Statistics extra questions + references

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For this course I will use Casella and Berger second edition as the standard reference [1].

1 Probability parts

Prove Slutky's theorem that if $X_n \to X$ in distribution and $Y_n \to c$ to a constant in probability, then $X_n + Y_n \to X + c$ and $X_n Y_n \to Xc$ in distribution. See Slutky's theorem on wiki or Theorem 5.5.17 in Casella and Berger (pg 239).

Section 5.4 of Casella and Berger deals with order statistics, and extends the course by considering the joint density of 2 order statistics.

I recommend checking out Theorem 7.3.9 (Cramer–Rao Inequality) on pg 335, as it provides a lower bound on the variance of the MLE in terms of Fisher information.

2 Asymptotic properties

The MLE is asymptotically normal under some strong conditions. A reference can be found in Asymptotic normality of MLE.

The delta method tells us that if our random sample is asymptotically normal then applying a C^1 function to it we recover asymptotic normality, see section 5.5.4 of Casella and Berger (pg 240).

References

[1] George. Casella and Roger L. Berger. *Statistical inference*. Duxbury advanced series. Duxbury/Thomson Learning, Pacific Grove, Calif, 2nd ed. edition, 2002.