Reading Course on Motivic Homotopy Theory

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Motivic Homotopy Theory ressembles ordinary homotopy theory of spaces (or anima) in the setting of smooth schemes. Many geometric concepts as K-Theory or complex cobordism have an analogue in the algebro-geometric setting and can be studied with tools of motivic homotopy theory. In this seminar/reading course we would like to learn the foundations of motivic homotopy theory and apply them to the study of algebraic cobordism **MGL**: Celebrated theorems as Quillens theorem on the homotopy groups of the complex cobordism spectrum **MU** or Landwebers exactness theorem also have motivic versions for **MGL** which we want to cover.

There will be a preliminary meeting

Thursday, 28th January at **14:00**

via Zoom with this link and Zoom coordinates:

Meeting ID: 628 0166 2910 **Password:** 586045

The seminar is completely online and planned to take place during block 3 running from mid-february until may on thursdays 14:00. Definite dates and times are going to be fixed on the 28th.

If you are interested in participating but are not available for the preliminary meeting or have technical difficulties, please do not hesistate to contact one of us.

Program

We will follow the Talbot seminar on motivic homotopy theory. We copied the topics and references for each talk, more detailed information can be found in the program of the Talbot workshop. Lecture notes are available here.

Prerequisites. Higher Category Theory and Higher Algebra, stable homotopy theory, basics on scheme theory, knowledge of complex oriented cohomology theories in the non-motivic context (Quillen's theorem, Landweber exactness) is useful (as a motivation) but not necessary

Talk 1 (Talbot.I.5)**.** Definition and universal property of the unstable and stable motivic homotopy category, see [11, Section 5.1, 5.2] + more stuff on Nisnevich, see [4, Lecture 12]

Talk 2 (Talbot.I.7)**.** Algebraic K-theory ([8, Chapter 4])

Talk 3 (Talbot II.1)**.** Purity theorem ([5, Chapter 3], [7] [8, Section 3.2])

Talk 4 (Talbot.II.2)**.** Morel's $\mathbb{A}^1$-connectivity theorem and homotopy t-structures ([6])

Talk 5 (Talbot.II.3)**.** Endomorphisms of the sphere spectrum ([6])

Talk 6 (Talbot.III.1)**.** Construction of motivic Eilenberg-MacLane spectra, algebraic K-theory spectrum, motivic complex bordism **MGL** ([14], [13], [15], see also [1, Section 16] for **MGL**)

Talk 7 (Talbot.III.2)**.** Universal property of **MGL** ([10])

Talk 8 (Talbot.III.3)**.** Landweber exactness ([9])

Talk 9 (Talbot.III.4)**.** Slices of a Landweber exact theory ([12])

Talk 10 (Talbot.III.5+6+7)**.** Hopkins-Morel theorem ([3]) This could be more than one talk.

One could also give a talk on the paper [2] (regarding modules over **MGL**).

References

[1] Tom Backmann and Marc Hoyois. *Norms in motivic homotopy theory.* 2020. arXiv: 1711.03061 [math.AG].

[2] Elden Elmanto et al. "Modules over algebraic cobordism". In: *Forum Math. Pi* 8 (2020), e14, e14.

[3] Marc Hoyois. "From algebraic cobordism to motivic cohomology". In: *J. Reine Angew. Math.* 702 (2015), pp. 173–226. ISSN: 0075-4102.

[4] Carlo Mazza, Vladimir Voevodsky, and Charles Weibel. *Lecture notes on motivic cohomology.* Vol. 2. Clay Mathematics Monographs. American Mathematical Society, Providence, RI; Clay Mathematics Institute, Cambridge, MA, 2006, pp. xiv+216.

[5] Fabien Morel. "An introduction to $\mathbb{A}^1$-homotopy theory". In: *Contemporary developments in algebraic K-theory.* ICTP Lect. Notes, XV. Abdus Salam Int. Cent. Theoret. Phys., Trieste, 2004, pp. 357–441.

[6] Fabien Morel. "On the motivic π_0 of the sphere spectrum". In: *Axiomatic, enriched and motivic homotopy theory.* Vol. 131. NATO Sci. Ser. II Math. Phys. Chem. Kluwer Acad. Publ., Dordrecht, 2004, pp. 219–260.

[7] Fabien Morel. "The stable $\mathbb{A}^1$-connectivity theorems". In: *K-Theory* 35.1-2 (2005), pp. 1–68.

[8] Fabien Morel and Vladimir Voevodsky. "$\mathbb{A}^1$-homotopy theory of schemes". In: *Inst. Hautes Études Sci. Publ. Math.* 90 (1999), 45–143 (2001).

[9] Niko Naumann, Markus Spitzweck, and Paul Arne Østvær. "Motivic Landweber exactness". In: *Doc. Math.* 14 (2009), pp. 551–593.

[10] Ivan Panin, Konstantin Pimenov, and Oliver Röndigs. "A universality theorem for Voevodsky's algebraic cobordism spectrum". In: *Homology Homotopy Appl.* 10.2 (2008), pp. 211–226.

[11] Marco Robalo. *Noncommutative Motives I: A Universal Characterization of the Motivic Stable Homotopy Theory of Schemes*. 2013. arXiv: 1206.3645 [math.AG].

[12] Markus Spitzweck. "Slices of motivic Landweber spectra". In: *J. K-Theory* 9.1 (2012), pp. 103–117.

[13] Vladimir Voevodsky. "A possible new approach to the motivic spectral sequence for algebraic K-theory". In: *Recent progress in homotopy theory (Baltimore, MD, 2000)*. Vol. 293. Contemp. Math. Amer. Math. Soc., Providence, RI, 2002, pp. 371–379.

[14] Vladimir Voevodsky. "$\mathbb{A}^1$-homotopy theory". In: *Proceedings of the International Congress of Mathematicians, Vol. I (Berlin, 1998)*. Extra Vol. I. 1998, pp. 579–604.

[15] Vladimir Voevodsky. "Open problems in the motivic stable homotopy theory. I". In: *Motives, polylogarithms and Hodge theory, Part I (Irvine, CA, 1998)*. Vol. 3. Int. Press Lect. Ser. Int. Press, Somerville, MA, 2002, pp. 3–34.