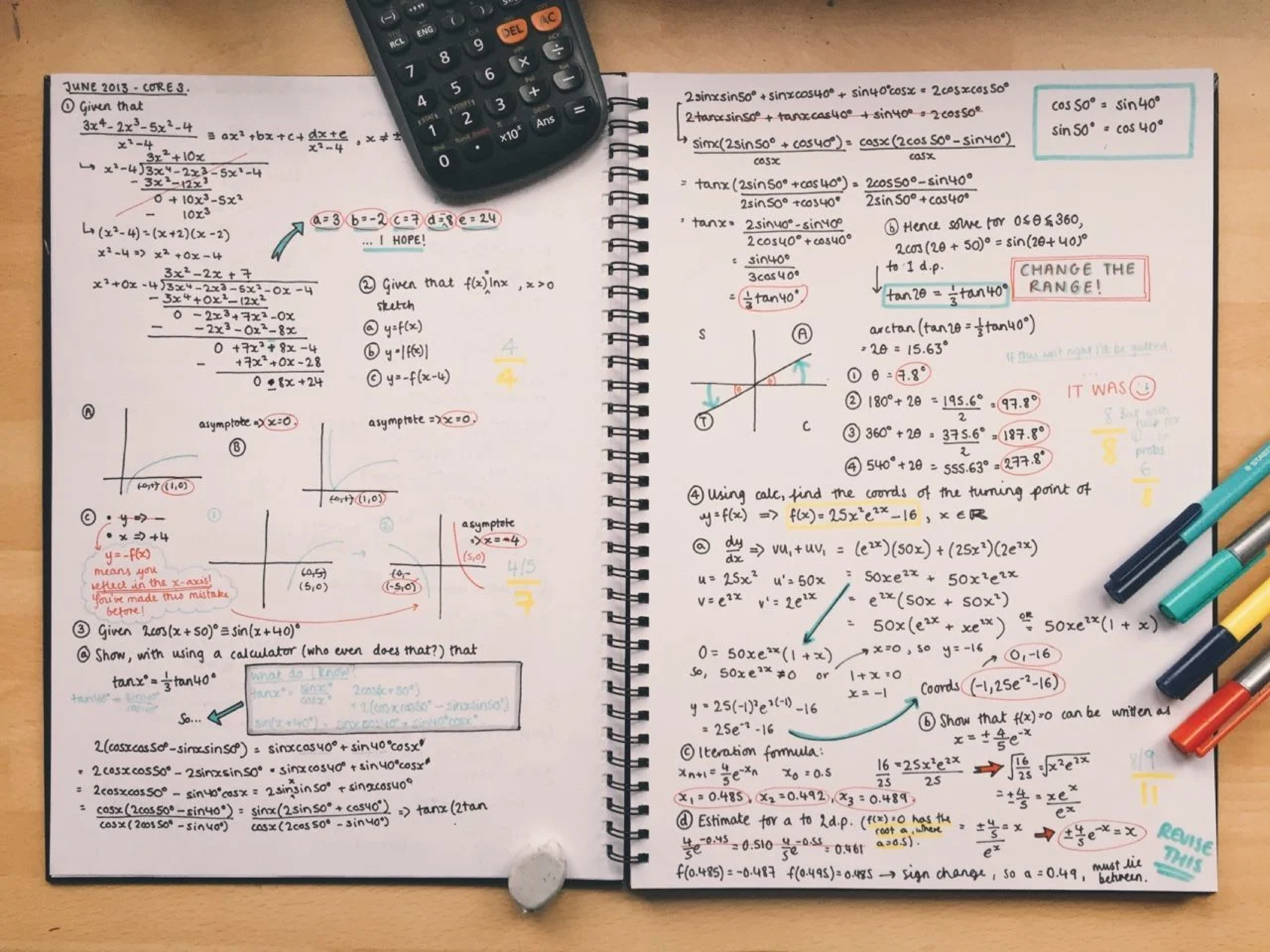

JUNE 2013 - CORE 3.

① Given that

$\frac{3x^4-2x^3-5x^2-4}{x^2-4} \equiv ax^2+bx+c+\frac{dx+e}{x^2-4}$, $x \neq \pm$

~~↳ $x^2-4 \overline{)3x^4-2x^3-5x^2-4}$ quotient $3x^2+10x$~~

~~$-\ 3x^2-12x^3$~~

~~$0+10x^3-5x^2$~~

~~$-\ 10x^3$~~

↳ $(x^2-4)=(x+2)(x-2)$

$x^2-4 \Rightarrow x^2+0x-4$

$x^2+0x-4 \overline{)3x^4-2x^3-5x^2-0x-4}$ quotient $3x^2-2x+7$

$-\ 3x^4+0x^2-12x^2$

$0\ -2x^3+7x^2-0x$

$-\ \ -2x^3-0x^2-8x$

$0\ +7x^2+8x-4$

$-\ \ +7x^2+0x-28$

$0\ \underset{-}{+}8x+24$

$\boxed{a=3}$ $\boxed{b=-2}$ $\boxed{c=7}$ $\boxed{d=8}$ $\boxed{e=24}$

... I HOPE!

② Given that $f(x)=\ln x$, $x>0$

sketch

ⓐ $y=f(x)$

ⓑ $y=|f(x)|$

ⓒ $y=-f(x-4)$

$\frac{4}{4}$

Ⓐ

asymptote $\Rightarrow \boxed{x=0}$.

~~(0,1)~~ $\boxed{(1,0)}$

Ⓑ

asymptote $\Rightarrow \boxed{x=0}$.

~~(0,1)~~ $\boxed{(1,0)}$

ⓒ • ~~$y \Rightarrow -$~~

• $x \Rightarrow +4$

$y=-f(x)$ means you reflect in the x-axis! You've made this mistake before!

①

~~(0,5)~~

(5,0)

②

~~(0,-~~

$\boxed{(-5,0)}$

asymptote $\Rightarrow \boxed{x=-4}$

(5,0)

$\frac{4/5}{7}$

③ Given $2\cos(x+50)^\circ \equiv \sin(x+40)^\circ$

ⓐ Show, with using a calculator (who even does that?) that

$\tan x^\circ = \frac{1}{3}\tan 40^\circ$

$\tan 40^\circ = \frac{\sin 40^\circ}{\cos 40^\circ}$

What do I know?

$\tan x^\circ = \frac{\sin x^\circ}{\cos x^\circ}$

$2\cos(x+50^\circ)$

$= 2(\cos x\cos 50^\circ - \sin x\sin 50^\circ)$

$\sin(x+40^\circ) = \sin x\cos 40^\circ + \sin 40^\circ\cos x^\circ$

So...

$2(\cos x\cos 50^\circ - \sin x\sin 50^\circ) = \sin x\cos 40^\circ + \sin 40^\circ\cos x^\circ$

$= 2\cos x\cos 50^\circ - 2\sin x\sin 50^\circ = \sin x\cos 40^\circ + \sin 40^\circ\cos x^\circ$

$= 2\cos x\cos 50^\circ - \sin 40^\circ\cos x = 2\sin x\sin 50^\circ + \sin x\cos 40^\circ$

$= \frac{\cos x(2\cos 50^\circ - \sin 40^\circ)}{\cos x(2\cos 50^\circ - \sin 40^\circ)} = \frac{\sin x(2\sin 50^\circ + \cos 40^\circ)}{\cos x(2\cos 50^\circ - \sin 40^\circ)} \Rightarrow \tan x(2\tan$

$2\sin x\sin 50^\circ + \sin x\cos 40^\circ + \sin 40^\circ\cos x = 2\cos x\cos 50^\circ$

~~$2\tan x\sin 50^\circ + \tan x\cos 40^\circ + \sin 40^\circ = 2\cos 50^\circ$~~

↳ $\frac{\sin x(2\sin 50^\circ + \cos 40^\circ)}{\cos x} = \frac{\cos x(2\cos 50^\circ - \sin 40^\circ)}{\cos x}$

$\cos 50^\circ = \sin 40^\circ$

$\sin 50^\circ = \cos 40^\circ$

$= \tan x\left(\frac{2\sin 50^\circ + \cos 40^\circ}{2\sin 50^\circ + \cos 40^\circ}\right) = \frac{2\cos 50^\circ - \sin 40^\circ}{2\sin 50^\circ + \cos 40^\circ}$

$= \tan x = \frac{2\sin 40^\circ - \sin 40^\circ}{2\cos 40^\circ + \cos 40^\circ}$

$= \frac{\sin 40^\circ}{3\cos 40^\circ}$

$= \boxed{\frac{1}{3}\tan 40^\circ}$

ⓑ Hence solve for $0 \leq \theta \leq 360$,

$2\cos(2\theta+50)^\circ = \sin(2\theta+40)^\circ$

to 1 d.p.

$\boxed{\tan 2\theta = \frac{1}{3}\tan 40^\circ}$

CHANGE THE RANGE!

S

Ⓐ

Ⓣ

C

$\arctan\left(\tan 2\theta = \frac{1}{3}\tan 40^\circ\right)$

$= 2\theta = 15.63^\circ$

If this isn't right I'll be gutted.

① $\theta = \boxed{7.8^\circ}$

② $180^\circ + 2\theta = \frac{195.6^\circ}{2} = \boxed{97.8^\circ}$

③ $360^\circ + 2\theta = \frac{375.6^\circ}{2} = \boxed{187.8^\circ}$

④ $540^\circ + 2\theta = 555.63^\circ = \boxed{277.8^\circ}$

IT WAS ☺

$\frac{8}{8}$ But with help for ④ in probs

$\frac{6}{8}$

④ Using calc, find the coords of the turning point of

$y=f(x) \Rightarrow f(x) = 25x^2e^{2x} - 16$, $x \in \mathbb{R}$

ⓐ $\frac{dy}{dx} \Rightarrow vu_1 + uv_1 = (e^{2x})(50x) + (25x^2)(2e^{2x})$

$u = 25x^2$ $u' = 50x$

$v = e^{2x}$ $v' = 2e^{2x}$

$= 50xe^{2x} + 50x^2e^{2x}$

$= e^{2x}(50x + 50x^2)$

$= 50x(e^{2x} + xe^{2x}) \overset{\text{OR}}{=} 50xe^{2x}(1+x)$

$0 = 50xe^{2x}(1+x)$

So, $50xe^{2x} \neq 0$ or $1+x=0$

$x=-1$

$\rightarrow x=0$, so $y=-16$ $\rightarrow \boxed{0,-16}$

$y = 25(-1)^2e^{2(-1)} - 16$

$= 25e^{-2} - 16$

Coords $\boxed{(-1, 25e^{-2}-16)}$

ⓑ Show that $f(x)=0$ can be written as

$x = \pm\frac{4}{5}e^{-x}$

$\frac{16}{25} = \frac{25x^2e^{2x}}{25} \Rightarrow \sqrt{\frac{16}{25}} = \sqrt{x^2e^{2x}}$

$= \pm\frac{4}{5} = \frac{xe^x}{e^x}$

$= \frac{\pm\frac{4}{5}}{e^x} = x \Rightarrow \boxed{\pm\frac{4}{5}e^{-x} = x}$

ⓒ Iteration formula:

$x_{n+1} = \frac{4}{5}e^{-x_n}$ $x_0 = 0.5$

$\boxed{x_1 = 0.485}$, $\boxed{x_2 = 0.492}$, $\boxed{x_3 = 0.489}$.

$\frac{8/9}{11}$

ⓓ Estimate for a to 2d.p. ($f(x)=0$ has the root a, where $a \approx 0.5$).

~~$\frac{4}{5}e^{-0.45} = 0.510$ $\frac{4}{5}e^{-0.55} = 0.461$~~

$f(0.485) = -0.487$ $f(0.495) = 0.485 \rightarrow$ sign change, so $a = 0.49$, must lie between.

REVISE THIS