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Lab 7 Extrapolation and Trapezoidal Methods

1. (pg. 191) Apply the extrapolation process described in Example 1 to determine $N_3(h)$, an approximation to $f'(x_0)$, for the following functions and stepsizes.

N1 is the approximation method, f and f is the function we wish to approximate the derivative of. For each value of the Richardson extrapolation we calculate it and store is value in a matrix.

a) $f(x) = ln(x),\ x_0 = 1.0,\ h = 0.4$

```
>> N1
N1 =

@(f, h, x0) (f (x0 + h) - f (x0 - h)) / (2 * h)

>> f
f =

@(x) log (x)

%  extrapolation(N1, f,  h, x0, n)
>> extrapolation(N1, f, 0.4, 1, 3)
i = 1
R =  1.0591

i = 2
R =

   1.05912   0.00000
   1.01366   0.99851


i = 3
```

```
R =

   1.05912   0.00000   0.00000
   1.01366   0.99851   0.00000
   1.00335   0.99992   1.00001


R =

   1.05912   0.00000   0.00000   0.00000
   1.01366   0.99851   0.00000   0.00000
   1.00335   0.99992   1.00001   0.00000
   1.00083   0.99999   1.00000   1.00000

ans = 1.0000108771
```

The true value of the derivative at $x_0 = 1.0$ $f'(1.0) = 1/1.0 = 1$ so error is $1.08771 * 10^{-5}$.

b) $f(x) = x + e^x,\ x_0 = 0.0,\ h = 0.4$

```
>> N1
N1 =

@(f, h, x0) (f (x0 + h) - f (x0 - h)) / (2 * h)

>> f
f =

@(x) x + e .^ x

%  extrapolation(N1, f,  h, x0, n)
>> extrapolation(N1, f, 0.4, 0, 3)
i = 1
R =  2.0269

i = 2
R =

   2.02688   0.00000
   2.00668   1.99995
```

```
i = 3
R =

   2.02688   0.00000   0.00000
   2.00668   1.99995   0.00000
   2.00167   2.00000   2.00000


R =

   2.02688   0.00000   0.00000   0.00000
   2.00668   1.99995   0.00000   0.00000
   2.00167   2.00000   2.00000   0.00000
   2.00042   2.00000   2.00000   2.00000

ans = 2.0000000127
```

The true value of the derivative at $x_0 = 0.0$ $f'(0.0) = 1 + e^0 = 2$ so error is $1.27 * 10^{-8}$.

c) $f(x) = 2^x sin(x),\ x_0 = 1.05,\ h = 0.4$

```
>> N1
N1 =

@(f, h, x0) (f (x0 + h) - f (x0 - h)) / (2 * h)

>> f
f =

@(x) 2 .^ x * sin (x)

%  extrapolation(N1, f,   h,   x0, n)
>> extrapolation(N1, f, 0.4, 1.05, 3)
i = 1
R =  2.2032

i = 2
R =
```

```
   2.20317   0.00000
   2.25724   2.27526


i = 3
R =

   2.20317   0.00000   0.00000
   2.25724   2.27526   0.00000
   2.27067   2.27515   2.27515


R =

   2.20317   0.00000   0.00000   0.00000
   2.25724   2.27526   0.00000   0.00000
   2.27067   2.27515   2.27515   0.00000
   2.27403   2.27515   2.27515   2.27515

ans = 2.2751459453
```

The true value of the derivative at $x_0 = 1.05$ is

$f'(1.05) = 2^{1.05}\,(cos(1.05) + ln(2)\,sin(1.05)) = 2.275145842$ so error is

$1.035705 * 10^{-7}$.

d) $f(x) = x^3 cos(x),\ x_0 = 2.3,\ h = 0.4$

```
>> N1
N1 =

@(f, h, x0) (f (x0 + h) - f (x0 - h)) / (2 * h)

>> f
f =

@(x) x .^ 3 * cos (x)

%  extrapolation(N1, f,   h,  x0, n)
>> extrapolation(N1, f, 0.4, 2.3, 3)
```

```
i = 1
R = -19.472

i = 2
R =

  -19.47176    0.00000
  -19.60622  -19.65104


i = 3
R =

  -19.47176    0.00000    0.00000
  -19.60622  -19.65104    0.00000
  -19.63685  -19.64706  -19.64680


R =

  -19.47176    0.00000    0.00000    0.00000
  -19.60622  -19.65104    0.00000    0.00000
  -19.63685  -19.64706  -19.64680    0.00000
  -19.64432  -19.64681  -19.64680  -19.64680

ans = -19.6467992179
```

The true value of the derivative at $x_0 = 2.3$

$f'(2.3) \; = 3(2.3)^2 cos(2.3) - (2.3)^3 sin(2.3) \; = \; -19.64679577$ so error is $3.443635 * 10^{-6}$

1. (pg. 202) Approximate the following integrals using the Trapezoidal rule.

a) $\int_{0.5}^{1} x^4 dx$

```
>> f
f =
```

```
@(x) x .^ 4

%  trapezoidal(f, x0, x1)
>> trapezoidal(f, 0.5, 1)
h =  0.50000
f0 =  0.062500
f1 =  1

ans = 0.2656250000
```

The true value of the integral on the interval is 0.91375 so error is 0.071875.

c) $\int_{1}^{1.5} x^2 ln(x) dx$

```
>> f = @(x) x.^2 * log(x)
f =

@(x) x .^ 2 * log (x)

%  trapezoidal(f, x0, x1)
>> trapezoidal(f, 1, 1.5)
h =  0.50000
f0 = 0
f1 =  0.91230

ans = 0.2280741233
```

The true value of the integral on the interval is 0.1922593577 so error is 0.0358147656.

e) $\int_{1}^{1.6} \frac{2x}{x^2-4} dx$

```
>> f
f =

@(x) (2 * x) / (x .^ 2 - 4)

%  trapezoidal(f, x0, x1)
>> trapezoidal(f, 1, 1.6)
h =  0.60000
f0 = -0.66667
f1 = -2.2222

ans = -0.8666666667
```

The true value of the integral on the interval is -.7339691751 so error is 0.1326974916.

g) $\int_{0}^{\pi/4} xsin(x)dx$

```
>> f
f =

@(x) x * sin (x)

%  trapezoidal(f, x0, x1)
>> trapezoidal(f, 0, pi/4)
h =  0.78540
f0 = 0
f1 =  0.55536

ans = 0.2180895062
```

The true value of the integral on the interval is 0.1517464139 so error is 0.0663430923.