Systematic Selection of N-Tuple Networks with Consideration of Interinfluence for Game 2048

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***Abstract*—The puzzle game 2048, a single-player stochastic game played on a 4×4 grid, is the most popular among similar slide-and-merge games. One of the strongest computer players for 2048 uses temporal difference learning (TD learning) on a set of N-tuples (called N*-tuple network*), where the N-tuple network is given by human based on characteristics of the game. In our previous work (Oka and Matsuzaki, 2016), the authors proposed a systematic method of selecting N-tuples under an assumption that the interinfluence among those N-tuples are negligible. Though the selected N-tuple networks worked fine, there were large gaps between those N-tuple networks and the human-designed network. In this paper, another systematic and game-characteristics-free method of selecting N-tuples is proposed for game 2048, in which the interinfluence among those N-tuples is coped with. The proposed method is effective and generic: the selected N-tuple networks are as good as human-designed one under the same setting, and we can obtain larger (or smaller) N-tuple networks in the same manner. We also report the experiment results when we combine the TD learning and expectimax search.**

I. Introduction

The puzzle game 2048 [1], a single-player stochastic game played on a 4×4 grid, is the most popular among similar slide-and-merge games like Threes and 1024. One of the reasons why the game attracts so many people is that it is very easy to learn but hard to master. The game also attracts researchers in the field of artificial intelligence and computational complexity. For instance, the difficulty of the game was discussed from the viewpoint of computational complexity by Abdelkader et al. [2] and Langerman and Uno [3]. As a testbed of artificial intelligence methods, there have been some competitions of computer players for the game 2048 [4], [5] and for a two-player version of 2048 [6]–[8].

One of the strongest computer players for 2048 uses the expectimax algorithm with an evaluation function learned from a large number of self-plays, where the evaluation function is designed based on an N*-tuple network*. An N-tuple network consists of a number of N-tuples: each N-tuple covers N cells on the grid and it contributes a number of features each for one distinct occurrence of tiles on the covered cells. Given an N-tuple network, the evaluation function simply calculates the summation of feature weights for all occurring features, where those weights were adjusted through the temporal difference learning (TD learning for short) [9] or its extended algorithm [10], [11].

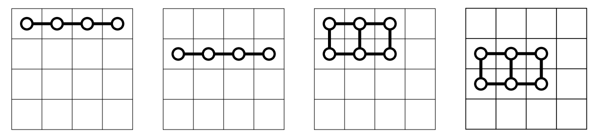


Fig. 1. The N-tuple network (with two 4-tuples and two 6-tuples) by Szubert and Jaśkowski [9]

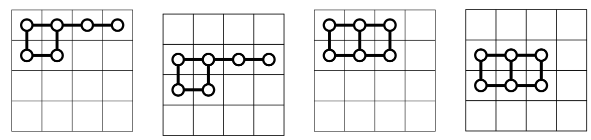


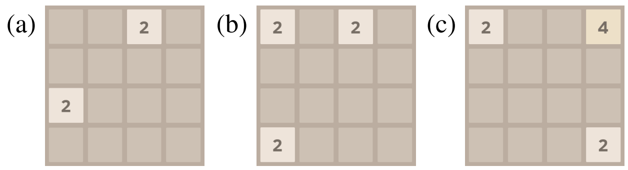
Fig. 2. The N-tuple network (with four 6-tuples) by Wu et al. [11]

In this approach, the design of N-tuple networks is an important issue. The first work by Szubert and Jaśkowski [9] used a network with two 6-tuples and two 4-tuples (Fig. 1), and Wu et al. [10], [11] extended the network to four 6-tuples (Fig. 2). As we can easily imagine, the more and the larger tuples we use the better evaluation functions we would obtain, but computing resources limit the number and/or size of tuples. So, the question we have is like "*what is the best* N*-tuple network that consists of at most eight 7-tuples*."

In our previous work [12], we proposed a systematic method of selecting those N-tuples without heuristics or characteristics of the game. Since the method deals the N-tuples independent, *i.e.* it does not take into account the interinfluence of N-tuples, the selected N-tuple networks are not good enough when the number of tuples is limited. In this paper, I propose another method of constructing N-tuple networks to resolve this problem. The idea is simple: starting from an empty network, we augment it by adding the best N-tuple greedily one by one. From the experiments, the proposed method successfully constructs N-tuple networks, and the constructed network with four 6-tuples is almost as good as the human-designed one [11].

The main contributions of the paper are summarized as follows:

- I propose a systematic way of constructing N-tuple networks. It does not rely on heuristics or human knowledge of the games, and thus we can also apply this technique to other games in a same manner.



(a) An example of the initial state. Two tiles are put randomly.
(b) After moving up. A new 2-tile appears at the lower-left corner.
(c) After moving right. Two 2-tiles are merged to a 4-tile, and score 4 is given.

Fig. 3. The process of the game 2048

- The method does not pose an assumption that the N-tuples are independent. Therefore the constructed N-tuple networks are as good as human-designed ones.
- The simple greedy player achieved average score 255,198 (with eight 7-tuples). The expectimax player achieved the maximum score 629,964 (with eight 7-tuples, look-aheads to 3 moves).

The rest of the paper is organized as follows. Section 2 briefly introduce the rule of the game 2048. Section 3 reviews the idea of applying N-tuple networks and TD learning to the game 2048. In Section 4, we review the related work focusing on TD-learning-based players for 2048. In Section 5, we propose a new systematic method of constructing N-tuple networks, and report the obtained N-tuple networks for the cases of $N = 6$ and $N = 7$. With the obtained N-tuple networks, we perform more experiments including the application of expectimax search, in Section 6. Finally Section 6 concludes the paper.

II. Rules of 2048

The game 2048 is played on a 4×4 grid. The objective of the original 2048 game is to reach a 2048 tile by moving and merging the tiles on the board according to the rules below. A new tile will be put randomly with number 2 (with probability of 90%) or 4 (with probability 10%). In the initial state, two tiles are put randomly (Fig. 3). The player selects a direction (either of up, left, down, and right), and then all the tiles will move in that direction. When two tiles of the same number combine they create a tile with the sum value and the player get the sum as the score. Here, the merges occur from the far side and a newly created tile do not merge again on the same move: moves to the right from $222␣$, $␣422$ and 2222 result in $␣␣24$, $␣␣44$, and $␣␣44$, respectively. Note that the player cannot select a direction in which no tiles move nor merge. After each move, a new tile appears at an empty cell with the same probability above. If the player cannot move the tiles, the game ends.

III. N-tuple Networks and Temporal Difference Learning for 2048

In this section, we review the idea of applying N-tuple networks and TD learning to the game 2048. The algorithm was first given by Szubert and Jaśkowski [9] and it was called TD-afterstate in their paper.

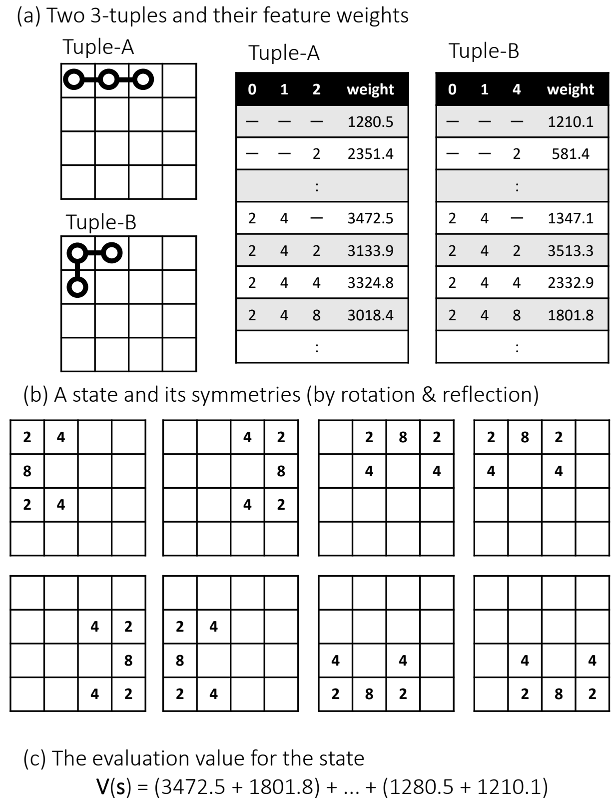


Fig. 4. An example for calculating an evaluation value of a state

A. Evaluation Function with N-tuple Networks

An N-tuple network consists of a number of N-tuples where each N-tuple covers N cells on the grid. In this paper, N denotes the number of cells in a tuple, and m the number of tuples in the network. If each cell in the tuple may have one of 16 values[1] an N-tuple contributes 16^N features, that is, we assign a feature weight for each of 16^N features. Note that 6- and 7-tuples require 64 MB and 1 GB, respectively, if we assign 32 bits for each feature weight.

Given an N-tuple network and corresponding set of feature weights, we calculate the value of an evaluation function of a state as follows. Since the board of the game 2048 is symmetric in terms of rotation and reflection, we can consider 8 sampling for each N-tuple. We take the feature weight for each sampling, and compute the sum of those values as the evaluation value of the state. Given a state s, the evaluation value $V(s)$ of the state is the sum of the feature weights for all N-tuple and all symmetric boards.

Let us see an example in Fig. 4 where we use an N-tuple network with two 3-tuples. We have eight symmetric boards for a state s, and each board has two feature weights for each tuple. Therefore, in this example, the evaluation value of a state is the sum of 16 feature weights. If we have a network with m N-tuples, then the evaluation value of a state is the sum of $8m$ feature weights.

[1]Empty cell, 2, 4, 8, 16, …, 32768.

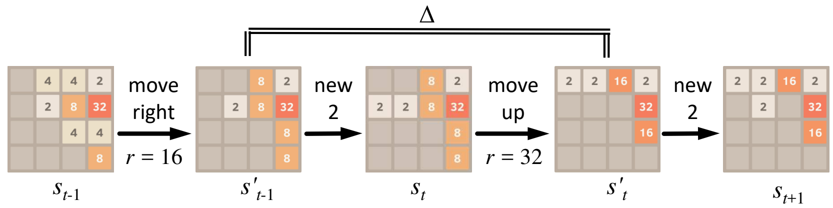


Fig. 5. Transition of states

The greedy 2048 player in this paper selects a move such that the sum of score and evaluation value is the maximum. For a state s, let the set of possible move be $A(s) \subseteq \{\mathsf{N}, \mathsf{E}, \mathsf{S}, \mathsf{W}\}$, the score given by move a be $R(s, a)$, and the next state by move a be $N(s, a)$, the player selects

$$\arg\max_{a \in A(s)} \left(R(s, a) + V(N(s, a))\right) \ .$$

B. Temporal Difference Learning

Temporal difference learning (TD learning) is one of the reinforcement learning algorithms. Though the idea of TD learning was introduced by Sutton [13], its origins reach back to the 1950's for the famous program for checkers [14]. TD learning has been adapted to several games such as backgammon [15], Othello [16], and Go [17].

In our algorithm, the evaluation values are adjusted by TD learning as follows (Fig. 5). Let s_t be a state at time t. The player selects a move a such that the sum of score and evaluation value of the state after the move is the maximum. Let $r = R(s_t, a)$ and s'_t be the score and the state after the move, respectively (note that $s'_t \neq s_{t+1}$ because s_{t+1} is given by putting a tile on s'_t). Then, the TD error Δ for the evaluation value is defined as follows.

$$\Delta = r + V(s'_t) - V(s'_{t-1})$$

To reduce the TD error, we update the evaluation values $V_j(s_{t-1})$ for all the N-tuples by a certain portion of Δ:

$$V'_j(s_{t-1}) = V_j(s_{t-1}) + \alpha\Delta$$

where the rate α is called *learning rate* and it was set to $\alpha = 2^{-10}$ throughout the experiments.

IV. Related Work

Several game-playing algorithms have been adapted to the game 2048 [9], [11], [18]–[22]. Among them, the state-of-the-art algorithm combines expectimax with TD learning or some other heuristics.

The first application of TD learning to the 2048 player was done by Szubert and Jaśkowski [9]. They utilized hand-selected two 4-tuples and two 6-tuples and the player learned with 1,000,000 self-play games achieved the average score 100,178. The two 4-tuples were extended to two 6-tuples by Wu et al. [11] and the extension increased the average score to 142,727. In our previous work [12], we developed a systematic method of selecting N-tuples under an assumption that the tuples are independent. The systematic selection worked fine if the number m of N-tuples is sufficiently large, but not enough for small m: the average score was 109,983 for four 6-tuples. Wu et al. [11] also proposed the multi-staged extension of the learning algorithm, and by the combination with expectimax search the player achieved the average score 328,946 (multi-staged TD, expectimax depth = 5). They recently achieved a 65536-tile [10].

The expectimax algorithm takes much more time when the depth is large. In the competition of computer players for the game 2048, it is often required to play a move in 1–10 milliseconds [4], [5]. For the execution times with expectimax and N-tuple networks, Yeh et al. [10] reports in details.

V. Augmentation-based Construction of N-Tuple Networks

Since an N-tuple covers a part of game board, it is straightforward to expect that a good N-tuple network should cover all the board. In fact, it is not enough to cover all the cells of the board, and we need to take into account the interinfluence between tuples to obtain a good N-tuple network. A possible way to do that is to define some degree of similarity between tuples, but it seems not so easy to define such a degree for the N-tuples in the game 2048. Therefore, in this paper, I propose another simple way, *augmentation-based construction* of N-tuple networks, that construct an N-tuple network by adding the best tuples one by one.

A. Algorithm

The following is the augmentation-based N-tuple network construction algorithm.

1) List up all the N-tuples and let $\mathcal{T}$ be the set of possible tuples. In this work, we expect all the tuples form a connected graph. We have 68 connected 6-tuples and 119 connected 7-tuples [12].
2) Initialized the network $\mathcal{N}$ to be an empty list.
3) Repeat the following two subtasks.
   a) For each tuple n_i in $\mathcal{T} - \mathcal{N}$, perform the TD learning over 1,000,000 self-play games with the network $\mathcal{N} \cup \{n_i\}$, and let the average of last 10,000 games be the score for n_i. To avoid variance due to the seeds of random numbers, we process this step 5 times and compute the average of those 5 runs.
   b) Pick up the tuple n_j with the largest score, and update the set $\mathcal{N}$ with $\mathcal{N} \cup \{n_j\}$.

B. Experiment Results

Tables I and II show the results of the augmentation-based construction algorithm in Section V-A for $N = 6$ and $N = 7$, respectively. Those tables show the best (with the highest score) four tuples and the worst (with the lowest score) tuple for each step of adding an N-tuple. We can obtain an N-tuple network with m-tuples just by selecting the first m tuples from the left in the *best* row.

We can find several interesting facts from these Tables.

- The four best tuples have a similar shape for each column (m). In detail, only one cell is different between the best tuple and the second best one in five of eight cases for $N = 6$ and six of eight cases for $N = 7$.
- For small m, the best tuples for m and those for $m + 1$ have rather different shapes. This is an important evidence that the interinfluence of N-tuples is not negligible.
- (Surprising result) In the $N = 7$ case, a tuple resulted in the worst one for $m = 2, \ldots, 6$, but the same tuple became the best one for the following $m = 7$.
- (Important result) The best tuples for the first three corresponds to those in the hand-designed N-tuple network by Wu et al. [11]. At $m = 4$, the right-most tuple in Fig. 2 was ranked at 43th of 65.

Figures 6 and 7 shows the variance of scores with respect to the number of N-tuples for $N = 6$ and $N = 7$, respectively. We can see that the average score increases almost linearly with a little stepping down. Even though only one tuple was different for the same m, the variance of scores was rather large. Scores for $m+1$ tuples sometimes became smaller than those for m tuples, which is because we stop the TD learning at 1,000,000 games, that is, the scores might not be converged yet.

VI. Evaluation of Selected Tuples

By using the N-tuple networks obtained in the previous section, I conducted two sets of experiments to evaluate them. The first set of experiments were to perform TD learning with more self-play games until the evaluation functions are converged. The second set of experiments were to apply the expectimax algorithm on top of the obtained N-tuple networks.

A. Learning from More Self-Play Games

I performed the TD learning with 10,000,000 self-play games for each of the constructed N-tuple networks.

Figures 8 and 9 shows the transition of scores (averaged for every 100,000 games) for $N = 6$ and $N = 7$ respectively. All the lines grew up smoothly without having significant dropping down. The lines for $m = 5, 6, 7, 8$ eventually reached to similar scores (about 230,000 for N=6 and about 260,000 for N=7). These scores are in fact a bit larger than our estimation of upper bounds of (averaged) scores in [12], and I consider that the scores do not increase even if we use more tuples.

I also compared the N-tuple networks obtained by the proposed method with the human-designed network [11] and the systematically generated ones in our previous work [12]. The other experiment conditions were the same except for the N-tuple networks.

Figure 10 shows the experiment results for $N = 6$, which includes the the proposed network with $m = 4, 8$, hand-designed one [11] with $m = 4$, and those by our previous work with $m = 4, 8, 16$. Figure 11 shows the experiment results for $N = 7$, which includes the proposed network with $m = 4, 8$, and those by our previous work with $m = 4, 8, 10$. From these results, we can clearly see that the N-tuple networks by the proposed method outperformed those by the previous work. Also, the obtained network with four 6-tuples is as good as human-designed one, and it outperformed until about 400,000 games.

B. Combining with Expectimax Algorithm

It is known by several researchers that the minimax algorithm does not work well but the expectimax algorithm works very well for the game 2048. I implemented the expectimax player with the evaluation functions obtained in the first set of experiments, and evaluated the performance with a lot of experiments. The search depths d of the expectimax algorithm were from $d = 1$ (choosing the maximum value without look-aheads) to $d = 7$ (with look-aheads of 3 moves). For each $N = 6, 7$ and $m = 1, \ldots, 8$, I calculated the average and the maximum of scores of 5,000 games. Figures 12 and 13 show the average scores for $N = 6$ and $N = 7$ respectively, and Figures 14 and 15 show the maximum scores for $N = 6$ and $N = 7$ respectively.

The average scores for $N = 6$ (Fig. 12) look reasonable. The gain of using expectimax search steps down with respect to the search depths. The average scores peak at $m = 5$ tuples. The maximum scores for $N = 6$ (Fig. 14) differ from our expectation: the peak of maximum scores are different for the search depths, but eventually at $m = 8$ the maximum scores are almost the same regardless of the search depths. To analyze the reasons of these results is left our future work.

The average scores for $N = 7$ (Fig. 13) also differ from our expectation: the simple greedy algorithm ($d = 1$) outperforms the expectimax algorithm for large m. A possible reason would be the overtraining to the greedy algorithm. In turn, the maximum scores for $N = 7$ (Fig. 15) look reasonable. The maximum scores 597,000 ($N = 6$, $m = 7$, $d = 7$) and 629,964 ($N = 7$, $m = 8$, $d = 7$) are larger than any other (simple) TD-learning players as far as the author knows. Note that the multi-staged algorithm [10], [11] could improve the scores for the networks 6-tuples[2].

Tables III and IV show the reaching ratios that the player successfully made the tiles $2048, \ldots, 32768$, the maximum scores, and the average scores. In general, by using the expectimax search with deeper look-aheads, the players made large tiles more successfully. In particular, the player with $N = 6$, $m = 8$, $d = 4$ can make a 16,384 tile at the probability 93.9%. This would be a reason why the average

[2]Due to the limitation of memory size, it is hard to apply the multi-staged algorithm for eight 7-tuples.

TABLE I
LIST OF THE FOUR BEST AND THE WORST 6-TUPLES IN THE AUGMENTATION-BASED N-TUPLE NETWORK CONSTRUCTION FOR $N = 6$.

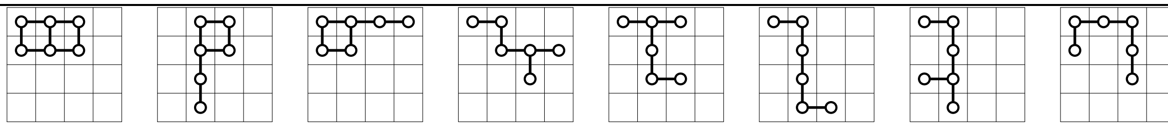
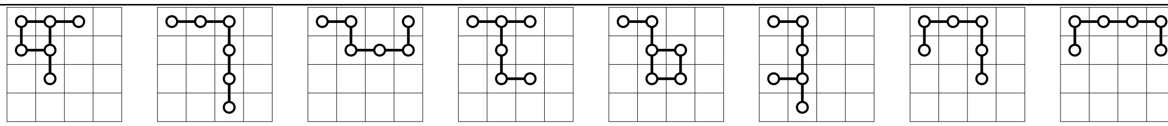
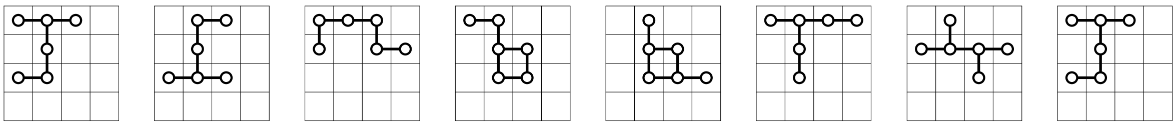
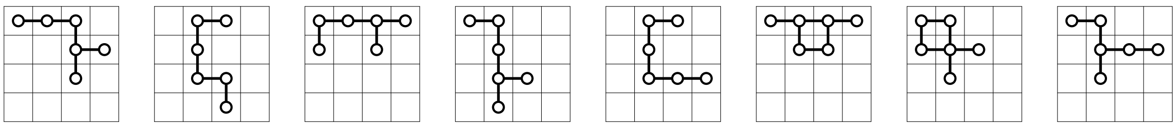
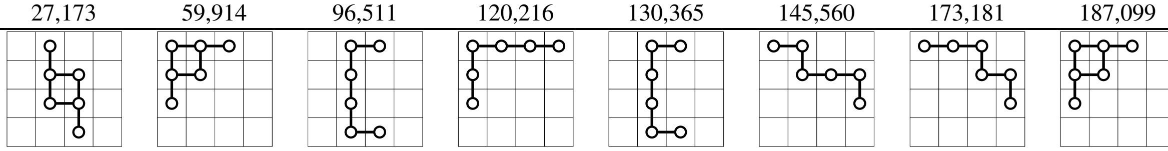
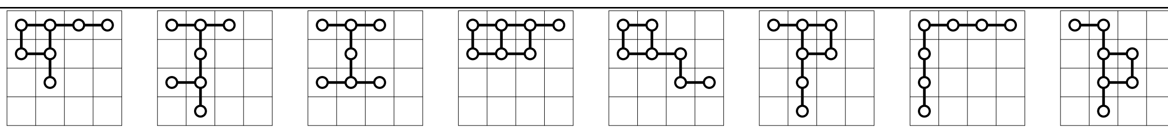
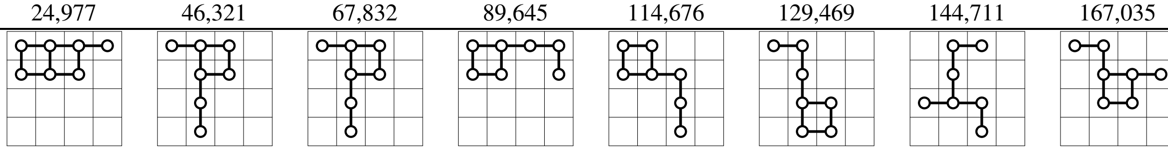
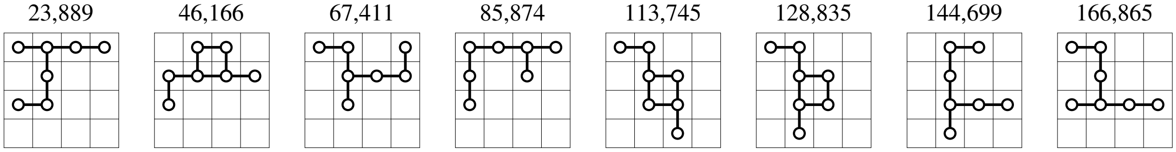
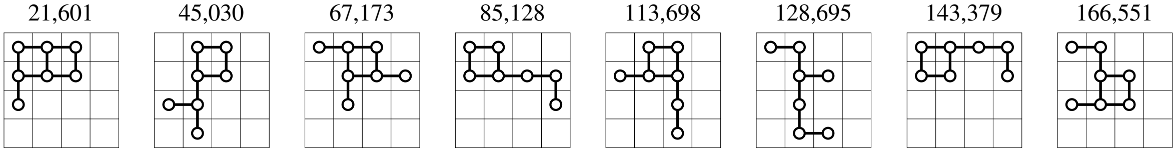
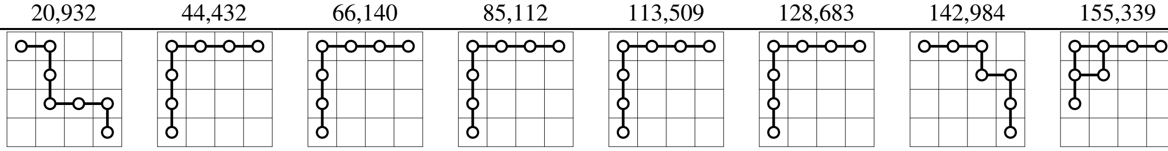
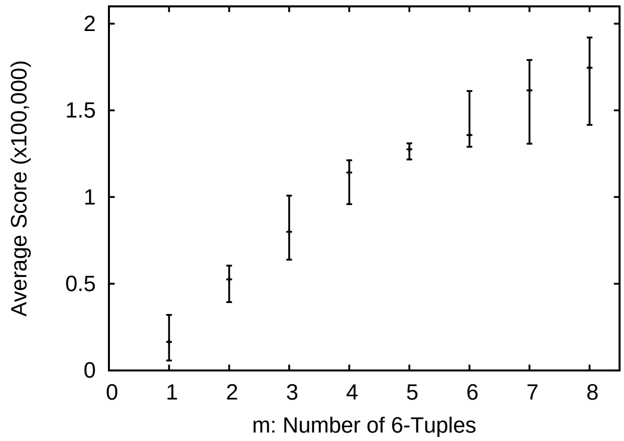
m	1	2	3	4	5	6	7	8
best 	32,019	60,402	100,790	121,199	130,946	161,103	179,072	192,009
2nd 	31,611	60,175	98,725	120,988	130,508	159,652	174,301	190,638
3rd 	28,364	60,109	98,011	120,912	130,438	149,805	174,176	187,811
4th 	27,173	59,914	96,511	120,216	130,365	145,560	173,181	187,099
worst 	5,671	39,360	64,024	95,881	12,1709	128,987	130,769	141,624

TABLE II
LIST OF THE FOUR BEST AND THE WORST 7-TUPLES IN THE AUGMENTATION-BASED N-TUPLE NETWORK CONSTRUCTION FOR $N = 7$.

m	1	2	3	4	5	6	7	8
best 	24,977	46,321	67,832	89,645	114,676	129,469	144,711	167,035
2nd 	23,889	46,166	67,411	85,874	113,745	128,835	144,699	166,865
3rd 	21,601	45,030	67,173	85,128	113,698	128,695	143,379	166,551
4th 	20,932	44,432	66,140	85,112	113,509	128,683	142,984	155,339
worst 	3,760	24,422	37,484	54,261	92,488	114,023	120,951	120,642



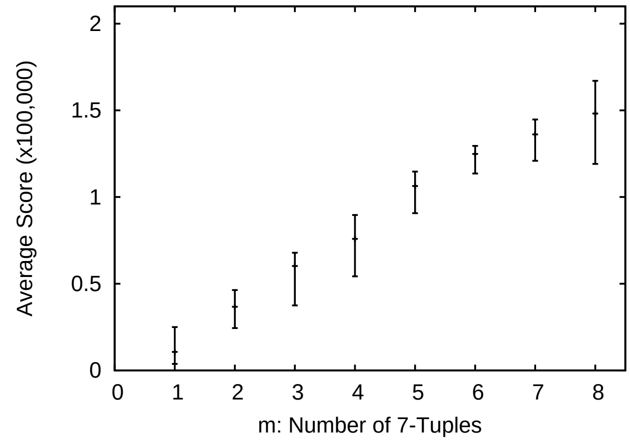
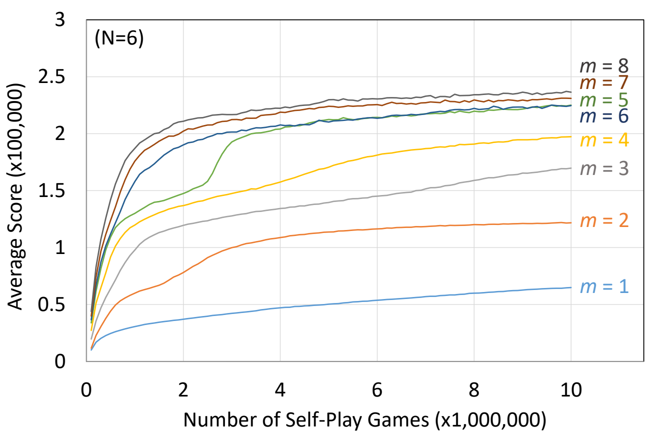


Fig. 6. The score of N-tuple networks in the augmentation-based construction for $N = 6$. The three points on the bar shows the score of the best and the worst networks and the average score.

Fig. 7. The score of N-tuple networks in the augmentation-based construction for $N = 7$. The three points on the bar shows the score of the best and the worst networks and the average score.



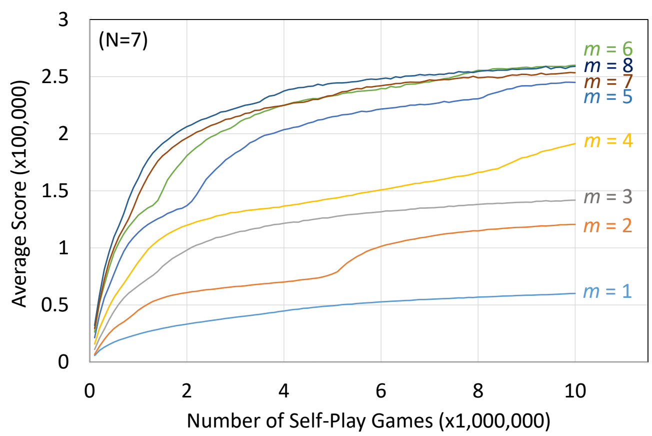
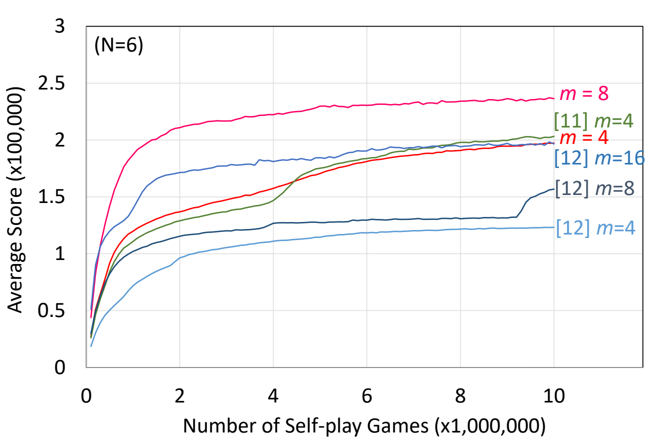


Fig. 8. Transitions of average scores over 10,000,000 self-play games for the constructed N-tuple networks for $N = 6$ and $m = 1, 2, \ldots, 8$.

Fig. 9. Transitions of average scores over 10,000,000 self-play games for the constructed N-tuple networks for $N = 7$ and $m = 1, 2, \ldots, 8$.



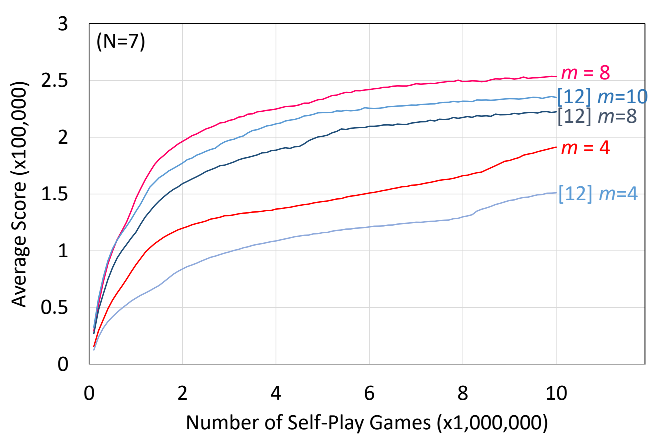


Fig. 10. Transitions of average scores over 10,000,000 self-play games compared with the N-tuple networks in our previous work [12] and the human-designed one by Wu et al. [11].

Fig. 11. Transitions of average scores over 10,000,000 self-play games compared with the N-tuple networks in our previous work [12].

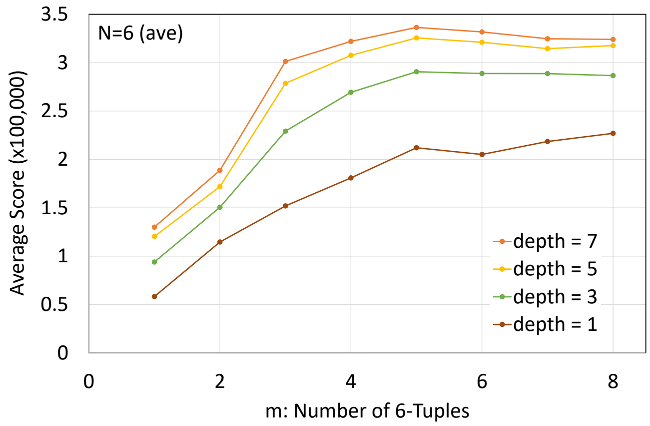


Fig. 12. The average scores of expectimax players (depth $d = 1, \ldots, 4$) based on the N-tuple network with $N = 6$ and $m = 1, \ldots, 8$.

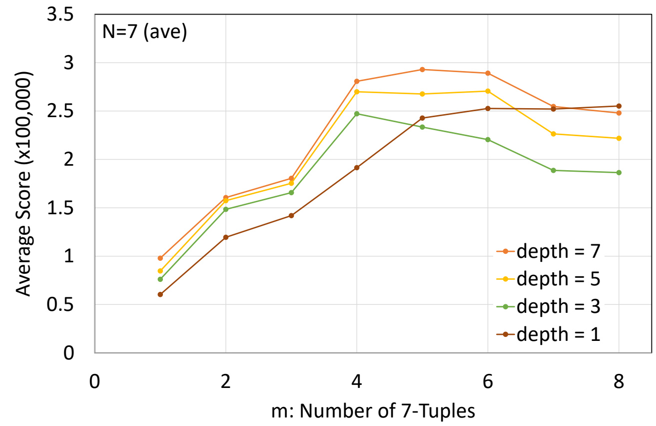


Fig. 13. The average scores of expectimax players (depth $d = 1, \ldots, 4$) based on the N-tuple network with $N = 7$ and $m = 1, \ldots, 8$.

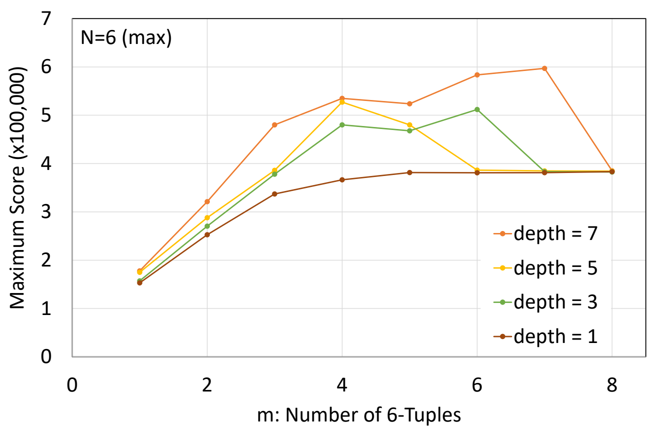


Fig. 14. The maximum scores of expectimax players (depth $d = 1, \ldots, 4$) based on the N-tuple network with $N = 6$ and $m = 1, \ldots, 8$.

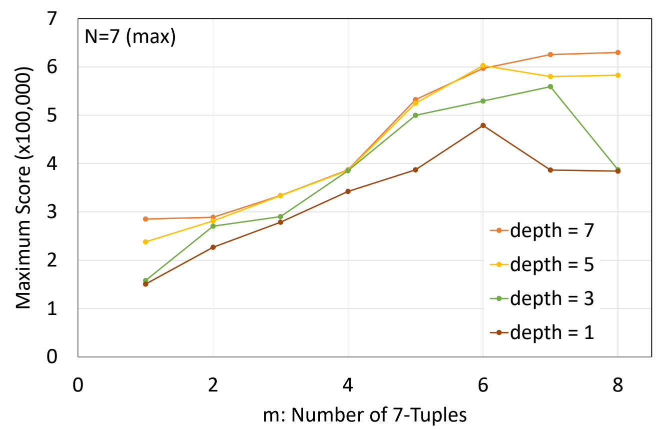


Fig. 15. The maximum scores of expectimax players (depth $d = 1, \ldots, 4$) based on the N-tuple network with $N = 7$ and $m = 1, \ldots, 8$.

scores increased for $N = 6$. In contrast, the results for $N = 7$, $m = 8$ looks very strange. The reaching ratios decreased significantly with the expectimax search, and the reaching ratios for 2048 were even lower than 90%. To analyze these results is left our future work.

VII. Conclusion

In this paper, a systematic method of constructing N-tuple networks was designed for game 2048. The proposed method does not require heuristics or knowledge of characteristics about the game, and constructs N-tuple networks for given size and number of tuples.

Compared with our previous method, the proposed algorithm can cope with tuples that may have interinfluence each other. The constructed N-tuple networks are as good as human-designed ones, as we saw in the comparison with four 6-tuples by Wu et al. (Fig. 10). With the constructed N-tuple networks, the greedy player and the expectimax player achieved high scores. For example, the simple greedy player achieved average score 255,198 (with eight 7-tuples) and the expectimax player achieved the maximum score 629,964 (with eight 7-tuples, look-aheads to 3 moves).

Our future work includes the following three. Firstly, we would like to carefully analyze the results obtained in this work, especially for the not good results in Section VI. Secondly, we would like to apply the extended learning methods, such as multi-stage TD learning [10], [11], to obtain better player. Thirdly, we would like to confirm the applicability of the proposed method for similar games.

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References

[1] G. Cirulli, "2048," http://gabrielecirulli.github.io/2048/, 2014.

[2] A. Abdelkader, A. Acharya, and P. Dasler, "On the complexity of slide-and-merge games," *CoRR*, vol. abs/1501.03837, 2015.

[3] S. Langerman and Y. Uno, "Threes!, Fives, 1024!, and 2048 are hard," *CoRR*, vol. abs/1505.04274, 2015.

[4] K.-H. Yeh, C.-C. Liang, K.-C. Wu, and I.-C. Wu, "2048-bot tournament in Taiwan," https://icga.leidenuniv.nl/wp-content/uploads/2015/04/2048-bot-tournament-report-1104.pdf, 2014.

[5] W. Jaśkowski and M. Szubert, "Game 2048 AI controller competition GECCO 2015," http://www.cs.put.poznan.pl/wjaskowski/pub/2015-GECCO-2048-Competition/GECCO-2015-2048-Competition-Results.pdf, 2015.

TABLE III

REACHING RATIOS, AVERAGE SCORES AND MAXIMUM SCORES OF THE EXPECTIMAX PLAYER (DEPTH $d = 1, \ldots, 4$) BASED ON THE N-TUPLE NETWORKS WITH $N = 6$ AND $m = 4, 8$.

$N = 6$	$m = 4$				$m = 8$			
	$d = 1$	$d = 3$	$d = 5$	$d = 7$	$d = 1$	$d = 3$	$d = 5$	$d = 7$
2048	97.6%	99.4%	99.8%	99.9%	98.3%	98.1%	99.6%	99.8%
4096	95.1%	99.0%	99.7%	99.8%	93.5%	96.6%	99.1%	99.6%
8192	83.0%	96.3%	99.0%	99.2%	86.4%	94.0%	98.2%	99.0%
16384	34.4%	72.3%	88.4%	92.6%	56.5%	81.3%	91.1%	93.9%
32768	0.0%	0.0%	0.0%	0.1%	0.0%	0.0%	0.0%	0.0%
average	180,851	269,348	307,520	322,022	226,958	286,676	317,648	324,019
maximum	366,448	479,996	527,172	534,920	383,232	382,388	384,428	385,032

TABLE IV

REACHING RATIOS, AVERAGE SCORES AND MAXIMUM SCORES OF THE EXPECTIMAX PLAYER (DEPTH $d = 1, \ldots, 4$) BASED ON THE N-TUPLE NETWORKS WITH $N = 7$ AND $m = 4, 8$.

$N = 7$	$m = 4$				$m = 8$			
	$d = 1$	$d = 3$	$d = 5$	$d = 7$	$d = 1$	$d = 3$	$d = 5$	$d = 7$
2048	98.7%	99.7%	99.8%	99.8%	98.9%	79.8%	84.4%	88.5%
4096	96.6%	99.2%	99.7%	99.7%	94.5%	60.7%	65.5%	71.4%
8192	87.7%	96.9%	98.9%	99.2%	88.6%	55.3%	64.3%	71.0%
16384	37.3%	69.9%	80.9%	86.4%	68.3%	49.9%	60.8%	68.1%
32768	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.1%	0.3%
average	191,488	247,213	269,962	280,759	255,198	186,346	221,792	247,964
maximum	342,308	385,244	385,824	387,060	384,416	387,260	582,732	629,964

[6] "GPCC (Games and Puzzles Competitions on Computers) problems for 2015." http://hp.vector.co.jp/authors/VA003988/gpcc/gpcc15.htm, 2015, (in Japanese).

[7] "GPCC (Games and Puzzles Competitions on Computers) problems for 2016." http://hp.vector.co.jp/authors/VA003988/gpcc/gpcc16.htm, 2015, (in Japanese).

[8] K. Oka and K. Matsuzaki, "An evaluation function for 2048 players: evaluation for the original game and for the two-player variant," in *Proceedings of the 57th Programming Symposium*, 2016, pp. 9–18, (in Japanese).

[9] M. Szubert and W. Jaśkowski, "Temporal difference learning of N-tuple networks for the game 2048," in *2014 IEEE Conference on Computational Intelligence and Games*, 2014, pp. 1–8.

[10] K. Yeh, I. Wu, C. Hsueh, C. Chang, C. Liang, and H. Chiang, "Multi-stage temporal difference learning for 2048-like games." arXiv, Tech. Rep. 1606.07374 [cs.LG], 2016.

[11] I.-C. Wu, K.-H. Yeh, C.-C. Liang, C.-C. Chang, and H. Chiang, "Multi-stage temporal difference learning for 2048," in *Technologies and Applications of Artificial Intelligence*, ser. Lecture Notes in Computer Science, vol. 8916, 2014, pp. 366–378.

[12] K. Oka and K. Matsuzaki, "Systematic selection of N-tuple networks for 2048," in *In Proceedings of 9th International Conference on Computers and Games (CG2016)*, 2016, to appear.

[13] R. S. Sutton, "Learning to predict by the methods of temporal differences," *Machine Learning*, vol. 3, no. 1, pp. 9–44, 1988.

[14] A. L. Samuel, "Some studies in machine learning using the game of Checkers," *IBM Journal of Research and Development*, vol. 44, no. 1, pp. 206–227, 1959.

[15] G. Tesauro, "TD-Gammon, a self-teaching backgammon program, achieves master-level play," *Neural computation*, vol. 6, no. 2, pp. 215–219, 1994.

[16] M. van der Ree and M. Wiering, "Reinforcement learning in the game of Othello: Learning against a fixed opponent and learning from self-play," in *IEEE Symposium on Adaptive Dynamic Programming And Reinforcement Learning (ADPRL)*, 2013, pp. 108–115.

[17] N. N. Schraudolph, P. Dayan, and T. J. Sejnowski, "Learning to evaluate Go positions via temporal difference methods," in *Computational Intelligence in Games*, 2001, pp. 77–98.

[18] A. Zaky, "Minimax and expectimax algorithm to solve 2048," http://informatika.stei.itb.ac.id/~rinaldi.munir/Stmik/2013-2014-genap/Makalah2014/MakalahIF2211-2014-037.pdf, 2014.

[19] R. Xiao, "nneonneo/2048-ai," https://github.com/nneonneo/2048-ai, 2015.

[20] P. Rodgers and J. Levine, "An investigation into 2048 AI strategies," in *2014 IEEE Conference on Computational Intelligence and Games*, 2014, pp. 1–2.

[21] K. Oka, K. Matsuzaki, and K. Haraguchi, "Exhaustive analysis and Monte-Carlo tree search player for two-player 2048," *Kochi University of Technology Research Bulletin*, vol. 12, no. 1, pp. 123–130, 2015.

[22] T. Chabin, M. Elouafi, P. Carvalho, and A. Tonda, "Using linear genetic programming to evolve a controller for the game 2048," http://www.cs.put.poznan.pl/wjaskowski/pub/2015-GECCO-2048-Competition/Treecko.pdf, 2015.