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Lab 2 Bisection Method

5. Use a fixed-point iteration method to determine a solution accurate to within 10^{-2} for $x^4 - 3x^2 - 3 = 0$ on [1, 2]. Use $p_0 = 1$.

```
>> g = @(x) (3*x.^2 + 3).^(1/4)
g =

@(x) (3 * x .^ 2 + 3) .^ (1 / 4)

>> fixedpoint(g, 1, 10.^-2, 100)
ITERATION 1
p =  1.5651
err =  0.56508
p0 =  1.5651

ITERATION 2
p =  1.7936
err =  0.22849
p0 =  1.7936

ITERATION 3
p =  1.8859
err =  0.092371
p0 =  1.8859

ITERATION 4
p =  1.9228
err =  0.036904
p0 =  1.9228

ITERATION 5
p =  1.9375
err =  0.014660
p0 =  1.9375
```

```
ITERATION 6
p =  1.9433
err =  0.0058094

ans =  1.9433
```

The answer is 1.9433

Verification by direct calculation x = 1.9471

So the result agrees with the direct calculation to the specified accuracy.

6. Use a fixed-point iteration method to determine a solution accurate to within 10^{-2} for $x^3 - x - 1 = 0$ on [1, 2]. Use $p_0 = 1$.

```
>> g
g =

@(x) (x + 1) .^ (1 / 3)

>> fixedpoint(g, 1, 10.^-2, 100)
ITERATION 1
p =  1.2599
err =  0.25992
p0 =  1.2599

ITERATION 2
p =  1.3123
err =  0.052373
p0 =  1.3123

ITERATION 3
p =  1.3224
err =  0.010060
```

```
p0 =  1.3224

ITERATION 4
p =  1.3243
err =  0.0019149

ans =  1.3243
```

The answer is 1.3243

Verification by direct calculation x = 1.3247

So the result agrees with the direct calculation to the specified accuracy.

10. Use a fixed-point iteration method to find an approximation to $25^{1/3}$ that is accurate to within 10^{-4}. Compare your result and the number of iterations required with the answer obtained in Exercise 13 of Section 2.1.

Fixed-point iteration

```
>> g
g =

@(x) 5 / (x .^ (1 / 2))

%  fixedpoint(function g, p0, TOL, nMax)
>> fixedpoint(g, 3, 10.^-4, 100)
ITERATION 1
p =  2.8868
err =  0.11325
p0 =  2.8868
```

```
ITERATION 2
p =  2.9428
err =  0.056080
p0 =  2.9428

ITERATION 3
p =  2.9147
err =  0.028175
p0 =  2.9147

ITERATION 4
p =  2.9287
err =  0.014053
p0 =  2.9287

ITERATION 5
p =  2.9217
err =  0.0070352
p0 =  2.9217

ITERATION 6
p =  2.9252
err =  0.0035155
p0 =  2.9252

ITERATION 7
p =  2.9234
err =  0.0017583
p0 =  2.9234

ITERATION 8
p =  2.9243
err =    8.7900e-04
p0 =  2.9243

ITERATION 9
p =  2.9239
err =    4.3953e-04
p0 =  2.9239

ITERATION 10
p =  2.9241
err =    2.1976e-04
p0 =  2.9241
```

```
ITERATION 11
p =  2.9240
err =    1.0988e-04
p0 =  2.9240

ITERATION 12
p =  2.9240
err =    5.4940e-05

ans =  2.9240
```

The answer is 2.9240

Verification by direct calculation x = 2.9240

So the result agrees with the direct calculation to the specified accuracy.

Bisection Method

```
>> f
f =

@(x) x .^ 3 - 25

%  bisection(function f, a, b, tol, nMax)
>> bisection(f, 2, 3, 10.^-4, 100)
iteration 1
c =  2.5000
err =  0.50000
a =  2.5000

iteration 2
c =  2.7500
err =  0.25000
a =  2.7500

iteration 3
c =  2.8750
err =  0.12500
```

```
a =  2.8750

iteration 4
c =  2.9375
err =  0.062500
b =  2.9375

iteration 5
c =  2.9062
err =  0.031250
a =  2.9062

iteration 6
c =  2.9219
err =  0.015625
a =  2.9219

iteration 7
c =  2.9297
err =  0.0078125
b =  2.9297

iteration 8
c =  2.9258
err =  0.0039062
b =  2.9258

iteration 9
c =  2.9238
err =  0.0019531
a =  2.9238

iteration 10
c =  2.9248
err =    9.7656e-04
b =  2.9248

iteration 11
c =  2.9243
err =    4.8828e-04
b =  2.9243

iteration 12
c =  2.9241
```

```
err =    2.4414e-04
b =  2.9241

iteration 13
c =  2.9240
err =    1.2207e-04
a =  2.9240

iteration 14
c =  2.9240
err =    6.1035e-05

ans =  2.9240
```

The fixed-point iteration method requires 12 iterations while the bisection method requires 14 iterations to to achieve the same accuracy for $25^{1/3}$.